SLUNCE

$M_\odot = (1,9891 \pm 0,0012) \times 10^{30}$ kg

$L_\odot = 3,86 \times 10^{26}$ W

$R_\odot = 695\,980$ km

$\langle \rho \rangle = 1400$ kg m^{-3}

$g_\odot = 27,4$ m s^{-2}

$T_{eff} = 5785$ K

$1'' = 726$ km ve vzdálenosti 1AU

$P_{rot,eq} = 26$ dní , $P_{rot,pol} = 35$ dní

věk $\sim 4,5 \times 10^9$ let

$v_{esc} = 6,17 \times 10^5$ m s^{-1}

úhlový moment $\mathcal{L}_\odot = 1,7 \times 10^{41}$ kg m^2 s^{-1}

ztráta hmoty zářením:

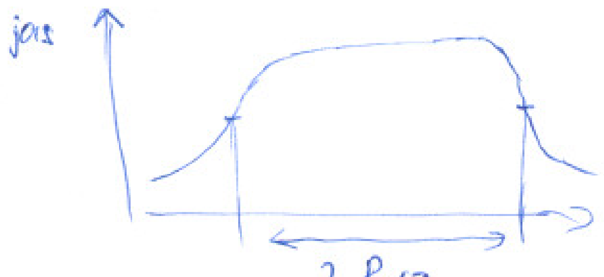
$$\frac{dM_\odot}{dt} = \frac{L_\odot}{c^2} \sim 4 \times 10^9 \text{ kg s}^{-1}$$

slunečním větrem:

$$\frac{dM_\odot}{dt} \sim 10^9 \text{ kg s}^{-1}$$

$\Rightarrow$ od počátku $7,5 \times 10^{26}$ kg ($0,04\%$ $M_\odot$)

Poloměr



měřeno limb–limb jasem, inflexní body definují okraj

$\hookrightarrow 960'' \sim (6,9626 \pm 0,0007) \times 10^8$ m referenční profil $\Rightarrow (6,9599 \pm 0,0007) \times 10^8$ m

→ helioseismicky

↳ povrchové f-vlny – frekvence (k_h, g)

$$k = \sqrt{l(l+1)}\,/R_\odot \; ; \; g = GM_\odot / R_\odot^2$$

$$\omega = \sqrt{gk} = \sqrt{GM_\odot [l(l+1)]^{1/2} / R_\odot^3}$$

↳ z disperzní relace $\omega(l)$ a $GM_\odot$

↳ $(6,9568 \pm 0,0003) \times 10^8$ m

↳ Schou et al., 1997, ApJ 489, L197

→ vývojová změna

$$\frac{dR_\odot}{dt} \sim 2,4 \text{ cm/rok}$$

∼ možná s cyklem aktivity

$\boxed{\text{Zploštění}}$

$$\frac{\Delta R}{R_\odot}$$

↳ rotace + mg. pole (?)

měření: $R_{surf}(\vartheta) = R_\odot \left[1 + \sum_{n=1}^{\infty} r_{2n} \underbrace{P_{2n}}_{\text{Legendre}} (\cos\vartheta) \right]$

↳ $r_2 \sim (-5,810 \pm 0,400) \times 10^{-6}$

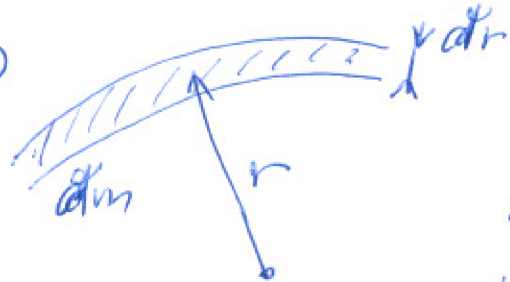
$r_4 \sim (-4,17 \pm 4,59) \times 10^{-7}$

$\boxed{\text{Rovnice nitra}}$

↳ hydrostatická rovnováha:
gravitace = gradient tlaku

↳ termální rovnováha:
tempo generace energie = svítivost

→



$$dm = 4\pi \rho r^2 dr$$

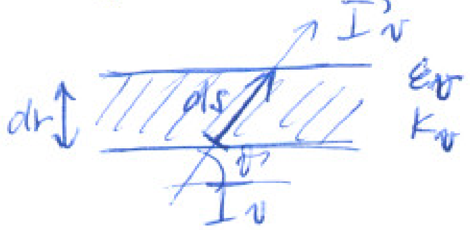
$$\hookrightarrow \boxed{\frac{dm}{dr} = 4\pi \rho r^2}$$

→ pohybová rovnice

$$4\pi r^2 dP = -\frac{G m\, dm}{r^2} = -Gm \frac{4\pi \rho r^2 dr}{r^2}$$

$$\hookrightarrow \boxed{\frac{dP}{dr} = -\frac{G m \rho}{r^2}}$$

→ přenos energie



$$I'_\nu = I_\nu + dI_\nu$$

$$dI_\nu(\vartheta) = \underbrace{-\kappa'_\nu I_\nu(\vartheta)\, ds}_{\text{absorpce}} + \underbrace{\varepsilon_\nu\, ds}_{\text{emise}}$$

$$ds = \frac{dr}{\cos\vartheta}$$

$$\cos\vartheta \frac{dI_\nu}{dr} = -\kappa'_\nu I_\nu(\vartheta) + \varepsilon_\nu = -\kappa'_\nu \Big(I_\nu - \underbrace{\frac{\varepsilon_\nu}{\kappa_\nu}}_{S_\nu}\Big)$$

Kirchhoffův zákon: v LTE

$$S_\nu = \frac{\varepsilon_\nu}{\kappa'_\nu} = B_\nu(T)$$

$$B_\nu(T) = \frac{2h\nu^3}{c^3} \frac{1}{\exp(h\nu/kT) - 1}$$

pro $I_\nu = B_\nu$ ... izotropní radiace $\Rightarrow$ žádný přenos energie

neizotropní $\rightarrow$ iteracemi: 1. iterace

$$\cos\vartheta \frac{dB_\nu}{dr} = -\varkappa'_\nu (I_\nu - B_\nu)$$

$$\hookrightarrow \quad I_\nu = B_\nu - \frac{\cos\vartheta}{\varkappa'_\nu} \frac{dB_\nu}{dr} \quad \leftarrow \varkappa_\nu = \varkappa_\nu/\rho$$

$$I_\nu = B_\nu - \frac{\cos\vartheta}{\rho\varkappa_\nu} \frac{dB_\nu}{dr}$$

tok záření:

$$F_\nu = \int_\Omega I_\nu \cos\vartheta \, d\Omega = -2\pi \frac{\int_0^1 \cos^2\vartheta \, d(\cos\vartheta)}{\varkappa_\nu \rho} \frac{dB_\nu}{dr} =$$

$$= -\frac{4\pi}{3} \frac{1}{\varkappa_\nu \rho} \frac{dB_\nu}{dr}$$

celkový tok:

$$F = \int_0^\infty F_\nu \, d\nu = -\int_0^\infty \frac{4\pi}{3\varkappa_\nu \rho} \frac{dB_\nu}{dr} d\nu = *$$

$$* = -\frac{4\pi}{3\varkappa\rho} \int_0^\infty \frac{dB_\nu}{dr} d\nu$$

$$\hookrightarrow \frac{1}{\varkappa} = \frac{\boxed{\int_0^\infty \frac{1}{\varkappa_\nu} \frac{dB_\nu}{dT} d\nu}}{\int_0^\infty \frac{dB_\nu}{dT} d\nu}$$

$\hookrightarrow$ střední Rosselandova opacita

$$* = -\frac{4\pi}{3\rho} \int_0^\infty \frac{1}{\varkappa_\nu} \frac{dB_\nu}{dT} \frac{dT}{dr} d\nu = -\frac{4\pi}{3\rho} \frac{dT}{dr} \boxed{\int_0^\infty \frac{1}{\varkappa_\nu} \frac{dB_\nu}{dT} d\nu} =$$

vypočteme: $\int_0^\infty B_\nu \, d\nu = \int_0^\infty \frac{2h}{c^2} \frac{\nu^3}{\exp(h\nu/(kT))-1} \, d\nu =$

$$= \frac{ac}{4\pi} T^4$$

$$a = \frac{8\pi^5 k^4}{15 c^3 h^3}$$

$\Rightarrow$

$$F = -\frac{4\pi}{3\rho} \frac{dT}{dr} \frac{1}{\varkappa} \frac{d}{dT} \int_0^\infty B_\nu \, d\nu =$$

$$= -\frac{4\pi}{3\rho\varkappa} \frac{dT}{dr} \frac{d}{dT}\left(\frac{ac}{4\pi} T^4\right) =$$

$$= -\frac{4ac}{3\rho\varkappa} T^3 \frac{dT}{dr}$$

totální energetický tok v kouli s poloměrem r

$$L = 4\pi r^2 F = -\frac{16\pi a c T^3}{3\varkappa\rho} \frac{dT}{dr}$$

$$\boxed{\frac{dT}{dr} = -\frac{3\varkappa\rho}{16\pi r^2 a c T^3} L}$$

→ produkce energie

$$\boxed{\frac{dL}{dr} = 4\pi \rho r^2 \epsilon}$$

→ stavová rovnice

ideální plyn: $P = nkT$

$n = n_H + n_{He} + n_Z + n_e$

↳ částicová hustota

→ lze vyjádřit v pomocných zastoupeních

$$X+Y+Z=1$$

$$n_H = \frac{\rho X}{M}, \quad n_{He} = \frac{\rho Y}{4M}, \quad n_Z = \frac{\rho Z}{AM}$$

$M = 6{,}67\times 10^{-27}$ kg, $A = \langle$ těžké ionty $\rangle \sim 16$

plná ionizace: $n_e = n_H + 2n_{He} + \frac{1}{2} A n_Z$

$$\Rightarrow n = 2n_H + 3n_{He} + \left(1+\frac{1}{2}A\right) n_Z =$$

$$= \frac{\rho}{M}\left(2X + \frac{3}{4}Y + \frac{1+A/2}{A} Z\right) \sim \frac{\rho}{M}\left(2X + \frac{3}{4}Y + \frac{1}{2}Z\right)$$

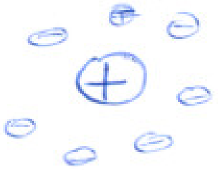
↳ $\boxed{P = nkT = \frac{k}{m}\rho T\left(2X + \frac{3}{4}Y + \frac{1}{2}Z\right) = \frac{\mathcal{R}\rho T}{\mu}}$

$$\mu = \frac{1}{2X + 3/4\,Y + 1/2\,Z}$$

↳ odhad elektrostatické síly:

$$\langle E_C\rangle \sim \frac{e^2}{\langle r\rangle} \Leftrightarrow \langle E_T\rangle \approx \frac{3}{2}kT$$

pro ☉ $\frac{E_C}{E_T} \le 0{,}1$ → málo, ale ne zanedbatelné!



teorie → částice mají distribuci Boltzmannovskou

$$n_i = n e^{-\frac{eU}{kT}}; \quad n_e = n e^{\frac{eU}{kT}}$$

U … střední potenciál v okolí iontu

↳ platí Poissonova rce

$$\Delta U = -4\pi\rho_e = 4\pi e(n_i - n_e) = 4\pi e n\left(e^{\frac{eU}{kT}} - e^{-\frac{eU}{kT}}\right) \Rightarrow$$

$$e^x \sim 1+x$$

$$\Rightarrow \Delta U \approx \frac{4\pi n e^2 U}{kT} = \frac{2U}{D^2}; \quad D = \sqrt{\frac{kT}{8\pi n_e e^2}}$$

Debye poloměr

1/6 =

ve sférických souřadnicích:

$$\Delta U = \frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial U}{\partial r}\right) = \frac{\partial^2 U}{\partial r^2} + \frac{2}{r}\frac{\partial U}{\partial r} = \frac{2}{D^2}U$$

rovnice tvaru: $$\frac{d^2U}{dr^2} + \frac{2}{r}\frac{dU}{dr} = K^2 U$$

řešení $$U(r) = C\frac{e^{-\sqrt{K^2}r}}{r} + \underbrace{C'\frac{e^{\sqrt{K^2}r}}{2r\sqrt{A^2}}}_{C'=0\ !\ U\to 0 \text{ pro } r\to\infty} \quad ; \quad K>0$$

$$U(r) = C\frac{e^{-Kr}}{r} = C\frac{e^{Ar}}{r} \quad ; \quad A=-K$$

zk: $$\frac{dU}{dr} = -\frac{e^{Ar}}{r^2} + \frac{Ae^{Ar}}{r}$$

$$\frac{d^2U}{dr^2} = -\left[\frac{-2e^{Ar}}{r^3} + \frac{Ae^{Ar}}{r^2}\right] + A\left[\frac{-e^{Ar}}{r^2} + \frac{Ae^{Ar}}{r}\right] =$$

$$= \frac{2e^{Ar}}{r^3} - \frac{Ae^{Ar}}{r^2} - \frac{Ae^{Ar}}{r^2} + \frac{A^2e^{Ar}}{r} =$$

$$= \frac{2e^{Ar}}{r^3} - \frac{2Ae^{Ar}}{r^2} + \frac{A^2e^{Ar}}{r}$$

$$\frac{d^2U}{dr^2} + \frac{2}{r}\frac{dU}{dr} = \cancel{\frac{2e^{Ar}}{r^3}} - \cancel{\frac{2Ae^{Ar}}{r^2}} + \frac{A^2e^{Ar}}{r} - \cancel{\frac{2e^{Ar}}{r^3}} + \cancel{\frac{2Ae^{Ar}}{r^2}} =$$

$$= \frac{A^2e^{Ar}}{r} \overset{!}{=} \frac{A^2e^{Ar}}{r}$$

řešení: $$U(r) = C\frac{e^{-Kr}}{r} \quad ; \quad K^2 = \frac{2}{D^2} \Rightarrow$$

$$\Rightarrow K = +\frac{\sqrt{2}}{D}$$

$$U(r) = C\frac{e^{-\frac{\sqrt{2}}{D}r}}{r}$$

$$U \sim E\cdot s \ ; \ E \sim \frac{Q}{r^2} \Rightarrow U \sim \frac{Q}{r}$$

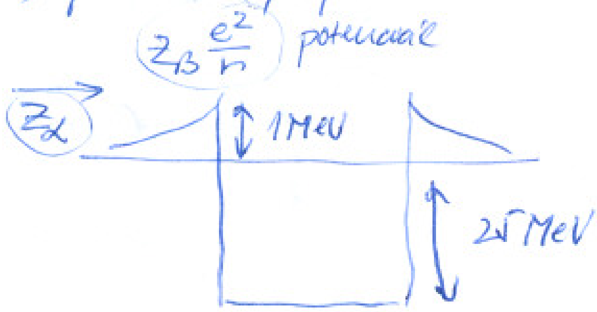
$$\Rightarrow C \sim Q \Rightarrow C \equiv e \Rightarrow \boxed{U(r) = \frac{e}{r}e^{-\frac{\sqrt{2}}{D}r}}$$

$$\delta E = -U(\sim D)\cdot \cancel{e} = -\frac{e^2}{D}$$

práce nutná k překonání potenciálu operujícího na Debye length

↳ jaderné reakce

→ pomalé, protože ⊕ + ⊕



bariéra ~ 1 MeV

tepelná energie ~ 1 keV

tempo reakcí částic α a B v jednotkové hmotnosti

$$r_{\alpha B} \sim n_\alpha n_B \langle \sigma v \rangle$$

v → relativní rychlost

σ → účinný průřez

α nalétá na B $\langle \sigma v \rangle = \int \sigma v \frac{dn}{n}$

Maxwell-Boltzmann

$$\frac{dn}{n} = \frac{2}{\sqrt{\pi}} \frac{1}{(kT)^{3/2}} e^{-E/kT} E^{1/2} dE$$

$$\Rightarrow r_{\alpha B} \sim \int e^{-E/kT} E \sigma dE$$

odhad → interakce na de Broglieho délce

$$\lambda_p = \frac{h}{p} = \frac{h}{\sqrt{2mE}}$$

$$\lambda = \frac{c}{\nu} \sim \frac{hc}{h\nu} = \frac{hc}{E} = \frac{h}{\frac{E}{c}} = \frac{h}{p}$$

pravděpodobnost penetrace Coulombovy bariéry

$$\sim e^{-\frac{E_c}{E}} \quad ; \quad E_c = \frac{Z_\alpha Z_B e^2}{\lambda_p}$$

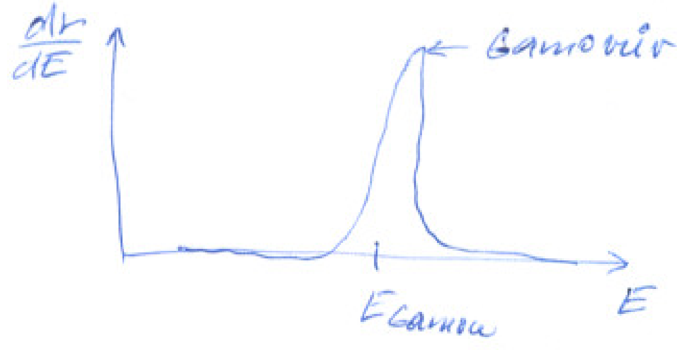
$$\sigma \sim \lambda_p^2 e^{\frac{-Z_\alpha Z_B e^2}{\lambda_p E}} \sim \frac{h^2}{2mE} \exp\left(\frac{-Z_\alpha Z_B e^2}{h}\sqrt{\frac{2m}{E}}\right) \sim$$

$$\sim \frac{1}{E} e^{-b/\sqrt{E}}$$

$$\Rightarrow r \sim \int e^{-\frac{E}{kT}} E \sigma \, dE \sim \int e^{-\frac{E}{kT}} E \cdot \frac{1}{E} e^{-\frac{b}{\sqrt{E}}} dE =$$

$$= \int e^{-\frac{E}{kT}} e^{-\frac{b}{\sqrt{E}}} dE = \int \underbrace{e^{-\frac{E}{kT} - \frac{b}{\sqrt{E}}}} dE$$

maximum v $E = (bkT)^{2/3}$



$$\Rightarrow r \sim e^{-\frac{a}{T^{1/3}}}$$

uvolněná energie termojaderné reakce

$$E \sim Q_{\alpha B} r_{\alpha B} \sim \rho T^n$$

$$n \sim \frac{a}{(kT)^{1/3}} - \frac{2}{3}$$

z teorie $n = \left(\frac{\pi^2 Z_\alpha^2 Z_B^2 e^4 m_{\alpha B}}{2h^2 kT}\right)^{1/3} - \frac{2}{3}$

$$m_{\alpha B} = \frac{m_\alpha m_B}{m_\alpha + m_B}$$

pro p-p $\Rightarrow E \sim X^2 \rho T^4$

n vyšší pro 3α proces, CNO cyklus, ...

Rovnice vnitřní struktury

$$\frac{dm}{dr} = 4\pi\rho r^2$$

$$\frac{dP}{dr} = -\frac{Gm\rho}{r^2}$$

$$\frac{dL}{dr} = 4\pi\rho r^2 \epsilon$$

$$\frac{dT}{dr} = -\frac{3\varkappa\rho}{16\pi r^2 acT^3} L$$

$$P = \frac{\mathcal{R}\rho T}{\mu} \quad \Big| \quad \mu = \frac{1}{2X + \frac{3}{4}Y + \frac{1}{2}Z}$$

$$\epsilon = \epsilon_0 X^2 \rho T^4$$

$$\varkappa = \varkappa_0 (X+1) Z \rho T^{-3,5}$$

Odhady:

vnitřní teplota: z hydrostatické rovnováhy a stavové rovnice od středu k povrchu

$$\langle\rho\rangle \sim \frac{M}{R^3}$$

$$\frac{P}{R} \sim \frac{GM}{R^2}\frac{M}{R^3} \sim \frac{GM^2}{R^5}$$

$$P \sim \frac{\mathcal{R}\rho T}{\mu_0} \sim \frac{\mathcal{R}MT}{\mu_0 R^3}$$

$X \sim 0{,}7$, $Y \sim 0{,}28$, $Z \sim 0{,}02$

$$\Rightarrow \mu_0 \sim 0{,}6$$

$$\Rightarrow \boxed{T = \frac{P\mu_0 R^3}{\mathcal{R}M} = \frac{GM^2}{R^4} \cdot \frac{\mu_0 R^3}{\mathcal{R}M} \sim \frac{GM\mu_0}{R\mathcal{R}} \sim 1{,}4 \times 10^7\,K}$$

$\hookrightarrow T \rightarrow \Delta T$ od centra k povrchu

$\boxed{\text{Vývoj na hlavní posloupnosti}}$

škálování: $m \to M$ transformace $m = M\,\tilde{m}\left(\frac{r}{R}\right)$

$r \to R$

$\rho \to \cancel{\rho}$

$$\Rightarrow \cancel{\rho} \sim \frac{M}{R^3}$$

$$\frac{P}{R} \sim \frac{GM}{R^2}\frac{M}{R^3} \sim \frac{GM^2}{R^5}$$

$$\frac{T}{R} \sim \frac{\varkappa \rho L}{R^2 T^3} \sim \Big|\ \varkappa \sim \varkappa_0 (X+1) Z \rho T^{-3,5} \Big/ \sim$$

$$\sim \frac{\varkappa_0 \rho^2 T^{-3,5} L}{R^{2} T^{3}} \sim \frac{\varkappa_0 M^2 L}{R^8 T^{-6,5}}$$

$$\frac{L}{R} \sim \rho r^2 \epsilon \sim \Big|\ \epsilon = \epsilon_0 X^2 \rho T^4 \Big/ \sim \frac{M^2}{R^6} R^2 \epsilon^0 T^4 \sim$$

$$\sim \frac{M^2 T^{4} \epsilon}{R^4}$$

- $\varkappa_0 \sim (X+1) \sim X^{\beta}$ $\quad \beta \sim 0,59$

  $\hookrightarrow \beta \sim \frac{1}{X+1}$

$$X^{\left(\frac{1}{X+1}\right)} \sim \ln \frac{1}{X+1} \cdot \exp X \sim 1 \cdot (1+X)$$

- $\epsilon_0 \sim X^2$

- $\mu_0 \sim X^{\eta} \longrightarrow \eta = \frac{d \log \mu}{d \log X}$

$$n = n_H + n_{He} + n_Z + n_e = \frac{\rho X}{M} + \frac{\rho(1-X-Z)}{4M} + \frac{\rho Z}{AM} +$$

$$+ \frac{2\rho(1-X-Z)}{4M} + \frac{\rho X}{M} + \frac{1}{2} A \frac{\rho Z}{AM} =$$

$$= \frac{\rho}{M}\left[\frac{5}{4}X + \frac{3}{4} - \cancel{\frac{\left(1 - \frac{1}{4}A\right)}{A}} Z\right]$$

$A \sim 16 \Rightarrow 0,2$

$$n = \frac{d \log \mu}{d \log X} = \frac{X}{\mu}\frac{d\mu}{dX} = \frac{-X\left(\frac{5}{4}X + \frac{3}{4}\right)\frac{5}{4}}{\left(5/4\, X + 3/4\right)^2} \sim -\frac{5X}{5X+3} \sim -0{,}54$$

škálování:

$$L \sim \frac{M^2 T^4 \varepsilon_0}{R^3} \; ; \quad T \sim \frac{\varkappa_0 M^2 L}{R^7 T^{6,5}}$$

$$\frac{M^2 T^4 \varepsilon_0}{R^3} = \frac{T^{7,5} R^7}{\varkappa_0 M^2}$$

$$T^{3,5} = \frac{M^4 \varepsilon_0}{R^{10} \varkappa_0}$$

$$T = \left(\varepsilon_0 \varkappa_0\right)^{2/7} \frac{M^{8/7}}{R^{20/7}}$$

$\downarrow$ z dvojčetu

$$T \sim \frac{M \mu_0}{R}$$

$$=$$

$$\frac{M\mu_0}{R} = \left(\varepsilon_0 \varkappa_0\right)^{2/7} \frac{M^{8/7}}{R^{20/7}}$$

$$\hookrightarrow R \sim \frac{\left(\varepsilon_0 \varkappa_0\right)^{2/13}}{\mu_0^{7/13}} M^{1/13}$$

$$\cancel{L \sim \frac{M^6}{R^7} \mu_0^4 \varepsilon_0}$$

$$L \sim \frac{M^2 T^4 \varepsilon_0}{R^3} \sim \frac{M^6}{R^7} \mu_0^4 \varepsilon_0 =$$

$$= \left[\frac{\mu_0^{7/13}}{\left(\varepsilon_0 \varkappa_0\right)^{2/13} M^{1/13}}\right]^7 M^6 \mu_0^4 \varepsilon_0 =$$

$$= \frac{M^{71/13} \mu_0^{\frac{101}{13}}}{\left(\varepsilon_0 \varkappa_0\right)^{14/13}} \sim X^{-4,78} M^{5,46}$$