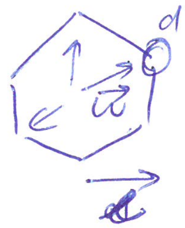
Sunspots

flux concentration by supergranulation:



equilibrium: decay rate $\sim \frac{d^2}{\eta}$ ← diffusivity

the same as advection towards edges in the steady state

$$\tau = \frac{d^2}{\eta} = \frac{l}{w} \Rightarrow d^2 = \frac{\eta l}{w} = \frac{l^2}{R_m}$$

↳ magnetic Reynolds number

how the field could be concentrated? → all the background flux conserved

$$B_0 l^2 = B d^2 \Rightarrow B = \frac{B_0 l^2}{d^2} = \frac{B_0 l^2}{l^2/R_m} = R_m B_0$$

characteristic time,

$l \sim 30\,\mathrm{Mm}$, $R_m \sim 10^4$, $B_0 \sim 0{,}1\,\mathrm{G}$, $w \sim 300\,\mathrm{m/s}$

$\Rightarrow d \sim 300\,\mathrm{km}$, $B \sim 10^3\,\mathrm{G}$ ← enough for the spot

$\tau \sim l/w = 10^5\,\mathrm{s} \sim 1\,\mathrm{day}$

acting against: pressure ballance

$$\frac{B_{max}^2}{2\mu} \sim \frac{\rho w^2}{2} \Rightarrow B_{max} \sim \sqrt{\mu\rho}\, w$$

for the photosphere: $\rho = 3\times 10^{-4}\,\mathrm{kg/m^3}$

$w \sim 300\,\mathrm{m/s}$

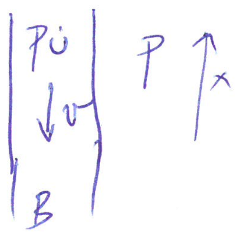
$\Rightarrow B_{max} = 60\,\mathrm{G}$

not enough for the spot

– different mechanism

convective collapse

– vertical flux tube → adiabatic cooling → downflow



pressure equilibrium: $P = p_i + \frac{B^2}{2\mu}$

motion equation in the tube

$$\frac{\partial v}{\partial t} + v\frac{\partial v}{\partial x} + \frac{1}{\rho}\frac{\partial P_i}{\partial x} = 0$$

$$\frac{\partial v}{\partial t} + \frac{\partial}{\partial x}\left[\frac{1}{2} v^2 + \frac{p}{\rho} - \frac{B^2}{2\rho\mu}\right] = 0$$

→ looking for a typical behavior ⇒ integrate over a piece of the flux tube $\int_{x_1}^{x_2} \dots dx = \langle \dots \rangle$

stationary solution: ⇒ $\frac{\partial}{\partial t}\langle v \rangle = 0$

⇒ $\left\langle p + \frac{1}{2}\rho v^2 - \frac{B^2}{2\mu} \right\rangle = \text{const} = \langle p_i \rangle$

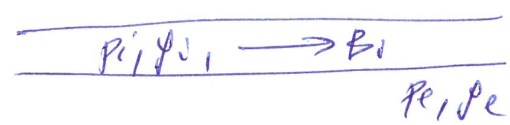
hence $\left\langle \frac{B^2}{2\mu} \right\rangle = \langle p \rangle + \frac{1}{2}\langle \rho v^2 \rangle - \langle p_i \rangle$

⇒ mg. field rises with increasing external pressure or with the flow along the flux tube

→ a detailed model: collapse stable for $B > 0{,}1\,T$

$\boxed{\text{Magnetic buoyancy}}$

→ horizontal flux tube at the bottom of the convection zone:



pressure ballance:

$$p_e = p_i + \frac{B_i^2}{2\mu}$$

EOS: $p = \frac{RT\rho}{\mu}$; $\mu = 1$

⇒ $RT(\rho_e - \rho_i) = \frac{B_i^2}{2\mu}$; for $\rho_i < \rho_e$ ... buoyancy by force $(\rho_e - \rho_i)g$

against magnetic tension

Lorentz force

$$f_L = j \times B = \frac{1}{\mu}(\nabla \times B) \times B = \underbrace{\frac{(B\cdot\nabla)B}{\mu}}_{\text{tension}} - \underbrace{\nabla \frac{B^2}{2\mu}}_{\text{work of mg. pressure}}$$

=D=

tension, $\frac{(B.\nabla)B}{\mu} \sim \frac{B_i^2}{l}$ .. l .. perturbation length

instability for: $(\rho_e - \rho_i) g > \frac{B_i^2}{\mu l}$ from the pressure ballance

$$\cancel{(\rho_e-\rho_i)} g > \frac{2RT}{l} \cancel{(\rho_e-\rho_i)}$$

$$\rightarrow l > \frac{2RT}{g} = 2H_p$$

$$\frac{dp}{dr} = -\rho g = p \frac{1}{p}\frac{dp}{dr} = p\frac{d\ln p}{dr} = -\frac{p}{H_p}$$

$$\frac{p}{\rho} = RT \Rightarrow \frac{p}{\rho g} = \frac{RT}{g} = \frac{p}{\frac{p}{H_p}} = H_p$$

thus if perturbation is large enough → the tube gets unstable and keeps rising

rising time: $\tau \sim \frac{d}{c_A}$; d... depth

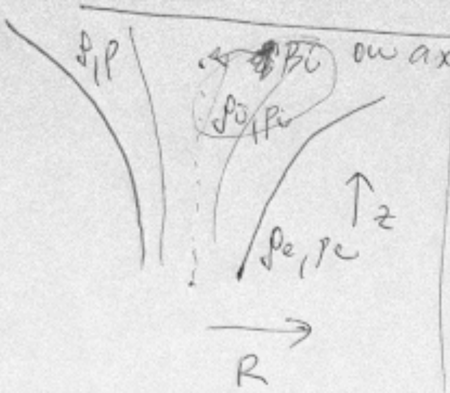
for 10kG field at the bottom of CZ

$(\langle\rho\rangle = 200\ kg/m^3$; $d = 200 Mm)$

$\tau \sim 2$ month (or larger due to the c_A profile)

if $\tau > \tau_{rot}$ → Coriolis force steps in → deflection

MHS model of sunspot



$$B = B(R)$$

$$\max(B) = B_i = B(R=0) \quad | \quad B_i \neq B_i(z)$$

$$B(R \to \infty) = 0$$

$$\rho_i = \rho_i(z) \; ; \; p_i = p_i(z)$$

$$\rho_e = \rho_e(z) \; ; \; p_e = p_e(z)$$

vertical equilibrium:

on axis: $\frac{dp_i}{dz} = -\rho_i g$; outside: $\frac{dp_e}{dz} = -\rho_e g$

+ need some energy equation to get temperature stratification

horizontal pressure ballance

$$p_i(z) + \frac{B_i^2}{2\mu} = p_e(z) \quad \text{on the axis}$$

$$p(R,z) + \frac{B^2(R)}{2\mu} = p_e(z) \quad \text{in general}$$

$\frac{d}{dz}$:

$$\frac{dp_i}{dz} + 0 = \frac{dp_e}{dz} \Rightarrow \rho_i = \rho_e$$

inside → density the same! pressure lower! ↳ temperature lower || ⇒ stable non-expanding magnetic tube

$\frac{1}{p_e}$:

$$\frac{p_i(z)}{p_e(z)} + \frac{B_i^2}{2\mu p_e(z)} = 1$$

EOS

$$\Rightarrow \frac{T_i(z)}{T_e(z)} = 1 - \frac{B_i^2}{2\mu p_e(z)}$$

$$\Rightarrow T_i < T_e$$

=F=

$\Rightarrow$ presence of mg. field does not affect density, but it affects ~~pres~~ temperature

only if the temperature deficit has this form — flux tube is stable

otherwise it expands with heights

$\rightarrow$ violation of temperature deficit

$$\Rightarrow \quad B_i = B_i(z)$$

in sunspot usually $\quad \frac{B_i^2}{2\mu} > p_e(z)$

differentiate: $\quad \frac{2B_i}{2\mu}\frac{dB_i}{dz} > \frac{dp_e}{dz}$

~~since $\frac{dp_e}{dz} < 0$~~

$B = 3000G \Rightarrow 2{,}4 \times 10^4\ N/m^2$

$1{,}4 \times 10^4\ N/m^2$ in photosphere

obvious problem

solution: B_i is measured deeper(!) than the photosphere, where also p_e is larger!

$$\Rightarrow B_i = B_i(z)\ , \quad \frac{dB_i}{dz} < 0$$

$p_i + \frac{B_i^2}{2\mu} = p_e \quad \Big| \frac{d}{dz}$; new conditions of expanding tube

$$\frac{dp_i}{dz} = \frac{dp_e}{dz} - \frac{2B_i}{2\mu}\frac{dB_i}{dz} < 0$$

$$\Rightarrow \frac{dp_i}{dz} > \frac{dp_e}{dz}$$

$\Rightarrow p_i$ drops faster in the expanding tube $\rightarrow$ density drops too

$\rightarrow$ Wilson depression

Does it expand?

$$\frac{dB_i}{dz} < 0 \rightarrow \frac{dB}{dz} < 0$$

$$\phi = \int B\,dS = \text{konst}$$

for $\frac{dB}{dz} < 0 \Rightarrow \frac{dS}{dz} > 0$

↳ expansion with height