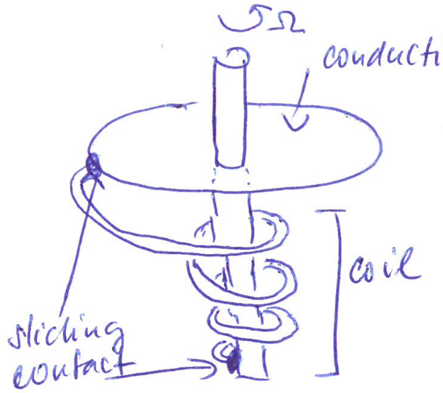
Homopolar dynamo



there is a current $I(t)$ flowing through a system. can $I(t)$ be rising?

inducted magnetic flux

$\phi = MI$, M... mutual conductance of wire & disc

rotation $\sim \frac{d}{dt}$ → origin of the electromotive ~~force~~ voltage $\mathcal{E}$

$$\mathcal{E} = \frac{d\phi}{dt} = \frac{\Omega}{2\pi}\phi = \frac{\Omega}{2\pi} MI$$

for the current: $L\frac{dI}{dt} + RI = \mathcal{E} = \frac{\Omega}{2\pi}MI$

solution seeking $I(t) = I_0 e^{\lambda t}$

$\Rightarrow \lambda = \frac{1}{L}\left(\frac{M}{2\pi}\Omega - R\right)$, L... inductance of the coil

rise for $\lambda > 0 \Rightarrow \Omega > \frac{2\pi R}{M}$

$\Rightarrow$ fast rotation = current generation

Mean-field dynamo

$$\frac{\partial B}{\partial t} = \nabla\times(v\times B) + \eta \Delta B = \nabla\times\left[v\times B - \eta\nabla\times B\right]$$

define: $B = \langle B\rangle + b$ ← fluctuations

↳ mean field $\quad \langle b\rangle = 0$

$v = \langle v\rangle + w \quad \langle w\rangle = 0$

then:

$$\frac{\partial}{\partial t}(\langle B\rangle + b) = \nabla\times\left[(\langle v\rangle + w)\times(\langle B\rangle + b) - \eta\nabla\times(\langle B\rangle + b)\right]$$

the mean part by $\langle\ \rangle$

$$\frac{\partial}{\partial t}\langle B\rangle = \nabla\times\left[\langle v\rangle\times\langle B\rangle + \langle w\times b\rangle - \eta\nabla\times\langle B\rangle\right]$$

fluctuating = original - mean

$$\frac{\partial}{\partial t} b = \nabla\times\left[\langle v\rangle\times b + w\times\langle B\rangle + w\times b - \langle w\times b\rangle - \eta\nabla\times b\right]$$

definition $\mathcal{E} = \langle w \times b \rangle$

$\mathcal{G} = w \times b - \langle w \times b \rangle$

→ electric field induced by fluctuations

~~Let~~'s assume: b and $\langle B \rangle$ in linear relation

$\mathcal{E}$ and b in linear relation

then $\mathcal{E}$ and $\langle B \rangle$ should also be linear

hence $\mathcal{E} = \alpha \langle B \rangle - \beta \nabla \times \langle B \rangle + \dots$

for an isotropic turbulence:

$$\alpha = 1/3 \, \langle w \cdot \nabla \times w \rangle \tau$$

$$\beta = 1/3 \, \langle w \cdot w \rangle \tau$$

τ... correlation time

kinetic helicity

then the mean-field equation:

$$\frac{\partial \langle B \rangle}{\partial t} = \nabla \times \Big[\underbrace{\langle v \rangle \times \langle B \rangle}_{\Omega \text{ effect}} + \underbrace{\alpha \langle B \rangle}_{\alpha \text{ effect}} - (\eta + \beta) \nabla \times \langle B \rangle \Big]$$

$\eta + \beta = \eta_t$ ... turbulent viscosity

α effect: toroidal → poloidal

Ω effect: poloidal → toroidal

how much is α? $\alpha = \pm \, l \, \Omega$ rotational speed

→ convective scale

α should be negative to Ω,

α... quenching

numerically: $\alpha = \dfrac{\langle w \times b \rangle \cdot B_H}{B_H^2}$ B_H... spots

$\alpha \in \langle \sim \mathrm{m/s}, \sim 100\, \mathrm{m/s} \rangle$

=B=