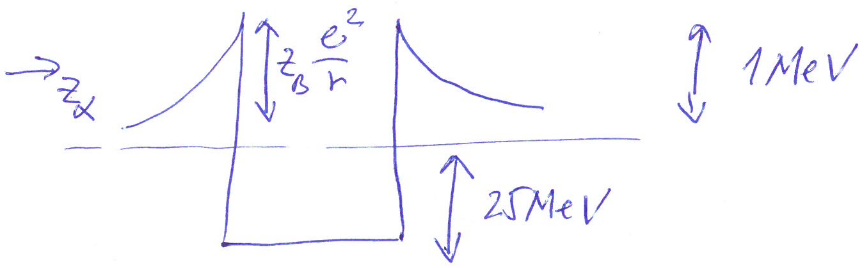
Gamow peak

→ slow reactions → yet ⊕ + ⊕ fuse



reaction rate per unit mass

$$r_{\alpha B} \propto n_\alpha n_B \langle \sigma v \rangle,$$

where $\langle \sigma v \rangle = \int \sigma v \frac{dn}{n}$

$\frac{dn}{n}$ → relative particle number in interval $[E, E+dE]$

the flux of particles α-type $(n_\alpha v)$ onto particles B-type with effective cross-section σ

$\frac{dn}{n}$ – from the distribution function, assuming equilibrium ← Maxwell-Boltzmann

$$\frac{dn}{n} = \frac{2}{\sqrt{\pi}} \frac{1}{(kT)^{3/2}} e^{-\frac{E}{kT}} E^{1/2} dE$$

then: $r_{\alpha B} \propto \int \sigma v e^{-E/kT} E^{1/2} dE \propto \left| E = \frac{1}{2} m v^2 \right|$

$$\propto \int \sigma E^{1/2} e^{-E/kT} E^{1/2} dE \propto \int \sigma E e^{-E/kT} dE$$

effective cross-section → estimate from the de Broglie wavelength and correct for the tunnelling effect

de Broglie: $\lambda_p = \frac{h}{p} = \frac{h}{\sqrt{2mE}}$

$\sigma \propto \lambda_p^2$ times propability of the tunnelling, which is $\propto \exp\left(\frac{E_e}{E}\right)$, where

$$E_e = \frac{Z_\alpha Z_B e^2}{\lambda_p}$$

then $\sigma \propto \lambda_p^2 e^{-\frac{Z_\alpha Z_B e^2}{\lambda_p E}} = \frac{h^2}{2mE} \exp\left[-\frac{Z_\alpha Z_B e^2}{hE}\sqrt{2mE}\right] =$

$$= \frac{h^2}{2mE} \exp\left[-\frac{Z_\alpha Z_B e^2}{h}\frac{\sqrt{2m}}{\sqrt{E}}\right] \propto \frac{1}{E} \exp\left[-\frac{b}{E^{1/2}}\right]$$

$$b = \frac{Z_\alpha Z_B e^2 \sqrt{2m}}{h}$$

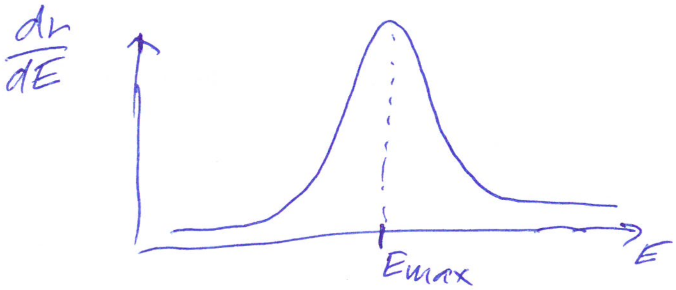
then the rate: $r \propto \int e^{\left(-E/kT - \frac{b}{E^{1/2}}\right)} dE$

$\frac{dr}{dE}$ – "efficiency" of the rates according to the energy

we search for the maximum

$$\frac{d}{dE}\left(\frac{dr}{dE}\right) = 0 = e^{\left(-\frac{E_{max}}{kT} - \frac{b}{E_{max}^{1/2}}\right)}\left[-\frac{1}{kT} + \frac{1}{2} b E_{max}^{-3/2}\right]$$

$$\Rightarrow \frac{2}{kTb} = E_{max}^{-3/2} \Rightarrow \underbrace{E_{max} \propto \left(\frac{bkT}{2}\right)^{2/3}}_{\text{gamov peak}}$$



estimate: Gamov peak narrow $\Rightarrow$ other energies almost do not contribute

$\rightarrow$ integrand can be replaced by narrow gaussian

then: $r \propto \frac{1}{T^{2/3}} e^{-\frac{a}{T^{1/3}}}, \quad a = \frac{3(b/2)^{2/3}}{k^{1/3}}$

from M–B distribution + factor from the integration

Estimates from the internal structure

→ central temperature:

hydrostatic equilibrium + equation of state

$$\langle \rho \rangle \propto \frac{M_\odot}{R_\odot^3}$$

$$\frac{dp}{dr} \sim \frac{0 - P_c}{R_\odot - 0} \sim - \frac{G M_\odot \langle \rho \rangle}{(R_\odot - 0)^2} \sim - \frac{G M_\odot^2}{R_\odot^5}$$

$$P_c \sim \frac{\mathcal{R} \langle \rho \rangle T_c}{\mu} \sim \frac{\mathcal{R} M_\odot \boxed{T_c}}{\mu R_\odot^3}$$

$$\Rightarrow \boxed{T_c} = \frac{P_c \mu R_\odot^3}{\mathcal{R} M_\odot} \sim \frac{G M_\odot^2}{R_\odot^4} \frac{\mu R_\odot^3}{\mathcal{R} M_\odot} \sim \frac{G M_\odot \mu}{\mathcal{R} R_\odot}$$

for $\mu = 0{,}6$ $\quad T_c \sim 1{,}4 \times 10^7\ K$