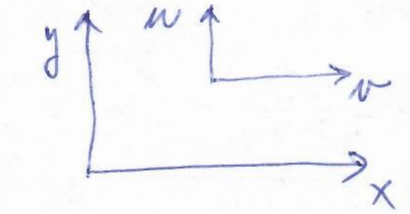
$\boxed{\text{Rossby waves}}$

→ effect of the Coriolis force

→ Coriolis parameter $f = 2\Omega \sin\varphi$
↑ latitude

Local Cartesian model:

$\rho \neq \rho(z)$, thin layer



~~[illegible]~~ considered forces – Coriolis & pressure gradient

$$\frac{du}{dt} = f v - \frac{\partial p}{\partial x} \quad (*) \qquad \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0$$

$$\frac{dv}{dt} = -f u - \frac{\partial p}{\partial y} \quad (**)$$

→ non-divergent velocity field

→ ignore mean flow in x (rotation). Adding constant v_0 not a problem, adding $v_0(y)$ – differential rotation – is a complication

non-divergent flow ⇒ has stream function

$$v = \frac{\partial \psi}{\partial x} \; ; \quad u = -\frac{\partial \psi}{\partial y}$$

← allows to relate ψ to isobars

$$\frac{\partial}{\partial x} : \quad \frac{d}{dt}\left(\frac{\partial u}{\partial x}\right) = f \frac{\partial v}{\partial x} - \frac{\partial^2 p}{\partial x^2} = f \frac{\partial^2 \psi}{\partial x^2} - \frac{\partial^2 p}{\partial x^2}$$

$$\frac{\partial}{\partial y} : \quad \frac{d}{dt}\left(\frac{\partial v}{\partial y}\right) = -f \frac{\partial u}{\partial y} - u \frac{df}{dy} - \frac{\partial^2 p}{\partial y^2} = f \frac{\partial^2 \psi}{\partial y^2} - \frac{\partial^2 p}{\partial y^2} + \frac{df}{dy}\frac{\partial \psi}{\partial y}$$

$+$

$$\frac{d}{dt}\underbrace{\left(\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y}\right)}_{=0} = f\left(\frac{\partial^2 \psi}{\partial x^2} + \frac{\partial^2 \psi}{\partial y^2}\right) - \left(\frac{\partial^2 p}{\partial x^2} + \frac{\partial^2 p}{\partial y^2}\right) + \frac{df}{dy}\frac{\partial \psi}{\partial y}$$

$$\Delta p = f \Delta \psi + \frac{df}{dy}\frac{\partial \psi}{\partial y}$$

= A =

$\Rightarrow$ if $f \neq f(y)$ $\rightarrow$ stream lines and isobars coincide $\Rightarrow$ balance between Coriolis force and pressure gradient force $\rightarrow$ no space for waves or oscillations $\rightarrow$ <u>geostrophic balance</u>.

$f = f(y)$ allows for <u>imbalance</u> which may drive oscillations

$$\frac{\partial}{\partial x}(**): \quad \frac{d}{dt}\frac{\partial v}{\partial x} = -f\frac{\partial w}{\partial x} - \frac{\partial p}{\partial x \partial y}$$

$$\frac{\partial}{\partial y}(*): \quad \frac{d}{dt}\left(\frac{\partial w}{\partial y}\right) = f\frac{\partial v}{\partial y} + \frac{df}{dy}v - \frac{\partial p}{\partial x \partial y}$$

$\Big\}$ $-$

$$\frac{d}{dt}\left(\frac{\partial v}{\partial x} - \frac{\partial w}{\partial y}\right) = -f\underbrace{\left(\frac{\partial w}{\partial x} + \frac{\partial v}{\partial y}\right)}_{=0} - \frac{df}{dy}v$$

$$\frac{d}{dt}\left(\frac{\partial^2 \psi}{\partial x^2} + \frac{\partial^2 \psi}{\partial y^2}\right) = -\frac{df}{dy}\frac{\partial \psi}{\partial x}$$

$$\frac{d}{dt}\Delta\psi = -\frac{df}{dy}\frac{\partial \psi}{\partial x}$$

for $f = f(y)$ relative (local) vorticity is not conserved — wave driving

$\hookrightarrow$ solution: $\psi = A\cos(ny)\,e^{ik(x - c_r t)}$, $-\frac{\pi}{2} \leq y \leq \frac{\pi}{2}$, $n \in \mathbb{Z}$

wave propagates $\parallel x$

$$\beta \equiv \frac{df}{dy} = 2\Omega\cos\varphi_0 / r_0 \quad r_0 \ldots \text{ tachocline radius}$$

$$\Rightarrow c_r = \ominus\frac{\beta}{k^2 + n^2}$$

$\hookrightarrow$ retrograde propagation with respect to co-rotating frame

$\hookrightarrow$ longer wavelength propagate faster westward since $\nabla^2 p = f \nabla^2 \psi + \frac{df}{dy}\frac{\partial \psi}{\partial y}$

$\Rightarrow p = A\left[f \cos ny + c_r n \sin ny \right] e^{ik(x - c_r t)}$

velocities $\rightsquigarrow$
$$\begin{cases} \frac{dx}{dt} = u = -\frac{\partial \psi}{\partial y} = An \sin ny \cos[k(x - c_r t)] \\ \frac{dy}{dt} = v = \frac{\partial \psi}{\partial x} = -Ak \cos ny \sin[k(x - c_r t)] \end{cases}$$

$x = x(t)$
$y = y(t)$

$\rightarrow$ counter-clockwise around lows, clockwise around highs

$\hookrightarrow$ the same as when nearly geostrophic