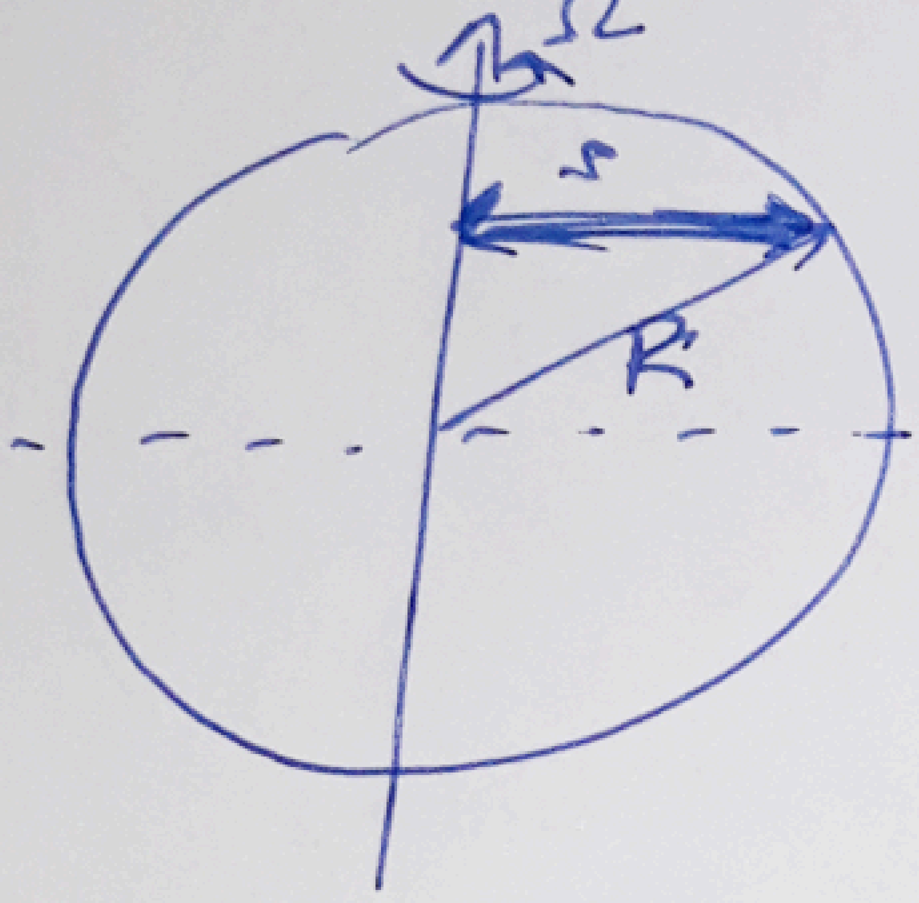
Solar rotation

- differential
- equator about 30% faster than the poles
- radiative zone rotates almost rigidly
- dif. rotation → key ingredients of the dynamo

Effect of the rotation on the interior structure

→ hydrostatic equilibrium must be modified — centrifugal force



$$\nabla p = -\rho \vec{g} + \rho \Omega^2 \vec{s}$$
$$= -\rho \nabla \phi + \rho \Omega^2 s \qquad (1)$$
$$= -\rho \underbrace{\nabla \psi}$$

modified potential, includes the action of centrifugal force

if Ω constant on cylinders ⇒ rotation has a separate potential
"conservative" rotation

$$\Omega^2 s = -\nabla V \Rightarrow V = -\int_0^s \Omega^2 s \, ds$$

$\nabla \phi \parallel \nabla V$ then ⇒ isosurfaces of ψ walign with surfaces of constant pressure
→ depends only on value of ψ.

for μ = const ⇒ T also function of ψ

$p = \frac{R \rho T}{\mu}$ if $\mu \neq$ const ⇒ T/μ function of ψ

Then: $\nabla p = -\rho \nabla \psi \quad | \nabla \times$

$$\underbrace{\nabla \times \nabla p}_{=0} = -\nabla \times (\rho \nabla \psi) = -(\nabla \rho) \times (\nabla \psi) - \rho \underbrace{(\nabla \times \nabla \psi)}_{=0}$$

$$\Rightarrow 0 = \nabla \rho \times \nabla \psi \Rightarrow \nabla \rho \parallel \nabla \psi$$

density function of ψ

back to the structure equations

→ continuity equation → Poisson equation

$\Delta\phi = 4\pi G\rho$ originally

$\Delta\psi = \Delta\phi + \Delta V = 4\pi G\rho - 2\Omega^2$ (2) now

→ radiative transfer equation:

$$L_R = -\frac{16\sigma T^3}{3\kappa\rho}\nabla T = -\frac{16\sigma T^3}{3\kappa\rho}\frac{dT}{d\psi}\nabla\psi$$

$$\rightarrow L_R = f(\psi)\nabla\psi \; ; \; f(\psi) = -\frac{16\sigma T^3}{3\kappa\rho}\frac{dT}{d\psi}$$

→ energy equilibrium

$\nabla \cdot L_R = \rho\epsilon$ (locally)

$$\nabla\cdot L_R = \frac{df}{d\psi}(\nabla\psi)^2 + f(\psi)\Delta\psi =$$

$$= \frac{df}{d\psi}\underbrace{(\nabla\psi)^2}_{(g_{eff})^2} + \underbrace{f(\psi)\left[4\pi G\rho - 2\Omega^2\right]}_{} = \underbrace{\rho\epsilon}_{}$$

constant of equipot. surfaces

not (!) constant on equipot. surfaces (g_{eff} larger on poles)

⇒ for rigidly rotating star – energy equilibrium equation cannot be fullfilled

→ <u>von Zeipel paradox</u> (1924)

→ need to modify the assumptions

⇝ aditional transport of energy from ~~the~~ poles (hotter) to equator (cooler)

→ <u>meridional circulation</u> necessary

characteristic time scale

$$\tau_{circ} \sim \frac{GM^2}{LR}\frac{1}{\xi} = \frac{\tau_{KH}}{\xi}$$ – K–H scale of contraction

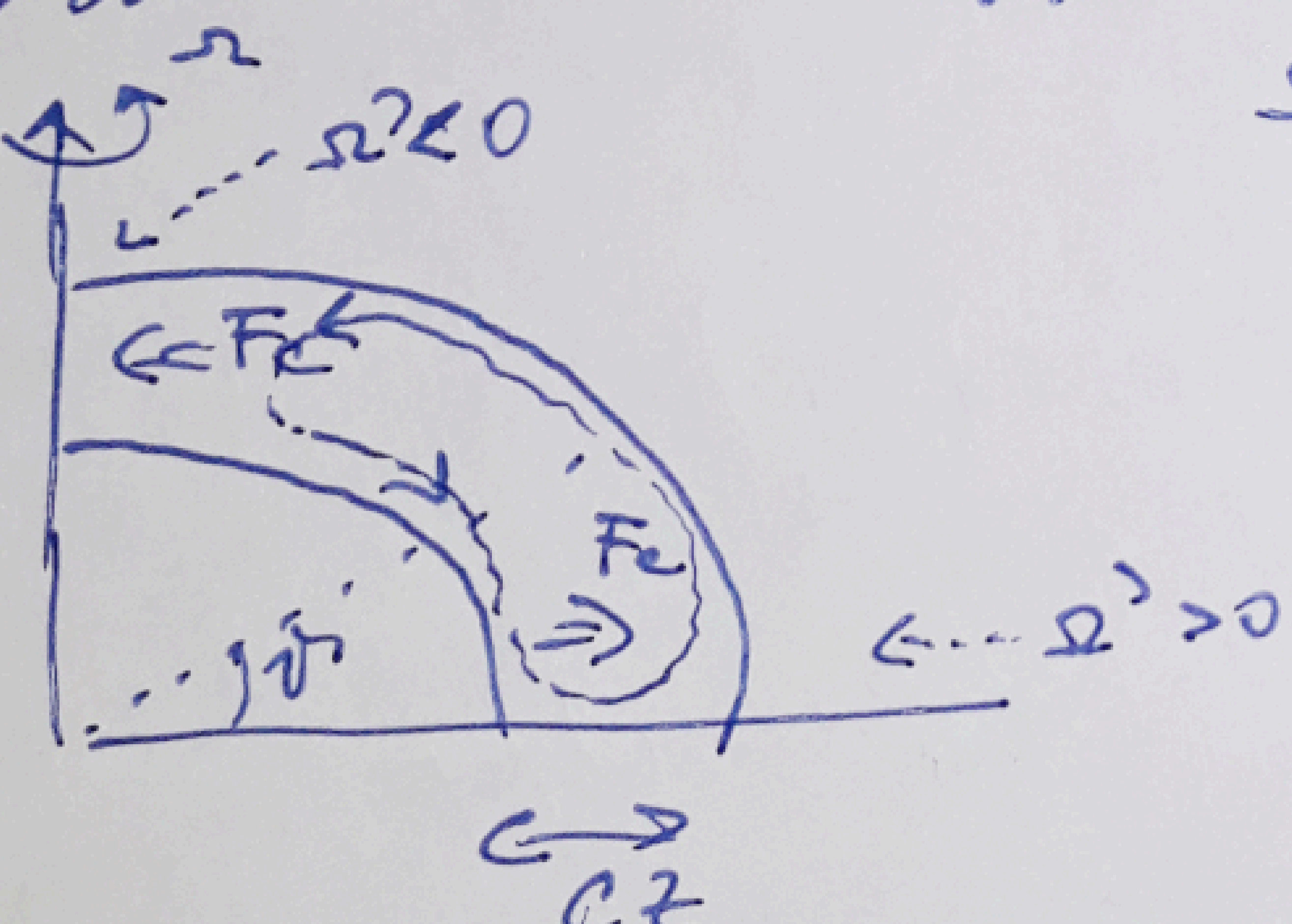
$$\xi = \frac{\Omega^2}{2\pi G\rho} = \frac{f_{centr}}{f_{grav}}$$ ... importance of the rotation

for the $\odot$: $\tau_{circ} \sim 10^{12}$ years (!)

How to interpret – heat incoming is not balanced by heat outgoing → an element (bubble) is either overheated or underheated compared to the surroundings. Buyoncy forces then lead to the circulation in the meridional plane.

Further (Baker & Kippenhahn, 1959) → circulation establishes for all kinds of rotation.
Roxburgh (1966) → it establishes for all forms of convection
↳ a stable solution can be found – both rotation and meridional circulation are not conservative → they do not have a potential

But → for the ☉ it is not important ($\tau_{circ} \sim 10^{12}$ yr)
in the ☉ a different form of the meridional circulation → ~~gyres~~ turbulent pumping



$\Omega' = \Omega(r) - \Omega_{mean}$

$F_c \propto -\Omega \times r$
↖ centrif. force
equator outwards
pole downwards
→ form a cell

Model of the differential rotation

→ basics: angular momentum conservation in the convection zone

$\mathcal{L} = r \times p = \text{const}$

$\vec{v} = \langle v_\varphi \rangle \vec{e}_\varphi + \vec{v}_m + \vec{w}$ ~ convection

differential rotation

$\vec{v}_m = (\langle v_r \rangle, \langle v_\vartheta \rangle, 0)$

} meridional circulation

$\langle \cdot \rangle$ = average over φ (longitude)

HD: $\vec{v} = \langle \vec{v} \rangle + \vec{w}$

$$\frac{\partial \vec{v}}{\partial t} + (\vec{v}\cdot\nabla)\vec{v} = -\frac{1}{\rho}\nabla p - \nabla\phi \quad (1)$$

$$\frac{\partial \rho}{\partial t} + \nabla\cdot(\rho\vec{v}) = 0 \quad (2)$$

azimuthal component $\langle v_\varphi \rangle = v_{rot} = \underbrace{r \sin\vartheta}_{s}\,\Omega = s\Omega$

we average (1) and (2) over φ

(1) $\frac{\partial v_\varphi}{\partial t} + (v\cdot\nabla)_\varphi v_\varphi = 0 \quad |\cdot s\rho$

(2) stationary : $\frac{\partial \rho}{\partial t} = 0 \Rightarrow \nabla\cdot(\rho\vec{v}) = 0$

„anelasticity"

(1) $s\rho\frac{\partial v_\varphi}{\partial t} + s\rho(\vec{v}\cdot\nabla)v_\varphi = 0$

$$(\rho\vec{v}\cdot\nabla)\vec{v} = \nabla\cdot(\rho\vec{v}\vec{v}) - (\nabla\cdot\rho\vec{v})\vec{v} = \nabla\cdot(\rho\vec{v}\vec{v})$$

$$\vec{v} = \begin{pmatrix} 0 \\ 0 \\ v_{rot} \end{pmatrix} + \begin{pmatrix} v_r \\ v_\vartheta \\ 0 \end{pmatrix} + \begin{pmatrix} w_r \\ w_\vartheta \\ w_\varphi \end{pmatrix}$$

rotation — meridional circulation — convection

$\frac{\partial \rho}{\partial t} = 0$ and $\langle\ \rangle_\varphi$

$$\frac{\partial}{\partial t}\left(s\rho\langle v_\varphi\rangle_\varphi\right) + \nabla\cdot\Big(s\rho\,\underbrace{\langle \vec{v}\, v_\varphi\rangle_\varphi}_{?}\Big) = 0$$

$$\langle \vec{v}\, v_\varphi \rangle_\varphi = \left\langle (\vec{v}_{rot} + \vec{v}_m + \vec{w})(v_{rot} + w_\varphi)\right\rangle_\varphi =$$

$$= \langle \vec{v}_{rot} v_{rot} + \vec{v}_m v_{rot} + \vec{w} v_{rot} + \vec{v}_{rot} w_\varphi + \vec{v}_m w_\varphi + \vec{w} w_\varphi \rangle_\varphi =$$

$$= v_{rot}^2 \frac{\vec{v}_{rot}}{v_{rot}} + v_{rot}\vec{v}_m + \underbrace{v_{rot}\langle \vec{w}\rangle_\varphi + \vec{v}_{rot}\langle w_\varphi\rangle_\varphi + \vec{v}_m\langle w_\varphi\rangle_\varphi}_{=0 \iff \langle w\rangle_\varphi = 0} + \langle \vec{w} w_\varphi\rangle_\varphi =$$

convection stochastic

$$\Rightarrow \frac{\partial}{\partial t}(s\rho v_{rot}) + \nabla\cdot\left(s\rho v_{rot}^2\,\vec{e}_\varphi + s\rho v_{rot}\vec{v}_m + \rho s\langle \vec{w} w_\varphi\rangle_\varphi\right) = 0$$

$v_{rot} = s\Omega$

$$\frac{\partial}{\partial t}\underbrace{(\rho s^2 \Omega)}_{= \mathcal{L} = \text{const}} + \nabla \cdot \left(\rho s^2 \Omega \vec{v}_m + \rho s \langle \vec{w} w_\varphi \rangle_\varphi\right) = 0$$

$$\Rightarrow \nabla \cdot \left(\rho s^2 \Omega \vec{v}_m + \rho s \langle \vec{w} w_\varphi \rangle_\varphi\right) = 0$$

for $\mathcal{L} =$ const the ballance is by meridional motions and convection

for $u_\varphi \nearrow$ (local acceleration) $\Rightarrow v_m \searrow$ to keep ballance. $\Rightarrow$ faster elements travel towards equator

for $u_\varphi \searrow$ (local deceleration) $\Rightarrow v_m \nearrow$

$\Rightarrow$ slower element travel to the poles

$\Rightarrow$ acceleration of the equator and deceleration of the poles

$\rightarrow$ sun-like differential rotation

$\rightarrow$ inconsistency: $\frac{\partial \mathcal{L}}{\partial t} \neq 0$, $\nabla \cdot (\rho w) \neq 0$, because it drives the convection