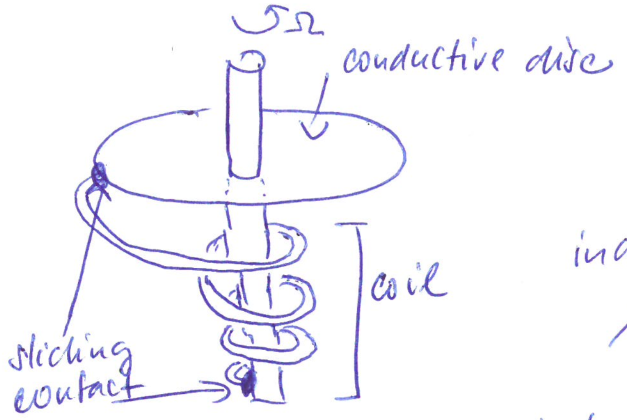
Homopolar dynamo



there is a current $I(t)$ flowing through a system. can $I(t)$ be rising?

inducted magnetic flux

$\phi = MI$, M... mutual conductance of wire & disc

rotation → origin of the electromotive ~~force~~ voltage $\mathcal{E}$

$$\mathcal{E} = \frac{d\phi}{dt} = \overset{\sim \frac{d}{dt}}{\frac{\Omega}{2\pi}}\phi = \frac{\Omega}{2\pi} M I$$

for the current: $L\frac{dI}{dt} + RI = \mathcal{E} = \frac{\Omega}{2\pi} M I$

solution seeking $I(t) = I_0 e^{\lambda t}$

$\Rightarrow \lambda = \frac{1}{L}\left(\frac{M}{2\pi}\Omega - R\right)$, L... inductance of the coil

rise for $\lambda > 0 \Rightarrow \Omega > \frac{2\pi R}{M}$

$\Rightarrow$ fast rotation = current generation

Mean-field dynamo

$$\frac{\partial B}{\partial t} = \nabla\times(v\times B) + \eta \Delta B = \nabla\times\left[v\times B - \eta\nabla\times B\right]$$

define: $B = \langle B\rangle + b$ ← fluctuations

↳ mean field $\quad \langle b\rangle = 0$

$v = \langle v\rangle + w \quad \langle w\rangle = 0$

then:

$$\frac{\partial}{\partial t}(\langle B\rangle + b) = \nabla\times\left[(\langle v\rangle + w)\times(\langle B\rangle + b) - \eta\nabla\times(\langle B\rangle + b)\right]$$

the mean part by $\langle\ \rangle$

$$\frac{\partial}{\partial t}\langle B\rangle = \nabla\times\left[\langle v\rangle\times\langle B\rangle + \langle w\times b\rangle - \eta\nabla\times\langle B\rangle\right]$$

fluctuating = original − mean

$$\frac{\partial}{\partial t} b = \nabla\times\left[\langle v\rangle\times b + w\times\langle B\rangle + w\times b - \langle w\times b\rangle - \eta\nabla\times b\right]$$

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definition $\mathcal{E} = \langle w \times b \rangle$

$G = w \times b - \langle w \times b \rangle$

→ electric field ~~ele~~ induced by fluctuations

~~Let~~'s assume: b and $\langle B \rangle$ in linear relation

$\mathcal{E}$ and b in linear relation

then $\mathcal{E}$ and $\langle B \rangle$ should also be linear

hence $\mathcal{E} = \alpha \langle B \rangle - \beta \nabla \times \langle B \rangle + \dots$

for an isotropic turbulence:

$\alpha = 1/3 \, \langle w \cdot \nabla \times w \rangle \tau$

$\beta = 1/3 \, \langle w \cdot w \rangle \tau$ $\quad \tau \dots$ correlation time

kinetic helicity

then the mean-field equation:

$$\frac{\partial \langle B \rangle}{\partial t} = \nabla \times \Big[\underbrace{\langle v \rangle \times \langle B \rangle}_{\Omega \text{ effect}} + \underbrace{\alpha \langle B \rangle}_{\alpha \text{ effect}} - (\xi + \beta) \nabla \times \langle B \rangle \Big]$$

$\xi + \beta = \xi_t$ ... turbulent viscosity

α effect: toroidal → poloidal

Ω effect: poloidal → toroidal

how much is α? $\quad \alpha = \pm \, l \, \Omega$ rotational speed

→ convective scale

α should be negative to Ω,

$\alpha \dots$ quenching

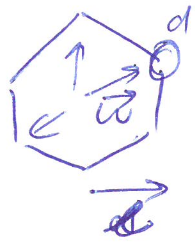
numerically: $\alpha = \dfrac{\langle w \times b \rangle \cdot B_H}{B_H^2}$ $\quad B_H \dots$ spots

$\alpha \in \langle \sim m/s, \ \sim 100 \, m/s \rangle$

$= B =$

$\boxed{\text{Sunspots}}$

flux concentration by supergranulation:



equilibrium: decay rate $\sim \frac{d^2}{\eta}$ ← diffusivity

the same as advection towards edges in the steady state

$$\tau = \frac{d^2}{\eta} = \frac{l}{w} \Rightarrow d^2 = \frac{\eta l}{w} = \frac{l^2}{R_m}$$

↳ magnetic Reynolds number

? how the field could be concentrated? → all the background flux conserved

$$B_0 l^2 = B d^2 \Rightarrow B = \frac{B_0 l^2}{d^2} = \frac{B_0 l^2}{l^2/R_m} = R_m B_0$$

characteristic time:

$l = 30$ Mm, $R_m \sim 10^4$, $B_0 \sim 0{,}1$ G, $w \sim 300$ m/s

$\Rightarrow d \sim 300$ km, $B \sim 10^3$ G ← enough for the spot

$\tau \sim l/w = 10^5$ s ~ 1 day

acting against: pressure ballance

$$\frac{B_{max}^2}{2\mu} \sim \frac{\rho w^2}{2} \Rightarrow B_{max} \sim \sqrt{\mu\rho}\, w$$

for the photosphere: $\rho = 3 \times 10^{-4}$ kg/m^3

$w \sim 300$ m/s

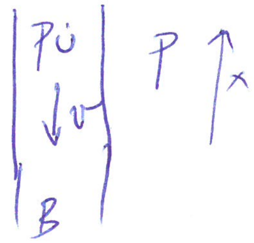
$\rightarrow \underline{B_{max} = 60\text{ G}}$

not enough for the spot

- different mechanism

<u>convective collapse</u>

- vertical flux tube → adiabatic cooling → downflow



pressure equilibrium: $p = p_i + \frac{B^2}{2\mu}$

motion equation in the tube

$$\frac{\partial v}{\partial t} + v\frac{\partial v}{\partial x} + \frac{1}{\rho}\frac{\partial p_i}{\partial x} = 0$$

$$\frac{\partial v}{\partial t} + \frac{\partial}{\partial x}\left[\frac{1}{2} v^2 + \frac{p}{\rho} - \frac{B^2}{2\rho\mu}\right] = 0$$

→ looking for a typical behavior ⇒ integrate over a piece of the flux tube $\int_{x_1}^{x_2} \dots dx = \langle \dots \rangle$

stationary solution: ⇒ $\frac{\partial}{\partial t}\langle v \rangle = 0$

⇒ $\left\langle p + \frac{1}{2}\rho v^2 - \frac{B^2}{2\mu} \right\rangle = \text{const} = \langle p_i \rangle$

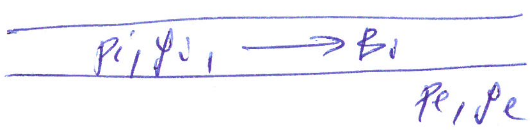
hence $\left\langle \frac{B^2}{2\mu} \right\rangle = \langle p \rangle + \frac{1}{2}\langle \rho v^2 \rangle - \langle p_i \rangle$

⇒ mg. field rises with increasing external pressure or with the flow along the flux tube

→ a detailed model: collapse stable for $B > 0{,}1\,T$

$\boxed{\text{Magnetic buoyancy}}$

→ horizontal flux tube at the bottom of the convection zone:



pressure ballance:

$$p_e = p_i + \frac{B_i^2}{2\mu}$$

EOS: $p = \frac{RT\rho}{\mu}$; $\mu = 1$

⇒ $RT(\rho_e - \rho_i) = \frac{B_i^2}{2\mu}$ | for $\rho_i < \rho_e$ ... buoyancy by force $(\rho_e - \rho_i)g$

against magnetic tension

Lorentz force

$$f_L = j \times B = \frac{1}{\mu}(\nabla \times B) \times B = \underbrace{\frac{(B\cdot\nabla)B}{\mu}}_{\text{tension}} - \underbrace{\nabla \frac{B^2}{2\mu}}_{\text{work of mg. pressure}}$$

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tension: $\frac{(B\cdot\nabla)B}{\mu_0} \sim \frac{B_i^2}{l}$ ... l ... perturbation length

instability for: $(\rho_e-\rho_i)\, g > \frac{B_i^2}{\mu_0 l}$ from the pressure ballance

$$\cancel{(\rho_e-\rho_i)}\, g > \frac{2RT\,\cancel{(\rho_e-\rho_i)}}{l}$$

$$\Rightarrow l > \frac{2RT}{g} = 2H_p$$

$$\frac{dp}{dr} = -\rho g = p\frac{1}{p}\frac{dp}{dr} = p\frac{d\ln p}{dr} = -\frac{p}{H_p}$$

$$\frac{p}{\rho} = RT \Rightarrow \frac{p}{\rho g} = \frac{RT}{g} = \frac{p}{\frac{p}{H_p}} = H_p$$

thus if perturbation large enough → the tube gets unstable and keep rising

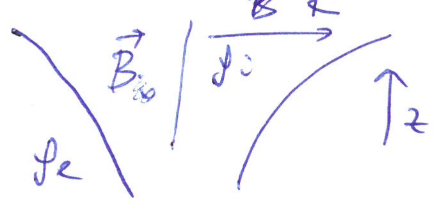
rising time: $\tau \sim \frac{d}{c_A}$; d ... depth

for 10 kG field at the bottom of CZ
($\langle\rho\rangle \approx 200\ \mathrm{kg/m^3}$, $d = 200\ \mathrm{Mm}$)

$\tau \sim 2$ month (or larger due to the c_A profile)

if $\tau > \tau_{rot}$ → Coriolis force steps in
→ deflection

$\boxed{\text{MHS model of the spot}}$



$B = B(R,\cancel{z})$

$\max(B) = B_i = B(R=0)$

$B(R\to\infty) = 0$

$\rho_i = \rho_i(z)$

$\rho_e = \rho_e(z)$

at depth pressure ballance:

$$p(R,z) + \frac{B^2(R,\cancel{z})}{2\mu_0} = p_e(z) \qquad \frac{\partial p}{\partial z} = -\rho(z)\, g$$

for $R\to\infty$: $\frac{dp_e}{dz} = -\rho_e(z)\, g$

for $R=0$: $p_i(z) + \frac{B_i^2(z)}{2\mu} = p_e(z)$

$$\frac{dp_i}{dz} = -\rho_i(z)\, g$$

→ ~~differentiate:~~ $\cancel{\frac{dp_i}{dz} = \frac{dp_e}{dz}}$

~~on $R=0$ $B_i =$ konst~~

~~$\Rightarrow \rho_i = \rho_e$~~

~~away from $R=0$~~ $p_i < p_e$ | $p = \frac{R \rho T}{\mu}$ | $\mu = 1$

$\cancel{p_i + \frac{B_i^2}{2\mu} = p_e}$ → $\frac{p_i}{p_e} + \frac{B_i^2}{2\mu p_e} = 1$

state equation

$B = 3000\,G - 2{,}4 \times 10^4\ N/m^2$

$1{,}4 \times 10^4\ N/m^2$

$$\underline{\frac{T_i(z)}{T_e(z)} = 1 - \frac{B_i^2}{2\mu p_e(z)}}$$

in the umbra, usually $\frac{B_i^2}{2\mu} > p_e(z)$

we differentiate, $\frac{2B_i}{2\mu}\frac{dB_i}{dz} > \frac{dp_e}{dz}$

$$\frac{dp_e}{dz} < 0 \Rightarrow \frac{dB_i}{dz} < 0$$

$$\phi = \int B\, dS = \text{konst} \Rightarrow \frac{dS}{dz} > 0$$

the flux tube diverges with height

since $p_i + \frac{B_i^2}{2\mu} = p_e$

$$\frac{dp_i}{dz} = \frac{dp_e}{dz} - \overbrace{\frac{2B_i}{2\mu}\frac{dB_i}{dz}}^{<0}$$

$$\Rightarrow \underset{\text{negative}}{\frac{dp_i}{dz}} > \underset{\text{negative}}{\frac{dp_e}{dz}}$$

$$\frac{dp_i}{dz} = -\rho_i(z)\, g$$

p_i drops faster in the tube ⇒ density drops too

→ partially responsible for the Wilson depression

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