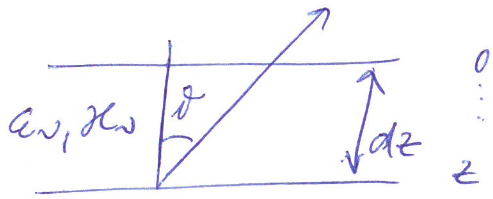
atmosphere

→ model = description of the changes in temperature, pressure, density, ... with height

→ RTE → how does the intensity change with height

$$\mu = \cos\vartheta$$



RTE: $\mu \dfrac{dI_\nu}{dz} = -\varkappa_\nu I_\nu + \varepsilon_\nu \quad / : \varkappa_\nu$

$$\mu \frac{dI_\nu}{\varkappa_\nu dz} = -I_\nu + \frac{\varepsilon_\nu}{\varkappa_\nu} \quad ; \quad \frac{\varepsilon_\nu}{\varkappa_\nu} \equiv S_\nu$$

$$d\tau_\nu \equiv -\varkappa_\nu dz$$

$$\Rightarrow \mu \frac{dI_\nu}{d\tau_\nu} = I_\nu - S_\nu$$

→ formal solution for a ~~semidefinite~~ semiinfinit atmosphere: $I_\nu(0,\mu) = \dfrac{1}{\mu}\displaystyle\int_0^\infty S_\nu(\tau_\nu)\, e^{-\tau_\nu/\mu}\, d\tau_\nu$

→ general solution for $\tau_1 \to \tau_2$

$$I_\nu = \frac{1}{\mu} I(\tau_1)\, e^{-\frac{\tau_2-\tau_1}{\mu}} + \frac{1}{\mu}\int_{\tau_1}^{\tau_2} S(\tau')\, e^{-\frac{\tau_2-\tau'}{\mu}}\, d\tau'$$

assume linear approximation:

$$S_\nu(\tau_\nu) = S_\nu(0) + b\,\tau_\nu$$

hence semiinfinite atmosphere:

$$I_\nu(0,\mu) = \int_0^\infty S_\nu(0)\, e^{-\frac{\tau_\nu}{\mu}} d\tau_\nu + b\int_0^\infty \tau_\nu \frac{1}{\mu} e^{-\frac{\tau_\nu}{\mu}} d\tau_\nu =$$

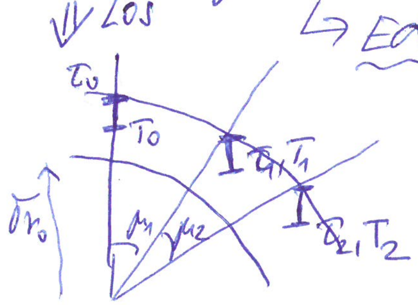
$$= \left| \int_0^\infty e^{-x} dx = \left[-e^{-x}\right]_0^\infty = 1 \; ; \; \int_0^\infty x\, e^{-x} dx = \left[e^{-x}(-x-1)\right]_0^1 = 1 \right| =$$

$$= S_\nu(0) + b\mu$$

⇒ intensity in direction μ equals to the source function at depth $\tau_\nu = \mu$

Formally: $I_\nu(0,\mu) = S_\nu(\tau_\nu)$

↳ Eddington-Barbier relation



$\tau_0 = \tau_1 = \tau_2$!

$\delta r_0 > \delta r_1 > \delta r_2$

for $T_0 > T_1 > T_2$

↳ limb darkening

→ photosphere approximately LTE $\Rightarrow S_\nu(\tau_\nu) \equiv B_\nu(T)$

$$\Rightarrow I_\nu(0,\mu) = B_\nu(T)$$

⇒ from the intensity across the disk (with various μ's) we scan the temperature profile in the atmosphere.

How?

$$d\tau_\nu = -\varkappa_\nu \, dz \qquad \Big| \text{ differentiate with } T$$

$$\frac{d\tau_\nu}{dT} = -\varkappa_\nu \frac{dz}{dT}$$

Eddington-Barbier

$$S_\nu(\tau_\nu) = B_\nu(T) \Rightarrow \frac{dS_\nu}{d\tau_\nu}\frac{d\tau_\nu}{dT} = \frac{dB}{dT}$$

$$\Rightarrow \frac{d\tau_\nu}{dT} = \frac{dB}{dT}\left(\frac{dS_\nu}{d\tau_\nu}\right)^{-1}$$

$$I_\nu(0,\mu) = B_\nu(T) \qquad \Big| \frac{d}{d\mu}$$

$$\frac{dI_\nu}{d\mu} = \frac{dB}{dT}\frac{dT}{d\mu} \Rightarrow \frac{dB}{dT} = \frac{dI_\nu}{d\mu}\left(\frac{dT}{d\mu}\right)^{-1}$$

$$\frac{d\tau_\nu}{dT} = \underbrace{\left(\frac{dI_\nu}{d\mu}\right)\left(\frac{dS_\nu}{d\tau_\nu}\right)^{-1}}_{=1,\ I_\nu(\mu) = S(\tau_\nu)}\left(\frac{dT}{d\mu}\right)^{-1}$$

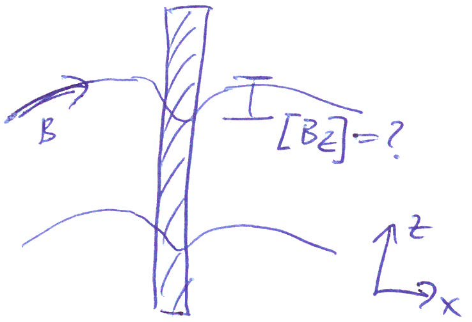
thus formally $\frac{dI_\nu}{d\mu} = \frac{dS_\nu}{d\tau_\nu}$

$$\Rightarrow -\varkappa_\nu \frac{dz}{dT} = \left(\frac{dT}{d\mu}\right)^{-1}$$

↳ ~~[illegible]~~ temperature profile with viewing angle

temperature profile with depth

Equilibrium in the prominence



$B = (B_x, B_y, B_z)$

$B_y \equiv 0, \ \frac{\partial}{\partial y} = 0$

$-\frac{dp}{dz} - \rho g + (j \times B)_z = 0$

→ negligible, mass supported by Lorentz force

↳ hydrostatic equilibrium

→ for $T \sim 10^4 K \rightarrow H_p \sim 300$ km

→ in conflict with observed heights

↳ integrate over x

$$g \int \rho \, dx = \int (j \times B)_z \, dx = \int \frac{1}{\mu} [(\nabla \times B) \times B]_z \, dx$$

$$j = \frac{1}{\mu} \begin{pmatrix} \partial_x & \partial_y & \partial_z \\ & \times & \\ B_x & B_y & B_z \end{pmatrix} = \left(\frac{\partial B_z}{\partial y} - \frac{\partial B_y}{\partial z}, \frac{\partial B_z}{\partial x} - \frac{\partial B_x}{\partial z}, \frac{\partial B_y}{\partial x} - \frac{\partial B_x}{\partial y} \right) =$$

$$= \left(0, \frac{\partial B_z}{\partial x} - \frac{\partial B_x}{\partial z}, 0 \right)$$

$$(j \times B)_z = \frac{1}{\mu} \left[\begin{pmatrix} 0 \\ \frac{\partial B_z}{\partial x} - \frac{\partial B_x}{\partial z} \\ 0 \end{pmatrix} \times \begin{pmatrix} B_x \\ 0 \\ B_z \end{pmatrix} \right]_z = \frac{1}{\mu} \left[-B_x \left(\frac{\partial B_z}{\partial x} - \frac{\partial B_x}{\partial z} \right) \right] =$$

$$= -\frac{1}{\mu} B_x \left(\frac{\partial B_z}{\partial x} - \frac{\partial B_x}{\partial z} \right)$$

slab is thin ⇒ B_x continuous over the slab

and $\frac{\partial B_z}{\partial x} \gg \frac{\partial B_x}{\partial z}$

then ⇒ $g \int_{x_1}^{x_2} \rho \, dx \sim -\frac{1}{\mu} \int_{x_1}^{x_2} B_x \frac{\partial B_z}{\partial x} dx \sim \frac{1}{\mu} B_x \int_{x_1}^{x_2} \frac{\partial B_z}{\partial x} dx =$

$= \frac{B_x}{\mu} \underbrace{[B_z]}_{\text{jump in } B_z}$

continuous, hence out of integral

for the prominence $[x] = 5000$ km, $\rho = 10^{-10}$ kg/m^3, $B_x = 10^{-3}$ T

$$\Rightarrow \underbrace{[B_z] \sim 2 \times 10^{-4} T}_{\text{small change, can't be measured}}$$