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Long-term tidal evolution of Kleopatra satellites

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ABSTRACT

Aims.
Methods.
Results.

Key words. Minor planets, asteroids: individual: (216) Kleopatra – Planets and satellites: individual: I Alexhelios – Planets and satellites: dynamical evolution and stability – Celestial mechanics – Methods: numerical

1. Introduction

2. Secular tides model

$$G_{20-4} = \frac{1}{576}e^8 + \frac{7}{2880}e^{10}\,, \tag{1}$$

$$G_{20-3} = \frac{1}{2304}e^6 + \frac{11}{18432}e^8\,, \tag{2}$$

$$G_{20-2} = 0\,, \tag{3}$$

$$G_{20-1} = \frac{1}{4}e^2 - \frac{1}{16}e^4 + \frac{13}{768}e^6 + \frac{113}{18432}e^8\,, \tag{4}$$

$$G_{200} = 1 - 5e^2 + \frac{63}{8}e^4 - \frac{155}{36}e^6 + \frac{2881}{2304}e^8\,, \tag{5}$$

$$G_{201} = \frac{49}{4}e^2 - \frac{861}{16}e^4 + \frac{21957}{256}e^6 - \frac{132635}{2048}e^8\,, \tag{6}$$

$$G_{202} = \frac{289}{4}e^4 - \frac{1955}{6}e^6 + \frac{83551}{144}e^8\,, \tag{7}$$

$$G_{203} = \frac{714025}{2304}e^6 - \frac{27483625}{18432}e^8 + 2986.78e^{10}\,, \tag{8}$$

$$G_{204} = \frac{284089}{256}e^8 - \frac{7369791}{1280}e^{10} + 12888.1e^{12}\,, \tag{9}$$

$$G_{205} = 3536.13e^{10} - 19815.8e^{12} + 49132.4e^{14}\,, \tag{10}$$

$$G_{206} = 10383.9e^{12} - 62736.2e^{14} + 170853e^{16}\,, \tag{11}$$

$$G_{207} = 28704.3e^{14} - 186401e^{16} + 553402e^{18}\,, \tag{12}$$

$$G_{208} = 75740.8e^{16} - 526830e^{18} + 1.69396\times 10^6 e^{20}\,, \tag{13}$$

$$s = \sin\frac{i}{2}\,, \tag{14}$$

$$c = \cos\frac{i}{2}\,, \tag{15}$$

$$F_{200} = -\frac{3}{2}s^2c^2\,, \tag{16}$$

$$F_{201} = -\frac{1}{2}c^4 + 2s^2c^2 - \frac{1}{2}s^4\,, \tag{17}$$

$$F_{202} = -\frac{3}{2}s^2c^2\,, \tag{18}$$

$$F_{210} = 3sc^3\,, \tag{19}$$

$$F_{211} = -3sc^3 + 3s^3c\,, \tag{20}$$

$$F_{212} = -3s^3c\,, \tag{21}$$

$$F_{220} = 3c^4\,, \tag{22}$$

$$F_{221} = 6s^2c^2\,, \tag{23}$$

$$F_{222} = 3s^4\,, \tag{24}$$

$$J(\omega) = \frac{1}{\mu} - \frac{\mathrm{i}}{\eta\omega} + \frac{\mu^{\alpha-1}}{(\mathrm{i}\zeta\eta\omega)^\alpha}\Gamma(1+\alpha)\,, \tag{25}$$

$$\bar{\mu} = \frac{1}{J}\,, \tag{26}$$

$$A_\ell = \frac{2\ell^2 + 4\ell + 3}{\ell}\frac{\bar{\mu}}{g\rho R}\,, \tag{27}$$

$$k_\ell = \frac{3}{2(\ell-1)}\frac{1}{1+A_l}\,, \tag{28}$$

$$\omega_{\ell mpq} = (l - 2p + q)n - m\omega\,, \tag{29}$$

$$\text{kvalitet} \equiv \Im\{k_l\}\,, \tag{30}$$

$$f_1 = \frac{(l-m)!}{(l+m)!}(2 - \delta_0^m)\,, \tag{31}$$

$$f_2 = f_1 F_{\ell mp}(i)^2 G_{\ell pq}(e)^2\,\text{kvalitet}\,, \tag{32}$$

$$[s_1, s_2, s_3] = \sum_{\ell=2}^{2}\sum_{m=\ell}^{\ell}\sum_{p=0}^{0}\sum_{q=-4}^{8}[(l-2p+q), (l-2p), m]\,f_2\,, \tag{33}$$

$$f = \frac{Gm_\oplus + Gm^\star}{Gm_\oplus}\frac{Gm^\star}{a}\left(\frac{R}{a}\right)^{2\ell+1}\,, \tag{34}$$

$$\frac{\partial \mathcal{R}}{\partial M} = f s_1 \,, \tag{35}$$

$$\frac{\partial \mathcal{R}}{\partial \omega} = f s_2 \,, \tag{36}$$

$$\frac{\partial \mathcal{R}}{\partial \Omega} = f s_3 \,, \tag{37}$$

$$\eta' = \sqrt{1 - e^2} \,, \tag{38}$$

$$\frac{\mathrm{d}a}{\mathrm{d}t} = \frac{2}{na} \frac{\partial \mathcal{R}}{\partial M} \,, \tag{39}$$

$$\frac{\mathrm{d}e}{\mathrm{d}t} = \frac{1 - e^2}{na^2 e} \frac{\partial \mathcal{R}}{\mathrm{d}M} - \frac{\eta'}{na^2 e} \frac{\partial \mathcal{R}}{\partial \omega} \,, \tag{40}$$

$$\frac{\mathrm{d}i}{\mathrm{d}t} = 0 \,, \tag{41}$$

$$f_T = \frac{1}{1 - e\cos E} \left(\frac{\mathrm{d}a}{\mathrm{d}t} \frac{n\eta'}{2} - \frac{\mathrm{d}e}{\mathrm{d}t} \frac{nae}{\eta'} \right) , \tag{42}$$

$$f_R = \frac{1}{\sin w} \left[\frac{\mathrm{d}e}{\mathrm{d}t} \frac{na}{\eta'} - f_T(\cos w + \cos E) \right] , \tag{43}$$

$$f_W = \frac{1}{\cos(\omega + w)} \frac{a}{|\boldsymbol{r}|} na\eta' \frac{\mathrm{d}i}{\mathrm{d}t} \,, \tag{44}$$

$$\boldsymbol{f}_{\mathrm{seculartides}} = f_R \hat{r} + f_T \hat{t} + f_W \hat{w} \,, \tag{45}$$

$$\vec{\Gamma} = \boldsymbol{r} \times m' \boldsymbol{f}_{\mathrm{seculartides}} \,. \tag{46}$$

a semimajor axis, e eccentricity, i inclination, Ω longitude of node, ω argument of pericentre, M mean anomaly, E eccentric anomaly, w true anomaly, n mean motion, $G_{\ell pq}$ Kaula eccentricity function, $F_{\ell mp}$ Kaula inclination function, J complex compliance for the Andrade rheology, $\bar{\mu}$ complex rigidity, μ rigidity, η viscosity, α slope of $Q(\omega)$, Q quality factor, ζ ratio Andrade-to-Maxwell time, k_ℓ complex Love number, $\omega_{\ell mpq}$ loading frequency (mode), $\mathcal{R}$ perturbing function, R radius of the Earth, $m_\oplus$ mass of the Earth, $m^\star$ mass of the Moon, m' mass of the test particle, $\hat{r}$ radial unitvector, $\hat{t}$ transversal unitvector, $\hat{w}$ normal unitvector,

Walterová (2021), Boué & Efroimsky (2019)

3. Long-term tidal evolution

4. Conclusions

Acknowledgements.

References

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