The Induction Equation

In solar MHD we eliminate the electric field, **E**, and the electric current density, **j** and work with the primary variable, **B**. To eliminate **E** we use (2.25) and (2.22) to get

$$\mathbf{E} = -\mathbf{v} \times \mathbf{B} + \frac{1}{\mu\sigma}\nabla \times \mathbf{B}, \tag{2.26}$$

and so (2.24) becomes

$$\frac{\partial \mathbf{B}}{\partial t} = \nabla \times (\mathbf{v} \times \mathbf{B}) - \nabla \times (\eta \nabla \times \mathbf{B}), \tag{2.27}$$

where

$$\eta = \frac{1}{\mu\sigma}, \tag{2.28}$$

is the *magnetic diffusivity*. Assuming η is constant (whereas it really depends on T and ρ), (2.27) simplifies to

$$\frac{\partial \mathbf{B}}{\partial t} = \nabla \times (\mathbf{v} \times \mathbf{B}) - \eta \nabla \times \nabla \times \mathbf{B},$$

$$= \nabla \times (\mathbf{v} \times \mathbf{B}) + \eta(\nabla^2 \mathbf{B} - \nabla(\nabla \cdot \mathbf{B})).$$

on using a vector identity. Using (2.23) we get the final form of the induction equation, namely

$$\frac{\partial \mathbf{B}}{\partial t} = \nabla \times (\mathbf{V} \times \mathbf{B}) + \eta \nabla^2 \mathbf{B}, \tag{2.29}$$

Example 2.3.1 What is the dimension of η?

$$\text{L.H.S.} \approx \frac{B}{\tau} \qquad \text{2nd term R.H.S.} \approx \eta \frac{B}{L^2}$$

$$\Rightarrow \eta \approx L^2 \tau^{-1}$$

In fact, η is given by Braginski (19) as

$$\eta = 10^9 T^{-3/2} m^2 s^{-1} \approx 1 . \text{in the solar corona}$$

The dimensions of η are the same as the kinematic viscosity in (2.19).

Example 2.3.2 The induction equation can be reduced to a dimensionless equation with only one parameter. This is done by setting $\mathbf{B} = B_0 \tilde{\mathbf{B}}$, $t = \tau \tilde{t}$, $x, y, z = L\tilde{x}, \tilde{y}, \tilde{z}$, $\mathbf{v} = (L/\tau)\tilde{\mathbf{v}}$. Substituting into the induction euqation gives

$$\frac{B_0}{\tau}\frac{\partial \tilde{\mathbf{B}}}{\partial \tilde{t}} = \frac{\tilde{\nabla}}{L} \times (\frac{L}{\tau}\tilde{\mathbf{v}} \times B_0 \mathbf{B}) + \eta \frac{B_0}{L^2}\nabla^2 \tilde{\mathbf{B}}.$$

$$\Rightarrow \frac{\partial \tilde{\mathbf{B}}}{\partial \tilde{t}} = \tilde{\nabla} \times (\frac{L}{\tau}\tilde{\mathbf{v}} \times \mathbf{B}) + \frac{\eta\tau}{L^2}\nabla^2 \tilde{\mathbf{B}}.$$

As we have seen above the combination $L^2/\eta\tau$ is a dimensionless number, R_m, called the *magnetic Reynold's number*.

$$R_m = \frac{L^2}{\eta\tau} = \frac{LV}{\eta}, \tag{2.30}$$

where V is a typical plasma velocity. On the Sun R_m is normally very large because L and V are typically large. R_m is a measure of the size of the

advection term, $\nabla \times (\mathbf{v} \times \mathbf{B})$, compared with the size of the diffusion term, $\eta \nabla^2 \mathbf{B}$.

Example 2.3.3 In sunspots, which are cooler than the coronal plasma by about a factor of 100, $\eta = 10^3 m^2 s^{-1}$, $L = 10^4 km$, $V = 1 km s^{-1} = 10^3 m s^{-1}$.

Therefore, the magnetic Reynold's number is

$$R_m = \frac{10^7 10^3}{10^3} = 10^7 \gg 1.$$

If $R_m \gg 1$ then the induction equation, (2.29), is approximated by

$$\frac{\partial \mathbf{B}}{\partial t} = \nabla \times (\mathbf{v} \times \mathbf{B}). \qquad (2.31)$$

However, if $R_m \ll 1$ then (2.29) reduces to

$$\frac{\partial \mathbf{B}}{\partial t} = \eta \nabla^2 \mathbf{B}. \qquad (2.32)$$

This can be thought of in terms of two different timescales with *(i)* changes due to fluid motions from (2.31)

$$\frac{B}{\tau_{motion}} \approx \frac{V}{L} B \qquad \Rightarrow \tau_{motion} \approx \frac{L}{V}.$$

and *(ii)* changes due to field diffusion from (2.32)

$$\frac{B}{\tau_{diffusion}} \approx \eta \frac{B}{L^2} \qquad \Rightarrow \tau_{diffusion} \approx \frac{L^2}{\eta}.$$

In general solar situations $\tau_{motion} \ll \tau_{diffusion}$ which corresponds to $R_m \gg 1$.

In this case the diffusion term in (2.3) is negligible. However, there are several important exceptions, for example in the neighbourhood of magnetic neutral points and lines, during magnetic reconnection and during solar flares.

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Remarks
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