Properties of the solution for B(x,t)

(i)

Figure 2.2: The self similar solution for various values of time.

In Figure (2.2) the values of x_1 and x_2 are related by the same value of ξ. Thus,

$$\frac{x_1}{(4\eta t_1)^{1/2}} = \frac{x_2}{(4\eta t_2)^{1/2}}.$$

(ii) For $|x| \ll (4\eta t)^{1/2}$, the value of ξ is small and the error function may be expanded in powers of ξ, since

$$\int_0^{\xi} e^{-u^2} du \approx \xi - \frac{1}{3}\xi^3 + \cdots$$

Thus,

$$B(x,t) \approx \frac{2B_0}{\sqrt{\pi}} \frac{x}{(4\eta t)^{1/2}}.$$

(iii) The current is $\mathbf{j} = \nabla \times \mathbf{B}/\mu$ and this implies that $\mathbf{j} = j\hat{\mathbf{y}}$ where

$$j = -\frac{1}{\mu}\frac{dB}{dx} = -\frac{B_0}{\mu(4\eta t)^{1/2}} e^{-x^2/4\eta t}.$$

The current has a gaussian shape and is concentrated near $x = 0$. The width of the current sheet (or more correctly the current layer) is $(4\eta t)^{1/2}$ and it increases with time. The total current in the plasma is

$$J = \int_{-\infty}^{\infty} j dx = -\frac{1}{\mu}\int_{-\infty}^{\infty} \frac{dB}{dx} dx = -\frac{2B_0}{\mu}.$$

The total current is constant and independent of time. However, intially all the current is at $x = 0$ but it spreads out as time proceeds. During the decay of $\mathbf{B}$ the magnetic energy lost is converted into ohmic heating that heats the plasma.

Example 2.4.2 Separation of variables

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