Per asteroides ad astra

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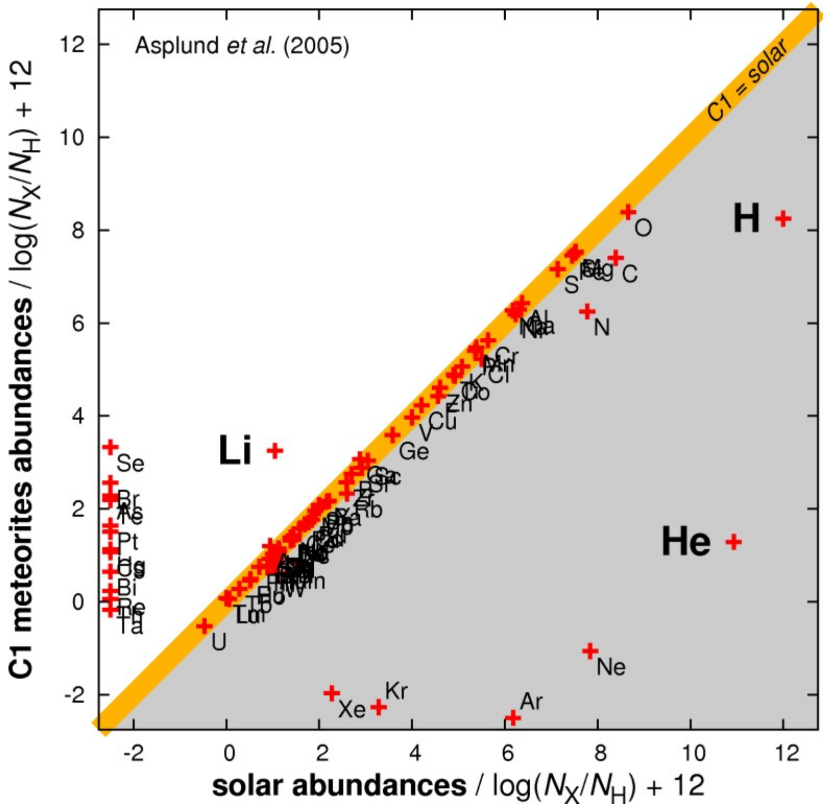
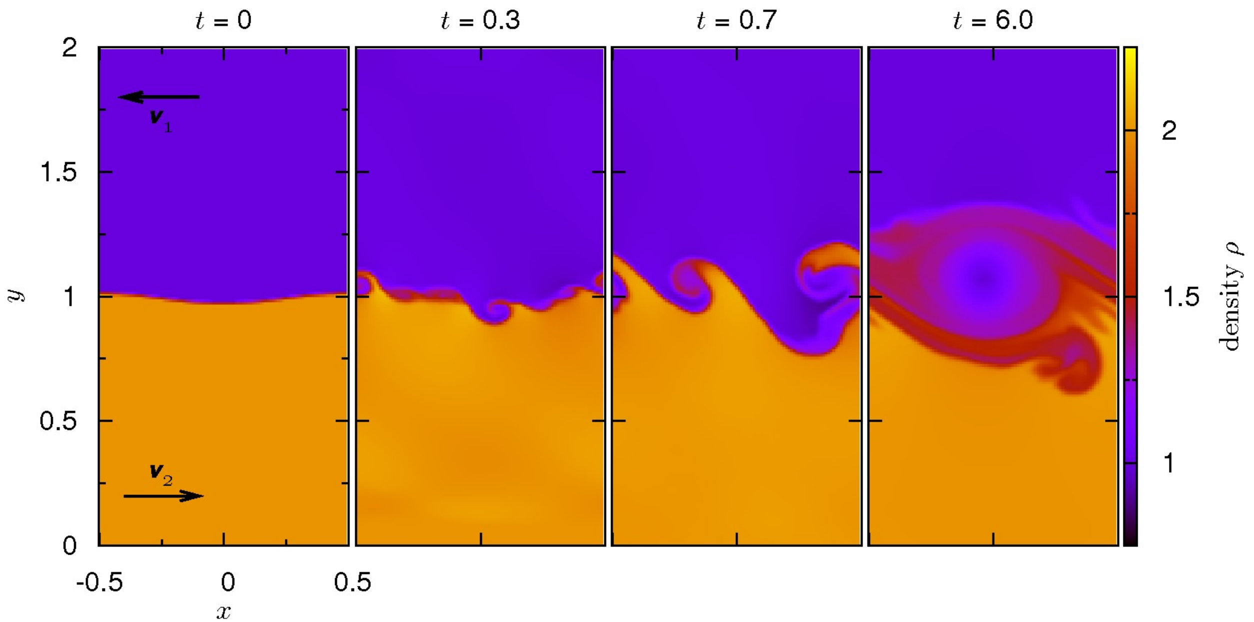
1. general problems
2. observations
3. our dynamical model (1 of)
4. asteroid families
5. future applications



serious
Five problems

- turbulence
- chaos
- irreversibility
- stochasticity
- $t = 0$



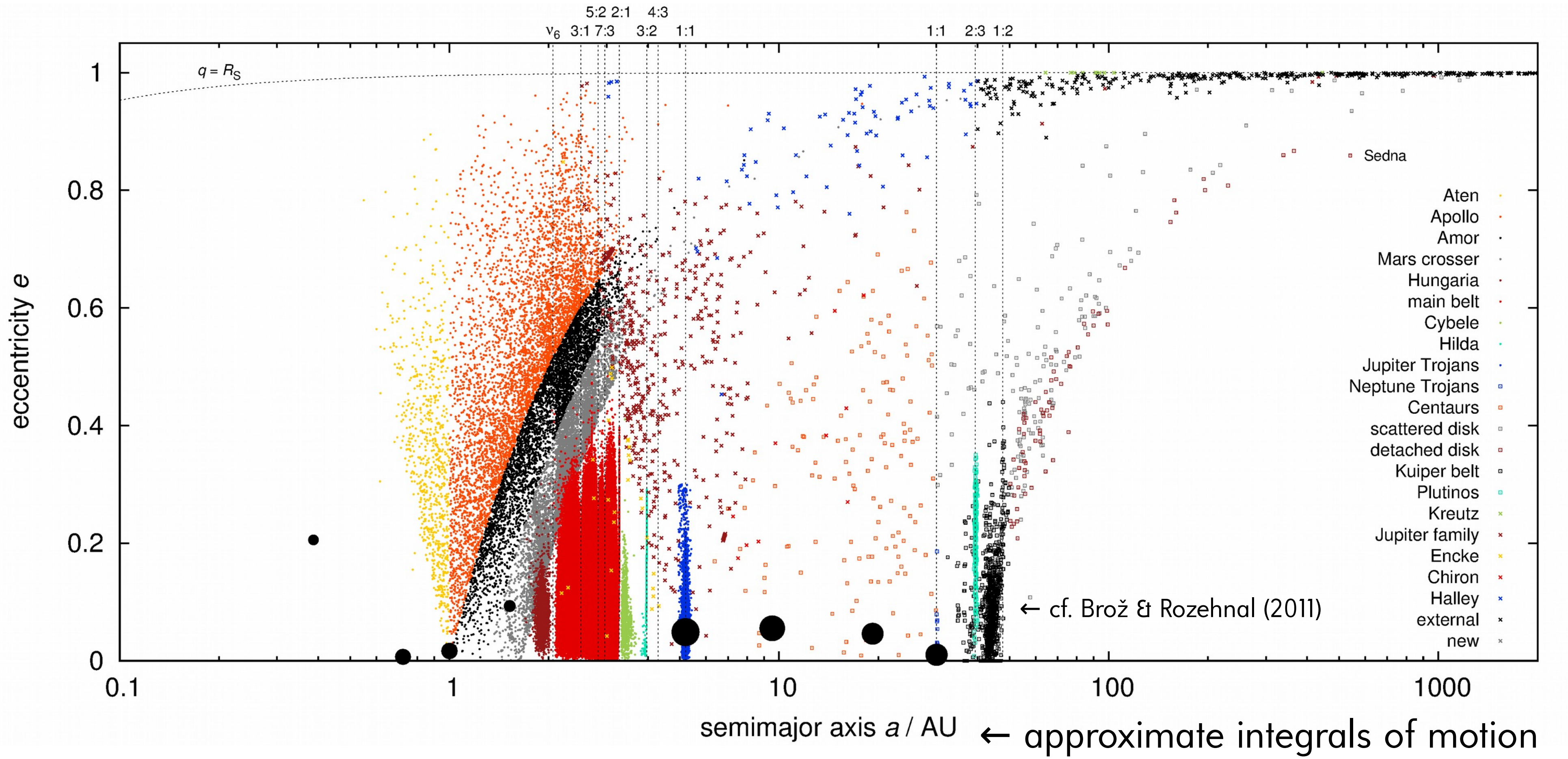
Kelvin–Helmholtz instability
Pluto code (Mignone et al. 2007)

↓
an **inverse** problem
(with exceptions)

additional instabilities:
Rayleigh–Taylor
magneto-rotational (Flock et al. 2013)
streaming (Johansen et al. 2007)

Observations ← *usually* taken @ 4.56 Gyr

- orbital distribution → families everywhere: MB, Hildas, Trojans of J & M, TNOs, irregular moons, ...

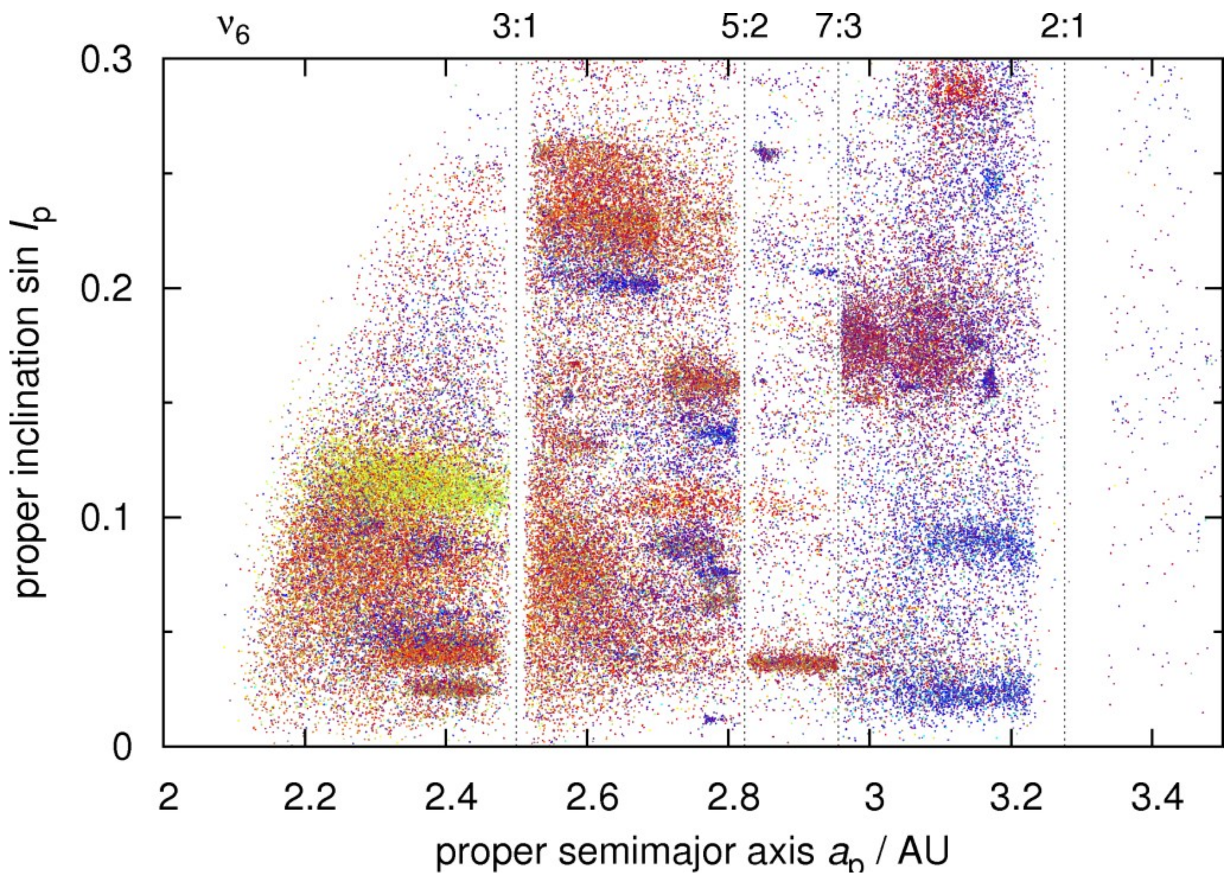


← cf. Brož & Rozehnal (2011)

← approximate integrals of motion

Observations (cont.)

- a detail of the Main Asteroid Belt, **124** families in total (according to our review in AIV, Nesvorný et al. 2015)



SDSS colours
Parker et al. (2008)

Eos family
Brož & Morbidelli (2013)

a “standard” for families → *N*-body model

We use a symplectic integration scheme (Levison and Duncan 1994), denoted as kick–drift–kick, where the ‘kick’ (actually, a perturbation) is performed as:

$$\dot{\mathbf{r}}_{n+1} = \dot{\mathbf{r}}_n + \ddot{\mathbf{r}}\frac{\Delta t}{2}\,, \tag{3}$$

and the ‘drift’ corresponds to an analytical solution of the two-body problem (the Sun–asteroid), which involves a numerical solution of the transcendent Kepler equation:

0. drift

$$\begin{aligned} M &= E - e\sin E\,, &(4)\\ \mathbf{r}_{n+1} &= p(E)\mathbf{r}_n + q(E)\dot{\mathbf{r}}_n\,, &(5)\\ \dot{\mathbf{r}}_{n+1} &= \dot{p}(E)\mathbf{r}_n + \dot{q}(E)\dot{\mathbf{r}}_n\,; &(6) \end{aligned}$$

we account for gravitational perturbations by planets, expressed in the heliocentric frame:

1. kick

$$\ddot{\mathbf{r}}_j = \sum_i \left[-\frac{Gm_i}{r_i^3}\mathbf{r}_i - \frac{Gm_i}{r_{ji}^3}\mathbf{r}_{ji}\right]\,, \tag{7}$$

possibly, the planetary migration, in an analytical way (Malhotra 1995), and also eccentricity damping (Morbidelli et al. 2010):

2. migration

$$\dot{\mathbf{r}}_{n+1} = \dot{\mathbf{r}}_n \left[1 + \frac{\Delta v}{\dot{r}}\frac{\Delta t}{\tau_{\rm mig}}\exp\left(-\frac{t-t_0}{\tau_{\rm mig}}\right)\right]\,, \tag{8}$$

N-body model (cont.)

as of Brož et al. (2011)

the Yarkovsky thermal effect (Vokrouhlický 1998, Vokrouhlický and Farinella 1999):

3. IR emission

$$f_X(\zeta) + \mathrm{i} f_Y(\zeta) = -\frac{8}{3\sqrt{3\pi}} \Phi\, t'_{1-1}(R';\zeta)\,, \tag{9}$$

$$f_Z(\zeta) = -\frac{4}{3}\sqrt{\frac{2}{3\pi}}\, \Phi\, t'_{10}(R';\zeta)\,, \tag{10}$$

$$\Phi \equiv \frac{(1-A)\mathcal{E}_\star \pi R^2}{m_j c_{\mathrm{vac}}}\,, \tag{11}$$

the YORP effect (Čapek and Vokrouhlický 2004):

4. YORP

$$\dot{\omega} = c f_k(\gamma)\,, \tag{12}$$

$$\dot{\gamma} = \frac{c g_k(\gamma)}{\omega}\,, \tag{13}$$

$$c \equiv c_{\mathrm{YORP}} \left(\frac{a}{a_0}\right)^{-2} \left(\frac{R}{R_0}\right)^{-2} \left(\frac{\rho}{\rho_0}\right)^{-1}, \tag{14}$$

mass shedding beyond the critical angular frequency (Pravec and Harris 2000):

5. mass shedding

$$\omega_{\mathrm{crit}} = \sqrt{\frac{4}{3}\pi G \rho}\,, \tag{15}$$

and random collisional reorientations with the time scale (Farinella et al. 1998):

6. collisions

$$\tau_{\mathrm{reor}} = B \left(\frac{\omega}{\omega_0}\right)^{\beta_1} \left(\frac{R}{R_0}\right)^{\beta_2}. \tag{16}$$

A number of unknowns...

i ... “mass-less” particles, *j* ... massive bodies

- $N_{\rm TP}$, $\boldsymbol{r}_{\rm i}$, $\boldsymbol{v}_{\rm i}$, $\boldsymbol{r}_{\rm j}$, $\boldsymbol{v}_{\rm j}$, $m_{\rm i}$, $m_{\rm j}$, $\tau_{\rm mig}$, Δv, $D_{\rm i}$, $\rho_{\rm i}$, $\rho_{\rm surf}$, K, C, $A_{\rm Bond}$, ϵ, $c_{\rm YORP}$, $\lambda_{\rm i}$, $\beta_{\rm i}$, $\omega_{\rm i}$, $f_{\rm k}$, $g_{\rm k}$, B, β_1, β_2, D_0, $D_{\rm PB}$, $\rho_{\rm PB}$, $v_{\rm imp}$, $\varphi_{\rm imp}$, $f_{\rm imp}$, $\omega_{\rm imp}$
- **32** (!) a-priori unknown ICs and parameters

- not speaking about Monte-Carlo or SPH models yet...
- time step Δt → discretisation error ← usually small(er)
- beware of (formal) uncertainties & (possible) systematics

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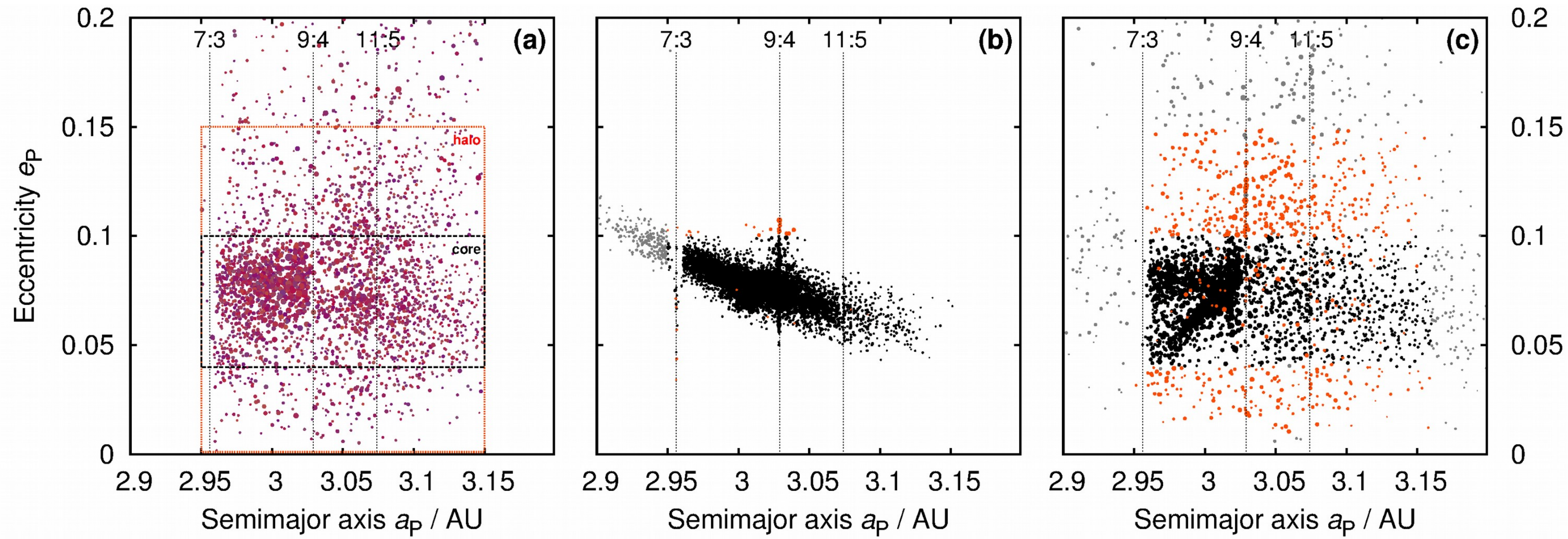
a similar *N*-body model for multiple stars, e.g. V505 Sgr (Brož et al. 2010), ξ Tau (Nemravová et al. in prep.) with χ^2 and simplex to fit minima timings (TTV), radial velocities RV & speckle-interferometry

going to C

Application A: Individual families

- Eos family (Brož & Morbidelli 2013) → *N*-body models are *essential* for family identifications!
- core vs halo, K-type taxonomy, distinct from background
- Yarkovsky drift d*a*/d*t* vs scattering in *e*, *i* by resonances

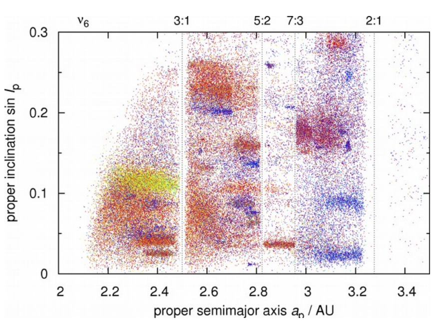
χ^2 metric → age 1.5 to 2.2 Gyr



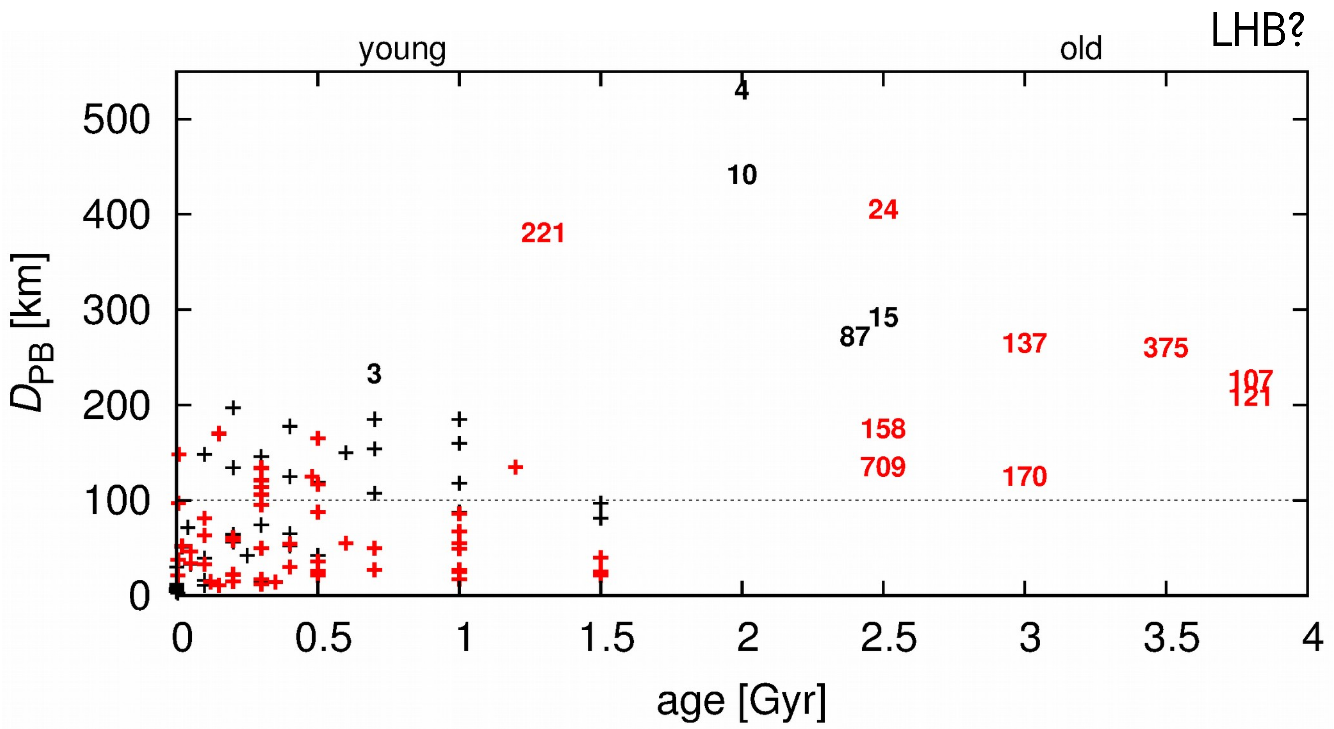
B: Statistics of families

all known, at least

- ages span 4 Gyr (Brož et al. 2013, Bottke et al. 2015) ← OK
- set of catastrophic disruptions $D_{PB} > 100$ km seems complete
- "new" families mostly $D_{PB} < 100$ km, or cratering events

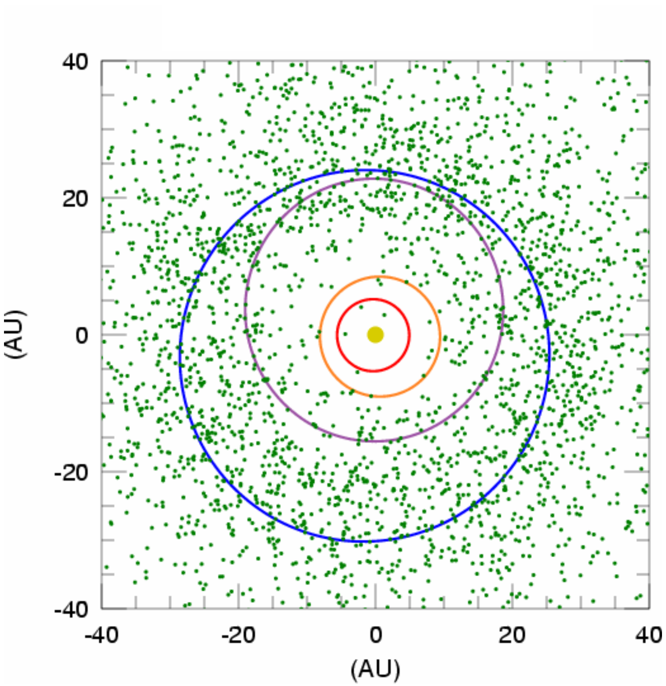


↑
parent-body sizes estimated by scaling (Durda et al. 2007)

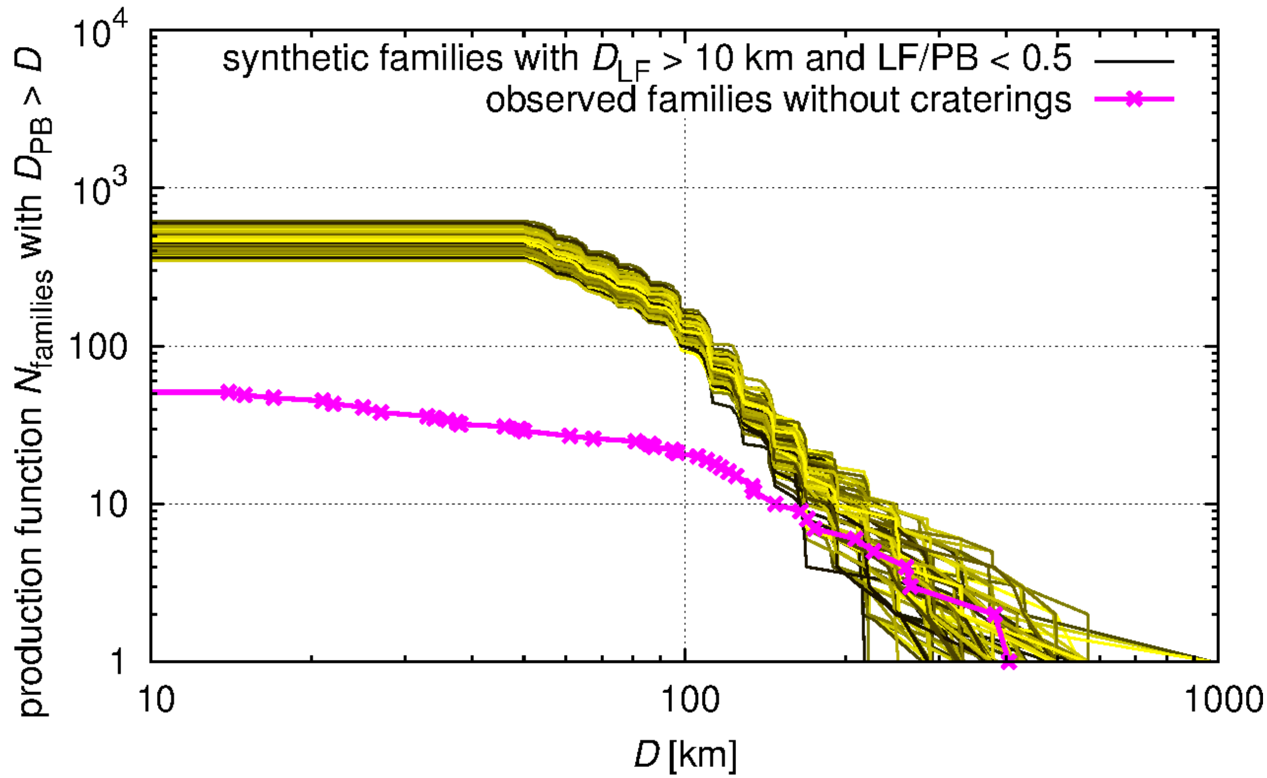


C: Late heavy bombardment of the MB

- no problems producing $D_{PB} > 200$ km families (Brož et al. 2013)
- *but* 5 times more $D_{PB} > 100$ km families ← breakups of trans-neptunian comets at low q & secondary collisions

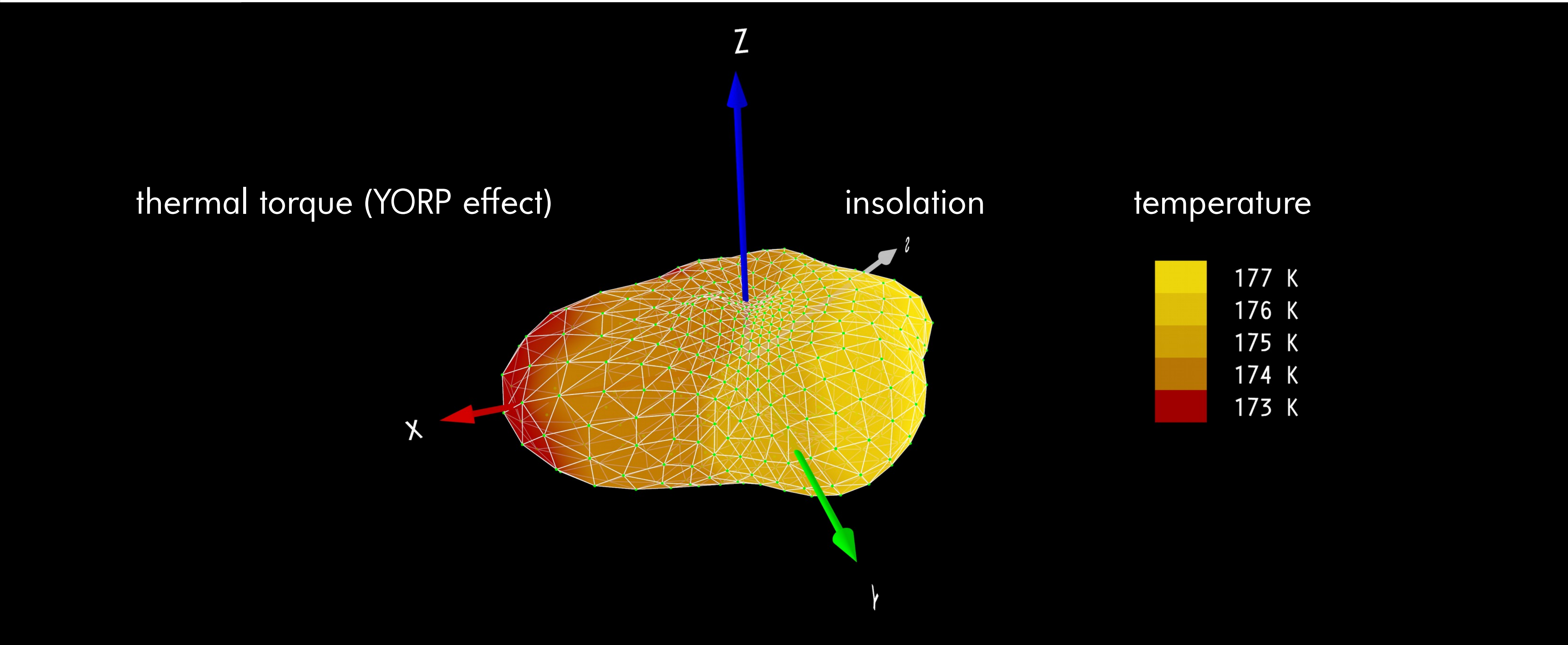


Levison et al. (2009)



Future applications

- 3-dimensional heat diffusion in meteoroids & boulders (FEM)
- important results for (25147) Itokawa (Ševeček et al. 2015)

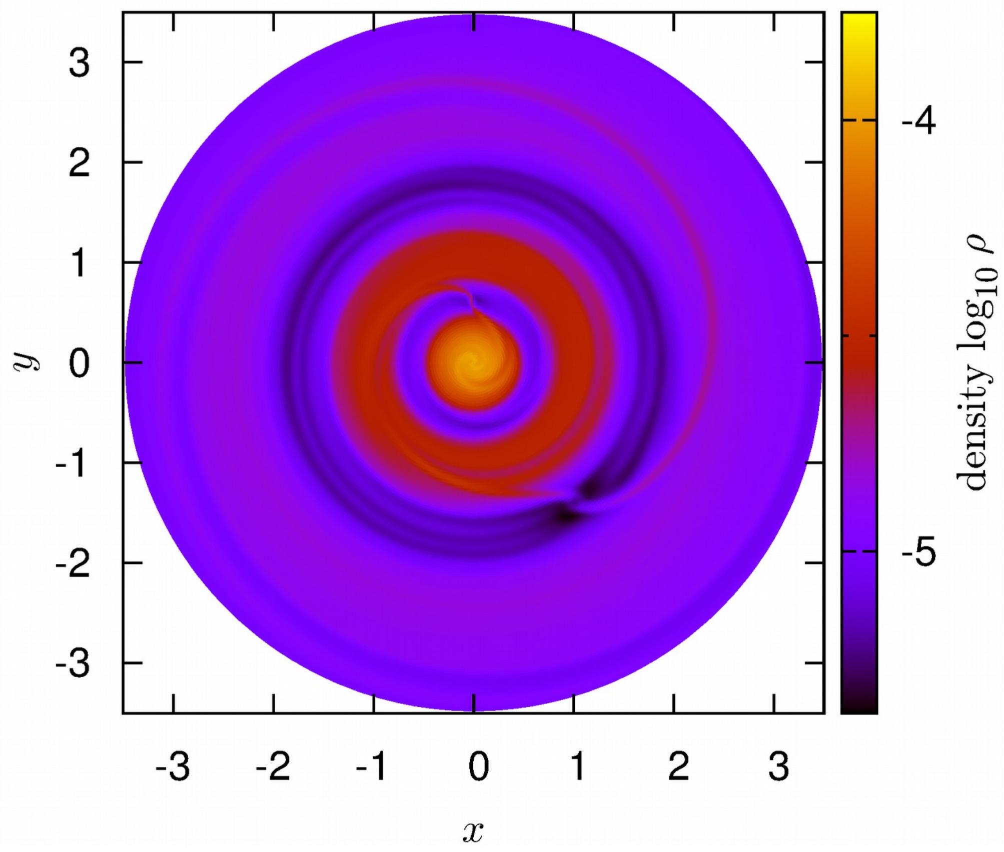


Future applications (cont.)

- protoplanetary disks & solid planetesimals *vs* resonances
- preliminary results in Chrenko & Brož (2015)

Jupiter →

Saturn →



minimum-mass solar nebula
$r/H = 0.05$
$\nu = 10^{-5}$

FARGO code
Masset (2000)

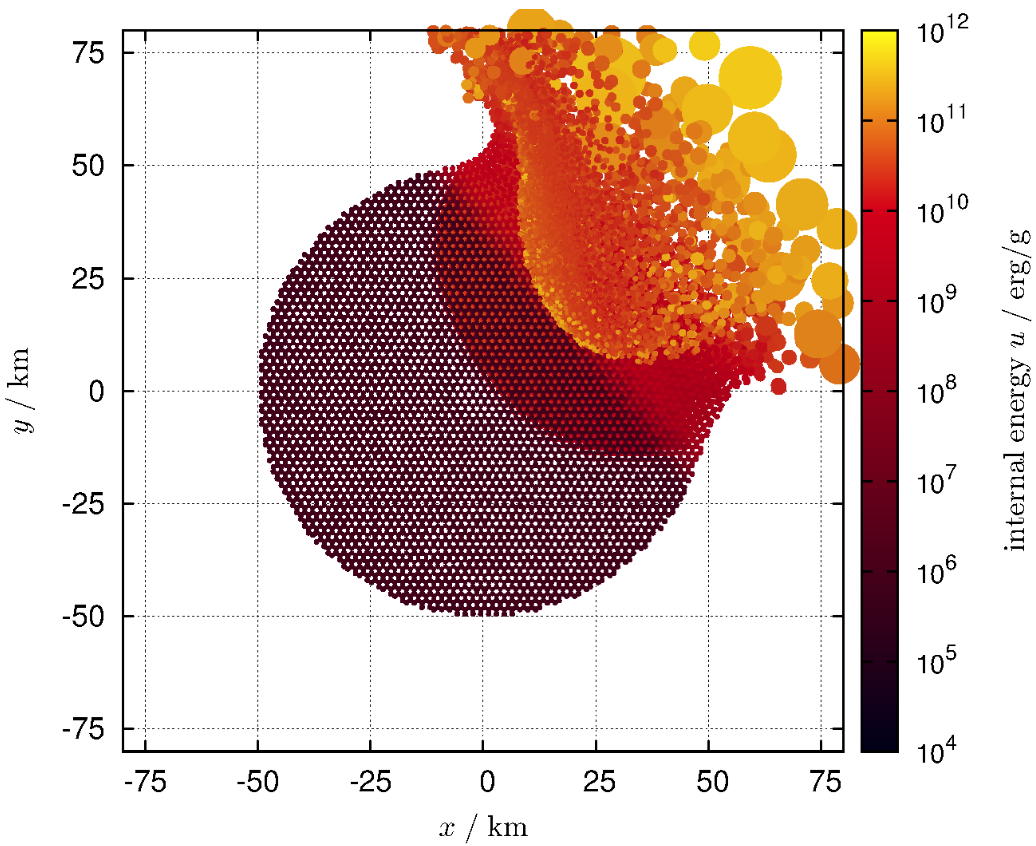
Future applications (end)

↓ smoothed particle

- SPH simulations of collisions (Rozehnal et al. submitted)
- improve scaling of SPH models (D_{PB} > and < 100 km)

projectile →

target →



D = 100 km
d = 30 km
v = 15 km/s
φ = 30°
N_{SPH} = 1.56 10^5

SPH5 code
Benz & Asphaug (1994)

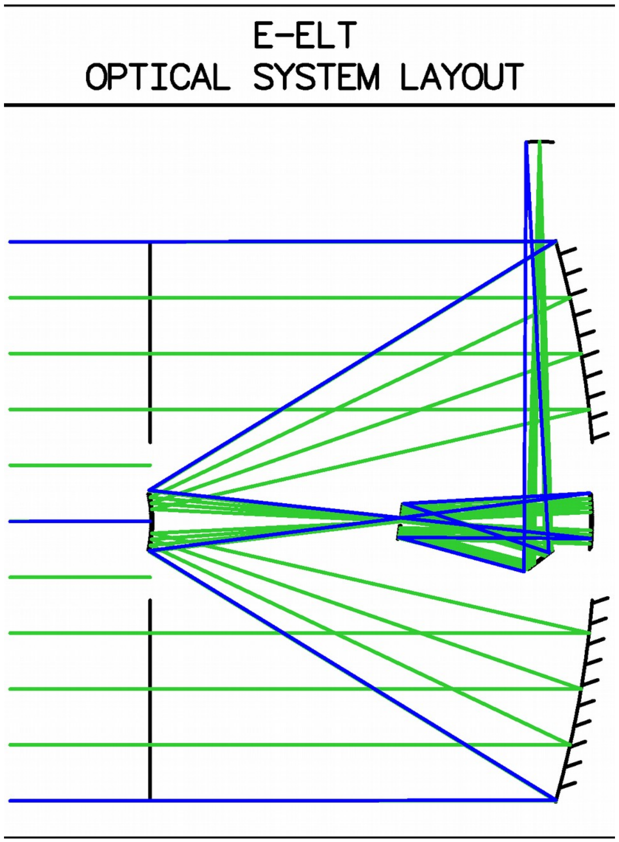
Textbooks (in prep.)

- **Hydrodynamics in Astronomy**

  protoplanetary disks (FVM), circumplanetary disks, asteroid collisions (SPH), cratering, heat diffusion (FEM), mount elasticity, ...

- **Astronomical Measurements**

  statistics, signal to noise, geometrical optics, diffraction, CCD electronics, superconductive detectors, polarimetry, interferometry, radiotelescopes, particle detectors, ...



Comments of the referees

- asteroids & stars
- details vs general
- a convex approximation

- paradigm shift (Brož & Rozehnal 2011, Brož & Morbidelli 2013)

- contradiction vs opportunity (Cibulková et al. 2014)

Jupiter Trojans

- hierarchical clustering (Zappalà et al. 1995), “randombox"
- families: Eurybates, Hektor, 1996 RJ, Arkesilaos & Ennomos, 2001 UV_{209}

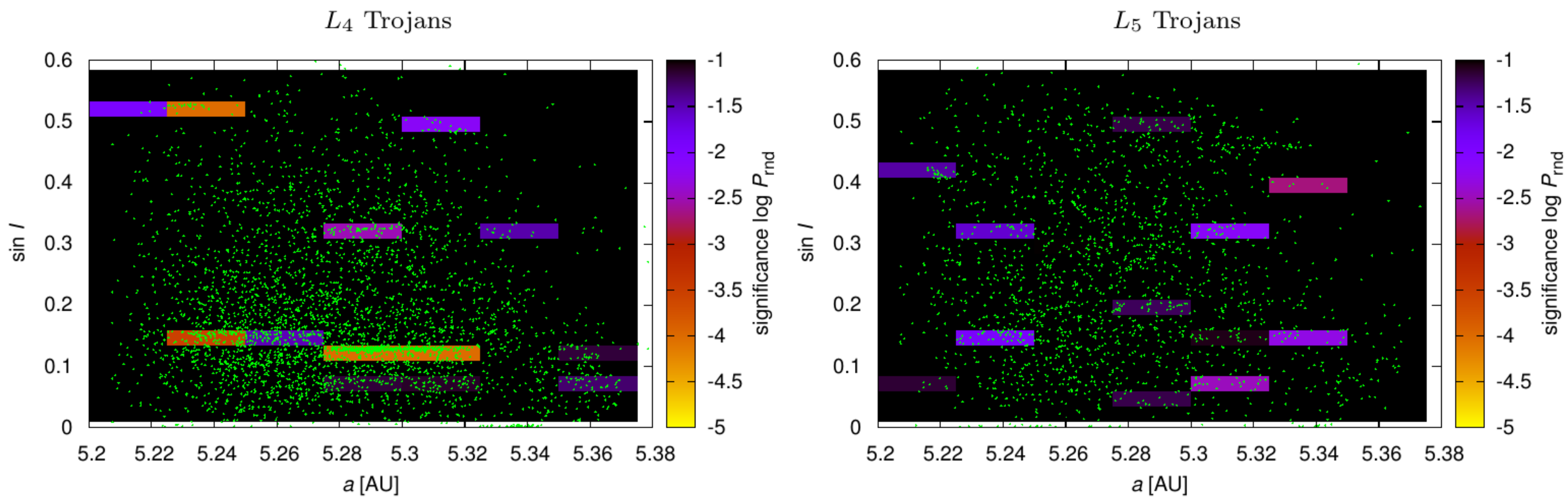
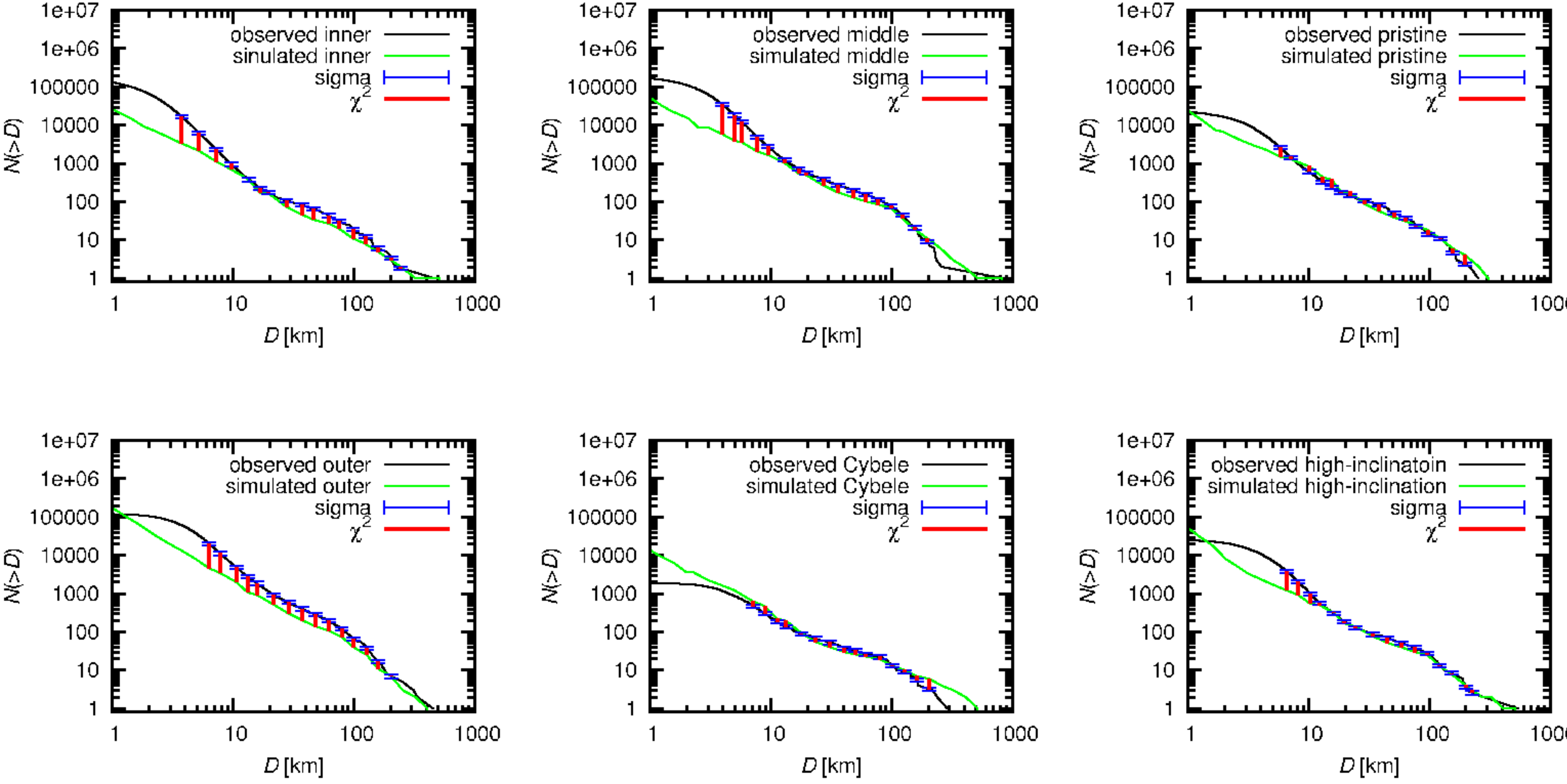


Figure 4. The statistical significance p expressed as colour on the logarithmic scale for observed asteroids in the proper semimajor axis vs proper inclination plane $(a_p, \sin I_p)$ (i.e. the same data as in Figure 1). L_4 Trojans are on the left, L_5 Trojans on the right. We computed the values of p for 7 times 18 boxes using our “randombox” method The range in proper eccentricity is 0.00 to 0.20. Statistically significant groups appear as orange boxes and they correspond to the families reported in Table 1.

Monte-Carlo collisional model of the MB

- macroporous rubble piles too weak (Cibulková et al. 2014)



Migration scenario

- jumping-Jupiter (Morbidelli et al. 2010), fifth giant planet (Nesvorný 2011)
- sufficient sampling ~1 yr for *x*, *y*, *z* interpolation
- uncertainties: M_{disk}
- systematics: different scenario, late phases, resonance sweeping, additional populations?

(E-belt, Bottke et al. 2011)

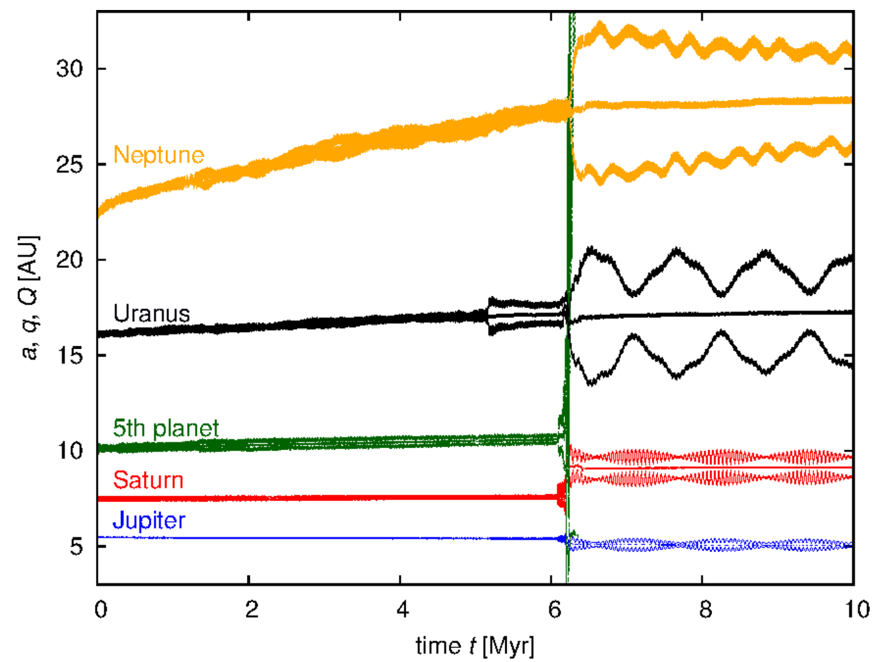


Figure 5. Orbital evolution of giant planets in the fifth giant planet scenario, adopted from Nesvorný & Morbidelli (2012), during the jumping Jupiter instability, as it was reproduced by our modified integrator. We plot time t vs the semimajor axis a, the pericentre q and the apocentre Q. Each evolutionary track is labeled with the name of the corresponding giant planet.

Chrenko et al. (2015)