Per asteroides ad astra

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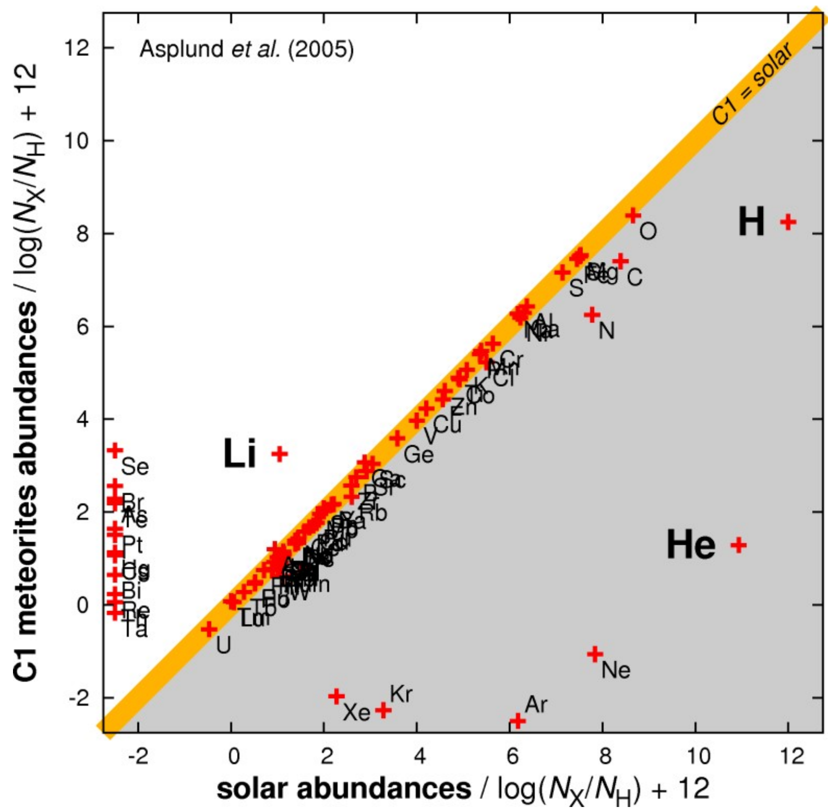
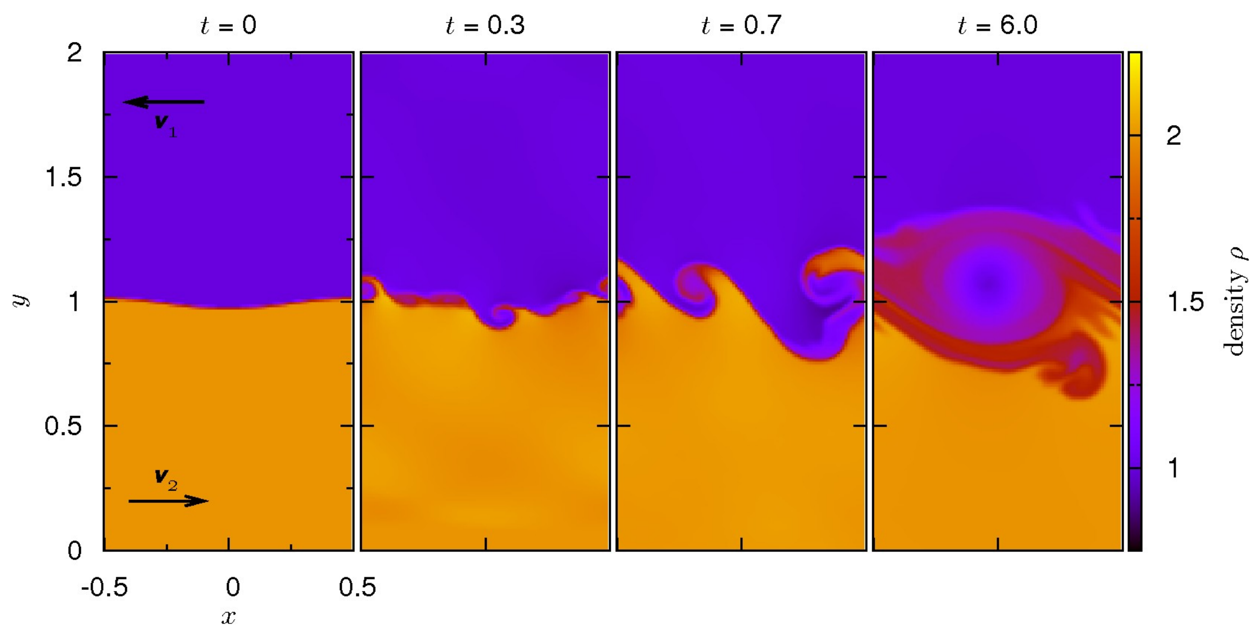
1. general problems
2. observations
3. our dynamical model (1 of)
4. asteroid families
5. future applications



serious Five problems

- turbulence
- chaos
- irreversibility
- stochasticity
- $t = 0$



Kelvin–Helmholtz instability
Pluto code (Mignone et al. 2007)

↓

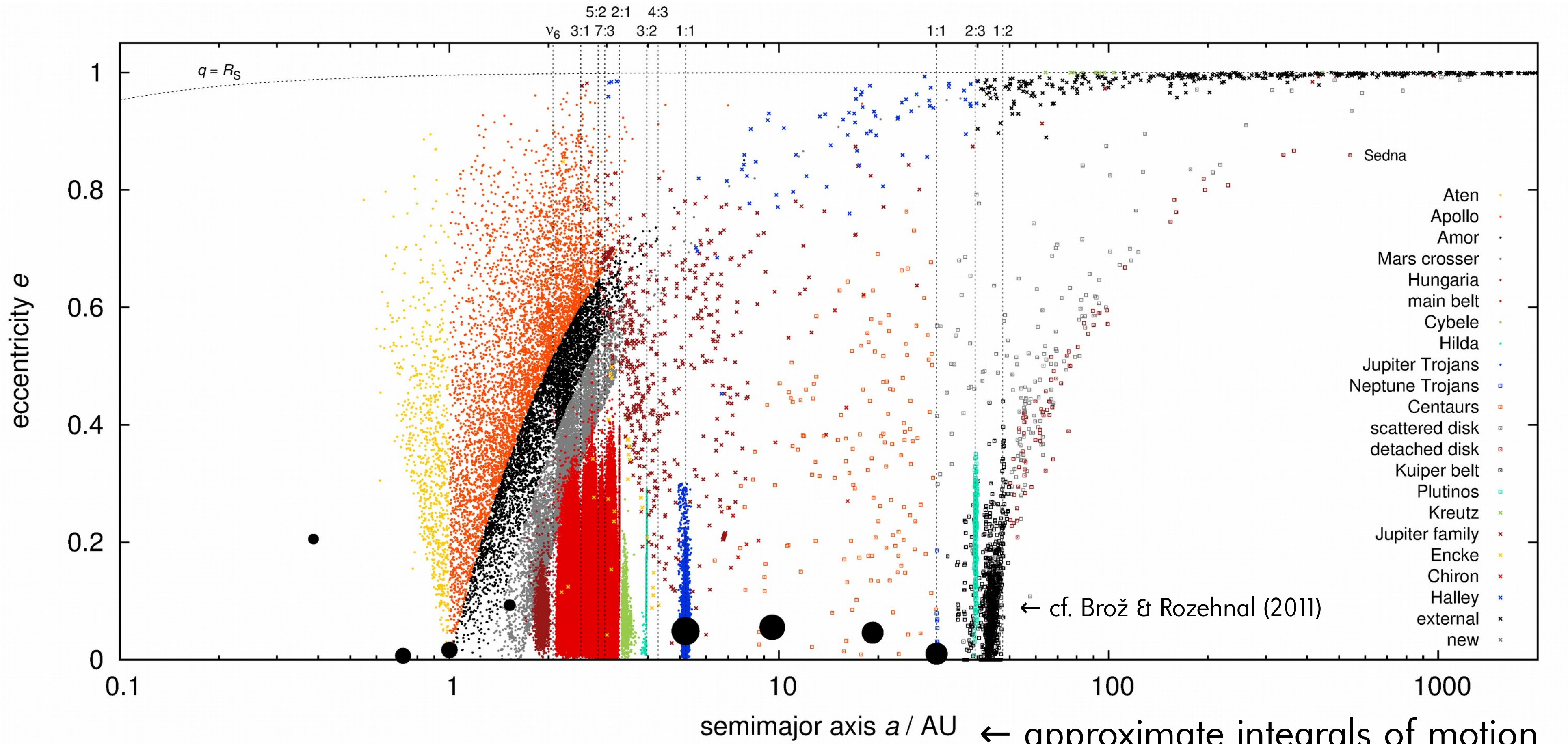
an **inverse** problem

(with exceptions)

additional instabilities:
Rayleigh–Taylor
magneto-rotational (Flock et al. 2013)
streaming (Johansen et al. 2007)

Observations ← *usually* taken @ 4.56 Gyr

- orbital distribution → families everywhere: MB, Hildas, Trojans of J & M, TNOs, irregular moons, ...

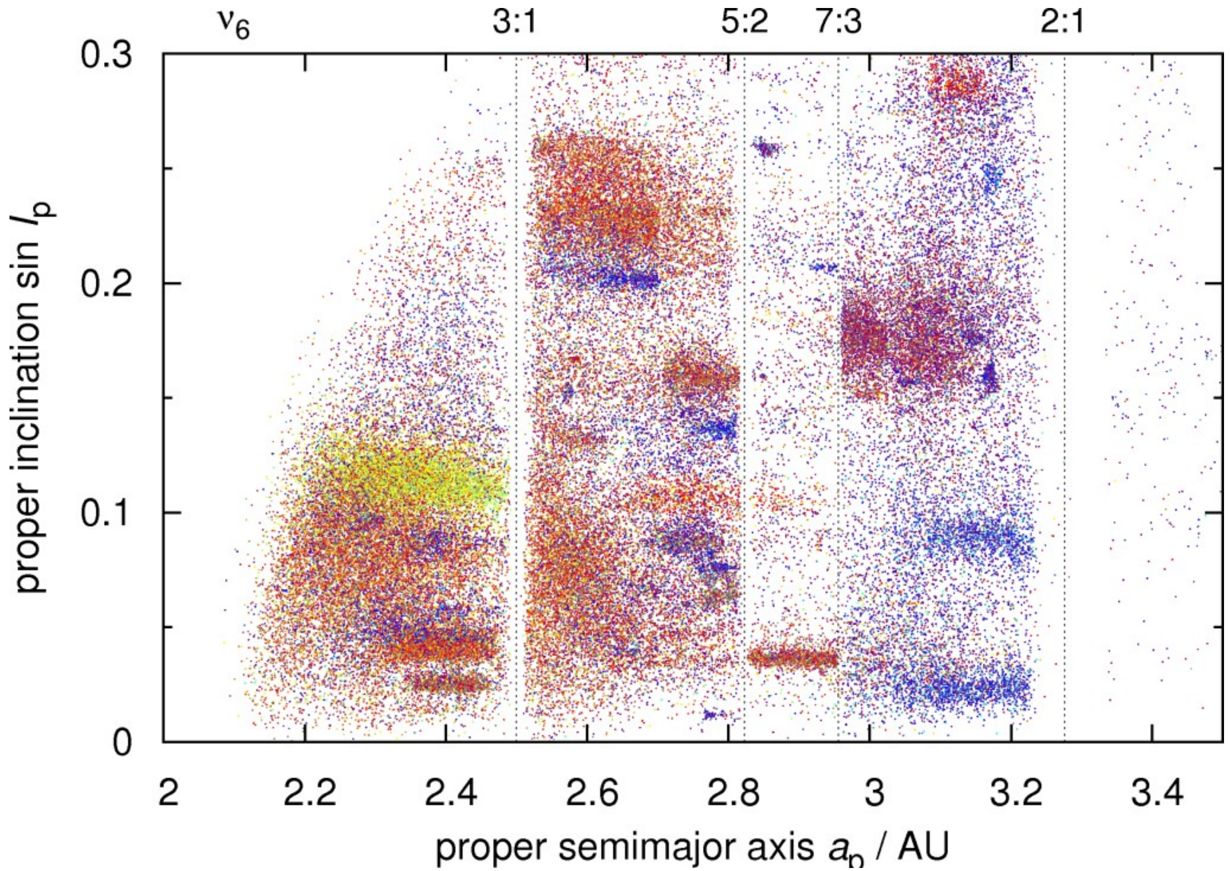


← cf. Brož & Rozehnal (2011)

← approximate integrals of motion

Observations (cont.)

- a detail of the Main Asteroid Belt, **124** families in total (according to our review in AIV, Nesvorný et al. 2015)



SDSS colours
Parker et al. (2008)

Eos family
Brož & Morbidelli (2013)

a “standard” for families → *N*-body model

We use a symplectic integration scheme (Levison and Duncan 1994), denoted as kick–drift–kick, where the ‘kick’ (actually, a perturbation) is performed as:

$$\dot{\mathbf{r}}_{n+1} = \dot{\mathbf{r}}_n + \ddot{\mathbf{r}}\frac{\Delta t}{2}\,, \tag{3}$$

and the ‘drift’ corresponds to an analytical solution of the two-body problem (the Sun–asteroid), which involves a numerical solution of the transcendent Kepler equation:

0. drift

$$M = E - e\sin E\,, \tag{4}$$

$$\mathbf{r}_{n+1} = p(E)\mathbf{r}_n + q(E)\dot{\mathbf{r}}_n\,, \tag{5}$$

$$\dot{\mathbf{r}}_{n+1} = \dot{p}(E)\mathbf{r}_n + \dot{q}(E)\dot{\mathbf{r}}_n\,; \tag{6}$$

we account for gravitational perturbations by planets, expressed in the heliocentric frame:

1. kick

$$\ddot{\mathbf{r}}_j = \sum_i \left[-\frac{Gm_i}{r_i^3}\mathbf{r}_i - \frac{Gm_i}{r_{ji}^3}\mathbf{r}_{ji}\right]\,, \tag{7}$$

possibly, the planetary migration, in an analytical way (Malhotra 1995), and also eccentricity damping (Morbidelli et al. 2010):

2. migration

$$\dot{\mathbf{r}}_{n+1} = \dot{\mathbf{r}}_n\left[1 + \frac{\Delta v}{\dot{r}}\frac{\Delta t}{\tau_{\rm mig}}\exp\left(-\frac{t-t_0}{\tau_{\rm mig}}\right)\right]\,, \tag{8}$$

N-body model (cont.)

as of Brož et al. (2011)

the Yarkovsky thermal effect (Vokrouhlický 1998, Vokrouhlický and Farinella 1999):

3. IR emission

$$f_X(\zeta) + \mathrm{i} f_Y(\zeta) = -\frac{8}{3\sqrt{3\pi}} \Phi\, t'_{1-1}(R';\zeta)\,, \tag{9}$$

$$f_Z(\zeta) = -\frac{4}{3}\sqrt{\frac{2}{3\pi}} \Phi\, t'_{10}(R';\zeta)\,, \tag{10}$$

$$\Phi \equiv \frac{(1-A)\mathcal{E}_\star \pi R^2}{m_j c_{\rm vac}}\,, \tag{11}$$

the YORP effect (Čapek and Vokrouhlický 2004):

4. YORP

$$\dot\omega = c f_k(\gamma)\,, \tag{12}$$

$$\dot\gamma = \frac{c g_k(\gamma)}{\omega}\,, \tag{13}$$

$$c \equiv c_{\rm YORP} \left(\frac{a}{a_0}\right)^{-2} \left(\frac{R}{R_0}\right)^{-2} \left(\frac{\rho}{\rho_0}\right)^{-1}, \tag{14}$$

mass shedding beyond the critical angular frequency (Pravec and Harris 2000):

5. mass shedding

$$\omega_{\rm crit} = \sqrt{\frac{4}{3}\pi G\rho}\,, \tag{15}$$

and random collisional reorientations with the time scale (Farinella et al. 1998):

6. collisions

$$\tau_{\rm reor} = B \left(\frac{\omega}{\omega_0}\right)^{\beta_1} \left(\frac{R}{R_0}\right)^{\beta_2}. \tag{16}$$

A number of unknowns...

i ... "mass-less" particles, j ... massive bodies

- N_{TP}, $\boldsymbol{r}_i$, $\boldsymbol{v}_i$, $\boldsymbol{r}_j$, $\boldsymbol{v}_j$, m_i, m_j, τ_{mig}, Δv, D_i, ρ_i, ρ_{surf}, K, C, A_{Bond}, ε, c_{YORP}, λ_i, β_i, ω_i, f_k, g_k, B, β_1, β_2, D_0, D_{PB}, ρ_{PB}, v_{imp}, φ_{imp}, f_{imp}, ω_{imp}
- **32** (!) a-priori unknown ICs and parameters

- not speaking about Monte-Carlo or SPH models yet...
- time step Δt → discretisation error ← usually small(er)
- beware of (formal) uncertainties & (possible) systematics

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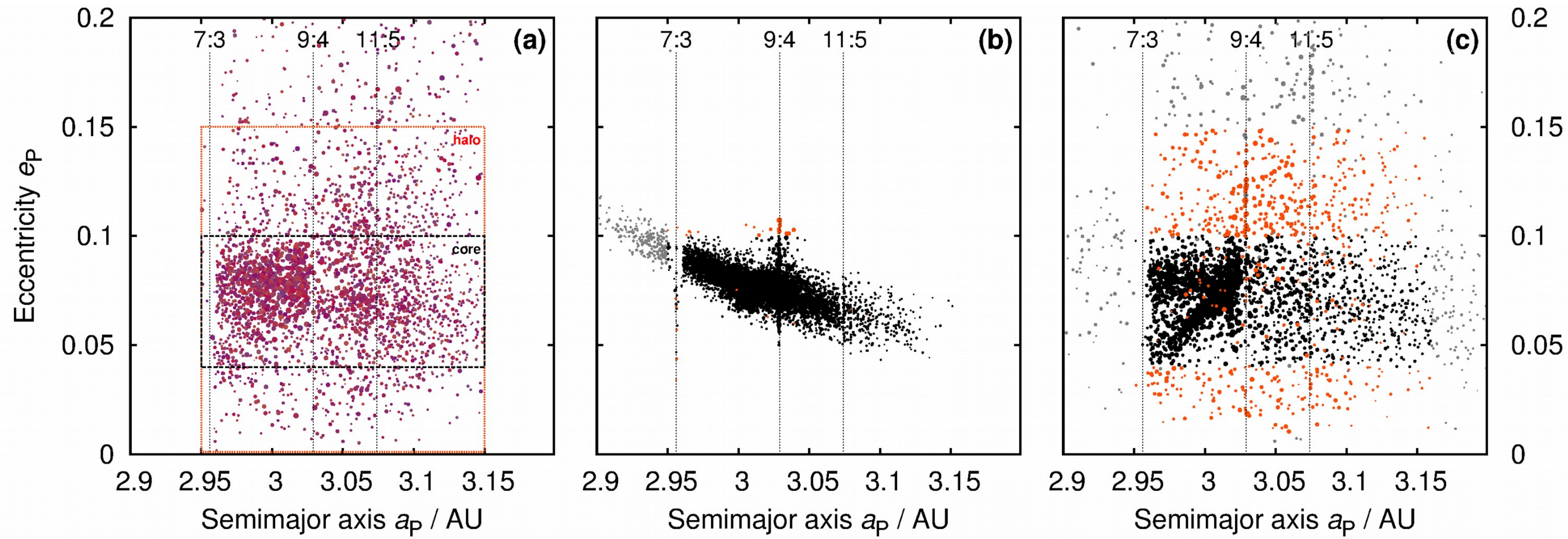
a similar N-body model for multiple stars, e.g. V505 Sgr (Brož et al. 2010), ξ Tau (Nemravová et al. in prep.) with χ^2 and simplex to fit minima timings (TTV), radial velocities RV & speckle-interferometry

going to C

Application A: Individual families

- Eos family (Brož & Morbidelli 2013) → *N*-body models are *essential* for family identifications!
- core vs halo, K-type taxonomy, distinct from background
- Yarkovsky drift d*a*/d*t* vs scattering in *e*, *i* by resonances

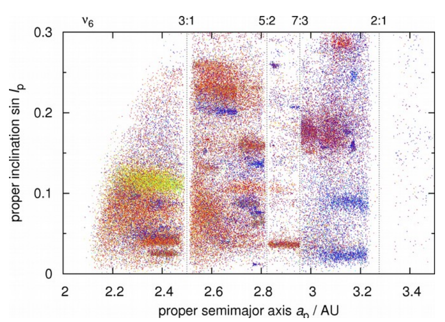
χ^2 metric → age 1.5 to 2.2 Gyr



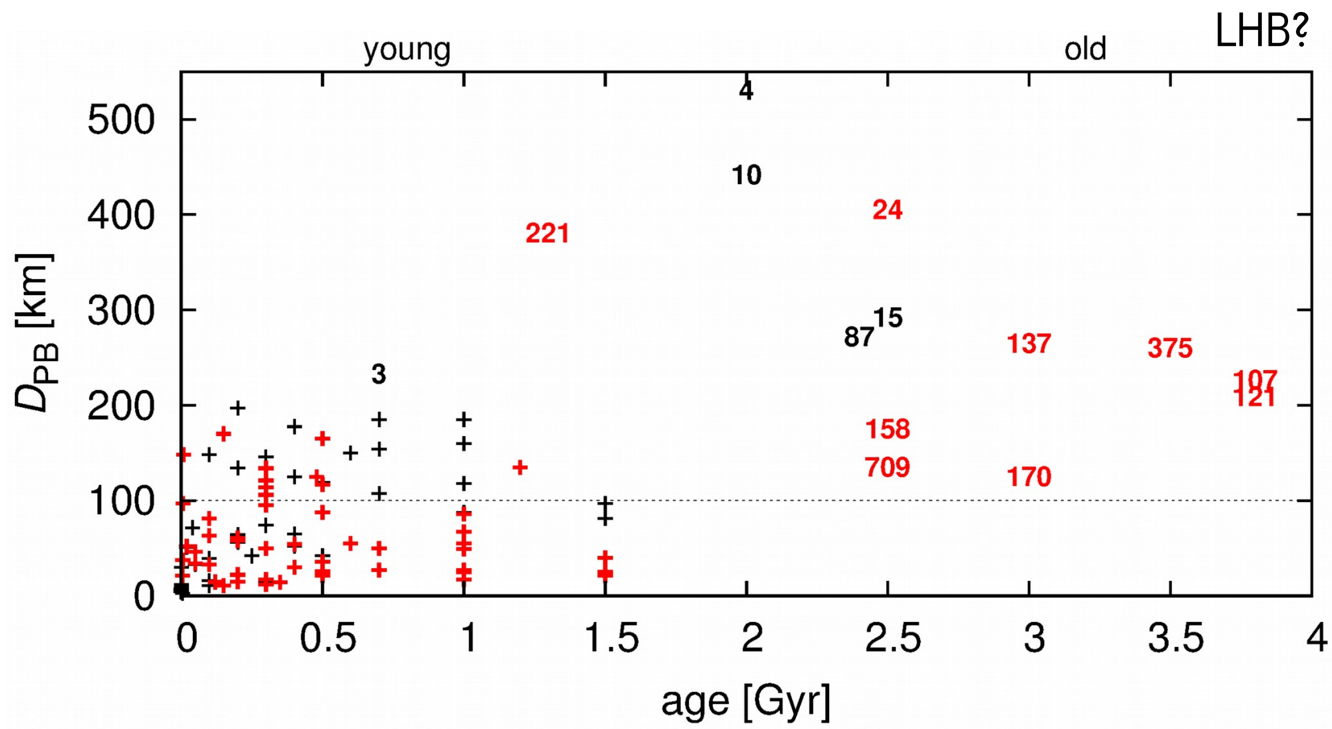
B: Statistics of families

all known, at least

- ages span 4 Gyr (Brož et al. 2013, Bottke et al. 2015) ← OK
- set of catastrophic disruptions $D_{PB} > 100$ km seems complete
- "new" families mostly $D_{PB} < 100$ km, or cratering events

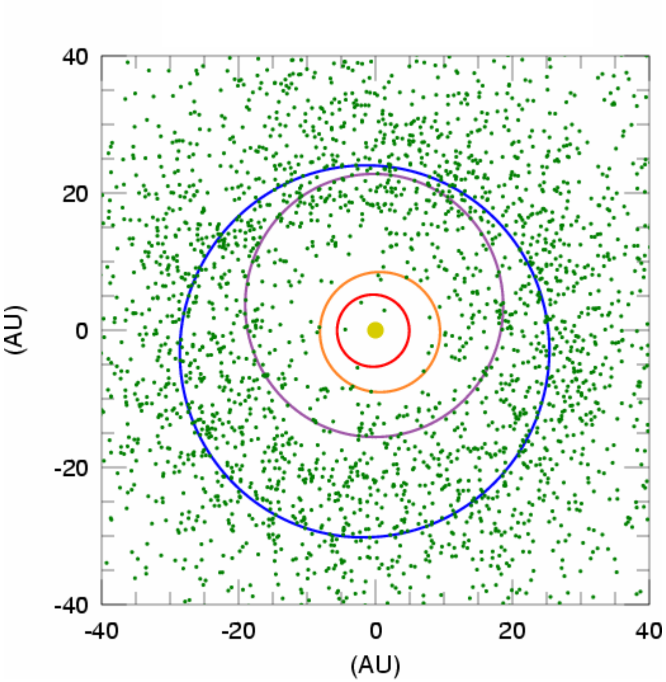


↑
parent-body sizes estimated by scaling (Durda et al. 2007)

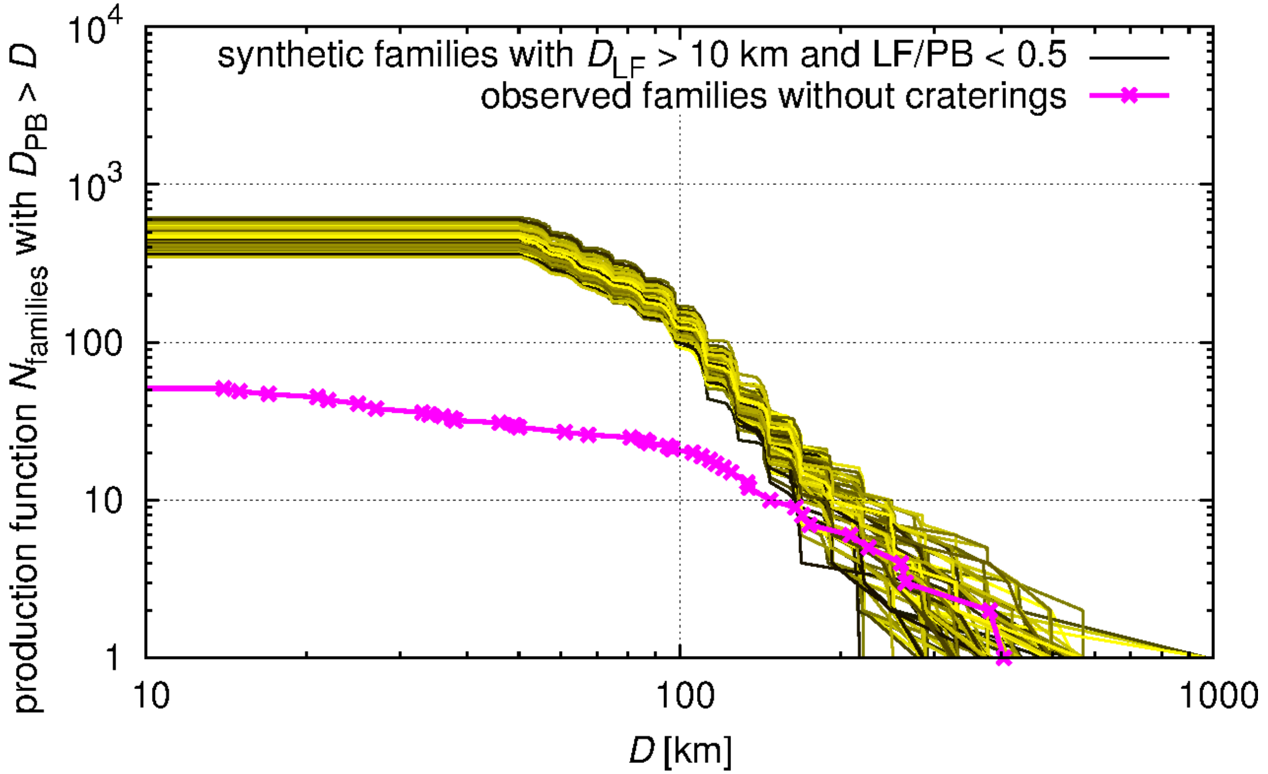


C: Late heavy bombardment of the MB

- no problems producing $D_{PB} > 200$ km families (Brož et al. 2013)
- *but* 5 times more $D_{PB} > 100$ km families ← breakups of trans-neptunian comets at low *q* & secondary collisions

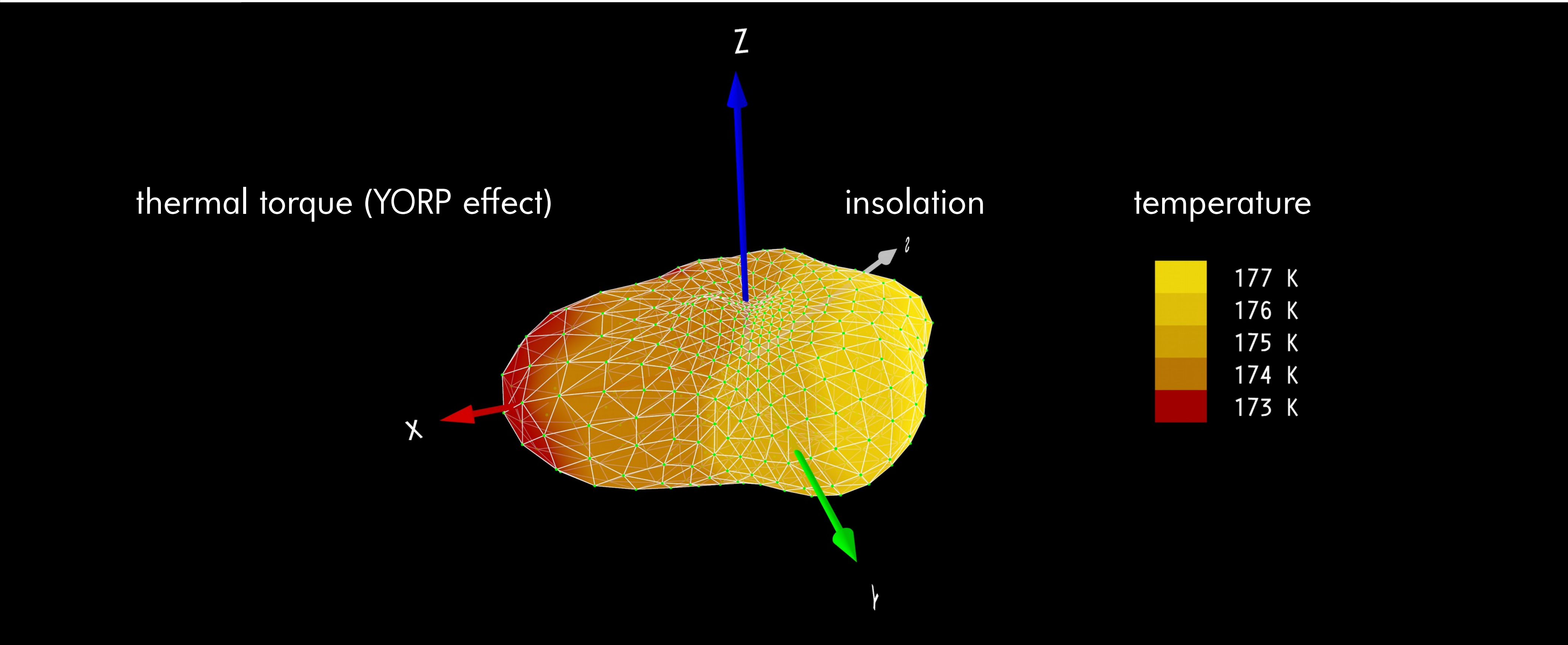


Levison et al. (2009)



Future applications

- 3-dimensional heat diffusion in meteoroids & boulders (FEM)
- important results for (25147) Itokawa (Ševeček et al. 2015)

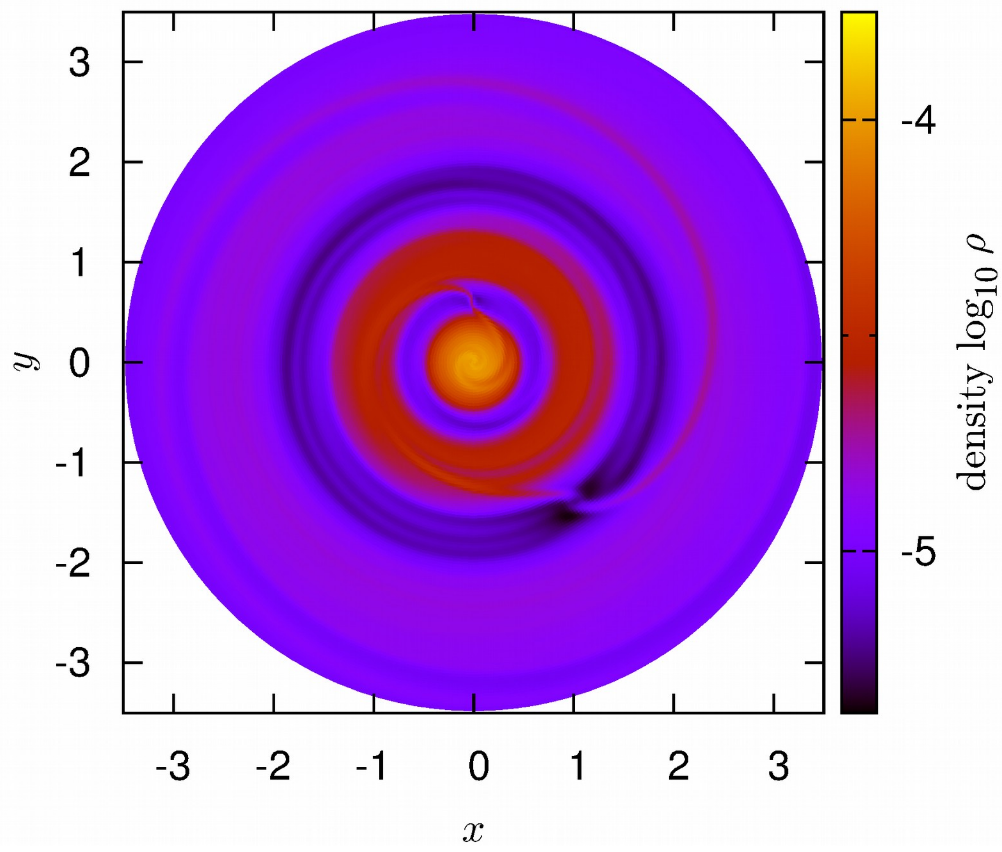


Future applications (cont.)

- protoplanetary disks & solid planetesimals *vs* resonances
- preliminary results in Chrenko & Brož (2015)

Jupiter →

Saturn →



minimum-mass solar nebula
$r/H = 0.05$
$\nu = 10^{-5}$

FARGO code
Masset (2000)

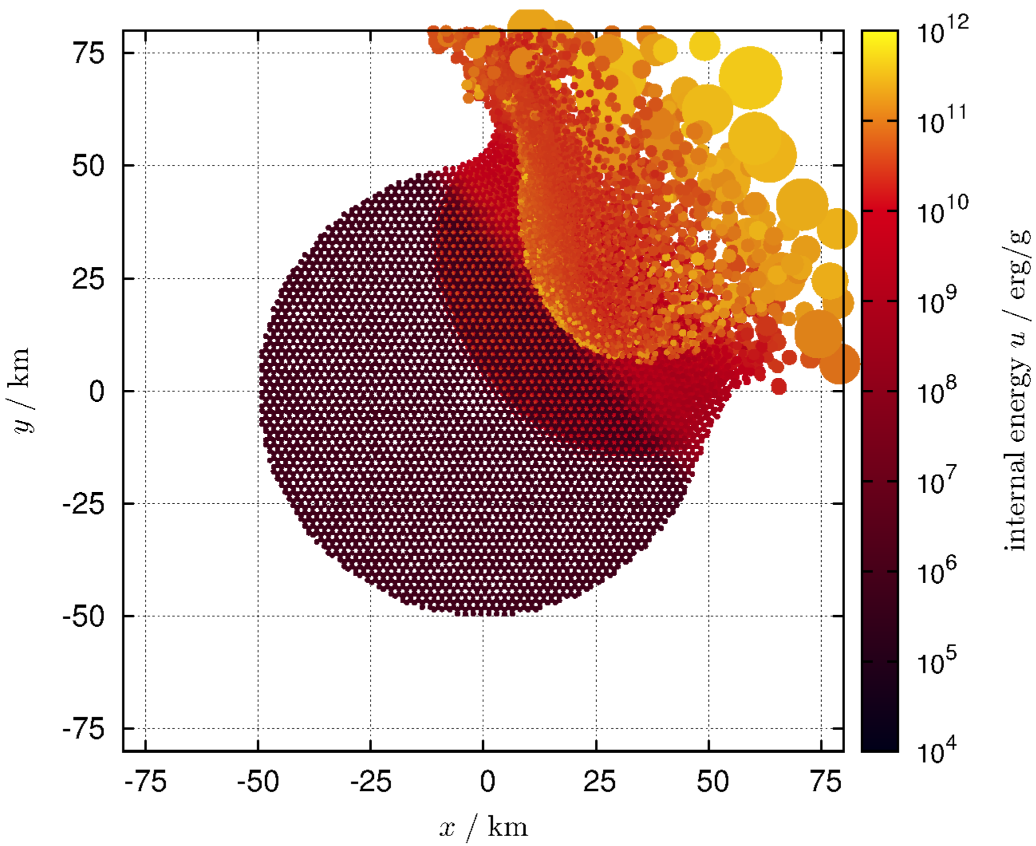
Future applications (end)

↓ smoothed particle

- SPH simulations of collisions (Rozehnal et al. submitted)
- improve scaling of SPH models (D_{PB} > and < 100 km)

projectile →

target →



D = 100 km
d = 30 km
v = 15 km/s
φ = 30°
N_{SPH} = 1.56 10^5

SPH5 code
Benz & Asphaug (1994)

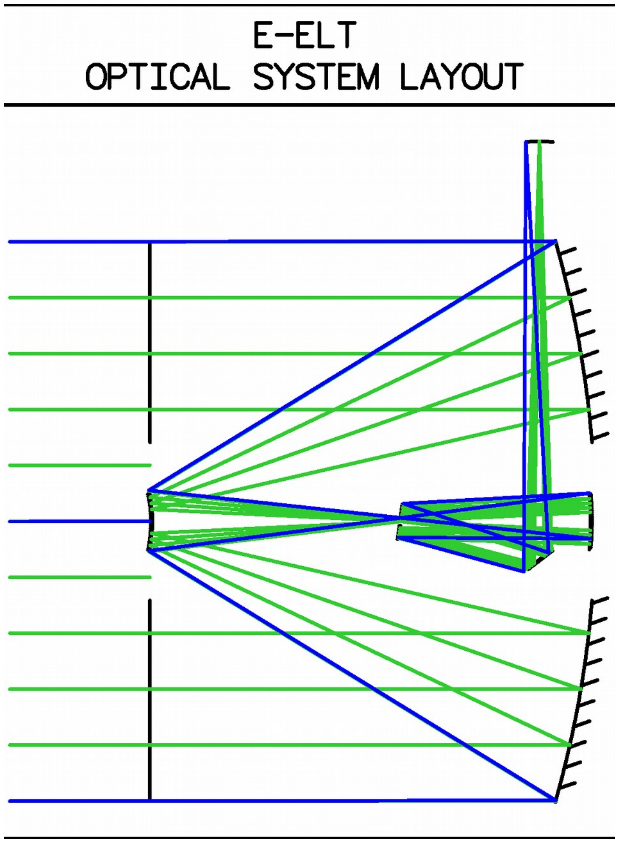
Textbooks (in prep.)

- **Hydrodynamics in Astronomy**

  protoplanetary disks (FVM), circumplanetary disks, asteroid collisions (SPH), cratering, heat diffusion (FEM), mount elasticity, ...

- **Astronomical Measurements**

  statistics, signal to noise, geometrical optics, diffraction, CCD electronics, superconductive detectors, polarimetry, interferometry, radiotelescopes, particle detectors, ...



Comments of the referees

- asteroids & stars
- details vs general
- a convex approximation

- paradigm shift (Brož & Rozehnal 2011, Brož & Morbidelli 2013)

- contradiction vs opportunity (Cibulková et al. 2014)