13 Binary stars evolution

According to what we already know about stellar evolution, it is clear that in a binary star, the primary, more massive component will evolve faster. If the orbital period, hence the distance between components, is smaller than a certain limit and the radius of the star will increase, it will overcome the limit of stability and gas from the star will start to flow towards the secondary, less massive component of the binary. It may happen during the evolution on the main sequence, or more likely during the transition from the main sequence to the giant branch, after hydrogen is exhauster in the centre. This process alters the equilibrium of the mass-loosing star and changes its further evolution. Let us first describe, how computations of the mass transfer are carried out.

13.1 Roche model and Lagrange points

Given the strong concentration of mass towards the centre of a star, it is possible to use the Roche model for studies of binary stars; even though it is more complicated compared to a single rotating star. Hereinafter, we assume that mass of the primary and the secondary components is concentrated in two points, with masses M_1 a M_2; we also introduce the *mass ratio*

$$q = \frac{M_2}{M_1}\,. \tag{13.1}$$

We shall use a *non-inertial* Cartesian coordinate system, rigidly connected with the binary star; ω denotes the angular orbital speed of the reference frame. The origin is in the point M_1 and the x axis points from M_1 to M_2, the y axis is perpendicular to x and in the orbital plane and the z axis is perpendicular to the orbital plane. The distance $A = 1$ between the two points is chosen as the unit of distance (Fig. 13.1). Let us denote the distance of the centre of mass from points M_1 and M_2 as x_1 and x_2. It holds $x_1/x_2 = M_2/M_1$ and $x_2 = 1 - x_1$, from which we obtain $x_1 = M_2/(M_1 + M_2)$.

There are three forces acting on an infinitesimal body with the mass m, located at an arbitrary point (x, y, z): a gravitational attraction of the two point masses and

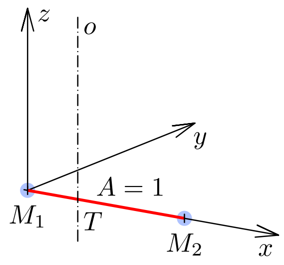

Figure 13.1: Definition of the coordinate system for computation of the potential in the vicinity of a binary star. The centre of mass T and the axis o of rotation are not in the origin.

a centrifugal force corresponding to the rotation of the coordinate system. These forces are expressed as

$$\begin{aligned}\mathbf{F}_1 &= -G\frac{mM_1}{|\mathbf{r}_1|^3}\mathbf{r}_1\,, \quad \mathbf{F}_2 = -G\frac{mM_2}{|\mathbf{r}_2|^3}\mathbf{r}_2\,, \quad \mathbf{F}_3 = m\omega^2\mathbf{r}_3\,,\\ \mathbf{r}_1 &= (x,y,z)\,, \quad \mathbf{r}_2 = (x-1,y,z)\,, \quad \mathbf{r}_3 = \left(x-\frac{M_2}{M_1+M_2},y,0\right)\,.\end{aligned}$$

Denoting also

$$r_1 = |\mathbf{r}_1|\,, \quad r_2 = |\mathbf{r}_2|\,, \quad r_3 = |\mathbf{r}_3|\,, \tag{13.2}$$

it is possible to write down a potential of these three forces ($F = m\nabla W$) as

$$W = \frac{GM_1}{r_1} + \frac{GM_2}{r_2} + \frac{1}{2}\omega^2 r_3^2\,. \tag{13.3}$$

The angular orbital speed ω can be expressed from the 3rd Kepler law (with $A = 1$)

$$\omega^2 A^3 = G(M_1+M_2) = GM_1(1+q) \tag{13.4}$$

and for a simpler notation we introduce use a scaled potential instead of W

$$\begin{aligned}\Omega &= \frac{W}{GM_1} = \frac{1}{r_1} + \frac{q}{r_2} + \frac{1}{2}(1+q)r_3^2 = (x^2+y^2+z^2)^{-\frac{1}{2}} + \\ &\quad + q((x-1)^2+y^2+z^2)^{-\frac{1}{2}} + \frac{1}{2}(1+q)(x^2+y^2) - qx + \frac{q^2}{2(1+q)}\,.\end{aligned} \tag{13.5}$$

The equation for equipotential surfaces is then

$$\Omega = C\,, \tag{13.6}$$

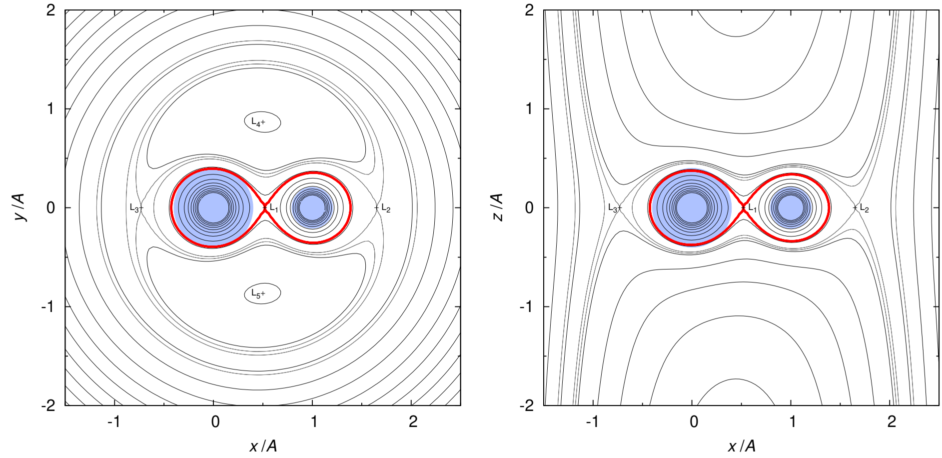

Figure 13.2: Potential $\Omega(x,y,0)$ and $\Omega(x,0,z)$ for a binary star with the masses $M_1 = 4{,}0\,M_\odot$, $M_2 = 3{,}2\,M_\odot$ ($q = 0{,}8$). The positions of the Lagrange libration points are plotted, as well as the critical equipotential, over which mass would be transferred (if overflown). The stars are plotted as spherical, with the radii R_1, R_2 as they would have on the zero-age main sequence (at the time $t = 0$). However, the shape of the surface would not remain spherical, it would rather adapt to a certain equipotential.

where C denotes a constant corresponding to a certain surface (Fig. 13.2). Let us note that the shape of equipotential surfaces is a function of a single variable, the mass ratio q.[1]

We are again interested in positions, where the resulting force acting on the test body is zero, or when

$$\nabla\Omega = \left(\frac{\partial\Omega}{\partial x}, \frac{\partial\Omega}{\partial y}, \frac{\partial\Omega}{\partial z}\right) = \mathbf{0}\,. \tag{13.7}$$

Expressed in coordinates

$$\frac{\partial\Omega}{\partial z} = -\frac{1}{\cancel{2}}\left(x^2+y^2+z^2\right)^{-\frac{3}{2}} \cdot \cancel{2}z + q\left(-\frac{1}{\cancel{2}}\right)\left((x-1)^2+y^2+z^2\right)^{-\frac{3}{2}} \cdot \cancel{2}z = 0\,. \tag{13.8}$$

This equation implies a solution $z = 0$. The second equation

$$\begin{aligned}\frac{\partial\Omega}{\partial y} &= -\frac{1}{2}\left(x^2+y^2+z^2\right)^{-\frac{3}{2}} \cdot 2y + q\left(-\frac{1}{2}\right)\left((x-1)^2+y^2+z^2\right)^{-\frac{3}{2}} \cdot 2y + \\ &\quad + \frac{1}{2}(1+q)\cdot 2y = 0\,,\end{aligned} \tag{13.9}$$

[1] A similar analysis is done in the celestial mechanics in the three-body problem. Apart from the centrifugal force, the Coriolis force ($\mathbf{F}_\mathrm{c} = -2m\omega \times \mathbf{v}$) arises during the transformation of coordinates. In our stationary case ($\mathbf{v} = 0$) it can be neglected. The equipotential surface are sometimes referred to as zero-velocity curves.

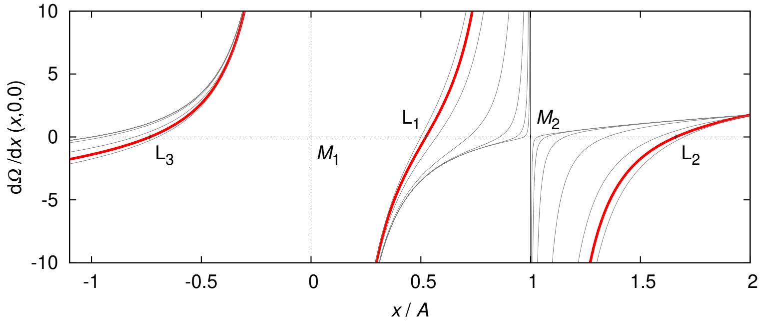

Figure 13.3: Function $\frac{\partial\Omega(x,0,0)}{\partial x}$ for the mass ratio $q = 3{,}2/4{,}0 = 0{,}8$ and its roots, which are the coordinates of the Lagrange libration points L_1, L_2, L_3. Functions for additional values of $q = 10^{-4}, 10^{-3}, 10^{-2}, 0{,}1, 0{,}5, 1{,}0$ are plotted by thin lines.

has *two* solutions. First, let us assume $y \neq 0$, after reduction of y we obtain

$$-\frac{1}{r_1^3} - \frac{q}{r_2^3} + 1 + q = 0\,, \tag{13.10}$$

which can be fulfilled only id

$$r_1 = r_2 = 1\,. \tag{13.11}$$

This corresponds to two points in the orbital plane, in the vertices of the equilateral triangles with the points M_1, M_2. They are called the Lagrange points L_4 and L_5.

If $y = 0$ (we are looking for collinear solutions on the x axis), then from the third equation

$$\begin{aligned}\frac{\partial\Omega(x,0,0)}{\partial x} &= -\frac{x}{|x|^3} + \frac{q(x-1)}{|x-1|^3} + (1+q)x - q \\ &= -\frac{\operatorname{sgn} x}{|x|^2} + \frac{q\operatorname{sgn}(x-1)}{|x-1|^2} + (1+q)x - q = 0 \end{aligned} \tag{13.12}$$

we obtain a polynomial of the degree 5 for x (with the parameter q), which has three real roots (Fig. 13.3). They are usually called L_1 (the point between M_1 and M_2); L_2 (located beyond M_2) a L_3 (located outwards of M_1).

As already mentioned for single stars, the importance of equipotential surfaces is that a star in equilibrium adopts a shape of one of those. The critical surface containing the point L_1 — often called the *Roche limit* — represents an important limit for stability of binary stars. A practical prescription how to compute the dimensions of the critical limit for a given mass ratio is included in the appendix of Harmanec (1990).

Physical classification of binary stars. The Roche model offers a criterion for contemporary physical classification of binary stars as:

1. *detached*, where both components have dimensions smaller than the critical surface;
2. *semi-detached*, in which one component is inside the critical surface and the other one is just filling it, so that mass is transferred across the point L_1;
3. *contact*, when both components fill of overfill the critical surface and have a common atmosphere. Alternatively, mass can be lost from the system across the point L_2.

13.2 Calculation of stellar evolution during mass transfer phase

Distance of binary components. If no mass is lost from the system, we can use the mass conservation law

$$M_1(t) + M_2(t) = K \tag{13.13}$$

and also the angular momentum conservation law (for its orbital part)

$$\mathcal{L} \doteq \mathcal{L}_{\mathrm{orb}} = \frac{M_1 M_2}{M_1 + M_2} A\, v_{\mathrm{K}} = \frac{M_1 M_2}{M_1 + M_2} A\, \frac{2\pi A}{P}\,. \tag{13.14}$$

The latter equation can be modified using the 3rd Kepler law

$$\frac{A^3}{P^2} = \frac{G(M_1 + M_2)}{4\pi^2} \tag{13.15}$$

to obtain

$$\mathcal{L}^2_{\mathrm{orb}} = G \frac{M_1^2 M_2^2}{M_1 + M_2} A\,, \tag{13.16}$$

which is further simplified for the constant total mass (13.13) as

$$A(t) M_1^2(t) M_2^2(t) = C\,. \tag{13.17}$$

We can now ask a logical question, when the distance between the two stars will be minimal. In the last Eq. (13.17) we eliminate the mass M_2 using Eq. (13.13)

$$A(M_1) = C M_1^{-2} (K - M_1)^{-2} \tag{13.18}$$

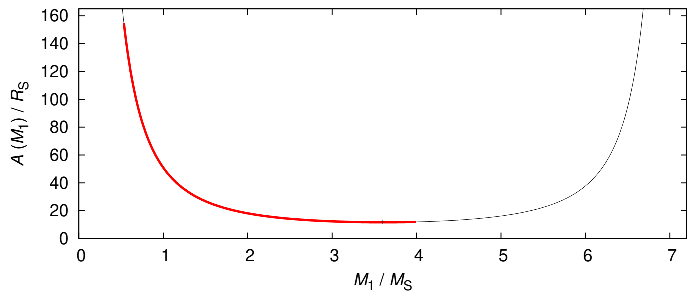

Figure 13.4: Distance of a binary star components with the initial masses $M_1 = 4\,M_\odot$, $M_2 = 3{,}2\,M_\odot$ during mass transfer, depending on the mass M_1 (according to Eq. (13.18)). Evolution of this binary computed for a realistic mass transfer is plotted in red — the original mass transfer $q = M_2/M_1 = 0{,}8$ has been more than reversed (to the value $q = 1{,}89$).

and we look for the mass M_1, when the derivative of this function with respect to M_1 is zero. We obtain

$$\frac{\mathrm{d}A(M_1)}{\mathrm{d}M_1} = -2CM_1^{-3}(K - M_1)^{-2} + 2CM_1^{-2}(K - M_1)^{-3} = 0\,, \tag{13.19}$$

which after simplifications leads to

$$-(K - M_1) + M_1 = -K + 2M_1 = 0 \tag{13.20}$$

or

$$M_1 = M_2\,. \tag{13.21}$$

We see that **the distance between the stars, in the case of conservative mass transfer, is minimal, when the masses of the two bodies are just equal.** If matter flows from the more massive components to the less massive ($M_1 > M_2$), the distance becomes smaller; if the flow is opposite ($M_1 < M_2$), or after the mass ratio has been reversed, respectively, the distance becomes larger (Fig. 13.4).

Non-conservative mass transfer.

Model of stellar interior.

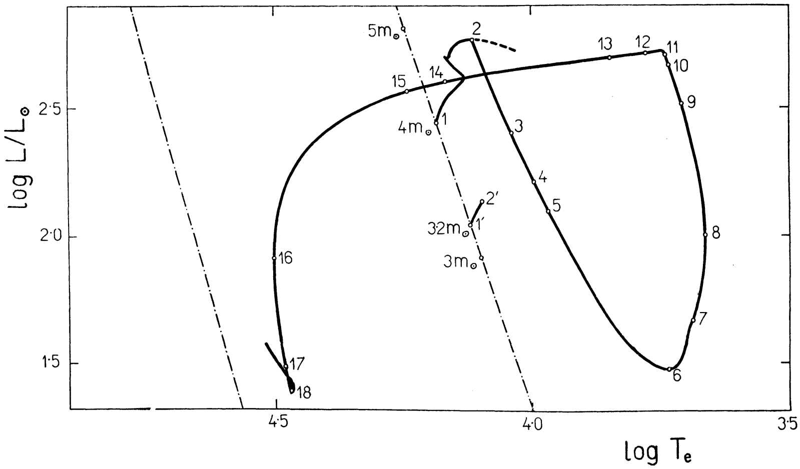

Figure 13.5: HR diagram for the *primary* component of a binary $4\,M_\odot$ and $3{,}2\,M_\odot$. Additional parameters of the binary at the beginning of mass transfer are: radii $R_1 = 4{,}78\,R_\odot$, $R_2 = 2{,}47\,R_\odot$, spectral types B7 III and B8 V, a distance between components $A = 11{,}95\,R_\odot$ and an orbital period $P = 1{,}785\,\mathrm{d}$. The zero-age main sequence (ZAMS, dash-dotted line) is indicated twice — for two different chemical compositions: $X = 0{,}602$, $Y = 0{,}354$ and $X = 0$, $Y = 0{,}956$. The evolution is shown before, during and also after mass transfer. The transfer proceeds between points 2 and 11; individual points are described in the text. Models ends in point 18, because the former secondary star would reach the Roche limit in this phase and a flow back to the former primary would start. Reprinted from Harmanec (1970).

13.3 Selected results of binary stars modelling

Example of a binary star $4\,M_\odot$ and $3{,}2\,M_\odot$.

13.4 Binary stars versus observations, evolutionary paradox

Evolutionary paradox. The key problem, which was necessary to explain by models of mass transfer in binaries, was the *evolutionary paradox of semi-detached systems*. When sufficient number of binaries were observed and their basic physical properties were determined, they were subsequently classified in 1950's to detached,

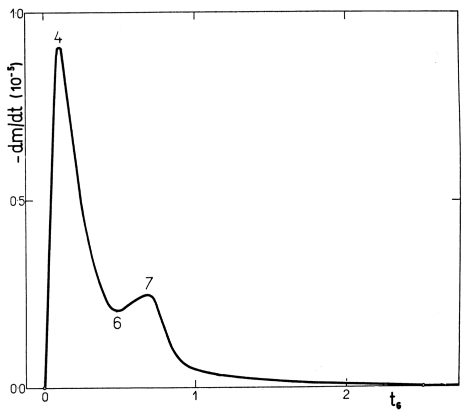

Figure 13.6: Rate of mass transfer $-\mathrm{d}M/\mathrm{d}t$ ($[\mathrm{d}M/\mathrm{d}t] = 10^{-5}\,M_\odot$) vs. time t ($[t] = 10^6$ years) for the binary $4\,M_\odot$ and $3{,}2\,M_\odot$. Reprinted from Harmanec (1970).

semi-detached and contact systems. It turned out that in all cases, the Roche limit in semi-detached systems is filled by the *less massive* secondary component (Fig. 13.7). However, it was known that according to the theory of stellar evolution, the more massive component should evolve and expand faster to the Roche limit. The observations seemed to indicate just the opposite.

With a great physicists intuition, Crawford (1955) suggested a possible solution to the paradox. He postulated that the more massive component will indeed evolve faster and mass will be exchanged, which reverses the original mass ratio. (His hypothesis was opposed by Czech astronomer Zdeněk Kopal.) It took more than 10 years, until the Crawford hypothesis was confirmed by calculations of mass transfer. The point is that the initial phase of mass transfer, during which the original mass ration reverses, proceeds so fast compared to other phases that we have only little chance to observe such a system.

Be stars.

Eccentric orbits.

Magnetic polars.

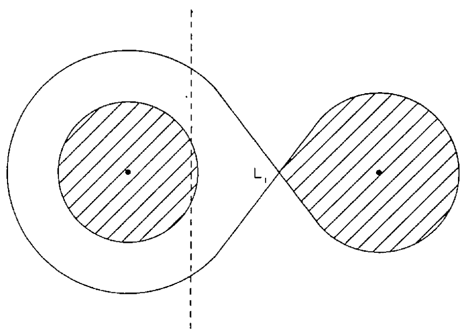

Figure 13.7: Scheme of a semi-detached system, in which the less massive secondary component fills its Roche lobe. Taken from the work of Crawford (1955).