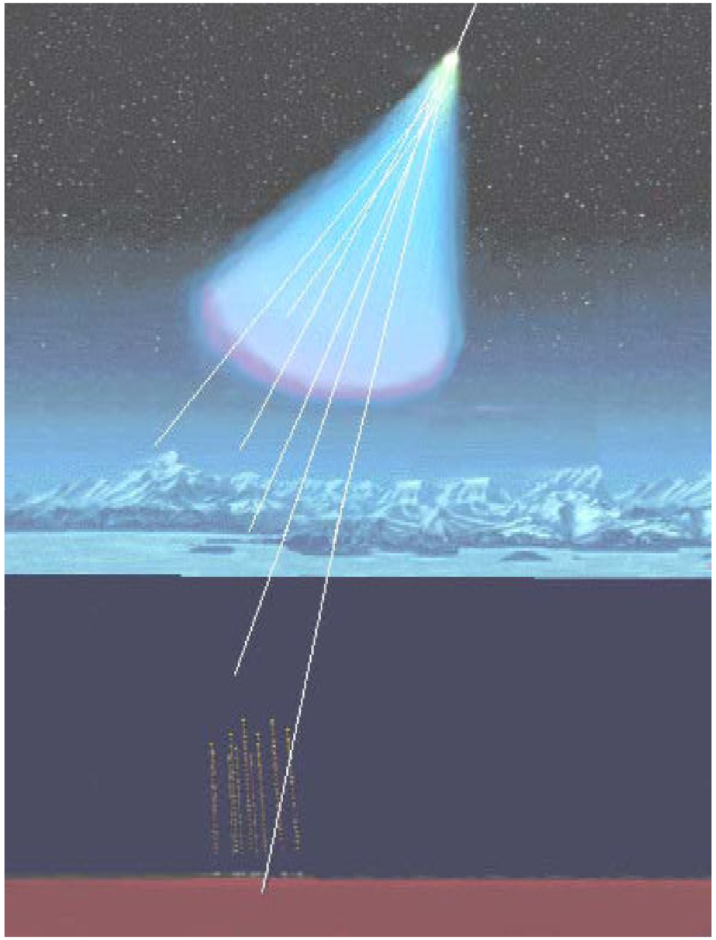
Přednáška 6.(5.11.2007)

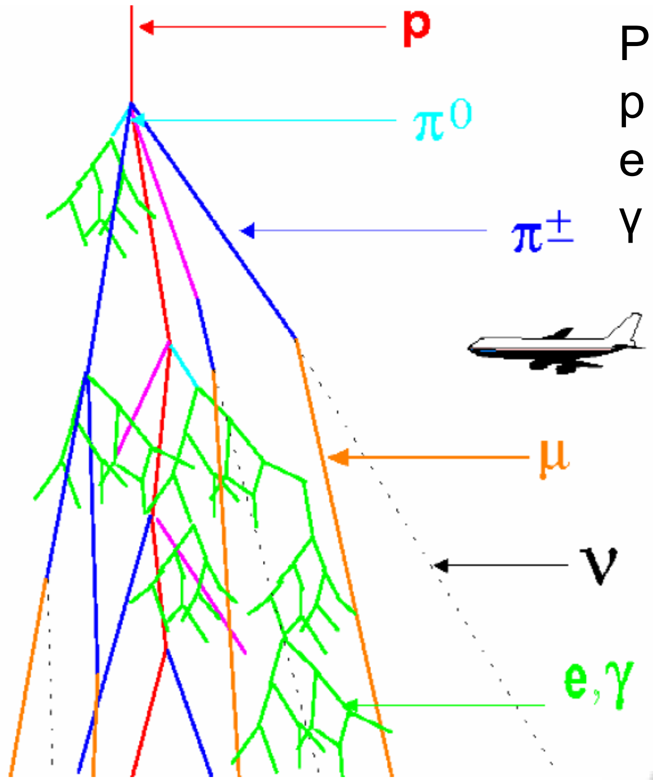
Objevy nových částic v kosmickém záření:positron, mion, pi mezon, podivné částice

Skládání momentů hybnosti
Fermiony a bosony

Klasifikace elementárních částic
Sakatův model
Kvarkový model hadronů

Kosmické záření:





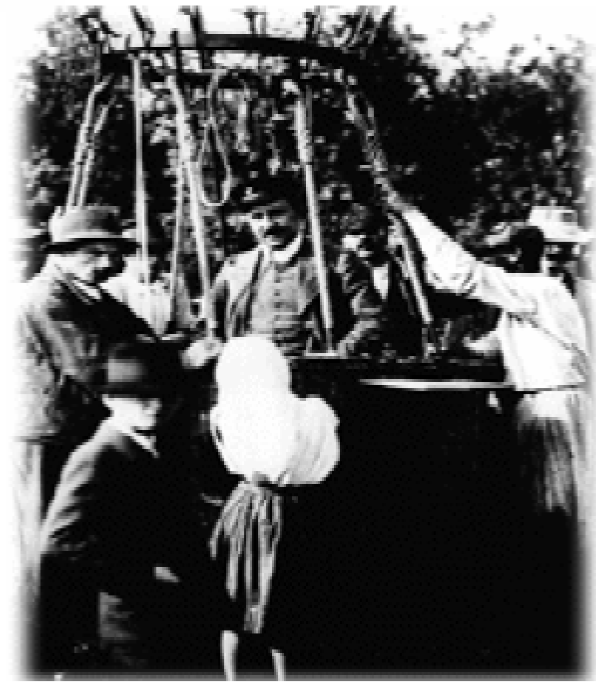
Primární částice:
p 80 %, α 9 %, n 8 %
e 2 %, těžká jádra 1 %
γ 0.1 %,

$$p + N/O \rightarrow \pi + \ldots$$

$$\pi^{\pm} \rightarrow \mu^{\pm} + \nu; \pi^{0} \rightarrow \gamma + \gamma$$

$$\mu^{\pm} \rightarrow e^{\pm} + \nu + \nu$$

Sekundární částice
na Zemi:
ν 68 %
μ 30 %
p, n, π ... 2 %



Objev positronu, Carl D. Anderson

Kladně nabitá částice.

P=23 MeV

6 mm Pb

P=63 MeV

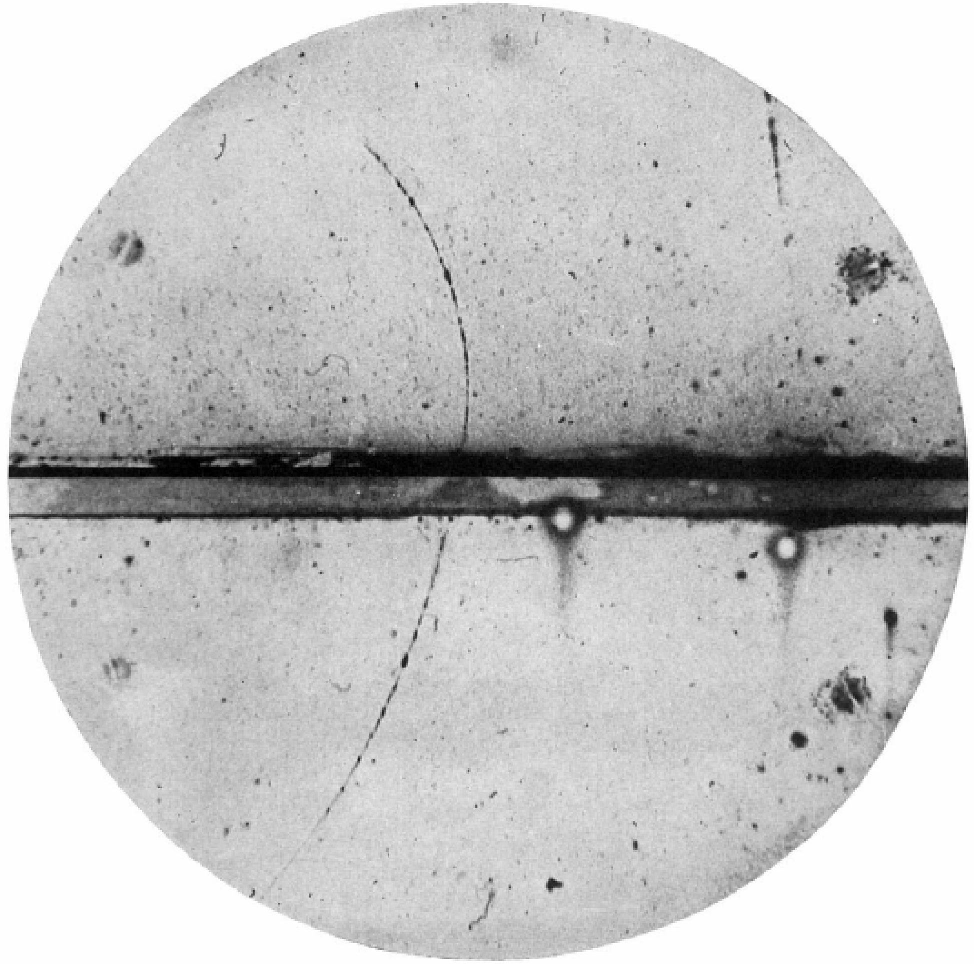


FIG. 1. A 63 million volt positron ($H\rho = 2.1 \times 10^5$ gauss-cm) passing through a 6 mm lead plate and emerging as a 23 million volt positron ($H\rho = 7.5 \times 10^4$ gauss-cm). The length of this latter path is at least ten times greater than the possible length of a proton path of this curvature.

proton: ionizace

$$T_k = \frac{(63MeV)^2}{2\cdot 938{,}27MeV} = 2{,}1MeV;$$

$$R_{Pb} = \frac{1}{\rho_{Pb}(dE/dx)_{\min}}\frac{T^2{}_k}{T_k + m_p} = \frac{1}{11{,}2gcm^{-3}1{,}15MeV/(gcm^{-2})}\frac{2{,}1^2\,MeV^2}{(2{,}1+938{,}27)MeV}$$

$$= 3{,}6\cdot 10^{-4}cm = 3{,}6\mu m$$

$$T_k = \frac{(23MeV)^2}{2\cdot 938{,}27MeV} = 0{,}28MeV$$

$$R_{Air} = \frac{1}{\rho_{Air}(dE/dx)_{\min}}\frac{T^2{}_k}{T_k + m_p} = \frac{1}{1{,}0\cdot 10^{-3}\,gcm^{-3}2MeV/(gcm^{-2})}\frac{0{,}28^2\,MeV^2}{(0{,}28+938{,}27)MeV}$$

$$= 0{,}042cm = 0{,}42mm$$

Kladný elektron: brzdné záření

$$E(L) = E_0 e^{-\frac{L}{X_0}} = 63e^{-\frac{6mm}{5{,}6mm}} = 21{,}6MeV$$

Částice s neznámou hmotou m - ionizace

$$\frac{1}{\rho_{Pb}(dE/dx)_{\min}}\frac{\left(\sqrt{p_1^2+m^2}-m\right)^2}{\sqrt{p_1^2+m^2}} - \frac{1}{\rho_{Pb}(dE/dx)_{\min}}\frac{\left(\sqrt{p_2^2+m^2}-m\right)^2}{\sqrt{p_2^2+m^2}} = t$$

$$\frac{1}{11{,}2\cdot 1{,}15}\frac{\left(\sqrt{63^2+m^2}-m\right)^2}{\sqrt{63^2+m^2}} - \frac{1}{11{,}2\cdot 1{,}15}\frac{\left(\sqrt{23^2+m^2}-m\right)^2}{\sqrt{23^2+m^2}} = 0{,}6cm$$

$$m = 61{,}5MeV$$

$$\Delta T \approx \frac{1}{\beta^2}$$

```
In[7]:= sol =
  Solve[ 1/(11.2×1.15) (√(63^2 + m^2) - m)^2/√(63^2 + m^2) - 1/(11.2×1.15) (√(23^2 + m^2) - m)^2/√(23^2 + m^2) ==
    0.6, m]
  ((23^2 + m^2) / 23^2) * (63^2 / (63^2 + m^2)) /. sol
Out[7]= {{m → -61.5412}, {m → 61.5412}}
Out[8]= {4.17526, 4.17526}
```

$$\Delta T_1/\Delta T_2 = \frac{\beta_2^2}{\beta_1^2} = \frac{\dfrac{p_2^2}{p_2^2+m^2}}{\dfrac{p_1^2}{p_1^2+m^2}} = \frac{p_2^2(p_1^2+m^2)}{p_1^2(p_2^2+m^2)} = \frac{23^2(63^2+61{,}5^2)}{63^2(23^2+61{,}5^2)} \cong 4{,}1$$

Yukawův mezon

The Nobel Prize in Physics 1949

"for his prediction of the existence of mesons on the basis of theoretical work on nuclear forces"

Hideki Yukawa

Japan

Kyoto Imperial University
Kyoto, Japan; Columbia University
New York, NY, USA

b. 1907
d. 1981

Krátký dosah jaderných sil je možné vysvětlit tak, že jsou zprostředkovány výměnou hmotného mezonu:

$$V_J(r) = \alpha_J \frac{\hbar c}{r} e^{-\frac{mr}{\hbar c}}$$

$$\frac{m \cdot 1\,fm}{\hbar c} \approx 1 \Rightarrow m \approx \frac{\hbar c}{1\,fm} = 197\,MeV$$

Takový mezon by měl ineragovat jadernou interakcí

Objev mionu – Street a Stevenson, Anderson a Nedermayer a další

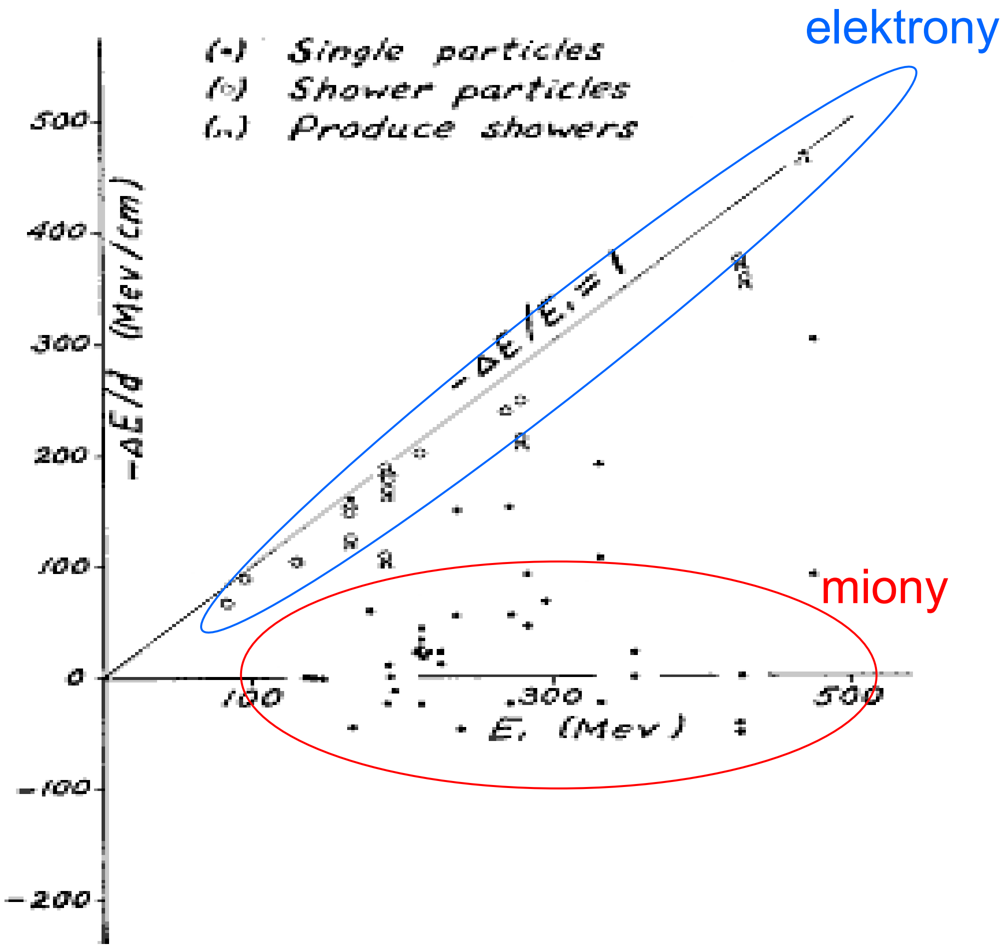


FIG. 1. Energy loss in 1 cm of platinum.

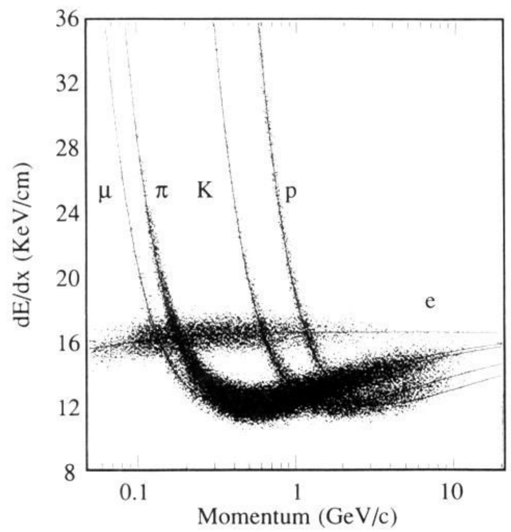


FIG. 2. Distribution in dE/dx vs momentum for particles in ıltihadron events. Lines indicate the predicted average $/dx$ as a function of momentum for different species.

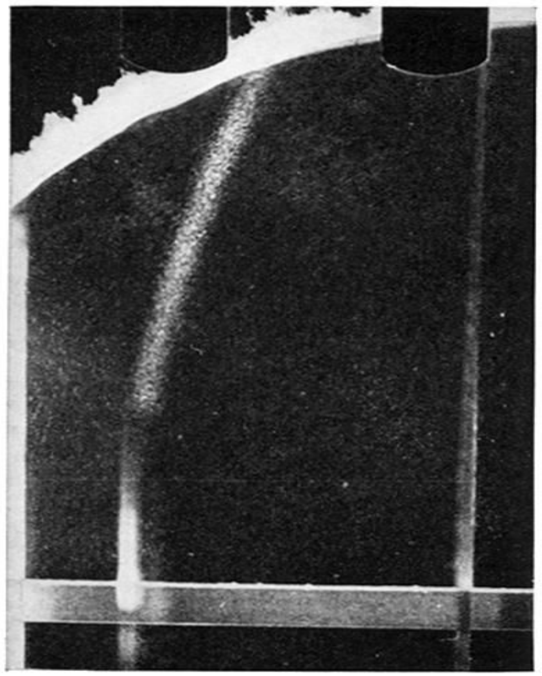


FIG. 3. Track B.

Určení hmoty částice podle měření ionizace a hybnosti

$$dE/dx \approx \frac{1}{\beta^2} \Rightarrow dE/dx = 4(dE/dx)_{\min} \Rightarrow \beta = 1/2$$

$$p = 200MeV$$

$$m\frac{v^2}{r} = evH \Rightarrow \frac{(mv)^2}{r} = e(mv)H \Rightarrow p = erH \Rightarrow \frac{pc}{e} = crH$$

$$p[eV] = 3\cdot 10^8[ms^{-1}]r[m]H[T] \Rightarrow p[GeV] = 0{,}3[ms^{-1}]r[m]H[T]$$

$$p[GeV] = 0{,}3r[m]H[T]$$

$$p[MeV] = 0{,}3r[mm]H[T]$$

$$p[TeV] = 0{,}3r[km]H[T]$$

$$M = \beta\gamma p = \frac{\beta}{\sqrt{1-\beta^2}}p = \frac{0{,}5}{\sqrt{1-0{,}5^2}}200MeV = 115{,}5MeV$$

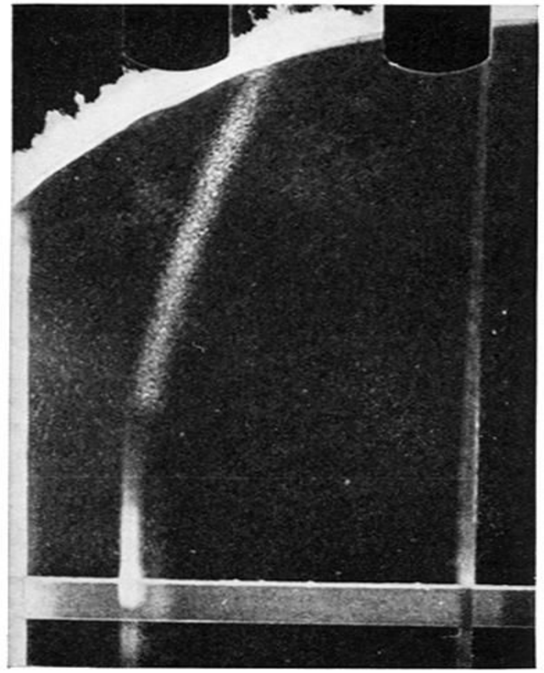


FIG. 3. Track B.

Určení hmoty částice podle měření doběhu a hybnosti

$$R(ange) = \frac{1}{\rho\cdot(dE/dx)_{\min}}\frac{T_k^2}{T_k + M} = \frac{1}{\rho\cdot(dE/dx)_{\min}}\frac{\left(\sqrt{p^2+M^2}-M\right)^2}{\sqrt{p^2+M^2}}$$

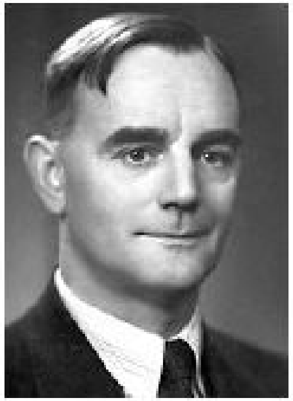
$$R/M = \frac{1}{\rho\cdot(dE/dx)_{\min}}\frac{\left(\sqrt{(p/M)^2+1^2}-1\right)^2}{\sqrt{(p/M)^2+1^2}}$$

Mion ale nebyl mezonem předpovězeným Yukawou, neinteragoval jadernou silou

Objev pionu a jeho rozpadu. Mion nebyl pionem předpovězeným Yukawou, tím byl až π mezon, objevený Powelem

The Nobel Prize in Physics 1950

"for his development of the photographic method of studying nuclear processes and his discoveries regarding mesons made with this method"



Cecil Frank Powell

United Kingdom

Bristol University
Bristol, United Kingdom

b. 1903
d. 1969

$$\pi^{\pm} \rightarrow \mu^{\pm} + \nu$$

$$\mu^{\pm} \rightarrow e^{\pm} + \nu + \nu$$

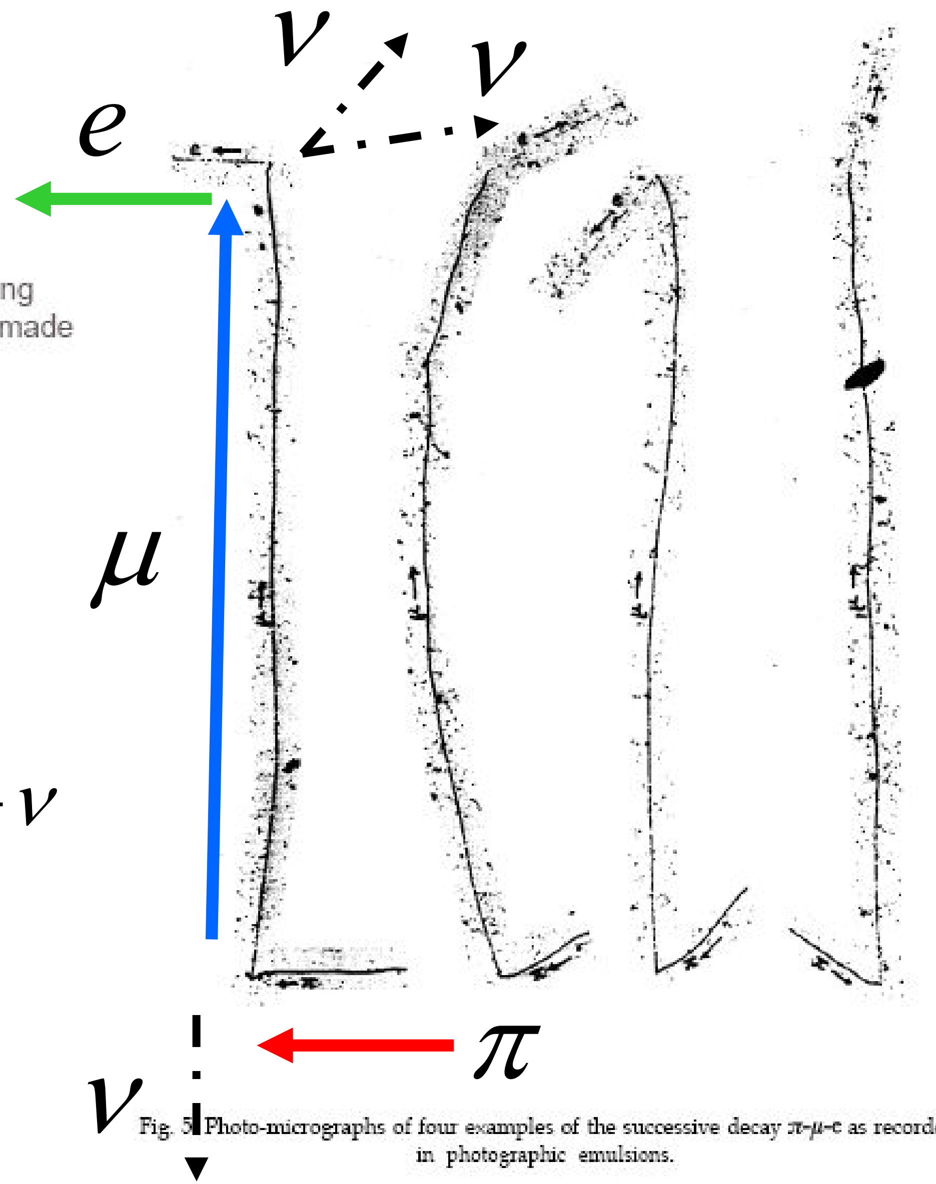


Fig. 5 Photo-micrographs of four examples of the successive decay π-μ-e as recorded in photographic emulsions.

Objev pionu a jeho rozpadu. Mion nebyl pionem předpovězeným Yukawou, tím byl až π mezon.

$$\pi^+ \to \mu^+ + \nu_\mu$$

$$p_{\mu^+} = \frac{M_{\pi^+}^2 - M_{\mu^+}^2}{2M_{\pi^+}}$$

$$e_{\mu^+} = \sqrt{\left(p_{\mu^+}\right)^2 + M_{\mu^+}^2} = \sqrt{\left(\frac{M_{\pi^+}^2 - M_{\mu^+}^2}{2M_{\pi^+}}\right)^2 + M_{\mu^+}^2} =$$

$$\sqrt{\left(\frac{M_{\pi^+}^2 + M_{\mu^+}^2}{2M_{\pi^+}}\right)^2} = \frac{M_{\pi^+}^2 + M_{\mu^+}^2}{2M_{\pi^+}}$$

$$\beta_{\mu^+} = p_{\mu^+} / e_{\mu^+} = \frac{M_{\pi^+}^2 - M_{\mu^+}^2}{M_{\pi^+}^2 + M_{\mu^+}^2}$$

$$T_{k,\mu^+} = \frac{M_{\pi^+}^2 + M_{\mu^+}^2}{2M_{\pi^+}} - M_{\mu^+} = \frac{\left(M_{\pi^+} - M_{\mu^+}\right)^2}{2M_{\pi^+}}$$

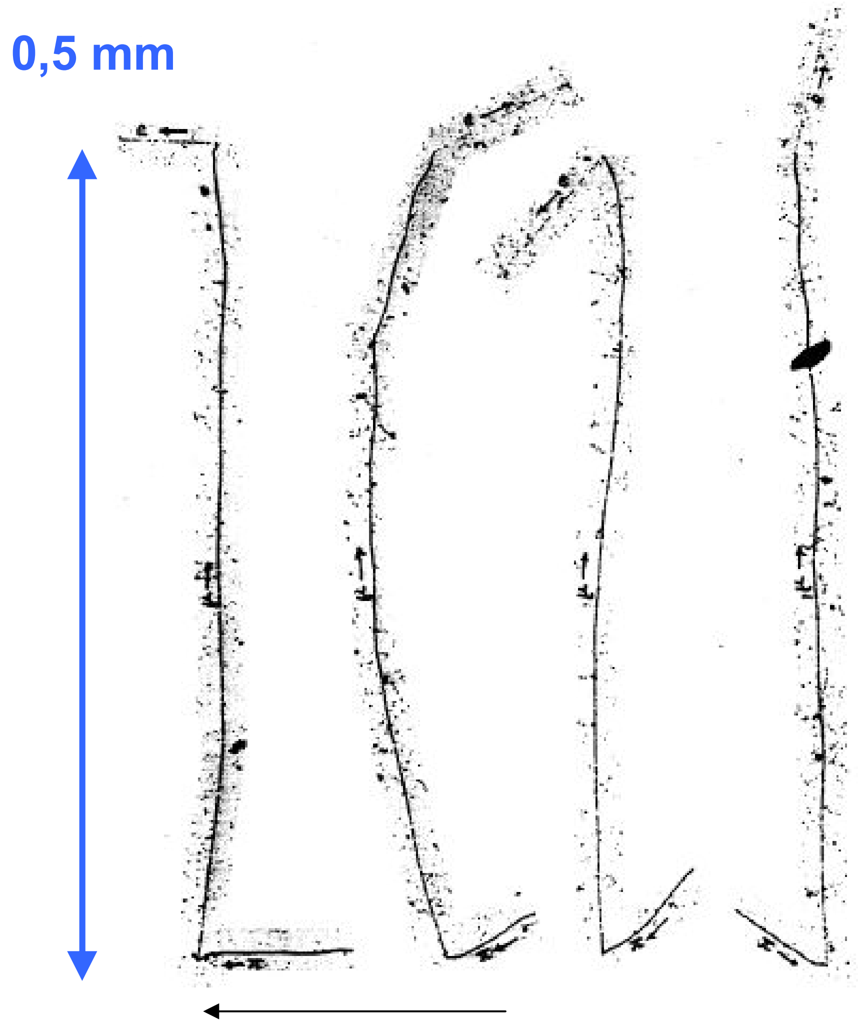


Fig. 5. Photo-micrographs of four examples of the successive decay π-μ-e as recorded in photographic emulsions.

Louis Leprince-Ringuet a Michel Heritier 1943
objev částice s hmotou M = 550MeV

$$H = 2500G = 0{,}25T$$

$$r_e = 1{,}6cm \Rightarrow p_e[MeV] = 0{,}3H[T]r_e[mm] = 1{,}2MeV$$

$$Hr_{prim} = 1{,}7 \cdot 10^6 Gcm \Rightarrow P_{prim} = 0{,}3H[T]r_{prim}[mm] = 510MeV$$

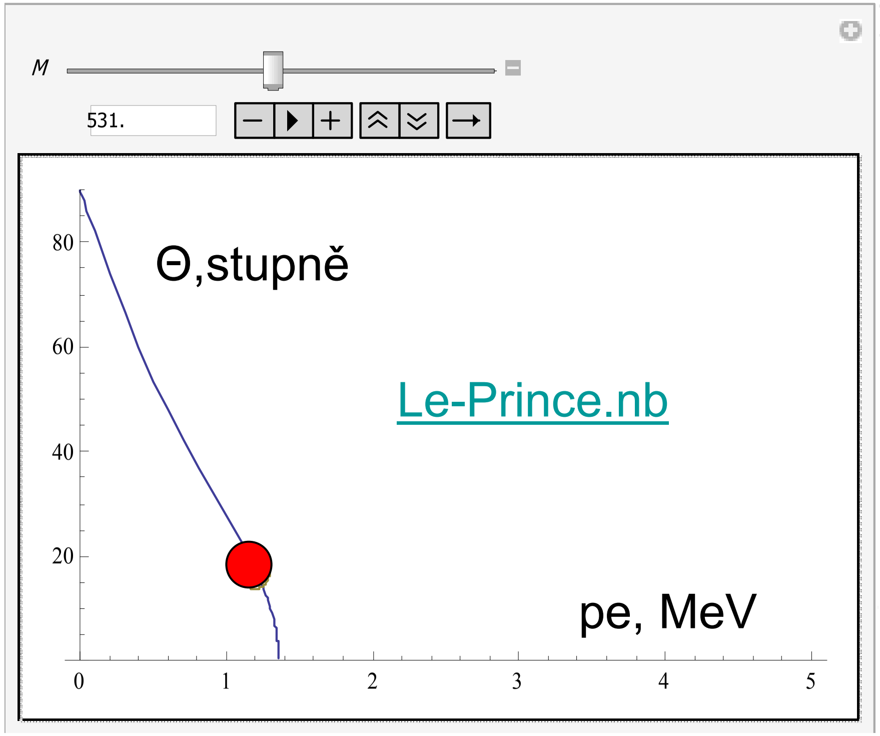
$$tg(\theta) = 0{,}32 \Rightarrow \theta = 17{,}75^o$$

$$\sqrt{\left|\vec{P}_{prim}\right|^2 + M^2} + m_e = \sqrt{\left|\vec{P}_{secondary}\right|^2 + M^2} + \sqrt{\left|\vec{p}_e\right|^2 + m^2{}_e}$$

$$\left|\vec{P}_{secondary}\right|^2 = \left(\sqrt{\left|\vec{P}_{prim}\right|^2 + M^2} + m_e - \sqrt{\left|\vec{p}_e\right|^2 + m^2{}_e}\right)^2 - M^2$$

$$\vec{P}_{prim} = \vec{P}_{secondary} + \vec{p}_e \Rightarrow \left|\vec{P}_{prim}\right|^2 = \left|\vec{P}_{secondary}\right|^2 + \left|\vec{p}_e\right|^2 + 2\left|\vec{P}_{secondary}\right|\left|\vec{p}_e\right|\cos(\theta)$$

Out[122]=



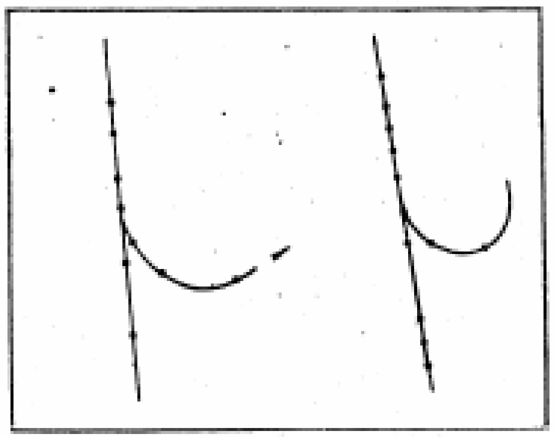
PHYSIQUE NUCLÉAIRE. — *Existence probable d'une particule de masse 990 m_e dans le rayonnement cosmique.* Note (¹) de MM. Louis Leprince-Ringuet et Michel Lhéritier.

Nous avons pris, au cours de l'année 1943, dans le laboratoire de Largentière (Hautes-Alpes) situé à 1000ᵐ d'altitude, une série de 10000 clichés de trajectoires cosmiques commandées par compteurs. Les rayons, filtrés par 10ᶜᵐ de plomb, traversaient une chambre de Wilson de 75ᶜᵐ de hauteur, placée dans un champ magnétique H de 2500 gauss environ. Nous nous sommes placés dans les conditions expérimentales les plus favorables [discutées précédemment (²), (³), (⁴)] pour profiter au mieux des clichés de collision entre particules pénétrantes et électrons du gaz de la chambre, dans le but de déterminer la masse au repos de la particule incidente.

Nous avons obtenu une dizaine de clichés intéressants. Le plus remarquable représente une collision dans le gaz pour laquelle d'excellentes conditions sont réalisées : le secondaire fait avec le plan médian de la chambre un angle ζ tel que tang ζ = 0,32 et son rayon de courbure projeté (1ᶜᵐ,6), ainsi que la flèche dont il s'écarte du primaire sont mesurables avec précision. Le (Hρ) du primaire = 1,7 × 10⁶ gauss × cm. La formule de collision élastique donne pour le primaire, qui est positif, la masse au repos

$$\mu_0 = 990 \pm 12\ \%\ \text{(limites extrêmes de l'erreur)}\quad (^5).$$

La masse ainsi obtenue peut surprendre. Les indications suivantes, qui donnent des garanties de la validité de la mesure, nous ont poussés à publier ce résultat.

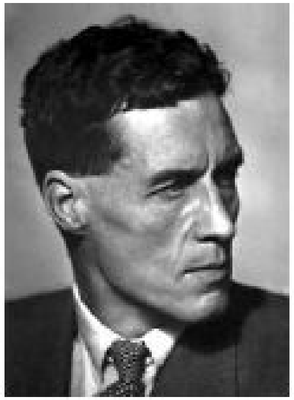


Dessin stéréoscopique de la collision.

Dráhy částic a interakce ve Wilsonově mlžné komoře

The Nobel Prize in Physics 1948

"for his development of the Wilson cloud chamber method, and his discoveries therewith in the fields of nuclear physics and cosmic radiation"



Patrick Maynard Stuart Blackett

United Kingdom

Victoria University
Manchester, United Kingdom

b. 1897
d. 1974

$$ {}^{4}_{2}\alpha + {}^{16}_{8}O \rightarrow {}^{16}_{8}O + {}^{4}_{2}\alpha $$

$$ {}^{16}_{8}O(\alpha,\alpha){}^{16}_{8}O $$

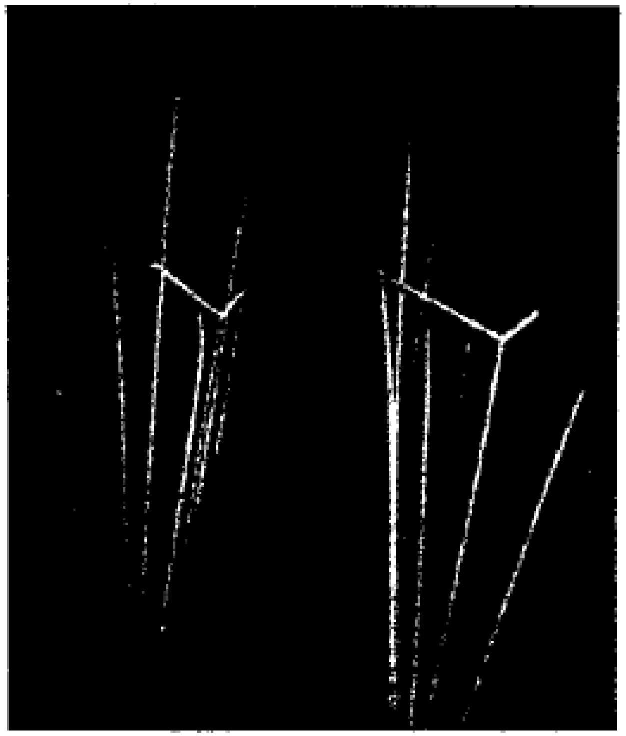


Fig. 1. Elastic collision of alpha particle with an oxygen nucleus.

Podivné částice

Two photographs taken by Rochester and Butler were of exceptional interest in that they seemed to suggest the existence of two new types of particles, one uncharged and one with a positive charge, and both of mass about 900 m. In one, Fig. 15, a forked track was observed in the gas, due to two particles, one positive and one negative with momenta of a few hundred

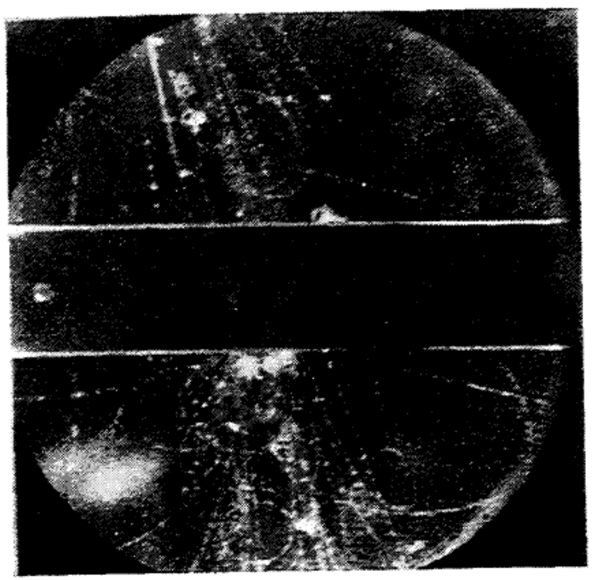


Fig. 15. Penetrating shower with anomalous forked track. A typical, but rare, type of penetrating shower showing several particles penetrating a g-cm lead plate together with some soft electronic component. On the right below the plate is a peculiar forked track, which for reasons given in the text, is considered to represent the spontaneous disintegration of a new type of neutral particle (π-meson) of mass about 900 m into a positive and negative particle of lower mass. H = 3,500 gauss. (Rochester and Butler, 1947)

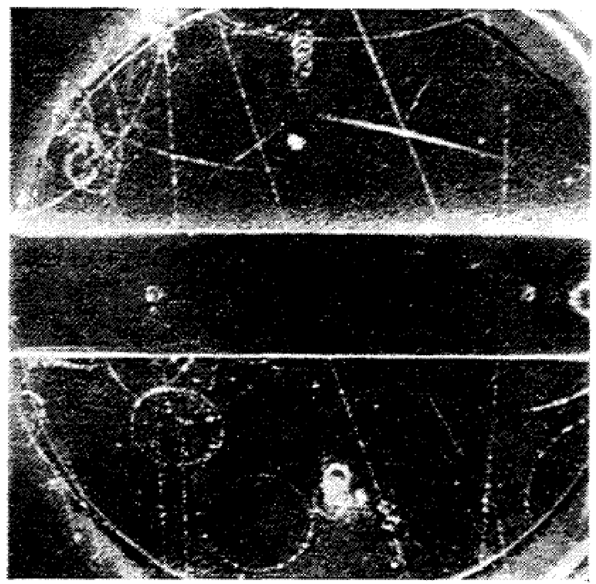


Fig. 16. Penetrating shower with anomalous bent track. A few penetrating particles pass through the plate. One of them, at the top right-hand corner of the photograph, makes an 18° deflection in the gas and then passes through the plate with little further deflection. This is interpreted as the spontaneous disintegration of a new type of positive particle (τ-meson) of mass about 900 m into a positive particle of lower mass together with a neutral unobserved particle. H = 7,000 gauss. (Rochester and Butler, 1947)

Podivné částice

Two photographs taken by Rochester and Butler were of exceptional interest in that they seemed to suggest the existence of two new types of particles, one uncharged and one with a positive charge, and both of mass about 900 m. In one, Fig. 15, a forked track was observed in the gas, due to two particles, one positive and one negative with momenta of a few hundred MeV/c. The simplest explanation was that a neutral particle had collided with a nucleus and ejected two mesons, but this was rejected since one would expect to find very many more of such cases occurring in the lead plate in the gas. As these were not found, it was concluded that the forked track did not represent a collision process at all, but a case of spontaneous integration of an unstable particle. From the momenta of the ejected particles the mass of the neutral particle was estimated as probably about 870 $\pm$ 200.

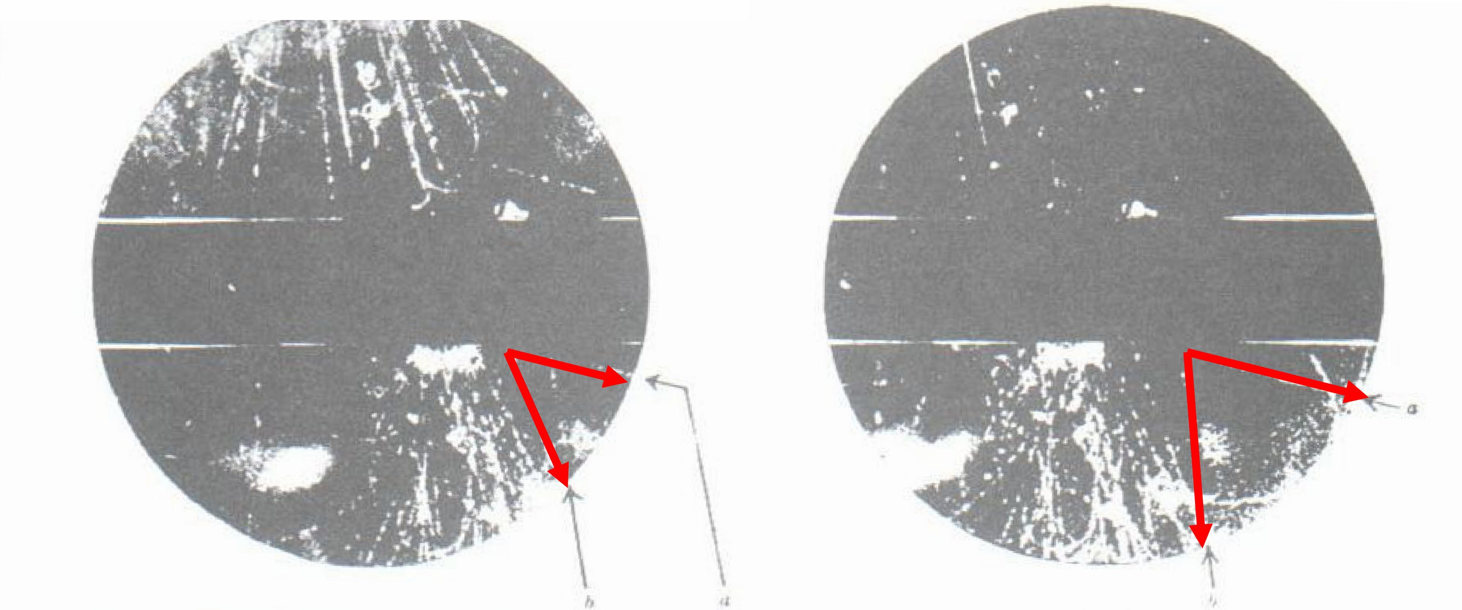


Fig. 1. STEREOSCOPIC PHOTOGRAPHS SHOWING AN UNUSUAL FORK (a b) IN THE GAS. THE DIRECTION OF THE MAGNETIC FIELD IS SUCH THAT A POSITIVE PARTICLE COMING DOWNWARDS IS DEVIATED IN AN ANTICLOCKWISE DIRECTION

Podivné částice

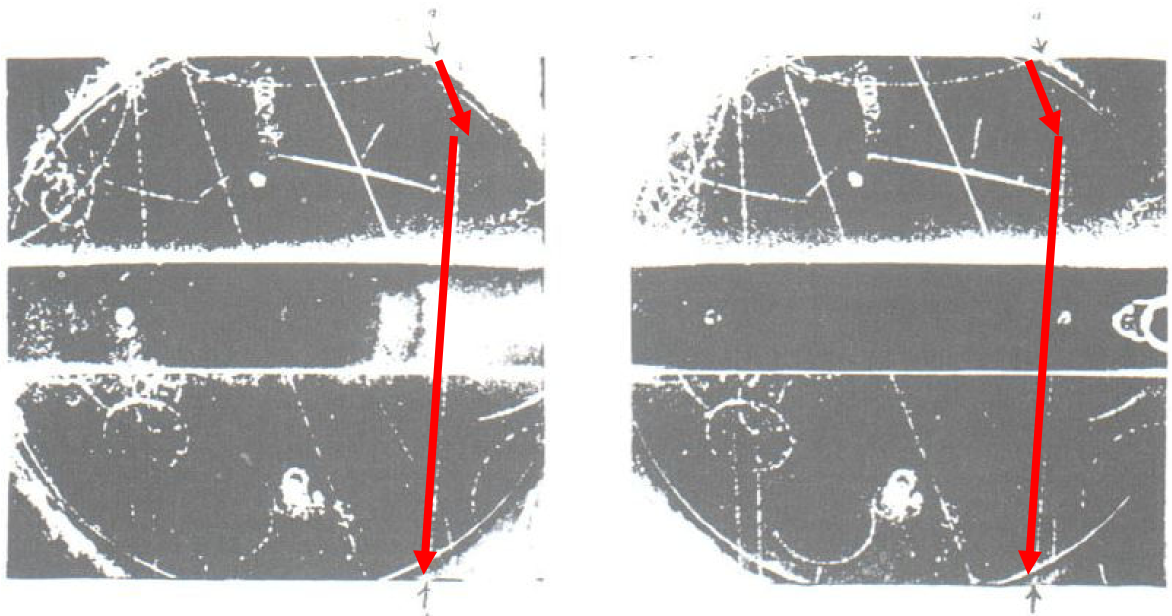
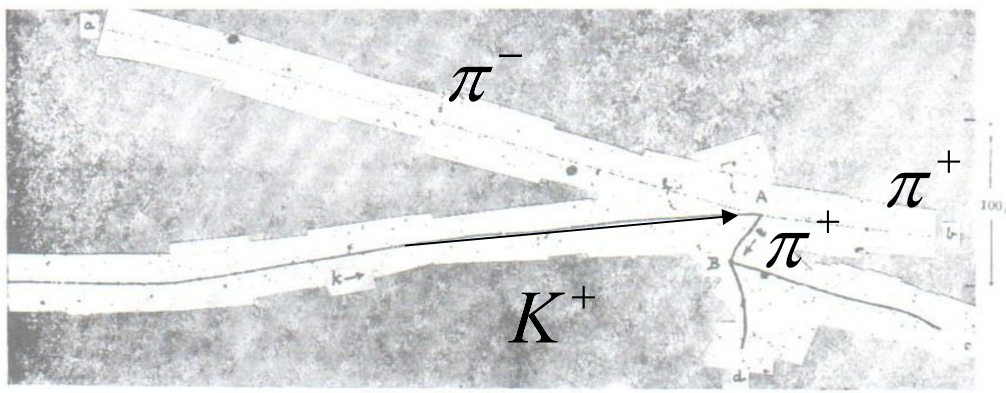


Fig. 2. STEREOSCOPIC PHOTOGRAPHS SHOWING AN UNUSUAL FORK (a b). THE DIRECTION OF THE MAGNETIC FIELD IS SUCH THAT A POSITIVE PARTICLE COMING DOWNWARDS IS DEVIATED IN A CLOCKWISE DIRECTION

A second photograph (Fig. 16) showed a positive particle which seemed to undergo a deflection of 18° in the gas, and then to pass through the 3-cm lead plate without appreciable further deflection or energy loss. Similar arguments to those used for the first photograph led to the interpretation that an unstable positive particle of mass about 1080 ± 100 had spontaneously transformed itself into a positive particle, probably a **μ-meson,** and into an unobserved neutral particle.

$$K^+ \rightarrow \pi^+ + \pi^+ + \pi^-$$



Observer: Mrs. W. J. van der Merwe
Fig. 8

Řešení záhady podivných částic

Podivné částice se rodí v silných interakcích

Ale rozpadají se velmi pomalu

Řešení:
Připsat částicím aditivní kvantové číslo **podivnost S**
Podivnost se zachovává v silných interakcích, tj. musí vzniknout více podivných částic tak aby součet jejich podivností byl roven nule

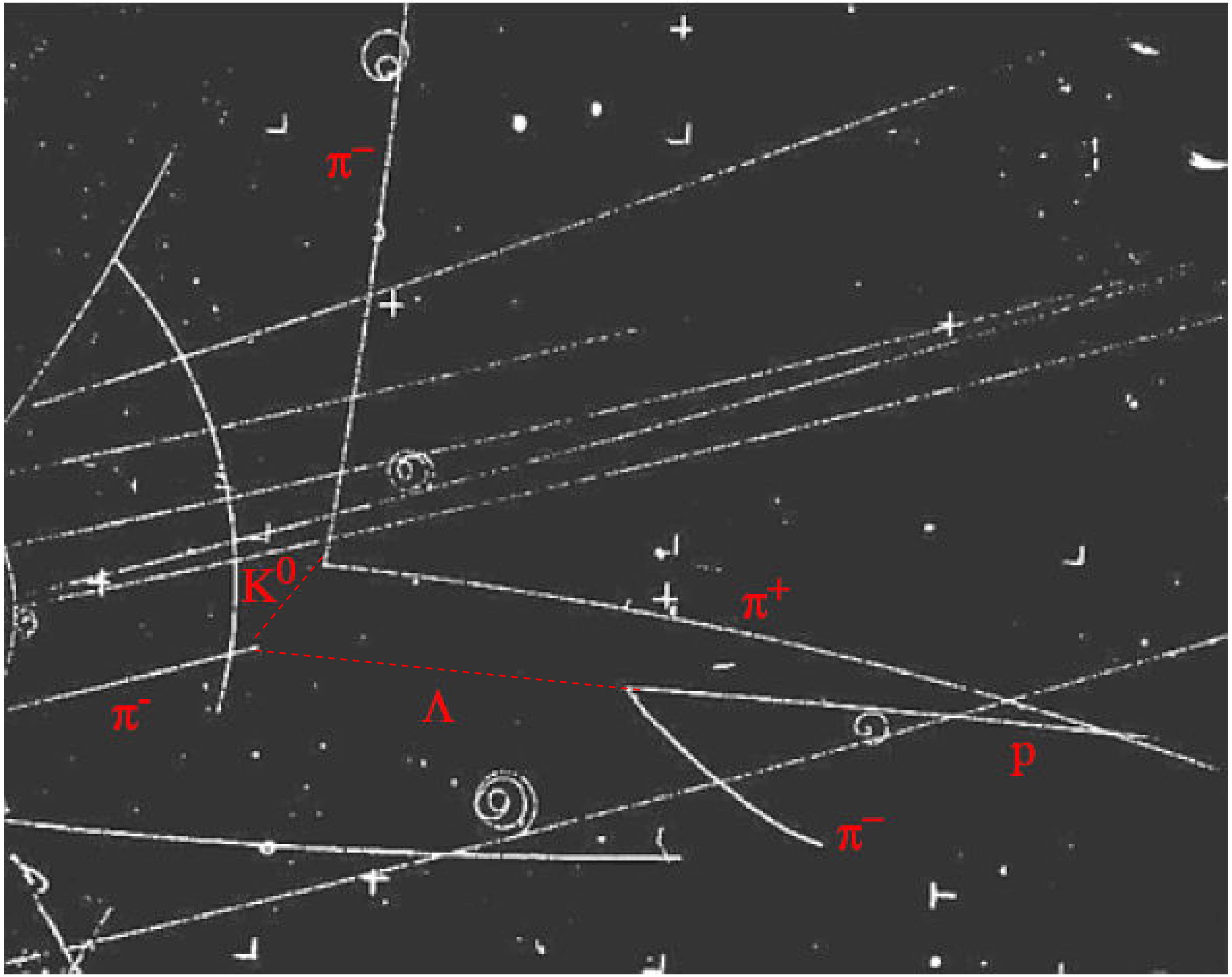
Při rozpadu se podivnost nezachovává podivné částice se proto rozpadají pomocí jiné interakce (slabé)

Párová produkce podivných částic:

$$\pi^- + p \rightarrow K^0 + \Lambda^0$$

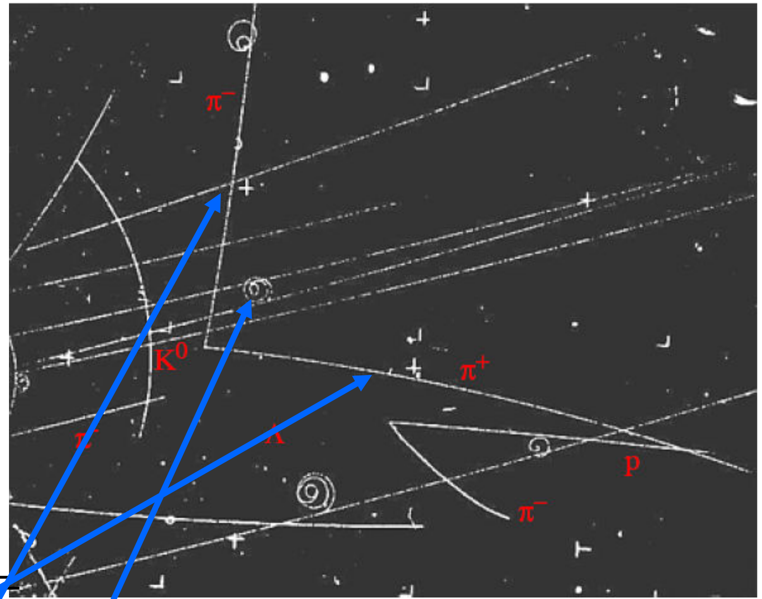
$$K^0 \rightarrow \pi^+ + \pi^-$$

$$\Lambda^0 \rightarrow p + \pi^-$$



$$S(\Lambda^0) = -1$$

$$S(K^0) = +1$$



$$K^0 \to \pi^+ + \pi^-$$

$$P_{K^0} = P_{\pi^+} + P_{\pi^-}$$

$$P_{K^0}{}^2 = E_{K^0}{}^2 - \left|\overrightarrow{P_{K^0}}\right|^2 = M_{K^0}^2 = \left(P_{\pi^+} + P_{\pi^-}\right)^2$$

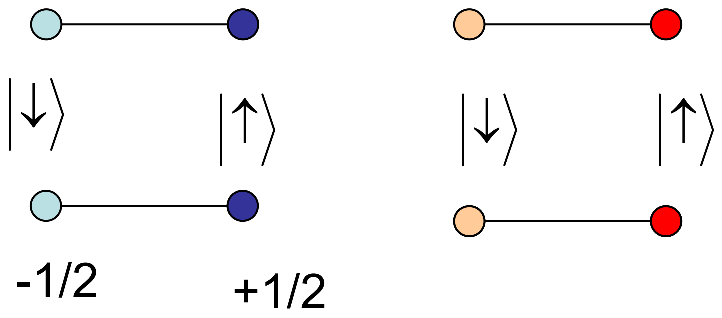
$$\left(P_{\pi^+} + P_{\pi^-}\right)^2 = \left(E_{\pi^+} + E_{\pi^-}\right)^2 - \left(\vec{P}_{\pi^+} + \vec{P}_{\pi^-}\right)^2 =$$

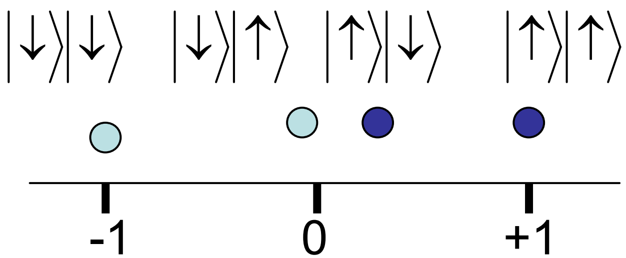
$$E_{\pi^+}^2 - \vec{P}_{\pi^+}^2 + E_{\pi^-}^2 - \vec{P}_{\pi^-}^2 + 2E_{\pi^+}E_{\pi^-} - 2\vec{P}_{\pi^+}\vec{P}_{\pi^-} =$$

$$2M_\pi^2 + 2\sqrt{\vec{P}_{\pi^+}^2 + M_\pi^2}\sqrt{\vec{P}_{\pi^-}^2 + M_\pi^2} - 2\left|\vec{P}_{\pi^+}\right|\left|\vec{P}_{\pi^-}\right|\cos(\theta_\pm) = M_{K^0}^2$$

Invariantní hmota – určení hmoty částice z měření produktů rozpadu

Skládání momentů hybnosti, ½ a ½





$$1/2 \otimes 1/2 = 1 \oplus 0$$

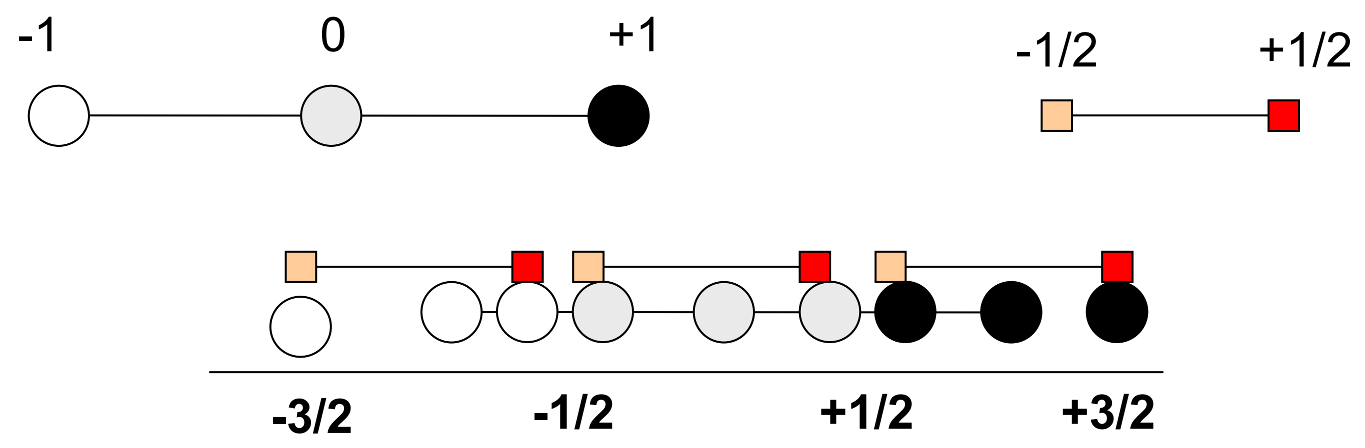
$$|1,-1\rangle = |\downarrow\downarrow\rangle = |1/2,-1/2\rangle|1/2,-1/2\rangle$$

$$|1,+1\rangle = |\uparrow\uparrow\rangle = |1/2,+1/2\rangle|1/2,+1/2\rangle$$

$$|1,0\rangle = \frac{1}{\sqrt{2}}\left(|\uparrow\downarrow\rangle + |\downarrow\uparrow\rangle\right) = \frac{1}{\sqrt{2}}\left(|1/2,+1/2\rangle|1/2,-1/2\rangle + |1/2,-1/2\rangle|1/2,+1/2\rangle\right)$$

$$|0,0\rangle = \frac{1}{\sqrt{2}}\left(|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle\right) = \frac{1}{\sqrt{2}}\left(|1/2,+1/2\rangle|1/2,-1/2\rangle - |1/2,-1/2\rangle|1/2,+1/2\rangle\right)$$

Skládání momentů hybnosti, 1 a ½



$$1 \otimes 1/2 = 3/2 \oplus 1/2$$

$$\left|3/2,-3/2\right\rangle = \left|1,-1\right\rangle \left|1/2,-1/2\right\rangle$$

$$\left|3/2,+3/2\right\rangle = \left|1,+1\right\rangle \left|1/2,+1/2\right\rangle$$

$$\left|3/2,-1/2\right\rangle = \sqrt{\frac{1}{3}}\left|1,-1\right\rangle \left|1/2,1/2\right\rangle + \sqrt{\frac{2}{3}}\left|1,0\right\rangle \left|1/2,-1/2\right\rangle$$

$$\left|3/2,1/2\right\rangle = \sqrt{\frac{1}{3}}\left|1,+1\right\rangle \left|1/2,-1/2\right\rangle + \sqrt{\frac{2}{3}}\left|1,0\right\rangle \left|1/2,+1/2\right\rangle$$

$$\left|1/2,-1/2\right\rangle = -\sqrt{\frac{2}{3}}\left|1,-1\right\rangle \left|1/2,1/2\right\rangle + \sqrt{\frac{1}{3}}\left|1,0\right\rangle \left|1/2,-1/2\right\rangle$$

$$\left|1/2,1/2\right\rangle = \sqrt{\frac{2}{3}}\left|1,+1\right\rangle \left|1/2,-1/2\right\rangle - \sqrt{\frac{1}{3}}\left|1,0\right\rangle \left|1/2,+1/2\right\rangle$$

Clebshovy Gordanovy koeficienty

$$\left[J_i, J_j\right] = \varepsilon_{ijk} i\hbar J_k \;\; ; \;\; J^2 \left| j,m \right\rangle = \hbar^2 j(j+1) \left| j,m \right\rangle \;\; ; \;\; J_3 \left| j,m \right\rangle = \hbar m \left| j,m \right\rangle$$

$$J_+ = J_1 + iJ_2 \;\; ; J_- = J_1 - iJ_2$$

$$J_+ \left| j,m \right\rangle :$$

$$J_3 \left| J_+ \left| j,m \right\rangle \right\rangle = J_3 \left(J_1 + iJ_2\right) \left| j,m \right\rangle = \left(J_3 J_1 + iJ_3 J_2\right) \left| j,m \right\rangle$$

$$J_3 J_1 - J_1 J_3 = \varepsilon_{312} i\hbar J_2 = +i\hbar J_2 \Rightarrow J_3 J_1 = +i\hbar J_2 + J_1 J_3$$

$$J_3 J_2 - J_2 J_3 = \varepsilon_{321} i\hbar J_1 = -i\hbar J_1 \Rightarrow J_3 J_2 = -i\hbar J_1 + J_2 J_3$$

$$J_3 \left| J_+ \left| j,m \right\rangle \right\rangle = \left(+i\hbar J_2 + J_1 J_3 + i\left(-i\hbar J_1 + J_2 J_3\right)\right) \left| j,m \right\rangle = \left(+i\hbar J_2 + J_1 \hbar m + i\left(-i\hbar J_1 + J_2 \hbar m\right)\right) \left| j,m \right\rangle$$

$$= i\hbar(m+1) J_2 + \hbar(m+1) J_1 \left| j,m \right\rangle = \hbar(m+1)\left(J_1 + iJ_2\right) \left| j,m \right\rangle = \hbar(m+1) \left| J_+ \left| j,m \right\rangle \right\rangle$$

$$J_3 \left| J_+ \left| j,m \right\rangle \right\rangle = \hbar(m+1) \left| J_+ \left| j,m \right\rangle \right\rangle \Rightarrow \left| J_+ \left| j,m \right\rangle \right\rangle = A_+ \left| j,m+1 \right\rangle$$

$$J_+ J_- = \left(J_1 + iJ_2\right)\left(J_1 - iJ_2\right) = J_1^{\,2} + J_2^{\,2} - i\left(J_1 J_2 - J_2 J_1\right) = J^2 - J_3^{\,2} - ii\hbar J_3 = J^2 - J_3^{\,2} + \hbar J_3$$

$$J_+ J_- \left| j,m \right\rangle = \hbar^2 j(j+1) - \hbar^2 m^2 + \hbar^2 m = \hbar^2 \left(j(j+1) - m(m-1)\right)$$

$$J_- J_+ = \left(J_1 - iJ_2\right)\left(J_1 + iJ_2\right) = J_1^{\,2} + J_2^{\,2} + i\left(J_1 J_2 - J_2 J_1\right) = J^2 - J_3^{\,2} + ii\hbar J_3 = J^2 - J_3^{\,2} - \hbar J_3$$

$$J_- J_+ \left| j,m \right\rangle = \hbar^2 j(j+1) - \hbar^2 m^2 - \hbar^2 m = \hbar^2 \left(j(j+1) - m(m+1)\right)$$

$$J_+ \left| j,m \right\rangle = A_+ \left| j,m+1 \right\rangle$$

$$\left(J_+ \left| j,m \right\rangle\right)^* = \left\langle \left\langle j,m \right| J_- \right| = A_+^{\,*} \left\langle j,m+1 \right|$$

$$A_+^{\,*} \left\langle j,m+1 \right| A_+ \left| j,m+1 \right\rangle = A_+^{\,2} = \left\langle j,m \right| J_- J_+ \left| j,m \right\rangle = \hbar^2 \left(j(j+1) - m(m-1)\right)$$

$$A_+ = \hbar \sqrt{j(j+1) - m(m+1)} \Rightarrow J_+ \left| j,m \right\rangle = \hbar \sqrt{j(j+1) - m(m+1)} \left| j,m+1 \right\rangle$$

$$A_- = \hbar \sqrt{j(j+1) - m(m-1)} \Rightarrow J_- \left| j,m \right\rangle = \hbar \sqrt{j(j+1) - m(m-1)} \left| j,m-1 \right\rangle$$

Clebshovy Gordanovy koeficienty

$$J_+\left| j,m\right\rangle = \hbar\sqrt{j(j+1)-m(m+1)}\left| j,m+1\right\rangle$$

$$J_-\left| j,m\right\rangle = \hbar\sqrt{j(j+1)-m(m-1)}\left| j,m-1\right\rangle$$

$$1\otimes 1/2$$

$$\left|3/2,+3/2\right\rangle = \left|1,+1\right\rangle\left|1/2,+1/2\right\rangle$$

$$J_-^{(3/2)} = J_-^{(1)} + J_-^{(1/2)}$$

$$J_-^{(3/2)}\left|3/2,+3/2\right\rangle = \left(J_-^{(1)} + J_-^{(1/2)}\right)\left|1,+1\right\rangle\left|1/2,+1/2\right\rangle$$

$$J_-^{(3/2)}\left|3/2,+3/2\right\rangle = \hbar\sqrt{3/2(3/2+1)-3/2(3/2-1)}\left|3/2,3/2-1\right\rangle =$$

$$\hbar\sqrt{15/4-3/4}\left|3/2,3/2-1\right\rangle = \hbar\sqrt{3}\left|3/2,1/2\right\rangle$$

$$\left(J_-^{(1)} + J_-^{(1/2)}\right)\left|1,+1\right\rangle\left|1/2,+1/2\right\rangle = J_-^{(1)}\left|1,+1\right\rangle\left|1/2,+1/2\right\rangle + \left|1,+1\right\rangle J_-^{(1/2)}\left|1/2,+1/2\right\rangle =$$

$$\hbar\sqrt{1(1+1)-1(1-1)}\left|1,+1-1\right\rangle\left|1/2,+1/2\right\rangle + \hbar\sqrt{1/2(1/2+1)-1/2(1/2-1)}\left|1,+1\right\rangle\left|1/2,+1/2-1\right\rangle =$$

$$\hbar\sqrt{2}\left|1,0\right\rangle\left|1/2,+1/2\right\rangle + \hbar\sqrt{1}\left|1,+1\right\rangle\left|1/2,-1/2\right\rangle$$

$$\hbar\sqrt{3}\left|3/2,1/2\right\rangle = \hbar\sqrt{2}\left|1,0\right\rangle\left|1/2,+1/2\right\rangle + \hbar\sqrt{1}\left|1,+1\right\rangle\left|1/2,-1/2\right\rangle$$

$$\left|3/2,1/2\right\rangle = \sqrt{\frac{2}{3}}\left|1,0\right\rangle\left|1/2,+1/2\right\rangle + \sqrt{\frac{1}{3}}\left|1,+1\right\rangle\left|1/2,-1/2\right\rangle$$

Clebshovy koeficienty - tabulka

35. CLEBSCH-GORDAN COEFFICIENTS, SPHERICAL HARMONICS, AND d FUNCTIONS

Note: A square-root sign is to be understood over *every* coefficient, *e.g.*, for $-8/15$ read $-\sqrt{8/15}$.

Notation:

m_1	m_2	J M	J M	...
m_1	m_2	Coefficients		
.	.			
.	.			

$$\langle j_1 j_2 m_1 m_2 | j_1 j_2 J M\rangle = (-1)^{J-j_1-j_2}\langle j_2 j_1 m_2 m_1 | j_2 j_1 J M\rangle$$

$$Y_1^0 = \sqrt{\frac{3}{4\pi}}\cos\theta$$

$$Y_1^1 = -\sqrt{\frac{3}{8\pi}}\sin\theta\, e^{i\phi}$$

$$Y_2^0 = \sqrt{\frac{5}{4\pi}}\Big(\frac{3}{2}\cos^2\theta - \frac{1}{2}\Big)$$

$$Y_2^1 = -\sqrt{\frac{15}{8\pi}}\sin\theta\cos\theta\, e^{i\phi}$$

$$Y_2^2 = \frac{1}{4}\sqrt{\frac{15}{2\pi}}\sin^2\theta\, e^{2i\phi}$$

$$Y_\ell^{-m} = (-1)^m Y_\ell^{m*}$$

$$d^\ell_{m,0} = \sqrt{\frac{4\pi}{2\ell+1}}\,Y_\ell^m\, e^{-im\phi}$$

1/2 × 1/2

m_1	m_2	J=1, M=+1
+1/2	+1/2	1

m_1	m_2	J=1, M=0	J=0, M=0
+1/2	−1/2	1/2	1/2
−1/2	+1/2	1/2	−1/2

m_1	m_2	J=1, M=−1
−1/2	−1/2	1

1 × 1/2

m_1	m_2	J=3/2, M=+3/2
+1	+1/2	1

m_1	m_2	J=3/2, M=+1/2	J=1/2, M=+1/2
+1	−1/2	1/3	2/3
0	+1/2	2/3	−1/3

m_1	m_2	J=3/2, M=−1/2	J=1/2, M=−1/2
0	−1/2	2/3	1/3
−1	+1/2	1/3	−2/3

m_1	m_2	J=3/2, M=−3/2
−1	−1/2	1

2 × 1

m_1	m_2	J=3, M=+3
+2	+1	1

m_1	m_2	J=3, M=+2	J=2, M=+2
+2	0	1/3	2/3
+1	+1	2/3	−1/3

m_1	m_2	J=3, M=+1	J=2, M=+1	J=1, M=+1
+2	−1	1/15	1/3	3/5
+1	0	8/15	1/6	−3/10
0	+1	2/5	−1/2	1/10

m_1	m_2	J=3, M=0	J=2, M=0	J=1, M=0
+1	−1	1/5	1/2	3/10
0	0	3/5	0	−2/5
−1	+1	1/5	−1/2	3/10

m_1	m_2	J=3, M=−1	J=2, M=−1	J=1, M=−1
0	−1	2/5	1/2	1/10
−1	0	8/15	−1/6	−3/10
−2	+1	1/15	−1/3	3/5

m_1	m_2	J=3, M=−2	J=2, M=−2
−1	−1	2/3	1/3
−2	0	1/3	−2/3

m_1	m_2	J=3, M=−3
−2	−1	1

1 × 1

m_1	m_2	J=2, M=+2
+1	+1	1

m_1	m_2	J=2, M=+1	J=1, M=+1
+1	0	1/2	1/2
0	+1	1/2	−1/2

m_1	m_2	J=2, M=0	J=1, M=0	J=0, M=0
+1	−1	1/6	1/2	1/3
0	0	2/3	0	−1/3
−1	+1	1/6	−1/2	1/3

m_1	m_2	J=2, M=−1	J=1, M=−1
0	−1	1/2	1/2
−1	0	1/2	−1/2

m_1	m_2	J=2, M=−2
−1	−1	1

3/2 × 1

m_1	m_2	J=5/2, M=+5/2
+3/2	+1	1

m_1	m_2	J=5/2, M=+3/2	J=3/2, M=+3/2
+3/2	0	2/5	3/5
+1/2	+1	3/5	−2/5

m_1	m_2	J=5/2, M=+1/2	J=3/2, M=+1/2	J=1/2, M=+1/2
+3/2	−1	1/10	2/5	1/2
+1/2	0	3/5	1/15	−1/3
−1/2	+1	3/10	−8/15	1/6

m_1	m_2	J=5/2, M=−1/2	J=3/2, M=−1/2	J=1/2, M=−1/2
+1/2	−1	3/10	8/15	1/6
−1/2	0	3/5	−1/15	−1/3
−3/2	+1	1/10	−2/5	1/2

m_1	m_2	J=5/2, M=−3/2	J=3/2, M=−3/2
−1/2	−1	3/5	2/5
−3/2	0	2/5	−3/5

m_1	m_2	J=5/2, M=−5/2
−3/2	−1	1

2 × 1/2

m_1	m_2	J=5/2, M=+5/2
+2	+1/2	1

m_1	m_2	J=5/2, M=+3/2	J=3/2, M=+3/2
+2	−1/2	1/5	4/5
+1	+1/2	4/5	−1/5

m_1	m_2	J=5/2, M=+1/2	J=3/2, M=+1/2
+1	−1/2	2/5	3/5
0	+1/2	3/5	−2/5

m_1	m_2	J=5/2, M=−1/2	J=3/2, M=−1/2
0	−1/2	3/5	2/5
−1	+1/2	2/5	−3/5

m_1	m_2	J=5/2, M=−3/2	J=3/2, M=−3/2
−1	−1/2	4/5	1/5
−2	+1/2	1/5	−4/5

m_1	m_2	J=5/2, M=−5/2
−2	−1/2	1

3/2 × 1/2

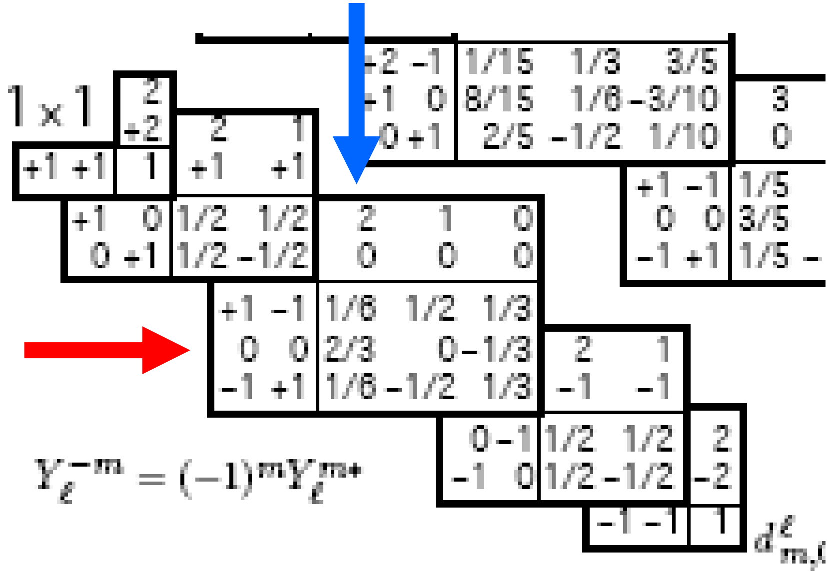
m_1	m_2	J=2, M=+2
+3/2	+1/2	1

m_1	m_2	J=2, M=+1	J=1, M=+1
+3/2	−1/2	1/4	3/4
+1/2	+1/2	3/4	−1/4

m_1	m_2	J=2, M=0	J=1, M=0
+1/2	−1/2	1/2	1/2
−1/2	+1/2	1/2	−1/2

m_1	m_2	J=2, M=−1	J=1, M=−1
−1/2	−1/2	3/4	1/4
−3/2	+1/2	1/4	−3/4

m_1	m_2	J=2, M=−2
−3/2	−1/2	1



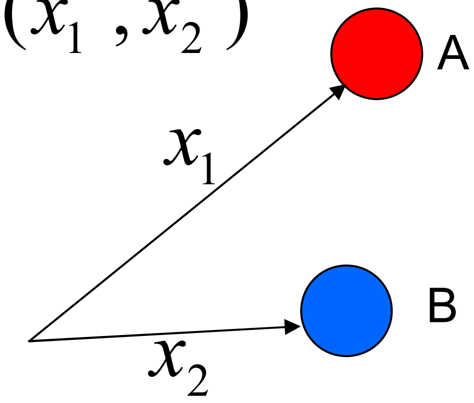
$$|2,0\rangle = \sqrt{\frac{1}{6}}|1,+1\rangle|1,-1\rangle + \sqrt{\frac{2}{3}}|1,0\rangle|1,0\rangle + \sqrt{\frac{1}{6}}|1,-1\rangle|1,+1\rangle$$

$$|1,0\rangle|1,0\rangle = \sqrt{\frac{2}{3}}|2,0\rangle + 0|1,0\rangle - \sqrt{\frac{1}{3}}|0,0\rangle$$

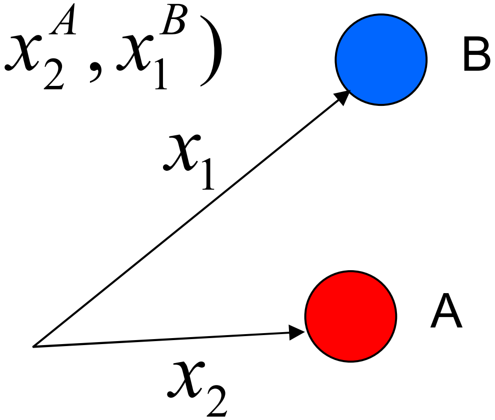
Fermiony a bosony

2 rozlišitelné částice:

$$\Psi(x_1^A, x_2^B)$$



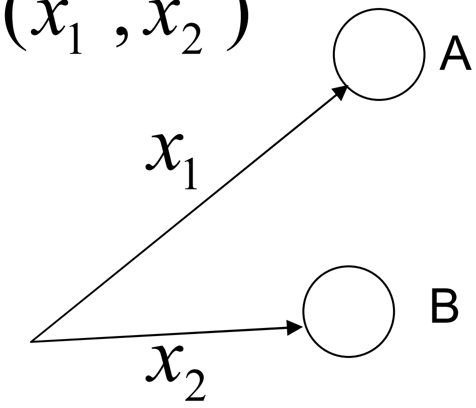
$$\Psi(x_2^A, x_1^B)$$



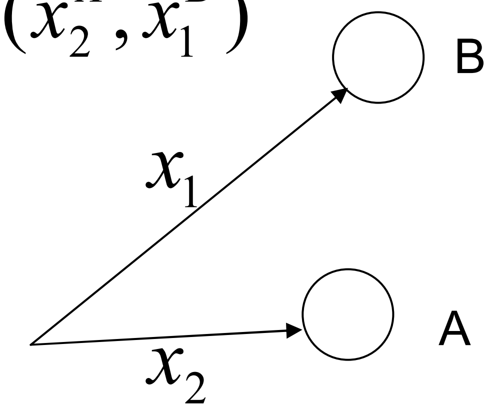
$$\Psi(x_1^A, x_2^B) \neq \Psi(x_2^A, x_1^B)$$

2 nerozlišitelné částice:

$$\Phi(x_1^A, x_2^B)$$



$$\Phi(x_2^A, x_1^B)$$



$$\left|\Phi(x_1, x_2)\right|^2 = \left|\Phi(x_2, x_1)\right|^2$$

$$\Rightarrow \Phi(x_2, x_1) = e^{i\delta} \cdot \Phi(x_1, x_2)$$

Fermiony a bosony

$$\Phi(x_1,x_2)\xrightarrow[1\leftrightarrow 2]{}\Phi(x_2,x_1)$$

$$|\Phi(x_1,x_2)|^2=|\Phi(x_2,x_1)|^2\Rightarrow\Phi(x_2,x_1)=e^{i\delta}\Phi(x_1,x_2)$$

$$\Phi(x_1,x_2)\xrightarrow[1\leftrightarrow 2]{}\Phi(x_2,x_1)\xrightarrow[1\leftrightarrow 2]{}\Phi(x_1,x_2)$$

$$\Phi(x_1,x_2)\xrightarrow[1\leftrightarrow 2]{}e^{i\delta}\Phi(x_1,x_2)\xrightarrow[1\leftrightarrow 2]{}e^{i\delta}e^{i\delta}\Phi(x_1,x_2)$$

$$\Phi(x_1,x_2)=\pm\Phi(x_2,x_1)$$

Máme-li vlnovou funkci pro rozlišitelné částice, např. součin dvou jednočásticových funkcí (pro neinteragující částice), můžeme z ní sestavit sym. a antisym. funkci:

$$\Psi(x_1,x_2)\ ;\Psi(x_1,x_2)=\psi_A(x_1)\psi_B(x_2)$$

$$\Phi_\pm(x_1,x_2)=\frac{1}{\sqrt{2}}\big(\Psi(x_1,x_2)\pm\Psi(x_2,x_1)\big)\Rightarrow\Phi_\pm(x_1,x_2)\xrightarrow[1\leftrightarrow 2]{}\pm\Phi_\pm(x_1,x_2)$$

$$\Phi_+(x_1,x_2)=\frac{1}{\sqrt{2}}\big(\Psi(x_1,x_2)+\Psi(x_2,x_1)\big)=B(x_1,x_2)$$ Bosony – mají celočíselný spin: 0, 1, …

$$\Phi_-(x_1,x_2)=\frac{1}{\sqrt{2}}\big(\Psi(x_1,x_2)-\Psi(x_2,x_1)\big)=F(x_1,x_2)$$ Fermiony – mají poločíselný spin: 1/ 2 , 3/2, ..

$$B(x_1,x_2,...,x_N)=\frac{1}{\sqrt{N!}}\sum_{perm1,2,..,N}\Psi(x_{i1},x_{i2},...,x_{iN})\Rightarrow N=3:$$

$$B(x_1,x_2,x_3)=\frac{1}{\sqrt{6}}\big(\Psi(x_1,x_2,x_3)+\Psi(x_1,x_3,x_2)+\Psi(x_2,x_1,x_3)+\Psi(x_2,x_3,x_1)+\Psi(x_3,x_1,x_2)+\Psi(x_3,x_2,x_1)\big)$$

$$F(x_1,x_2,...,x_N)=\frac{1}{\sqrt{N!}}\sum_{perm1,2,..,N}(-1)^p\Psi(x_{i1},x_{i2},...,x_{iN})\Rightarrow N=3:$$

$$F(x_1,x_2,x_3)=\frac{1}{\sqrt{6}}\big(\Psi(x_1,x_2,x_3)-\Psi(x_1,x_3,x_2)-\Psi(x_2,x_1,x_3)+\Psi(x_2,x_3,x_1)+\Psi(x_3,x_1,x_2)-\Psi(x_3,x_2,x_1)\big)$$

Dvě (neinteragující) částice v jednorozměrné nekonečně hluboké potenciálové jámě

$$-\frac{(\hbar c)^2}{2M}\psi''(x) = E\psi(x); \quad \psi(-L/2) = \psi(L/2) = 0$$

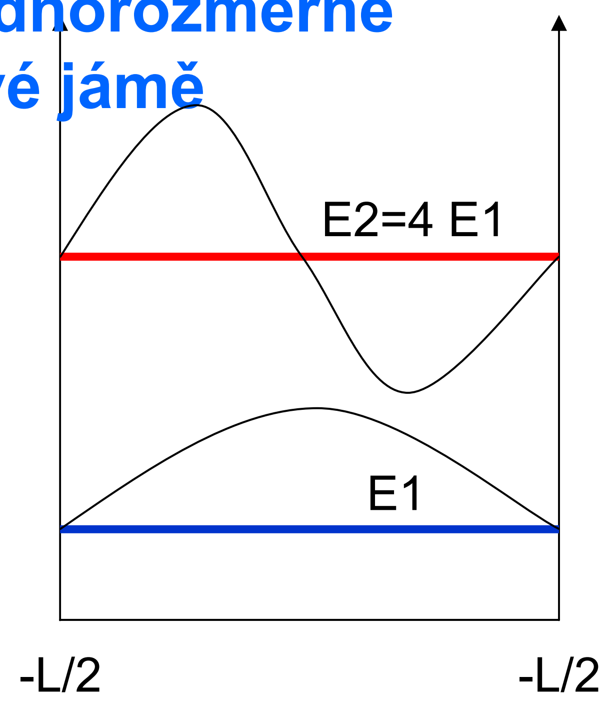
$$\psi''(x) = -\frac{2ME}{(\hbar c)^2}\psi(x)$$

$$\psi_1(x) = Cos\left(\frac{\sqrt{2ME_1}}{\hbar c}x\right)\ ;\ \psi_1(L/2) = Cos\left(\frac{\sqrt{2ME_1}}{\hbar c}\frac{L}{2}\right) = 0 \Rightarrow \frac{\sqrt{2ME_1}}{\hbar c}\frac{L}{2} = \frac{\pi}{2}$$

$$\Rightarrow E_1 = \frac{(\pi\hbar c/L)^2}{2M}\ ;\ \frac{\sqrt{2ME_1}}{\hbar c} = \frac{\pi}{L} \Rightarrow \psi_1(x) = Cos\left(\frac{\sqrt{2ME_1}}{\hbar c}x\right) = Cos\left(\frac{\pi}{L}x\right)$$

$$\psi_2(x) = Sin\left(\frac{\sqrt{2ME_2}}{\hbar c}x\right)\ ;\ \psi_2(L/2) = Sin\left(\frac{\sqrt{2ME_2}}{\hbar c}\frac{L}{2}\right) = 0 \Rightarrow \frac{\sqrt{2ME_2}}{\hbar c}\frac{L}{2} = \pi$$

$$\Rightarrow E_2 = 4\frac{(\pi\hbar c/L)^2}{2M}\ ;\ \frac{\sqrt{2ME_2}}{\hbar c} = \frac{2\pi}{L} \Rightarrow \psi_2(x) = Sin\left(\frac{\sqrt{2ME_2}}{\hbar c}x\right) = Sin\left(\frac{2\pi}{L}x\right)$$



Dvě rozlišitelné částice:

$$\Psi(x_1,x_2) = \psi_1(x_1)\psi_2(x_2) = Cos\left(\frac{\pi}{L}x_1\right)Sin\left(\frac{2\pi}{L}x_2\right)$$

Dva nerozlišitelné fermiony:

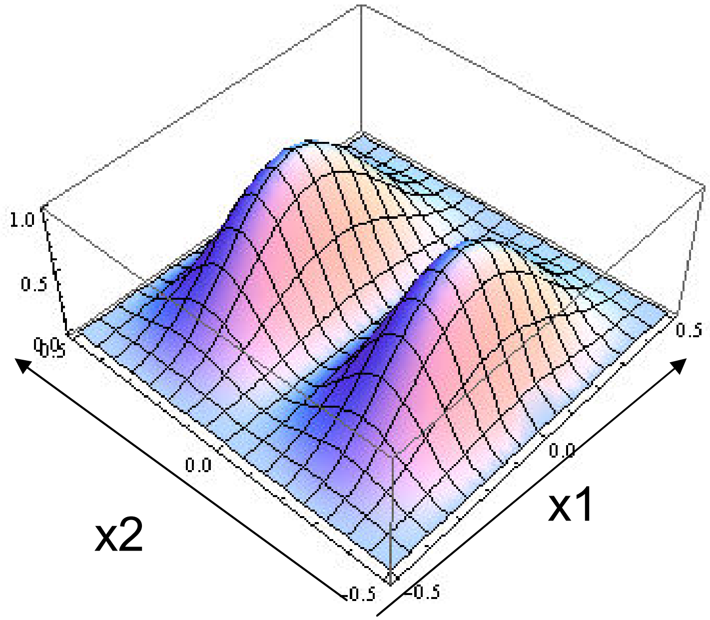
$$F(x_1,x_2) = \frac{1}{\sqrt{2}}\left(\Psi(x_1,x_2) - \Psi(x_2,x_1)\right) = \frac{1}{\sqrt{2}}\left(\psi_1(x_1)\psi_2(x_2) - \psi_1(x_2)\psi_2(x_1)\right) = \frac{1}{\sqrt{2}}\left(Cos\left(\frac{\pi}{L}x_1\right)Sin\left(\frac{2\pi}{L}x_2\right) - Cos\left(\frac{\pi}{L}x_2\right)Sin\left(\frac{2\pi}{L}x_1\right)\right)$$

Dva nerozlišitelné bosony:

$$B(x_1,x_2) = \frac{1}{\sqrt{2}}\left(\Psi(x_1,x_2) + \Psi(x_2,x_1)\right) = \frac{1}{\sqrt{2}}\left(\psi_1(x_1)\psi_2(x_2) + \psi_1(x_2)\psi_2(x_1)\right) = \frac{1}{\sqrt{2}}\left(Cos\left(\frac{\pi}{L}x_1\right)Sin\left(\frac{2\pi}{L}x_2\right) + Cos\left(\frac{\pi}{L}x_2\right)Sin\left(\frac{2\pi}{L}x_1\right)\right)$$

$$|\Psi(x_1,x_2)|^2 = \left(Cos\left(\frac{\pi}{L}x_1\right)Sin\left(\frac{2\pi}{L}x_2\right)\right)^2$$

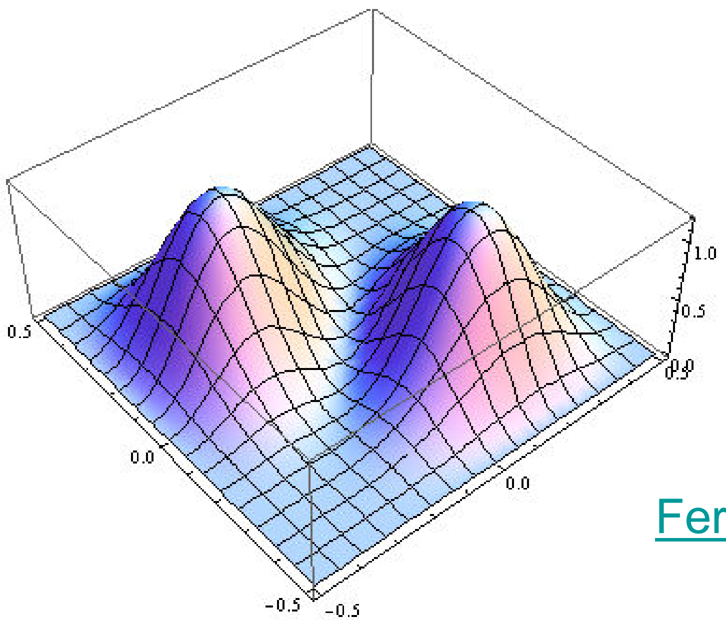
Dvě rozlišitelné částice $|\Psi(x_1,x_2)|^2$



$$B(x_1,x_2) = \left(\frac{1}{\sqrt{2}}\left(Cos\left(\frac{\pi}{L}x_1\right)Sin\left(\frac{2\pi}{L}x_2\right) + Cos\left(\frac{\pi}{L}x_2\right)Sin\left(\frac{2\pi}{L}x_1\right)\right)\right)^2$$

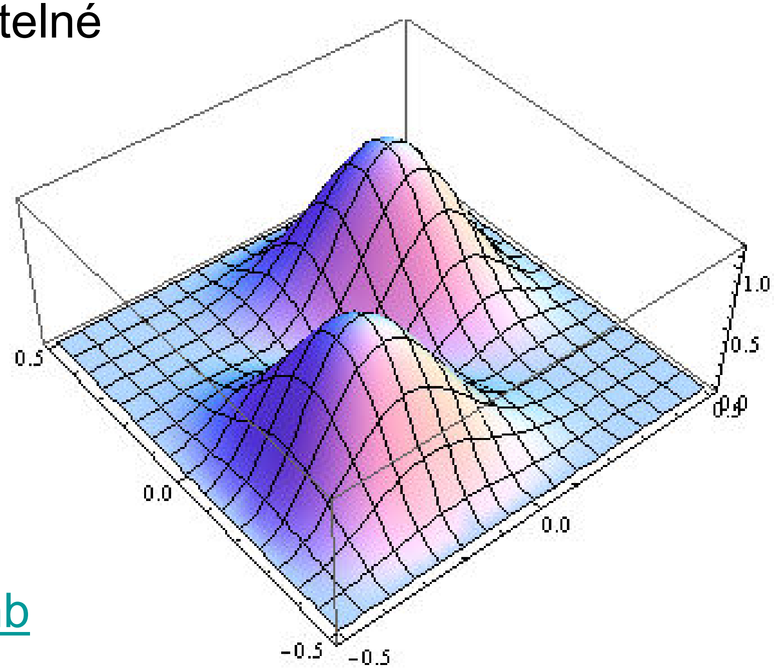
$$F(x_1,x_2) = \left(\frac{1}{\sqrt{2}}\left(Cos\left(\frac{\pi}{L}x_1\right)Sin\left(\frac{2\pi}{L}x_2\right) - Cos\left(\frac{\pi}{L}x_2\right)Sin\left(\frac{2\pi}{L}x_1\right)\right)\right)^2$$

Dva nerozlišitelné fermiony $|F(x_1,x_2)|^2$



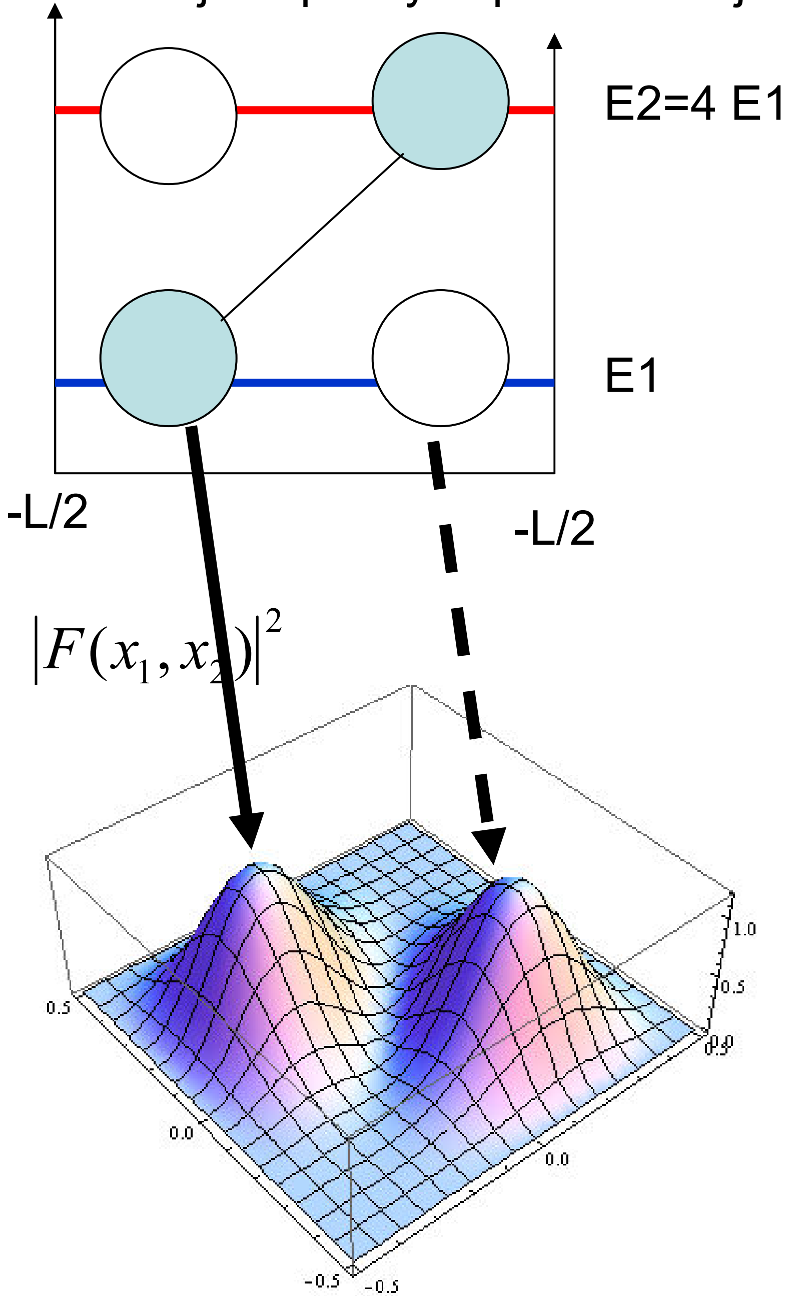
Dva nerozlišitelné bosony

$|B(x_1,x_2)|^2$

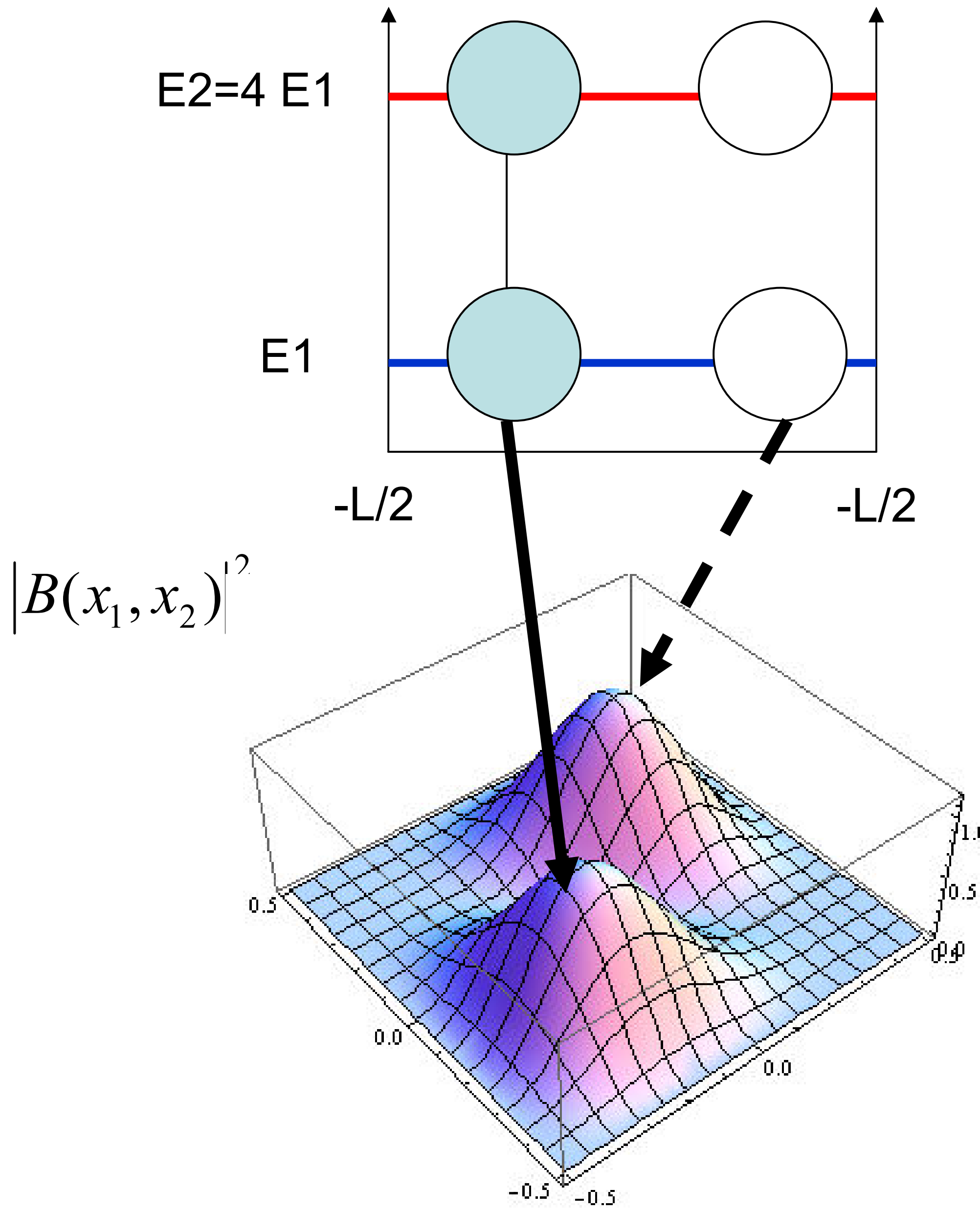


Fermions-Bosons.nb

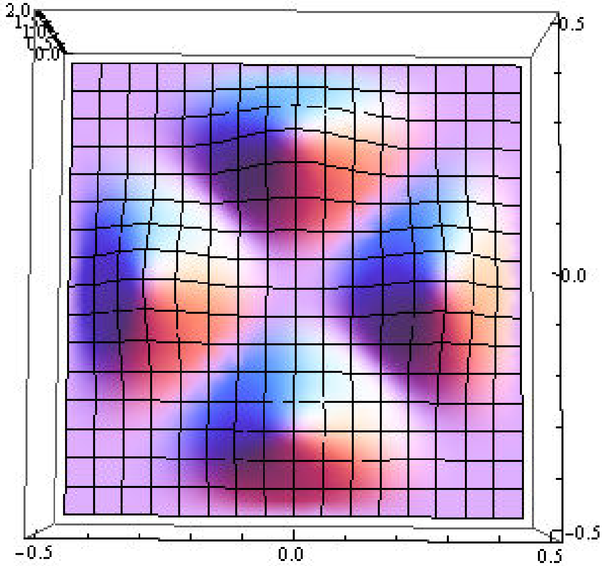
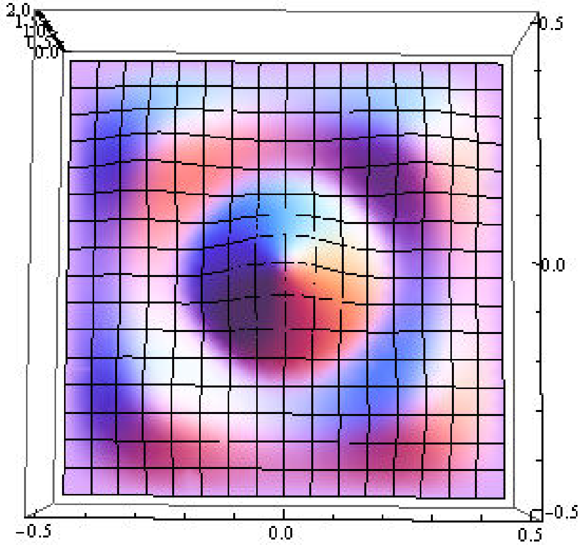
Dva nerozlišitelné Fermiony se převážně nacházejí v opačných polovinách jámy



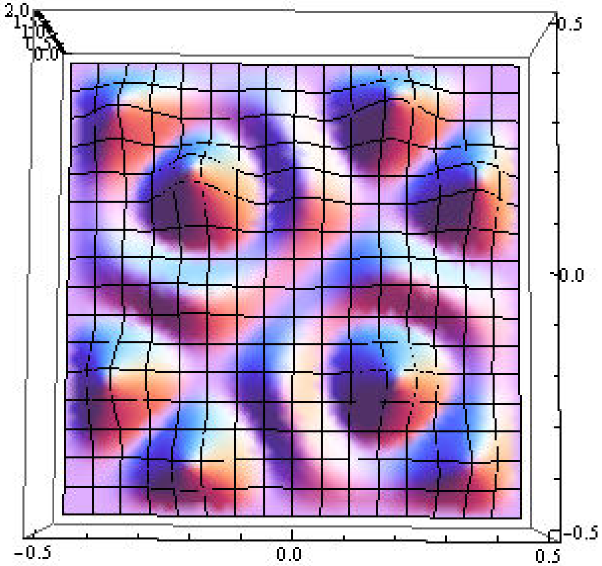
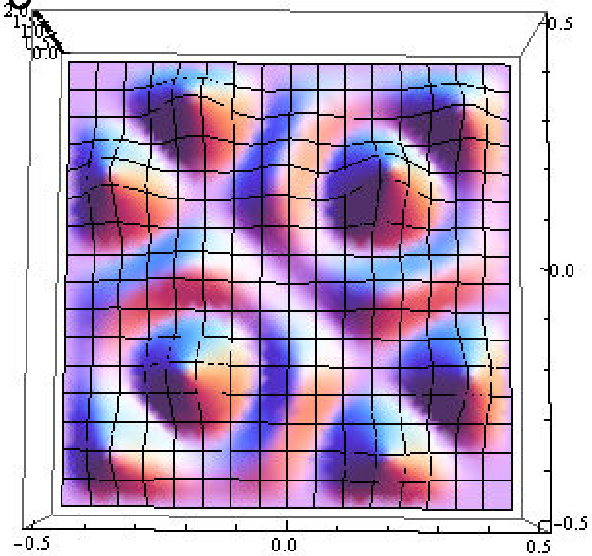
Dva nerozlišitelné Bosony se převážně nacházejí ve stejných polovinách jámy



Totéž pro hladiny 1 a 3

Totéž pro hladiny 2 a 5

Zkuste sami poznat dvojici fermionů a bosonů

Obě částice na stejné hladině:

$$\Psi(x_1,x_2) = Cos\left(\frac{\pi}{L}x_1\right)Cos\left(\frac{\pi}{L}x_2\right)$$

Rozlišitelné

Nerozlišitelné bosony:

$$B(x_1,x_2) = \frac{1}{\sqrt{2}}\left(Cos\left(\frac{\pi}{L}x_1\right)Cos\left(\frac{\pi}{L}x_2\right) + Cos\left(\frac{\pi}{L}x_2\right)Cos\left(\frac{\pi}{L}x_1\right)\right) = \sqrt{2}\Psi(x_1,x_2)$$

Nerozlišitelné fermiony:

$$F(x_1,x_2) = \frac{1}{\sqrt{2}}\left(Cos\left(\frac{\pi}{L}x_1\right)\cdot Cos\left(\frac{\pi}{L}x_2\right) - Cos\left(\frac{\pi}{L}x_2\right)\cdot Cos\left(\frac{\pi}{L}x_1\right)\right) = 0$$

Pauliho vylučovací princip

Fermiony ale mají spin:

$$F(x_1,x_2) = \frac{1}{\sqrt{2}}\left(Cos\left(\frac{\pi}{L}x_1\right)|\uparrow\rangle\cdot Cos\left(\frac{\pi}{L}x_2\right)|\downarrow\rangle - Cos\left(\frac{\pi}{L}x_2\right)|\downarrow\rangle\cdot Cos\left(\frac{\pi}{L}x_1\right)|\uparrow\rangle\right) =$$

$$Cos\left(\frac{\pi}{L}x_1\right)Cos\left(\frac{\pi}{L}x_2\right)\frac{1}{\sqrt{2}}\left(|\uparrow\rangle|\downarrow\rangle - |\downarrow\rangle|\uparrow\rangle\right) \quad 1/2\otimes 1/2 = 1\oplus 0 \quad |0,0\rangle = \frac{1}{\sqrt{2}}\left(|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle\right)$$

Mohou být dva elektrony s celkovým spinem = 0, prostorová část vlnové funkce je symetrická, spinová část antisymetrická, celková funkce pak antisymetrická

Klasifikace elementárních částic:

Fermiony $S = 1/2$		Elektomagneticka/Slaba	Silna
leptony	$e^-, \nu_e, \mu^-, \nu_\mu, \tau^-, \nu_\tau$	ANO	NE
	$e^+, \bar{\nu}_e, \mu^+, \bar{\nu}_\mu, \tau^+, \bar{\nu}_\tau$		
kvarky	u, d, s, c, b, t	ANO	ANO
	$\bar{u}, \bar{d}, \bar{s}, \bar{c}, \bar{b}, \bar{t}$	ANO	ANO

Leptony je odvozeno od řeckého leptos: *The name "lepton" (from Greek leptos) was first used by physicist Léon Rosenfeld in 1948: Following a suggestion of Prof. C. Møller, I adopt — as a pendant to "nucleon" — the denomination "lepton" (from λεπτός, small, thin, delicate) to denote a particle of small mass.*

Kvarky vymyslel M. Gell-Man, název pochází z knihy Jamese Joyce: *Finnegans Wake.*
The word quark was originally coined by Murray Gell-Mann as a nonsense word rhyming with "pork". Later, he found the same word in James Joyce's book Finnegans Wake, where seabirds give "three quarks", akin to three cheers (probably onomatopoeically imitating a seabird call, like "quack" for ducks, as well as making a pun on the relationship between Munster and its provincial capital, Cork) in the passage "Three quarks for Muster Mark!/Sure he has not got much of a bark/And sure any he has it's all beside the mark." Further explanation for the use of the word "quark" may be derived from the fact that, at the time, there were only three known quarks in existence.

Klasifikace elementárních částic:

Bosony $S = 1$		Elektomagneticka/Slaba	Silna
gama	γ	ANO	NE
intermedialni bosony	W^+, W^-, Z^0	ANO	NE
gluony	g	NE	ANO

Gama zprostředkovává elektromagnetickou interakci,

W a Z slabou interakci,

gluony pak silnou interakci (gluon = glue+’-on’).

Klasifikace elementárních částic:

Kvarky se nevyskytují volně, tvoří vázané stavy silně interagujících částic, tzv. **HADRONY** (hadros=silný)

Mezony = $q\,\bar{q}$ mají celočíselný spin a jsou tedy bosony (mezony je historický název = částice s hmotou mezi elektronem a protonem)

Baryony = $q\,q\,q$, **Antibaryony** = $\bar{q}\,\bar{q}\,\bar{q}$ mají poločíselný spin, tj. fermiony (název baryony od baryos=těžký). Podivným baryonům se také říká **hyperony**

Sakatův model

Předchůdce kvarkového modelu. Základními konstituenty byly tzv. Sakatony:

p – odpovídal protonu, tj. náboj $Q = +1$, podivnost $S=0$, baryonové číslo $B=1$
n – odpovídal neutronu, $Q= 0$, $S=0$, $B=1$
λ – odpovídal hyperonu Λ, $Q = 0$, $S=-1$, $B=1$

Mesony = vázaný stav Sakatonu a antisakatonu,
např. π^+ = p, anti-n
K^+ = p, anti- λ, atd.

Baryony = vázaný stav dvou Sakatonů a jednoho anti-Sakatonu:
$\Sigma^+ = \lambda\ p$ anti-n
$\Xi^- = \lambda\ \lambda$ anti-p

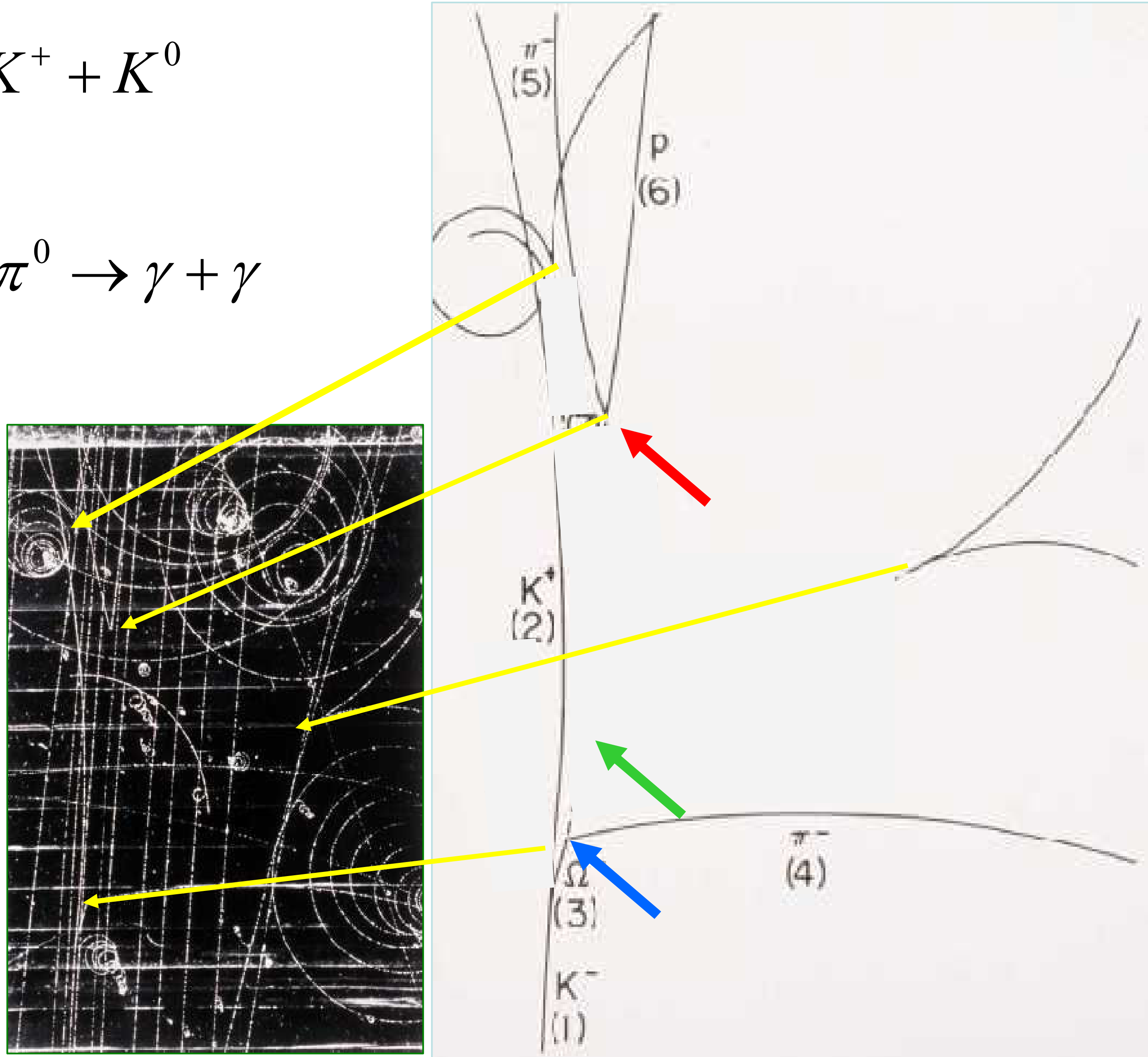
Objev Ω baryonu a konec Sakatova modelu

$$K^- + p \rightarrow \Omega^- + K^+ + K^0$$

$$\Omega^- \rightarrow \Xi^0 + \pi^-$$

$$\Xi^0 \rightarrow \Lambda^0 + \pi^0 \; ; \; \pi^0 \rightarrow \gamma + \gamma$$

$$\Lambda^0 \rightarrow p + \pi^-$$



Podivnost Omega Hyperonu je -3, ale

v Sakatově modelu může být podivost max. -2:
(λ , λ, anti-n/p)

Izotopický spin (Izospin) a hypernáboj

Izospin zavedl Werner Heisenberg pro objasnění několika jevů:
-Blízké hodnoty hmoty neutronu a protonu
-Nábojová nezávislost jaderných sil – jsou stejné mezi neutronem a protonem, pp i nn
-Blízké hodnoty hmoty nabitých a neutrálního pionu

$$N = \begin{pmatrix} p \\ n \end{pmatrix}$$ Izotopický dublet

$$T = 1/2$$
$$T_3 = +1/2 \Leftrightarrow p$$
$$T_3 = -1/2 \Leftrightarrow n$$

$$Q = T_3 + \frac{B}{2}$$
$$N \Rightarrow B = 1$$
$$\pi \Rightarrow B = 0$$

B - baryonové číslo

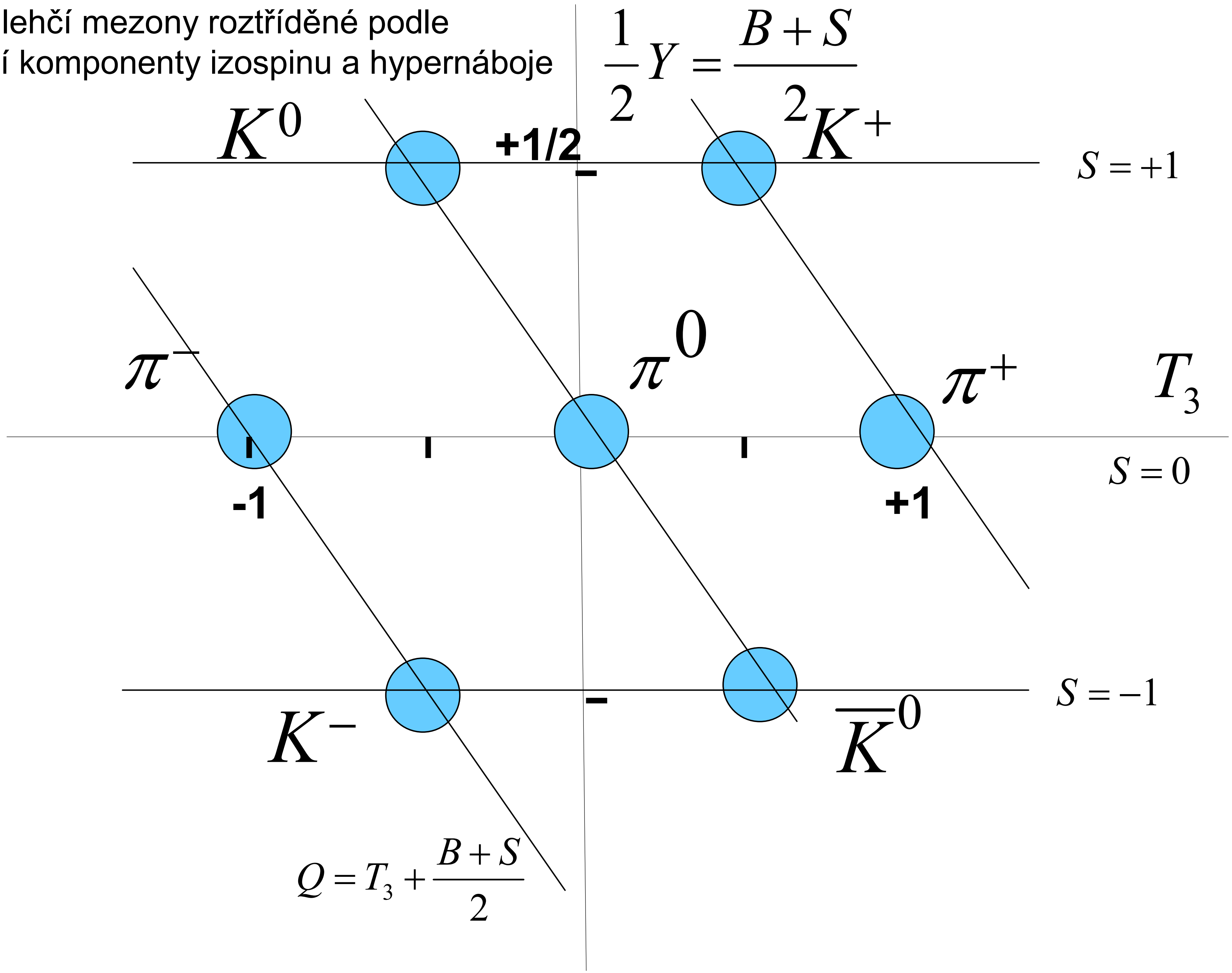
$$\pi = \begin{pmatrix} \pi^+ \\ \pi^0 \\ \pi^- \end{pmatrix}$$ Izotopický triplet

$$T_3 = +1 \Leftrightarrow \pi^+$$
$$T_3 = 0 \Leftrightarrow \pi^0$$
$$T_3 = -1 \Leftrightarrow \pi^-$$

$$Q = T_3 + \frac{B+S}{2}$$
$$Y = B + S$$

Zobecnění platné i pro podivné mezony a baryony
Y - hypernáboj

Nejlehčí mezony roztříděné podle
třetí komponenty izospinu a hypernáboje



Gell-Mann a Zweig a kvarky

The Nobel Prize in Physics 1969

"for his contributions and discoveries concerning the classification of elementary particles and their interactions"

Murray Gell-Mann

USA

California Institute of Technology (Caltech) Pasadena, CA, USA

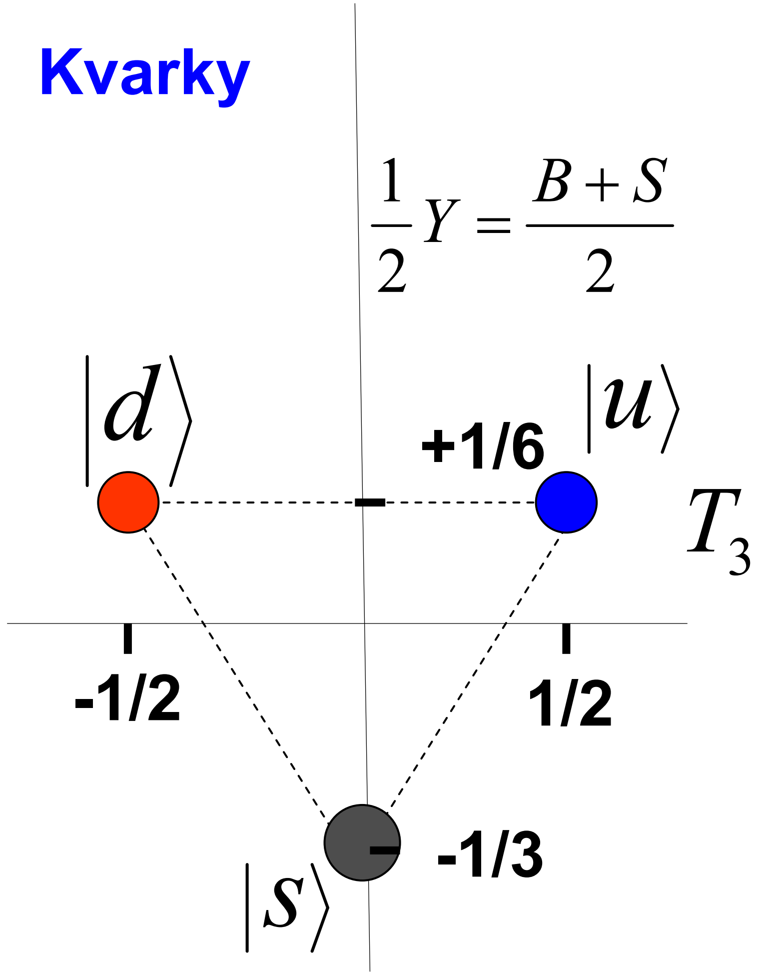
b. 1929

Bylo zřejmé, že existuje baryon s podivností S=-3 (Omega hyperon)

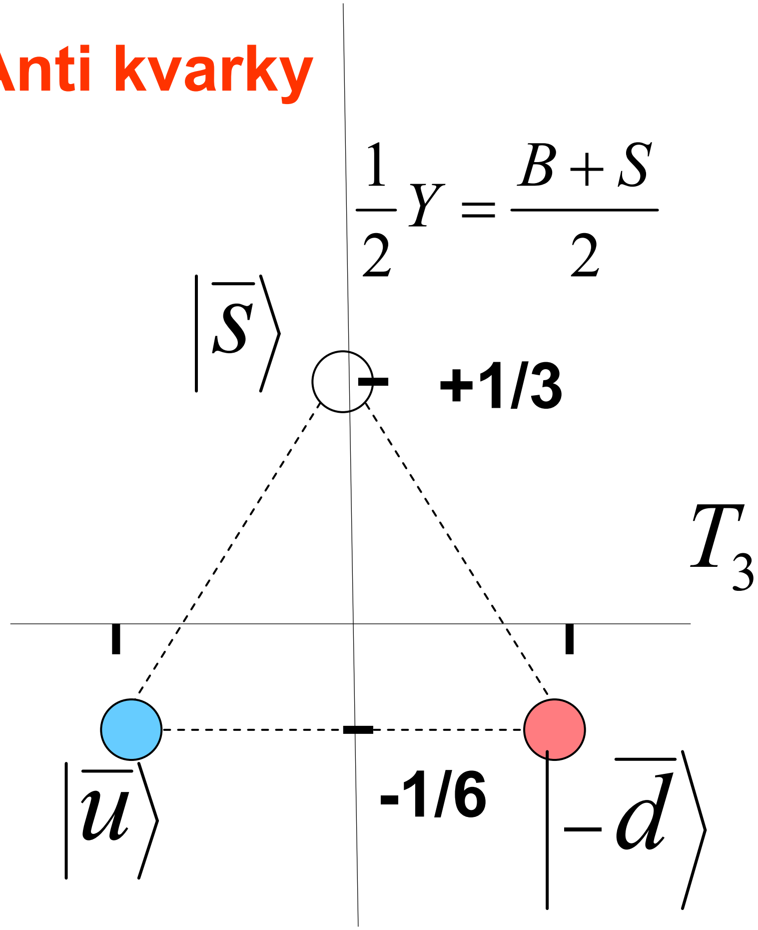
Gell-Mann a Zweig navrhli, že baryony jsou složeny ze tří kvarků (3 kvarky s podivností -1 = Omega)

Kvarky ale musejí mít B=1/3 a také třetinové náboje

Kvarky



Anti kvarky

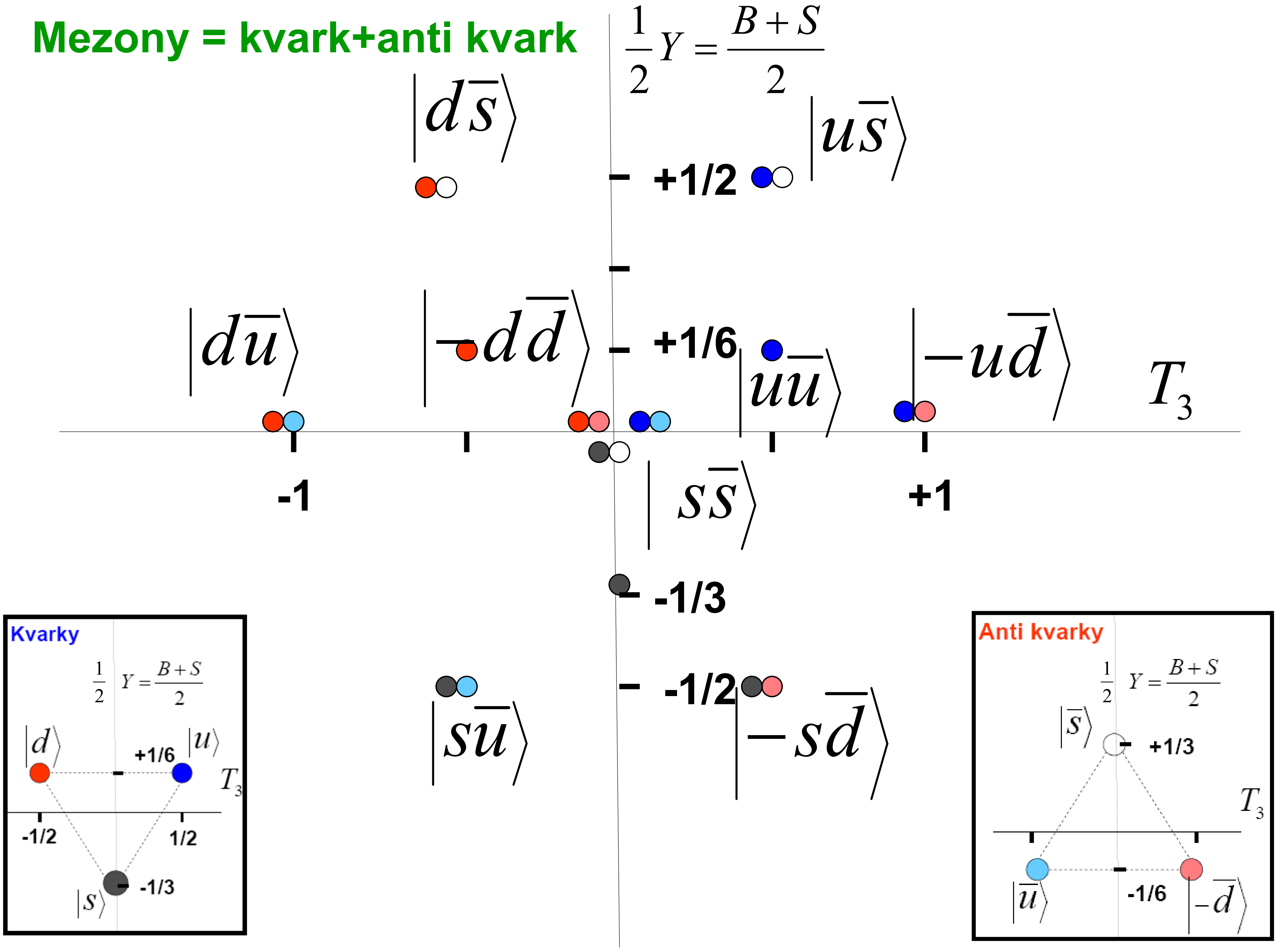


$$u: T_3 = +1/2; S = 0; B = 1/3 \Rightarrow Q = +1/2 + \frac{1/3+0}{2} = +\frac{2}{3}$$

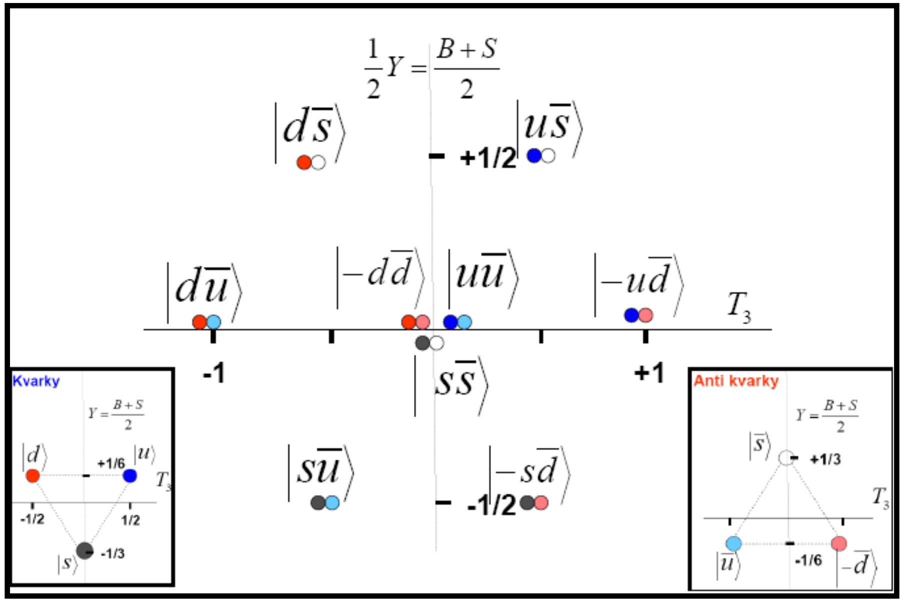
$$d: T_3 = -1/2; S = 0; B = 1/3 \Rightarrow Q = -1/2 + \frac{1/3+0}{2} = -\frac{1}{3}$$

$$s: T_3 = 0; S = -1; B = 1/3 \Rightarrow Q = 0 + \frac{1/3-1}{2} = -\frac{1}{3}$$

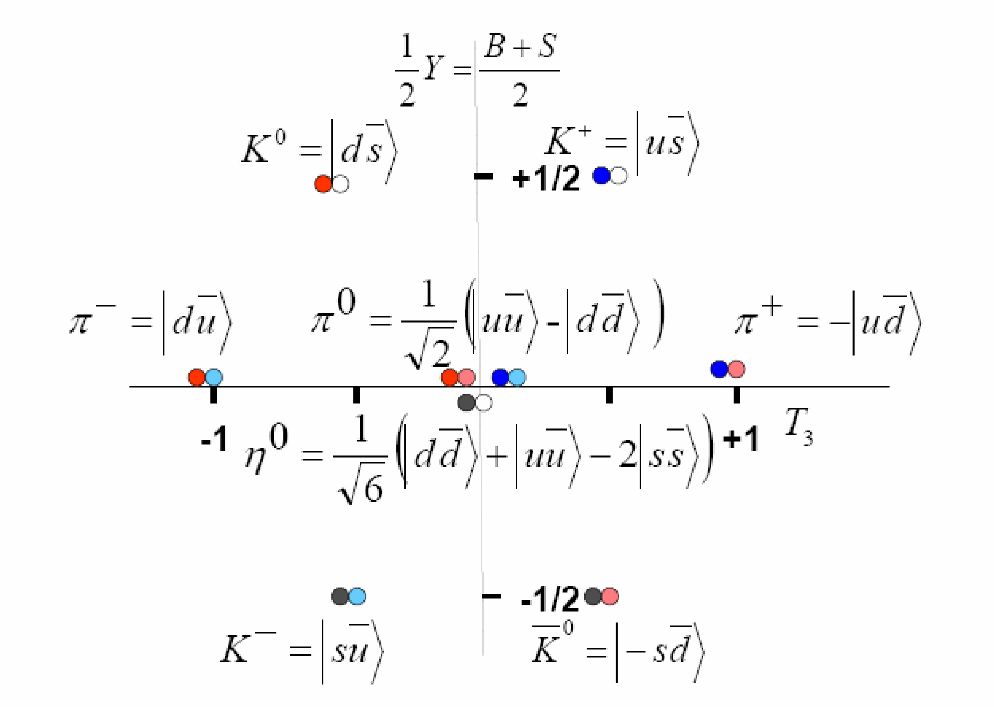
Mezony = kvark+anti kvark



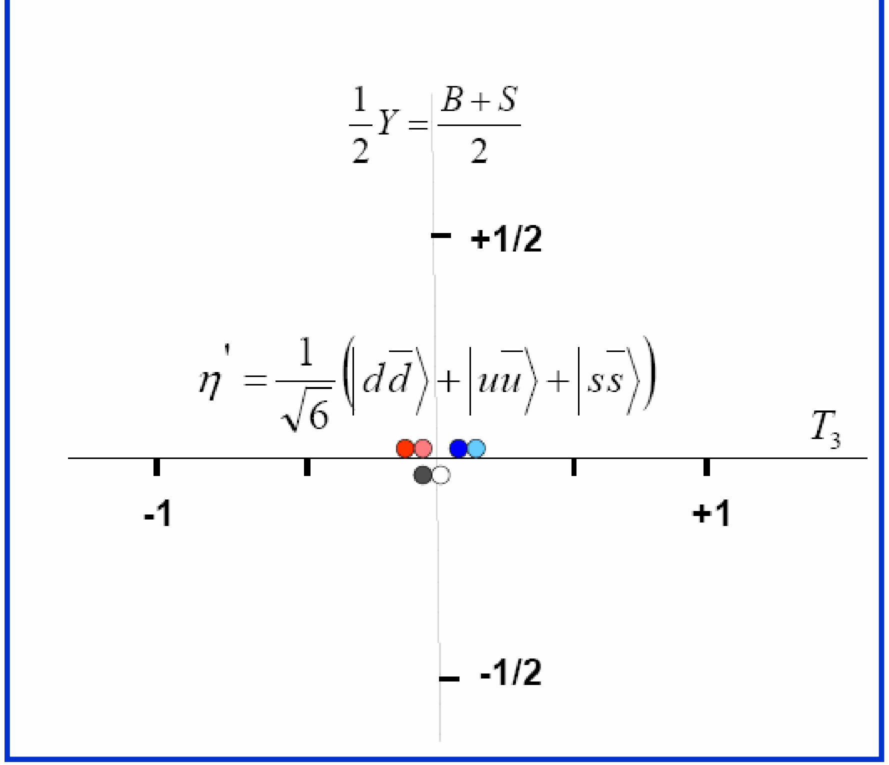
$$3 \otimes \bar{3} = 8 \oplus 1$$



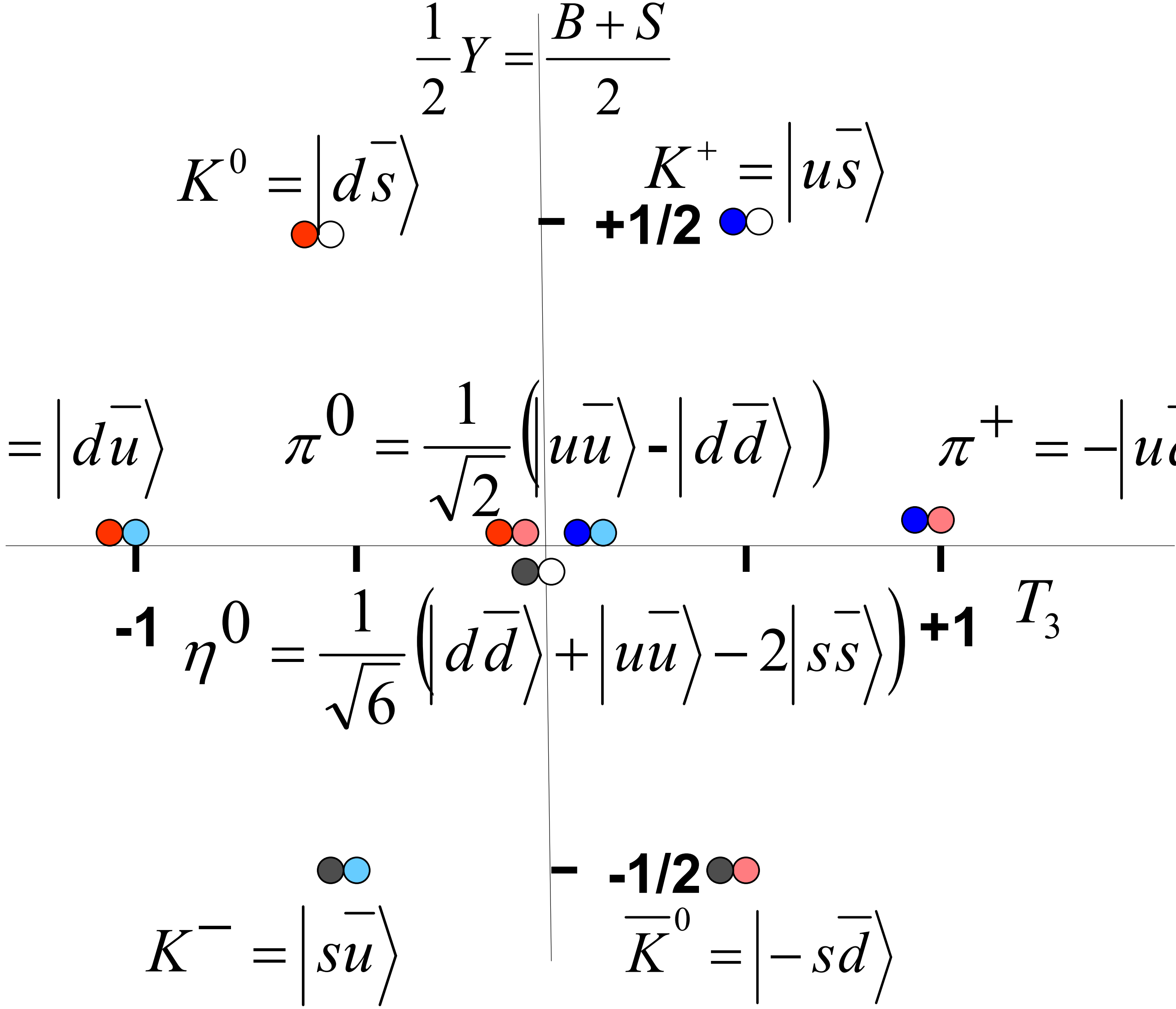
Oktet pseudoskalárních mezonů



Singlet pseudoskalárních mezonů



Oktet pseudoskalárních mezonů



$$\frac{1}{2}Y = \frac{B+S}{2}$$

$$K^0 = |d\bar{s}\rangle \qquad K^+ = |u\bar{s}\rangle$$

$$\pi^- = |d\bar{u}\rangle \qquad \pi^0 = \frac{1}{\sqrt{2}}\left(|u\bar{u}\rangle - |d\bar{d}\rangle\right) \qquad \pi^+ = -|u\bar{d}\rangle$$

$$\eta^0 = \frac{1}{\sqrt{6}}\left(|d\bar{d}\rangle + |u\bar{u}\rangle - 2|s\bar{s}\rangle\right)$$

$$K^- = |s\bar{u}\rangle \qquad \bar{K}^0 = |-s\bar{d}\rangle$$

Singlet pseudoskalárních mezonů

$$\frac{1}{2}Y = \frac{B+S}{2}$$

$$\eta' = \frac{1}{\sqrt{6}}\left(\left|d\bar{d}\right\rangle + \left|u\bar{u}\right\rangle + \left|s\bar{s}\right\rangle\right)$$