

Oznámení: Proseminář jaderné a subjaderné fyziky,
každé Úterý od 9:00 Troja, 9.patro

Přednáška 4.

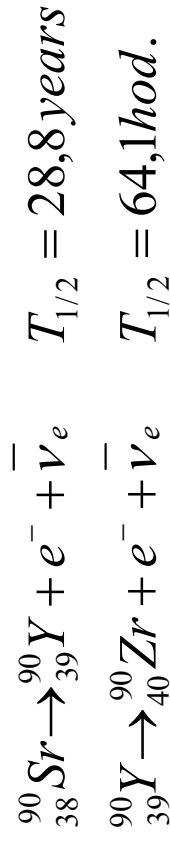
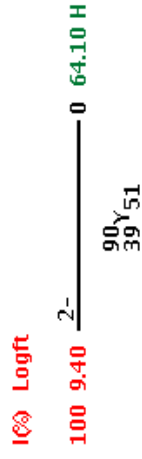
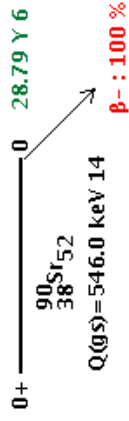
Přirozená radioaktivita, využití pro datování

Kinematika, Mandelstamovy proměnné,

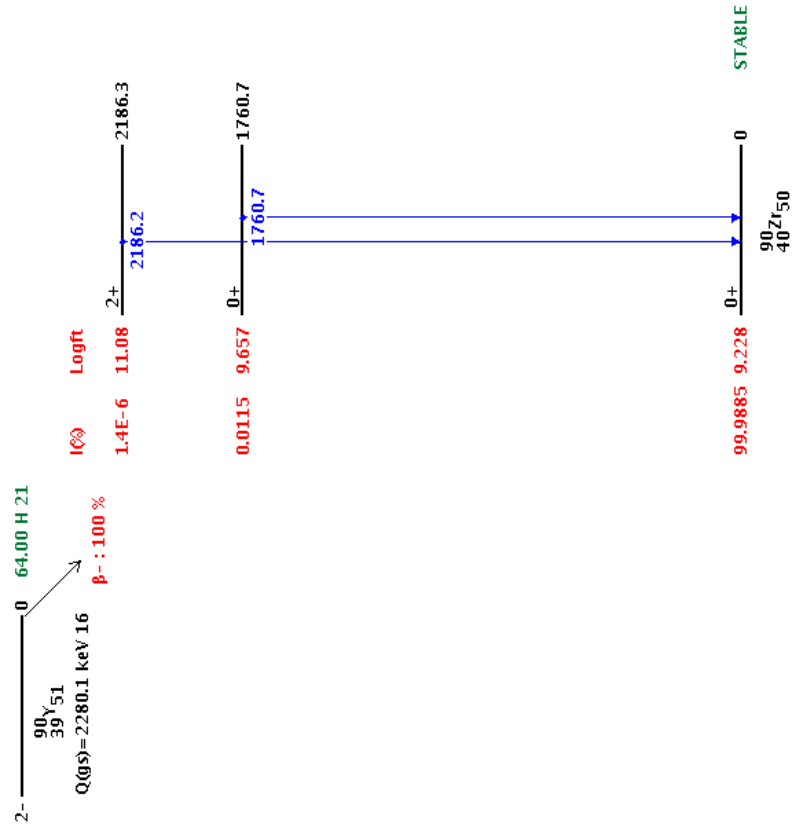
Gama přechody, použití

Interakce jaderného záření s prostředím

Účinky radiace



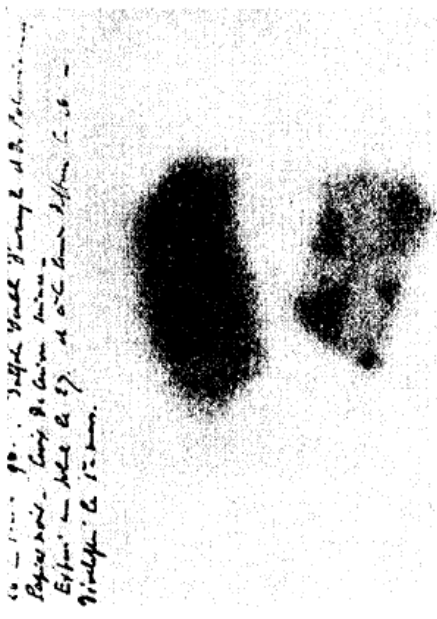
Příklad beta zářiče



Objev přirozené radioaktivity:

Antoine Henri Becquerel – objevil, že uranové soli produkují pronikavé záření

Marie a Pierre Curie zkoumali toto záření a Marie vyřešila záhadu chybějících prvků mezi Bi a U, jako první izolovala Radium a Polonium ve smolinci (pitchblende) z Jáchymova



The Nobel Prize in Physics 1903

"in recognition of the extraordinary services he has rendered by his discovery of spontaneous radioactivity"



Antoine Henri Becquerel

🏆 1/2 of the prize

France

École Polytechnique
Paris, France

b. 1852
d. 1908



Pierre Curie

🏆 1/4 of the prize

France

École municipale de physique et de chimie industrielles (Municipal School of Industrial Physics and Chemistry)
Paris, France

b. 1859
d. 1906



Marie Curie, née Skłodowska

🏆 1/4 of the prize

France

b. 1867
(in Warsaw, Poland, then Russian Empire)
d. 1934



The Nobel Prize in Chemistry 1911

"in recognition of her services to the advancement of chemistry by the discovery of the elements radium and polonium, by the isolation of radium and the study of the nature and compounds of this remarkable element"



Marie Curie, née Skłodowska

France

Sorbonne University
Paris, France

b. 1867
(in Warsaw, Poland, then Russian Empire)
d. 1934

Pierre Curie: "It can even be thought that radium could become very dangerous in criminal hands, and here the question can be raised whether mankind benefits from knowing the secrets of Nature, whether it is ready to profit from it or whether this knowledge will not be harmful for it. The example of the discoveries of Nobel is characteristic, as powerful explosives have enabled man to do wonderful work. They are also a terrible means of destruction in the hands of great criminals who are leading the peoples towards war. I am one of those who believe with Nobel that mankind will derive more good than harm from the new discoveries."

Rozpadové řady:

Thoriová:

$$Mod(A,4)=0:$$



$$T_{1/2} = 13,9 \cdot 10^9 \text{ years}$$

Neptuniová:

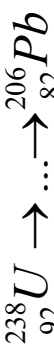
$$Mod(A,4)=1:$$



$$T_{1/2} = 0,00255 \cdot 10^9 \text{ years}$$

Uranová:

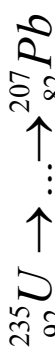
$$Mod(A,4)=2:$$



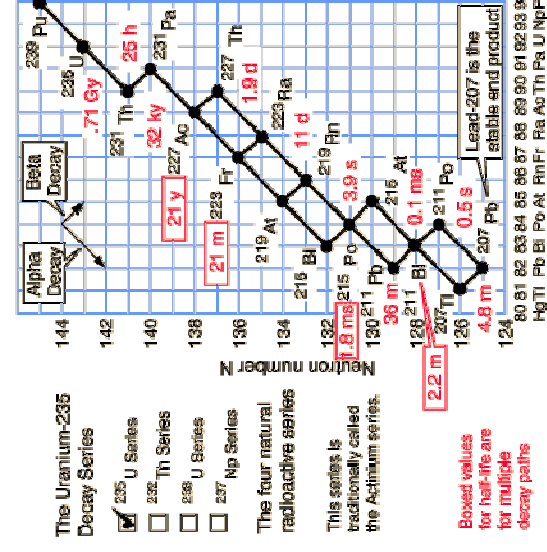
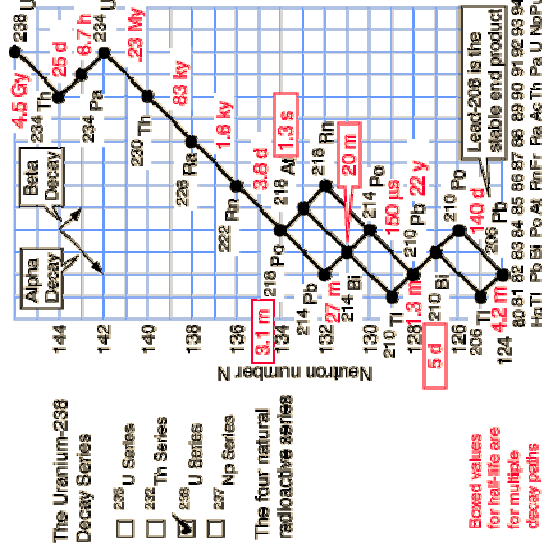
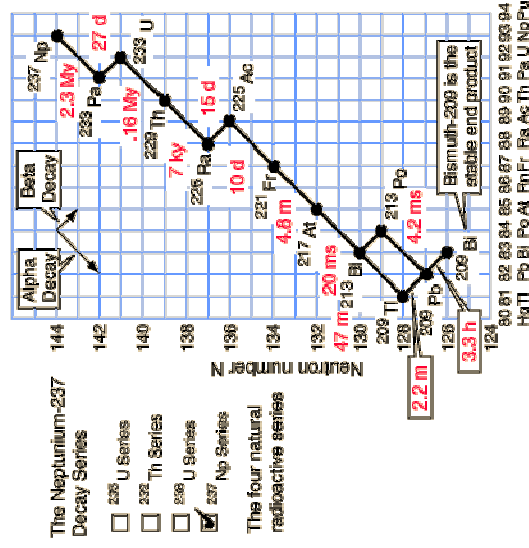
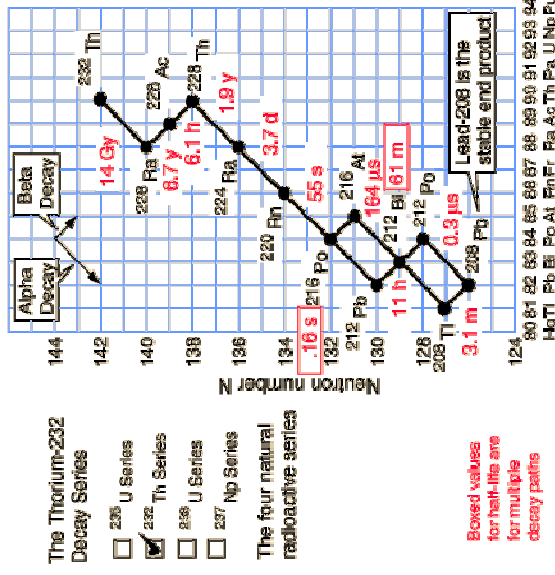
$$T_{1/2} = 4,51 \cdot 10^9 \text{ years}$$

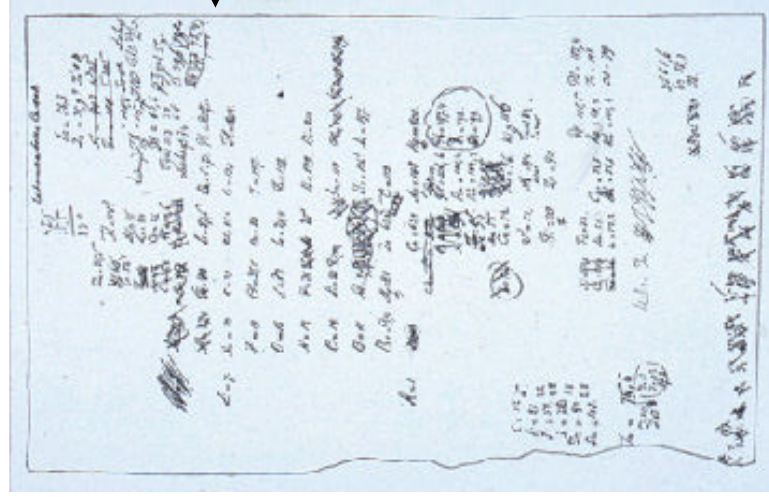
Aktiniová:

$$Mod(A,4)=3:$$



$$T_{1/2} = 0,707 \cdot 10^9 \text{ years}$$





Periodická tabulka prvků

Mendelejev 1869

1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18
1 H	2 He																
3 Li	4 Be	5 Na	6 Mg									7 B	8 C	9 N	10 O	11 F	12 Ne
11 K	12 Ca	13 Sc	14 Ti	15 V	16 Cr	17 Mn	18 Fe	19 Co	20 Ni	21 Cu	22 Zn	23 Al	24 Si	25 P	26 S	27 Cl	28 Ar
19 K	20 Ca	21 Sc	22 Ti	23 V	24 Cr	25 Mn	26 Fe	27 Co	28 Ni	29 Cu	30 Zn	31 Ga	32 Ge	33 As	34 Se	35 Br	36 Kr
37 Rb	38 Sr	39 Y	40 Zr	41 Nb	42 Mo	43 Tc	44 Ru	45 Rh	46 Pd	47 Ag	48 Cd	49 In	50 Sn	51 Sb	52 Te	53 I	54 Xe
55 Cs	56 Ba	57 La	58 Ce	59 Pr	60 Nd	61 Pm	62 Sm	63 Eu	64 Gd	65 Tb	66 Dy	67 Ho	68 Er	69 Tm	70 Yb	71 Lu	
87 Fr	88 Ra	89 Ac	90 Th	91 Pa	92 U	93 Np	94 Pu	95 Am	96 Cm	97 Bk	98 Cf	99 Es	100 Fm	101 Md	102 No	103 Lr	

68	Ce	Pr	Nd	Pm	Sm	Eu	Gd	Tb	Dy	Ho	Er	Tm	Yb	Lu
71	Th	Pa	U	Np	Pu	Am	Cm	Bk	Cf	Es	Fm	Md	No	Lr

	Ti=50	Zr=90	?=180
	V=51	Nb=94	Ta=182
	Cr=52	Mo=96	W=186
	Mn=55	Rh=104.4	Pt=197.4
	Fe=56	Ru=104.4	Ir=198
	Ni=Co=59	Pd=106.6	Os=199
H=1	Cu=63.4	Ag=108	Hg=200
	Be=9.4	Mg=24	Zn=65.2
	B=11	Al=27.4	?=68
	C=12	Si=28	?=70
	N=14	P=31	As=75
	O=16	S=32	Se=79.4
	F=19	Cl=35.5	Br=80
Li=7	Na=23	K=39	Rb=85.4
	Ca=40	Sr=87.6	Ba=137
	?=45	Ce=92	
	?Er=56	La=92	
	?Yt=60	Di=95	
	?		
	In=75.6	Th=118?	

Datování pomocí radioaktivních rozpadů

Isotopes of Uranium (Z=92)

Click on an isotope to get more information about its decay

Isotope	Half-life	Spin Parity	Decay Mode(s) or Abundance
218U	1.5 ms	0+	%A=100
219U	42 us		%A=?
220U			
221U			
222U	1.0 us	0+	%A=100
223U	18 us		%A=?
224U	0.9 ms	0+	%A=100
225U	95 ms		%A=100
226U	0.35 s	0+	%A=100
227U	1.1 m	(3/2+)	%A=100, %EC+%B+ < 0.001
228U	9.1 m	0+	%EC<5, %A>95
229U	58 m	(3/2+)	%EC+%B+ ~ 80, %A ~ 20
230U	20.8 d	0+	%A=100, %SF < 1.4E-10
231U	4.2 d	(5/2-)	%EC=100, %A ~ 0.0055
232U	68.9 y	0+	%A=100, %24NE=9E-11 7
233U	1.592e+5 y	5/2+	%A=100, %SF < 6E-11, %24NE < 9.5E-11
234U	2.455e+5 y	0+	%Abundance=0.0055 5, %A=100, %SF=1.64E-9 22, %NE=9E-12 7, %MG=1.4E-11 3
235U	7.038e+8 y	7/2-	%Abundance=0.7200 12, %A=100, %SF=7.0E-9 21, %20NE=8E-10 4
235m1U	25 m	1/2+	%IT=100
236U	2.342e7 y	0+	%A=100, %SF=9.4E-8 4
237U	6.75 d	1/2+	%B=100
238U	4.468e+9 y	0+	%Abundance=99.2745 60, %A=100, %SF=5.45E-5 7, %BB=2.2E-10 7
238m1U	225 ms	0+	

Zjednodušený příklad:

$$N_{235}(t) = N_{235}(0) \cdot e^{-\frac{t}{T_{235}/\ln(2)}}$$
$$N_{238}(t) = N_{238}(0) \cdot e^{-\frac{t}{T_{238}/\ln(2)}}$$
$$\frac{N_{235}(t)}{N_{238}(t)} = \frac{N_{235}(0)}{N_{238}(0)} e^{-\left(\frac{1}{T_{235}/\ln(2)} - \frac{1}{T_{238}/\ln(2)}\right)t}$$
$$\frac{0.7\%}{99.3\%} = 1 \cdot e^{-\left(\frac{1}{0,70 \cdot 10^9 [y]/\ln(2)} - \frac{1}{4,47 \cdot 10^9 [y]/\ln(2)}\right)t [y]}$$

$$t = 5,97 \cdot 10^9 \text{ y}$$

[Uran.nb](#)

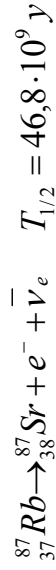
Určování stáří hornin

Metoda Uran-Olovo

^{238}U	miliardy let 4.468	^{206}Pb	<p>-V uranových dolech: olovo vzniklo z uranu, nebzlo tam žádné na začátku, protože se chemicky nesnese s uranem</p> <p>-Stáří Země určeno jako 4,2 miliardy let.</p>
^{235}U	0.704	^{207}Pb	
^{40}K	1.251	^{40}Ar	
^{87}Rb	48.8	^{87}Sr	

Metoda Rubidium-Stroncium

Metoda Rubidium-Stroncium



$$t = 0 : \left(\frac{^{87}\text{Sr}}{^{86}\text{Sr}} \right)_{t=0} ; \left(\frac{^{87}\text{Rb}}{^{86}\text{Sr}} \right)_{t=0}$$

$$\left(\frac{^{87}\text{Rb}}{^{86}\text{Sr}} \right)_t = \frac{\left(\frac{^{87}\text{Rb}}{^{86}\text{Sr}} \right)_{t=0} \cdot e^{\frac{t}{T_{1/2}/\ln(2)}}}{\left(\frac{^{86}\text{Sr}}{^{86}\text{Sr}} \right)_{t=0}} = \left(\frac{^{87}\text{Rb}}{^{86}\text{Sr}} \right)_{t=0} \cdot e^{\frac{t}{T_{1/2}/\ln(2)}}$$

$$\left(\frac{^{87}\text{Sr}}{^{86}\text{Sr}} \right)_t = \frac{\left(\frac{^{87}\text{Sr}}{^{86}\text{Sr}} \right)_{t=0} + \left(\frac{^{87}\text{Rb}}{^{86}\text{Sr}} \right)_{t=0} \cdot \left(1 - e^{\frac{t}{T_{1/2}/\ln(2)}} \right)}{\left(\frac{^{86}\text{Sr}}{^{86}\text{Sr}} \right)_{t=0}} =$$

$$\left(\frac{^{87}\text{Sr}}{^{86}\text{Sr}} \right)_{t=0} + \left(\frac{^{87}\text{Rb}}{^{86}\text{Sr}} \right)_{t=0} \cdot e^{\frac{t}{T_{1/2}/\ln(2)}} \cdot \left(e^{\frac{t}{T_{1/2}/\ln(2)}} - 1 \right) =$$

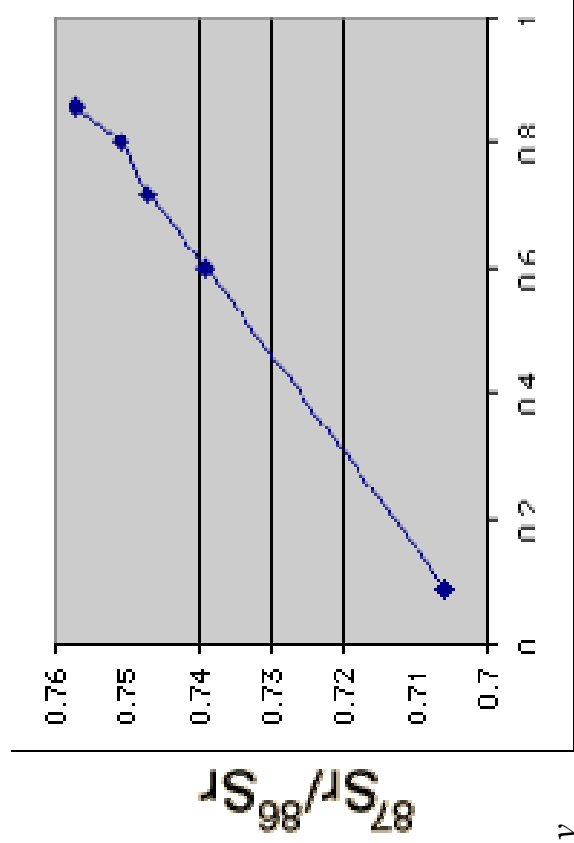
$$\frac{\left(\frac{^{87}\text{Sr}}{^{86}\text{Sr}} \right)_{t=0} + \left(\frac{^{87}\text{Rb}}{^{86}\text{Sr}} \right)_{t=0} \cdot \left(e^{\frac{t}{T_{1/2}/\ln(2)}} - 1 \right)}{\left(\frac{^{86}\text{Sr}}{^{86}\text{Sr}} \right)_{t=0}} =$$

$$\left(\frac{^{87}\text{Sr}}{^{86}\text{Sr}} \right)_{t=0} + \left(\frac{^{87}\text{Rb}}{^{86}\text{Sr}} \right)_{t=0} \cdot \left(e^{\frac{t}{T_{1/2}/\ln(2)}} - 1 \right)$$

$$\left(e^{\frac{t}{T_{1/2}/\ln(2)}} - 1 \right) = \frac{0,049}{0,77} \Rightarrow t = \frac{T_{1/2} = 46,8 \cdot 10^9 \text{ y}}{\ln(2)} \ln \left(1 + \frac{0,049}{0,77} \right) = 4,17 \cdot 10^9 \text{ y}$$

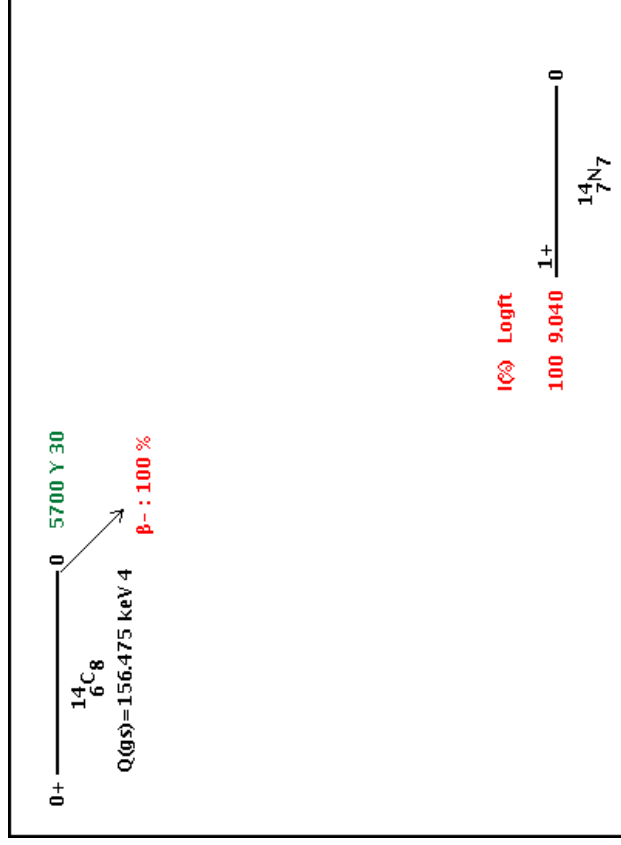
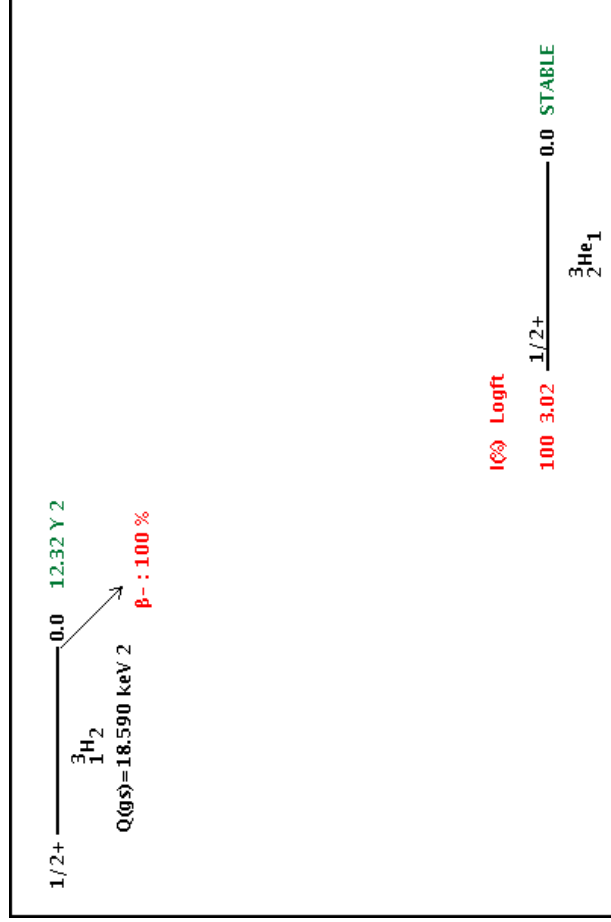
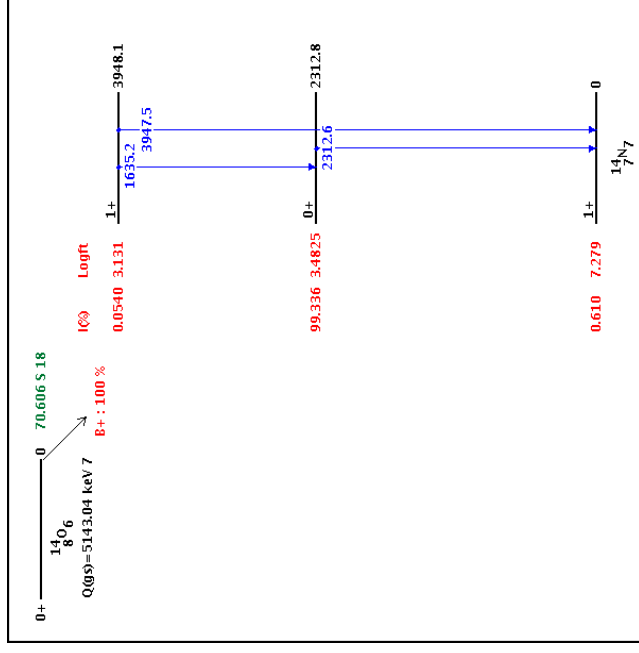
Meteorit	$^{87}\text{Rb}/^{86}\text{Sr}$	$^{87}\text{Sr}/^{86}\text{Sr}$
Modoc	0.86Sr	0.757
Homestead	0.8Sr	0.751
Bruderheim	0.72Sr	0.747
Kyushu	0.6Sr	0.739
Buth Furnace	0.09Sr	0.706

4.6 x 10^9 years



$^{87}\text{Rb}/^{86}\text{Sr}$

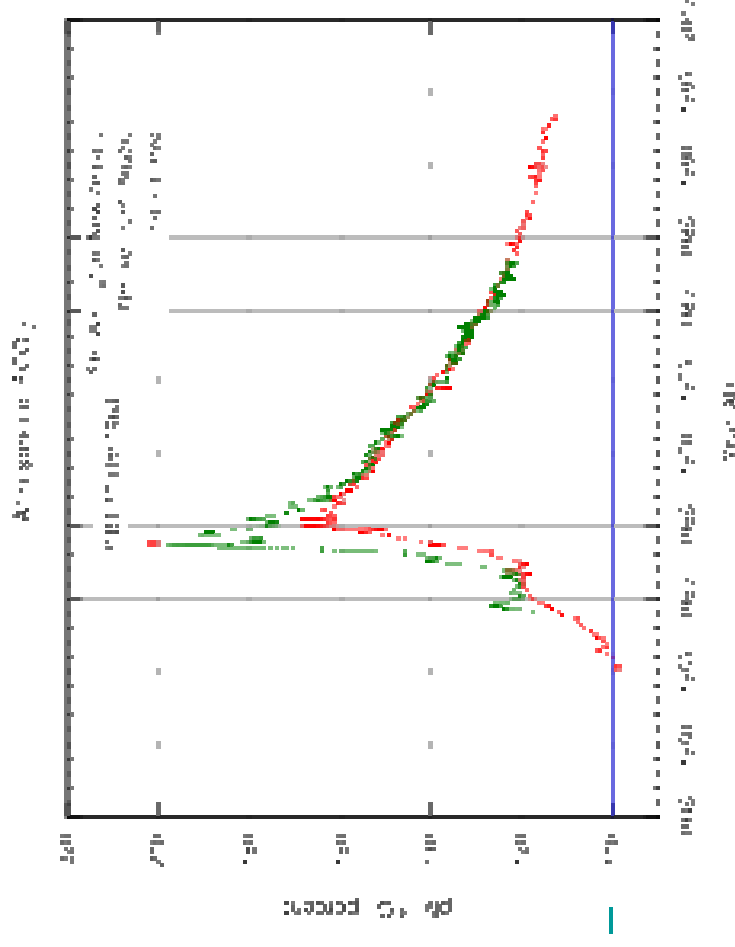
Radioaktivní izotopy vznikající v atmosféře působením kosmického záření:



Datování stárí živých organismů pomocí uhlíku 14

- Uhlík 14 tvoří CO2 a ten se dostává do zelených rostlin
- Živočiškové, kteří se živí zelenými rostlinami a-nebo jinými živočichy, kteří požívají zelenou stravu dostávají do těla C14, ten je v rovnováze s C12
- Jakmile umřou, začne se obsah C14 v jejich tělech snižovat

Je třeba brát do úvahy změny obsahu uhlíku 14 v atmosféře



Atmospheric ^{14}C , [New Zealand\[1\]](#) and [Austria\[2\]](#). The New Zealand curve is representative for the Southern Hemisphere, the Austrian curve is representative for the Northern Hemisphere. Atmospheric nuclear weapon tests almost doubled the concentration of ^{14}C in the Northern Hemisphere [\[3\]](#).

Kinematika – 4 vektory

4 – vektor

$$(\vec{V}_0, \vec{V}) = (V_0, V_x, V_y, V_z)$$

$$V_0 \rightarrow \gamma(V_0 - \beta V_x)$$

$$\vec{V}$$

$$V_x \rightarrow \gamma(V_x - \beta V_0)$$

Součin dvou 4-vektorů:

$P \cdot Q = P_0 Q_0 - \vec{P} \cdot \vec{Q}$ Součin dvou 4-vektorů je invariantní při Lorentzově transformaci:

$$P \cdot Q \rightarrow \gamma(P_0 - \beta P_x) \gamma(Q_0 - \beta Q_x) - \gamma(P_x - \beta P_0) \gamma(Q_x - \beta Q_0) - P_y Q_y - P_z Q_z =$$

$$P_0 Q_0 (\gamma^2 - \gamma^2 \beta^2) + P_0 Q_x (-\gamma^2 \beta + \gamma^2 \beta) + P_x Q_0 (-\gamma^2 \beta^2 + \gamma^2 \beta^2) - P_x Q_x (-\gamma^2 \beta^2 + \gamma^2) - P_y Q_y - P_z Q_z =$$

$$P_0 Q_0 (\gamma^2 - \gamma^2 \beta^2) - P_x Q_x (\gamma^2 - \gamma^2 \beta^2) - P_y Q_y - P_z Q_z = P_0 Q_0 - P_x Q_x - P_y Q_y - P_z Q_z = P \cdot Q$$

Příkladem 4-vektoru je:

(ct, r)

4-vektor energie a hybnosti, 4-impuls:

$$P = (E, \vec{p})$$

$$P \cdot P = E \cdot E - \vec{p} \cdot \vec{p} = E^2 - p^2 = M^2$$

Použití 4-hybností pro výpočet prahu reakcí. Příklad - objev antiprotonu

$$p + p \rightarrow p + p + \bar{p}$$

$$(E, \vec{p}) + (m_p, \vec{0}) = \left(\sum_{i=1}^4 E_i, \sum_{i=1}^4 \vec{p}_i \right)$$

$$P_{init} = (E + m_p, \vec{p})$$

$$P_{init}^2 = (E + m_p)^2 - |\vec{p}|^2 = E^2 - |\vec{p}|^2 + m_p^2 + 2m_p E$$

$$P_{thr} = \left(\sum_{i=1}^4 m_i, \vec{0} \right) = (4m_p, \vec{0})$$

$$P_{thr}^2 = 16m_p^2$$

$$P_{init}^2 = P_{thr}^2$$

$$2m_p^2 + 2m_p E_{thr} = 16m_p^2$$

$$E_{thr} = 7m_p$$

$$T_{k,thr} = E_{thr} - m_p = 6m_p$$

Kinematika - Mandelstamovy invarianty s a t

$$s = (P_1 + P_2)^2$$

cms:

$$P_1 = (E_1, \vec{p}^*), P_2 = (E_2, -\vec{p}^*)$$

$$s = (E_1 + E_2)^2 - (\vec{p}^* - \vec{p}^*)^2 = (E_1 + E_2)^2 = E_{cms}^2 > 0$$

$$t = (P_3 - P_1)^2$$

cms:

$$P_1 = (E_1, \vec{p}_1^*), P_3 = (E_3, \vec{p}_3^*)$$

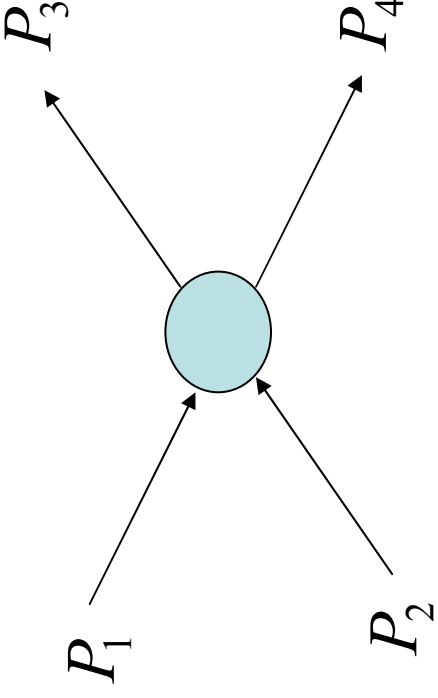
$$t = (E_1 - E_3)^2 - (\vec{p}_1^* - \vec{p}_3^*)^2 = M_1^2 + M_3^2 - 2(E_1 E_3 - \vec{p}_1^* \cdot \vec{p}_3^*)$$

$$M_1 = M_3 = M \Rightarrow E_1 = E_3 = E;$$

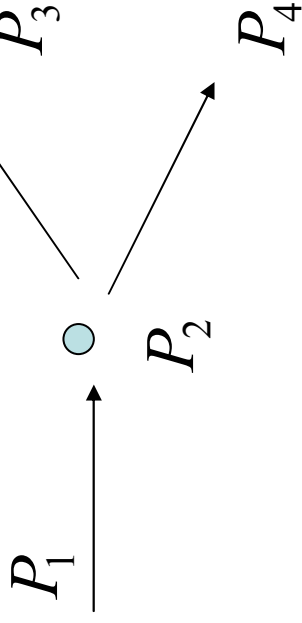
$$M_1 = M_3 \text{ \& } M_2 = M_4 \Rightarrow \left| \vec{p}_1^* \right| = \left| \vec{p}_3^* \right| = \left| \vec{p}^* \right|$$

$$t = 2M^2 - 2 \left(E^2 - \left| \vec{p}^* \right|^2 \cos(\theta^*) \right) = -2 \left((E^2 - M^2) - \left| \vec{p}^* \right|^2 \cos(\theta^*) \right) =$$

$$-2 \left| \vec{p}^* \right|^2 (1 - \cos(\theta_{13})) = -4 \left| \vec{p}^* \right|^2 \sin^2(\theta^* / 2) = -\left| \vec{q} \right|^2$$



Kinematika - Mandelstamovy invarianty s a t



$$t = (P_3 - P_1)^2 = (P_4 - P_2)^2$$

lab :

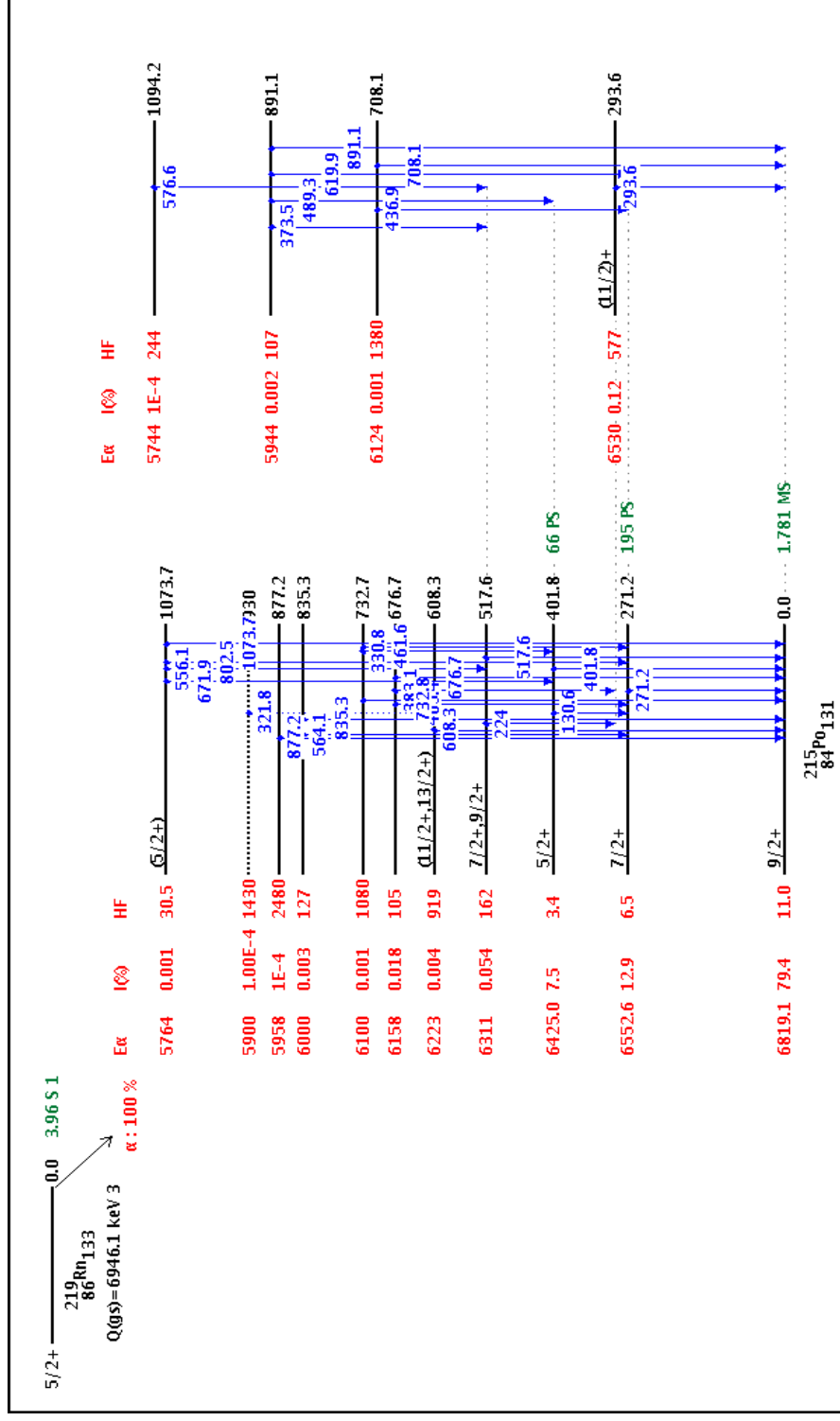
$$P_4 = (E, \vec{p}), P_2 = (m, \vec{0})$$

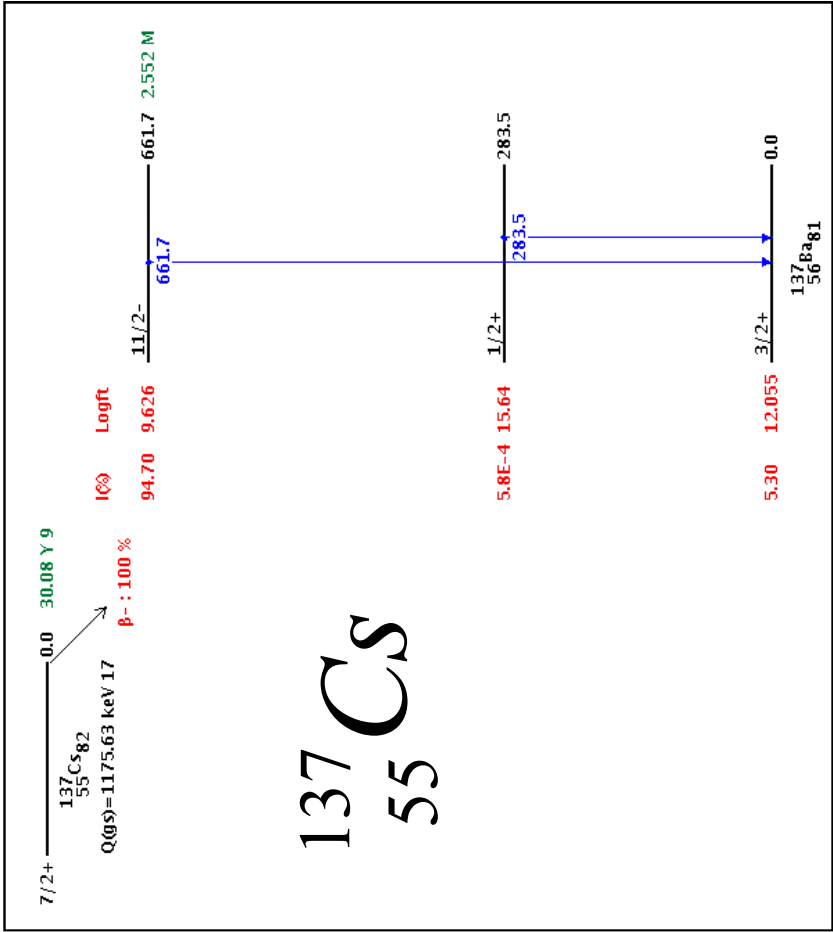
$$t = (E - m)^2 - (\vec{p})^2 = m^2 - 2Em + m^2 = -2m(E - m) = -2mT_k$$

$$T_k = \frac{-t}{2m}$$

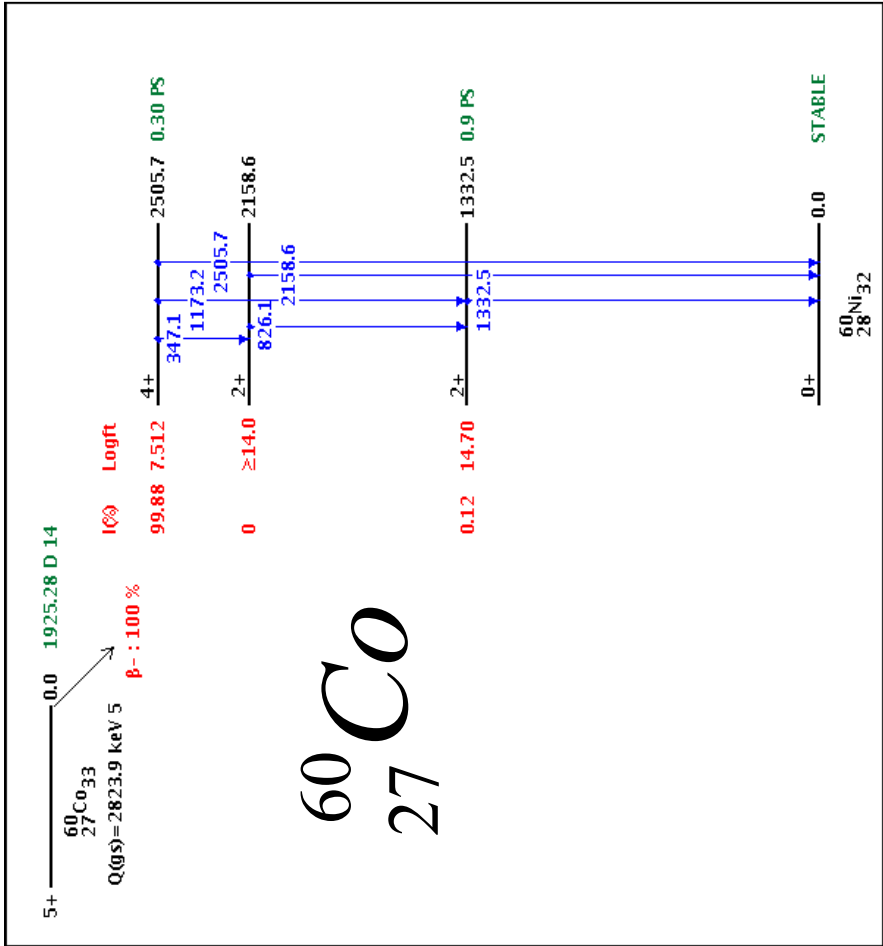
$$\left(cf : T_k = \frac{p^2}{2m} \right)$$

Vzbuzené stavy jader a gama záření





Příklady známých gama zářičů





The Nobel Prize in Physics 1961

"for his pioneering studies of electron scattering in atomic nuclei and for his thereby achieved discoveries concerning the structure of the nucleons"



Robert Hofstadter

1/2 of the prize

USA

Stanford University
Stanford, CA, USA

b. 1915
d. 1990

"for his researches concerning the resonance absorption of gamma radiation and his discovery in this connection of the effect which bears his name"



Rudolf Ludwig
Mössbauer

1/2 of the prize

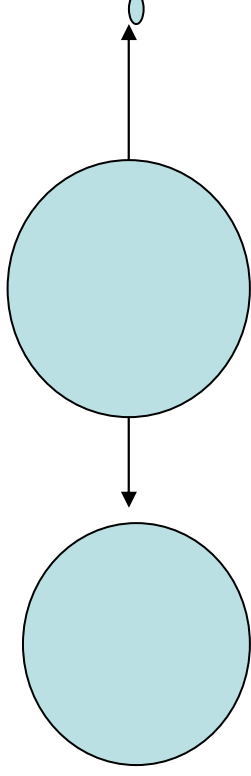
Federal Republic of
Germany

Technical University
Munich, Federal Republic
of Germany; California
Institute of Technology
(Caltech)
Pasadena, CA, USA

b. 1929

Měření šířek gama přechodů a Mössbauerův jev

Při gama přechodu jádro odnáší část energie a tak je energie gama o něco menší

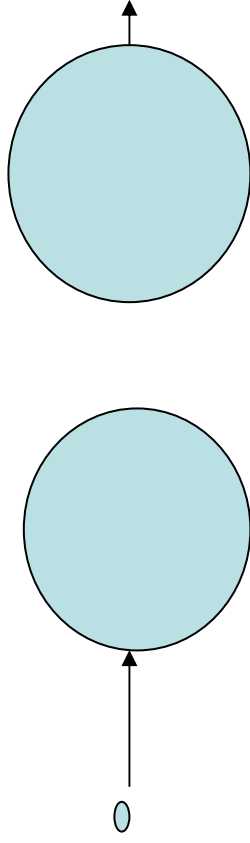


$$(M + E_\gamma) \rightarrow M + E'_\gamma$$

$$E'_\gamma = \frac{(M + E_\gamma)^2 - M^2}{2(M + E_\gamma)} = \frac{2ME_\gamma + E_\gamma^2}{2(M + E_\gamma)} =$$

$$\frac{2E_\gamma(M + E_\gamma) - E_\gamma^2}{2(M + E_\gamma)} = E_\gamma - \frac{E_\gamma^2}{2(M + E_\gamma)}$$

Naopak pro vybuzení jádra do excitovaného stavu je třeba energie o něco málo větší než odpovídá energii gama přechodu

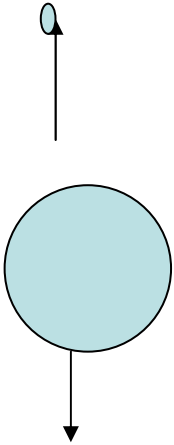


$$E''_{\gamma} + M \rightarrow (M + E_{\gamma})$$

$$(E''_{\gamma} + M)^2 - (E''_{\gamma})^2 = (M + E_{\gamma})^2$$

$$2E''_{\gamma}M = 2E_{\gamma}M + E_{\gamma}^2$$

$$E''_{\gamma} = E_{\gamma} + \frac{E_{\gamma}^2}{2M}$$



$$(M + E_{\gamma}) \rightarrow M + E'_{\gamma}$$

$$E'_{\gamma} = E_{\gamma} - \frac{E_{\gamma}^2}{2(M + E_{\gamma})}$$

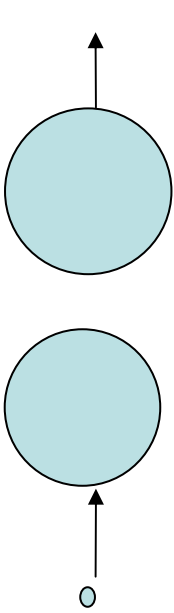
$$({}^{191}_{77}\text{Ir} + 129\text{keV}) \rightarrow + E'_{\gamma}$$

$$M_{{}^{191}_{77}\text{Ir}} = (77 \cdot 938,27 + (191 - 77) \cdot 939,57 - 191 \cdot 8,5)\text{MeV} = 177734\text{MeV}$$

$$E'_{\gamma} - 129\text{keV} = -\frac{(129 \cdot 10^3)^2 \text{eV}^2}{2 \cdot 177734 \cdot 10^6 \text{eV}} = 46,8 \cdot 10^{-3} \text{eV}$$

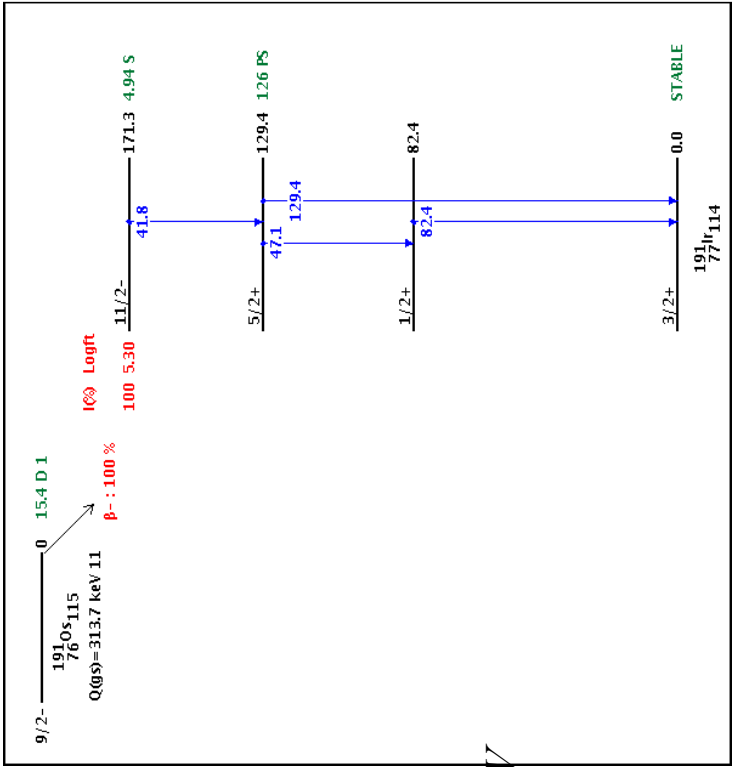
$$E''_{\gamma} - 129\text{keV} = \frac{(129 \cdot 10^3)^2 \text{eV}^2}{2 \cdot (177734 \cdot 10^6 + 129 \cdot 10^3) \text{eV}} = 46,8 \cdot 10^{-3} \text{eV}$$

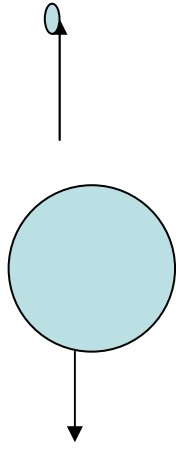
$$\Gamma = \frac{\hbar c}{c \tau} = \frac{\hbar c}{c(T_{1/2} / \ln(2))} = \frac{197 \cdot 10^6 \text{eV} \cdot 10^{-13} \text{cm}}{30 \text{cm} / \text{ns} \cdot 0,126 \text{ns} / 0,69} = 3,6 \cdot 10^{-6} \text{eV}$$



$$E''_{\gamma} + M \rightarrow (M + E_{\gamma})$$

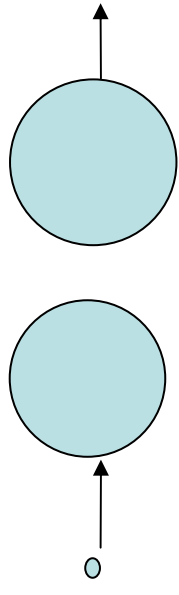
$$E''_{\gamma} = E_{\gamma} + \frac{E_{\gamma}^2}{2M}$$





$$(M + E_{\gamma}) \rightarrow M + E'_{\gamma}$$

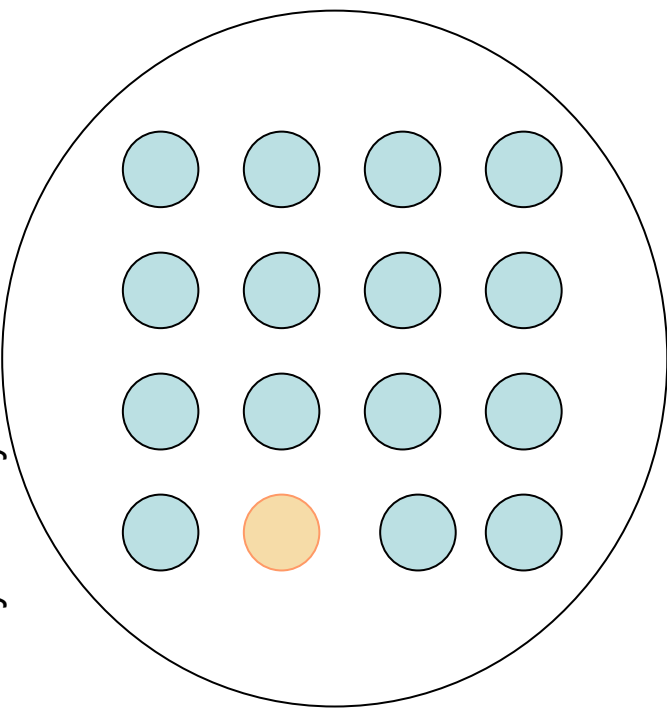
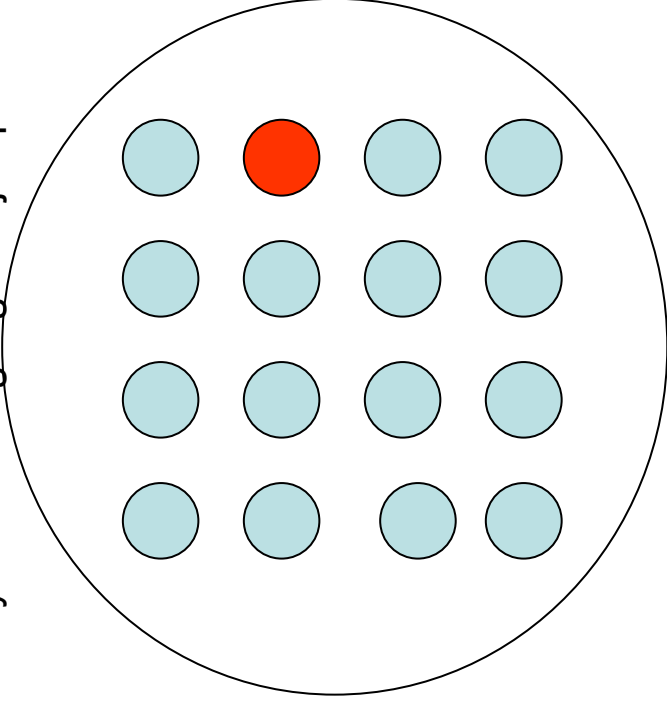
$$E'_{\gamma} = E_{\gamma} - \frac{E_{\gamma}^2}{2(M + E_{\gamma})}$$



$$E''_{\gamma} + M \rightarrow (M + E_{\gamma})$$

$$E''_{\gamma} = E_{\gamma} + \frac{E_{\gamma}^2}{2M}$$

Je-li atom zabudován do krystalu ($M \rightarrow M$ krystal), pak při nízké teplotě je odražen celý krystal a energie gama je správná na to aby v jiném takovém krystalu byla absorbována



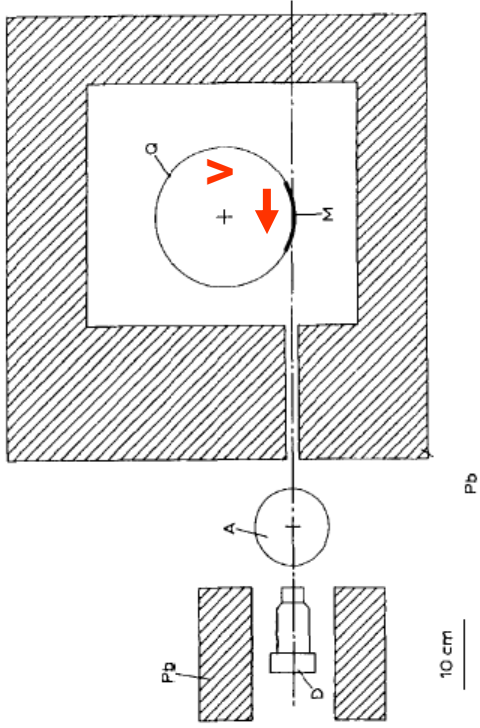


Fig. 7. Experimental arrangement. A, absorber-cryostat; Q, rotating cryostat with source; D, scintillation detector.



$$\Gamma = 3,6 \cdot 10^{-6} \text{ eV}$$

$$129 \cdot 10^3 \text{ } \beta \text{eV} = 3,6 \cdot 10^{-6} \text{ eV}$$

$$\beta = \frac{3,6 \cdot 10^{-6} \text{ eV}}{129 \cdot 10^3 \text{ eV}} = 2,8 \cdot 10^{-11}$$

$$\nu = 2,8 \cdot 10^{-11} \cdot 3 \cdot 10^{10} \text{ cm/s} = 0,84 \text{ cm/s}$$

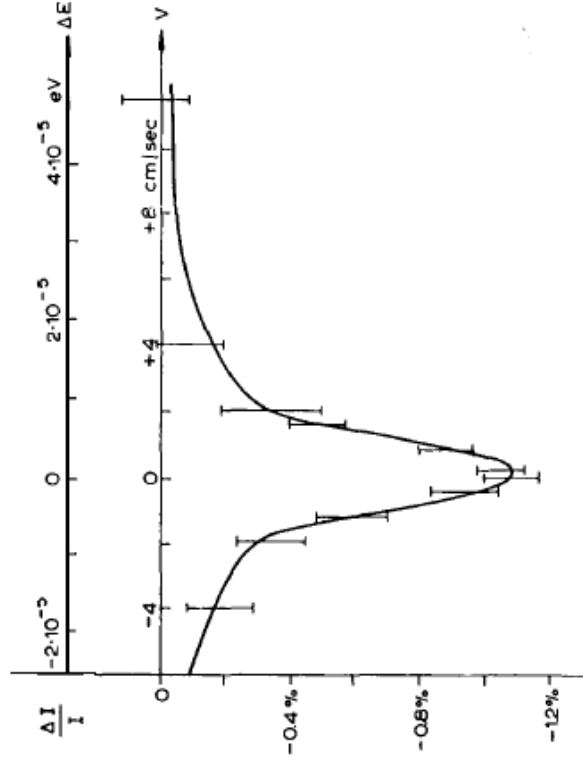


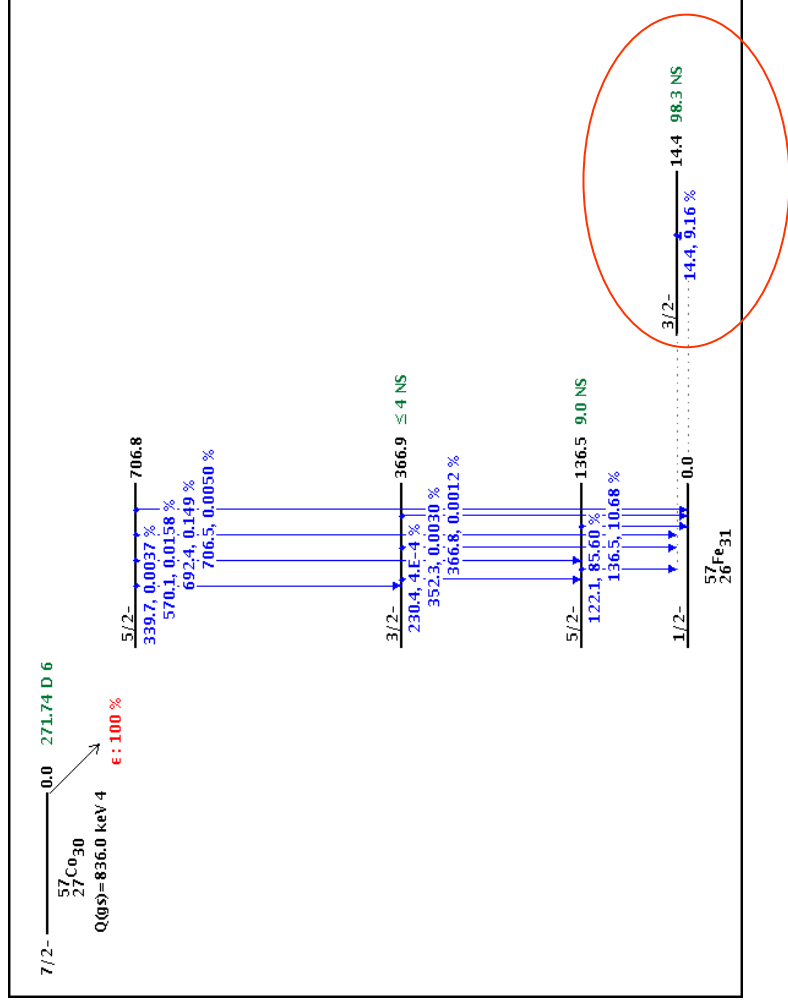
Fig. 8. Relative intensity ratio $\Delta I/I$ of gamma radiation measured behind the resonant iridium absorber, in comparison with intensities measured behind a nonresonant absorber.

$^{57}_{26}Fe$

$$M_{^{57}_{26}Fe} = (26 \cdot 938,27 + (57 - 26) \cdot 939,57 - 57 \cdot 8,8) MeV =$$

53020 MeV

$$\Gamma = \frac{\hbar c}{c \tau} = \frac{\hbar c}{c(T_{1/2} / \ln(2))} = \frac{197 \cdot 10^6 eV 10^{-13} cm}{30 cm / ns \cdot 98,3 ns / 0,69} = 46 \cdot 10^{-10} eV$$



Detektor



β

Absorbátor

Rezonanční absorbce 14,4 keV gama. Rudý posuv gama je kompenzován pohybem absorbátoru. Experiment Repky

$$E'_\gamma = E_\gamma - \frac{E_\gamma}{c^2} gh$$

$$E''_\gamma = \gamma(E'_\gamma + \beta E'_\gamma) = \frac{1}{\sqrt{1-\beta^2}}(E'_\gamma + \beta E'_\gamma) \cong (1 + \frac{\beta^2}{2})(E'_\gamma + \beta E'_\gamma) \cong E'_\gamma(1 + \beta)$$

$$E_\gamma(1 - \frac{gh}{c^2})(1 + \beta) \frac{1}{\sqrt{1-\beta^2}} \cong E_\gamma(1 - \frac{gh}{c^2})(1 + \beta) \cong E_\gamma(1 - \frac{gh}{c^2} + \beta) = E_\gamma$$

$$\beta = \frac{gh}{c^2} = \frac{10m/s^2 \cdot 20m}{9 \cdot 10^{16}m^2/s^2} = 2,2 \cdot 10^{-15} \Rightarrow v = 2,2 \cdot 10^{-15} \cdot 3 \cdot 10^8 m/s =$$

$$6,6 \cdot 10^{-4} mm/s = 0,66 \mu m/s$$

Interakce jaderného záření s prostředím

Ionizace – nabité částice

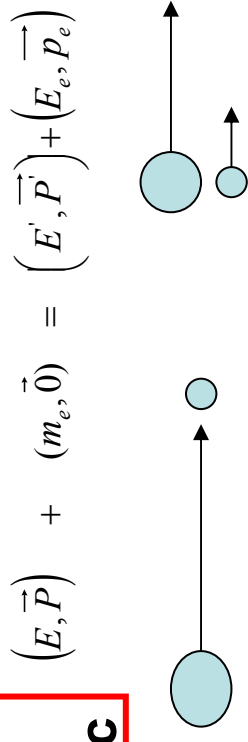
Brzdné záření – elektrony

Interakce gama záření

Ionizační ztráty

- týkají se všech nabitých částic

T_{max}



$$E + m_e = E' + E_e \quad ; \quad \vec{P} = \vec{P}' + \vec{p}_e$$

$$E' = E + m_e - E_e \quad ; \quad |\vec{P}'| = |\vec{P}| - |\vec{p}_e|$$

$$M^2 = E'^2 - |\vec{P}'|^2 = (E + m_e - E_e)^2 - (|\vec{P}| - |\vec{p}_e|)^2$$

$$M^2 = \left(E^2 - |\vec{P}|^2 \right) + \left(E_e^2 - |\vec{p}_e|^2 \right) + m_e^2 + 2Em_e - 2EE_e - 2m_eE_e + 2|\vec{P}||\vec{p}_e|$$

$$0 = 2m_e^2 + 2Em_e - 2EE_e - 2m_eE_e + 2|\vec{P}||\vec{p}_e|$$

$$\left(|\vec{P}||\vec{p}_e| \right)^2 = (E_e - m_e)^2 (E + m_e)^2$$

$$|\vec{P}|^2 (E_e^2 - m_e^2) = |\vec{P}|^2 (E_e - m_e)(E + m_e) = (E_e - m_e)^2 (E + m_e)^2$$

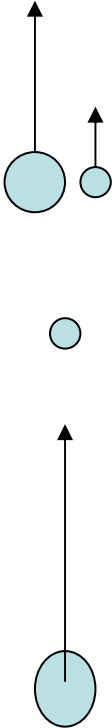
$$|\vec{P}|^2 (\tau_{k,\max} + 2m_e) = (E + m_e)^2 \tau_{k,\max} \quad ; \quad \tau_{k,\max} ((E + m_e)^2 - |\vec{P}|^2) = |\vec{P}|^2 2m_e$$

$$\tau_{k,\max} = \frac{2m_e |\vec{P}|^2}{M^2 + 2Em_e + m_e^2} = \frac{2m_e |\vec{P}|^2 / M^2}{1 + 2(E/M)(m_e/M) + (m_e/M)^2}$$

$$\tau_{k,\max} = \frac{2m_e \beta^2 \gamma^2}{1 + 2\gamma \frac{m_e}{M} + \left(\frac{m_e}{M} \right)^2}$$

Tmax

$$\tau_{k,\max} = \frac{2m_e\beta^2\gamma^2}{1+2\gamma\frac{m_e}{M}+\left(\frac{m_e}{M}\right)^2}$$



Tmin

Ionizační ztráty - týkají se všech nabitých částic

$$\frac{\pi(r_{\min}^2/8)}{\sin^4(\theta/2)} = \frac{d\sigma}{d\cos(\theta)} = \frac{d\sigma}{d(1-\cos(\theta))} = \frac{2P^2 d\sigma}{d(4P^2(1-\cos(\theta))/2)} = \frac{2P^2 d\sigma}{d(4P^2 \sin^2(\theta/2))} = \frac{2P^2 d\sigma}{dq^2}$$

$$\frac{2P^2 d\sigma}{dq^2} = \frac{\pi(r_{\min}^2/8)}{\sin^4(\theta/2)} = \frac{(2P)^4 \pi(r_{\min}^2/8)}{(2P)^4 \sin^4(\theta/2)} = \frac{(2P)^4 \pi(r_{\min}^2/8)}{q^4}$$

$$\frac{d\sigma}{dq^2} = \frac{P^2 \pi r_{\min}^2}{q^4} \Rightarrow \frac{d\sigma}{d|t|} = \frac{P^2 \pi r_{\min}^2}{|t|^2}$$

$$r_e = \frac{\alpha(\hbar c)}{m_e}$$

$$\frac{d\sigma}{2m_e d\frac{|t|}{2m_e}} = \frac{P^2 \pi r_{\min}^2}{4m_e^2 \left(\frac{|t|}{2m_e}\right)^2}$$

$$\frac{d\sigma}{d\tau_k} = \frac{P^2 \pi r_{\min}^2}{2m_e} \frac{1}{\tau_k^2} = \frac{m_e P^2 \pi \alpha^2 (\hbar c)^2}{2m_e^2 T_k^2} \frac{1}{\tau_k^2} = \frac{P^2}{2T_k^2} m_e \pi r_e^2 \frac{1}{\tau_k^2} =$$

$$\frac{(\gamma\beta)^2 M^2}{2(\gamma-1)^2} m_e \pi r_e^2 \frac{1}{\tau_k^2} = \frac{\gamma^2-1}{2(\gamma-1)^2} m_e \pi r_e^2 \frac{1}{\tau_k^2} = \frac{\gamma+1}{2(\gamma-1)} m_e \pi r_e^2 \frac{1}{\tau_k^2} =$$

$$\frac{(\gamma+1)^2}{2(\gamma^2-1)} m_e \pi r_e^2 \frac{1}{\tau_k^2} = \frac{(\gamma+1)^2}{2(\gamma\beta)^2} m_e \pi r_e^2 \frac{1}{\tau_k^2} = 2 \left(\frac{\gamma+1}{2} \right)^2 \frac{1}{(\gamma\beta)^2} m_e \pi r_e^2 \frac{1}{\tau_k^2}$$

$$\frac{d\sigma}{d\tau_k} = 2 \left(\frac{\gamma+1}{2} \right)^2 \frac{1}{(\gamma\beta)^2} m_e \pi r_e^2 \frac{1}{\tau_k^2}$$

$$p = \sigma \cdot \Delta x \cdot \rho \cdot \frac{N_A Z}{A}$$

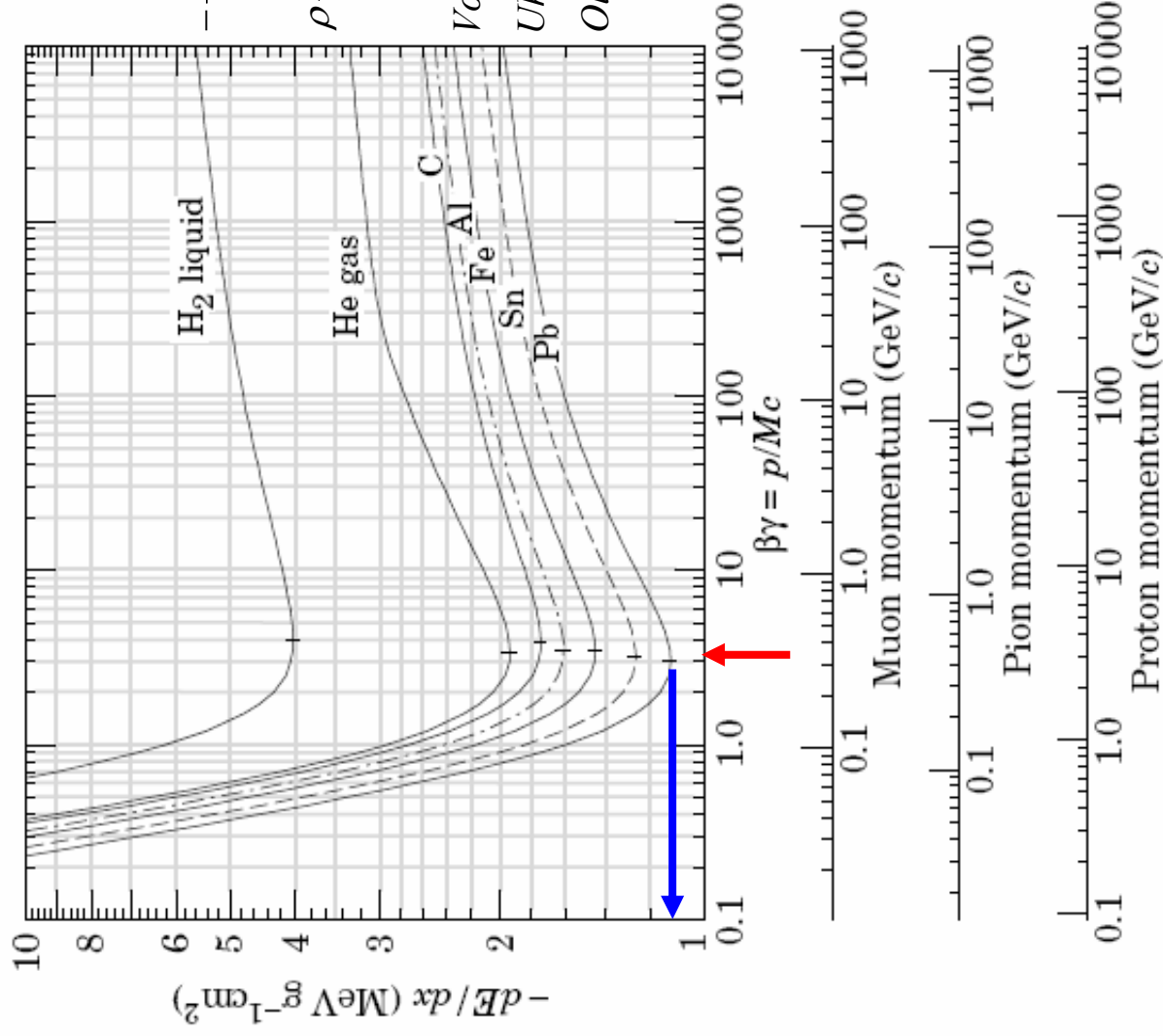
$$\frac{dp}{d\tau_k} = \Delta x \cdot \rho \cdot \frac{N_A}{A} Z \cdot 2 \left(\frac{\gamma+1}{2} \right)^2 \frac{1}{(\gamma\beta)^2} m_e \pi r_e^2 \frac{1}{\tau_k^2}$$

$$\langle \tau_k \rangle = \int_{\tau_{\min}}^{\tau_{\max}} \tau_k \frac{dp}{d\tau_k} d\tau_k = \Delta x \cdot \rho \cdot \frac{N_A}{A} Z \cdot 2 \left(\frac{\gamma+1}{2} \right)^2 \frac{1}{(\gamma\beta)^2} m_e \pi r_e^2 \int_{\tau_{\min}}^{\tau_{\max}} \tau_k \frac{1}{\tau_k^2} d\tau_k =$$

$$\Delta x \cdot \rho \cdot \frac{N_A}{A} Z \cdot 2 \left(\frac{\gamma+1}{2} \right)^2 \frac{1}{(\gamma\beta)^2} m_e \pi r_e^2 \ln \left(\frac{\tau_{\max}}{\tau_{\min}} \right)$$

$$-\frac{\Delta T_k}{\Delta x} = \rho \cdot \frac{N_A}{A} Z \cdot 2 \left(\frac{\gamma+1}{2} \right)^2 \frac{1}{(\gamma\beta)^2} m_e \pi r_e^2 \ln \left(\frac{\tau_{\max}}{\tau_{\min}} \right)$$

$$-\frac{dE}{dx} = K z^2 \frac{Z}{A} \frac{1}{\beta^2} \left[\frac{1}{2} \ln \frac{2m_e c^2 \beta^2 \gamma^2 T_{\max}}{I^2} - \beta^2 - \frac{\delta(\beta\gamma)}{2} \right]$$



$$-\frac{dE}{dx} [\text{MeV} / \text{cm}] = \rho [\text{gcm}^{-3}] \cdot 4 [\text{MeV} / (\text{gcm}^{-2})] \left[\frac{Z}{A} \frac{1}{\beta^2} - \right]$$

$$\rho \frac{(dE / dx)_{\min}}{\beta^2}$$

$$\text{Vodík} : (dE / dx)_{\min} = 4 \text{MeV} / (\text{gcm}^{-2})$$

$$\text{Uhlík, voda} : (dE / dx)_{\min} = 2 \text{MeV} / (\text{gcm}^{-2})$$

$$\text{Olovo} : (dE / dx)_{\min} = 1,12 \text{MeV} / (\text{gcm}^{-2})$$

Doběh :

$$-\frac{dE}{dx}=\frac{(dE/dx)_{\min}\cdot\rho}{\beta^2}=\frac{(dE/dx)_{\min}\cdot\rho}{(E^2-m^2)/E^2}$$

$$-dE\frac{E^2-m^2}{E^2}=(dE/dx)_{\min}\cdot\rho\cdot dx$$

$$-\int\limits_{E_0}^mdE\left(1-\frac{m^2}{E^2}\right)=(dE/dx)_{\min}\cdot\rho\int\limits_0^Rdx$$

$$-(m-E_0)-m^2\left(\frac{1}{m}-\frac{1}{E_0}\right)=(dE/dx)_{\min}\cdot\rho\cdot R$$

$$E_0-2m+\frac{m^2}{E_0}=(dE/dx)_{\min}\cdot\rho\cdot R$$

$R=\frac{E_0-m}{(dE/dx)_{\min}}\cdot\rho$

$\frac{E_0-m}{(dE/dx)_{\min}}=\frac{T_k}{(dE/dx)_{\min}}\cdot\rho$

$\frac{T_k}{(dE/dx)_{\min}}\cdot\rho=T_k+m$

$$T_k<<m:$$

$$R=\frac{T_k}{(dE/dx)_{\min}}/\rho\approx\frac{T_k}{m}$$

$$T_k>>m:$$

$$R=\frac{T_k}{(dE/dx)_{\min}}/\rho\approx T_k$$

Př.:

$$\alpha, m = (2 \cdot 938,27 + 2 \cdot 939,57 - 28) \text{MeV} = 3725,68 \text{MeV}$$

$$T_k = 7,7 \text{MeV}$$

vzduch:

$$(dE/dx)_{\min} = 2,0 \text{MeV} / (\text{g} \cdot \text{cm}^{-2})$$

$$\rho = 0,0012 \text{g} \cdot \text{cm}^{-3}$$

$$R = \frac{T_k}{(dE/dx)_{\min} \cdot \rho} \cdot \frac{T_k}{T_k + m} = \frac{7,7 \text{MeV}}{2,0 \frac{\text{MeV}}{\text{g} \cdot \text{cm}^{-2}} \cdot 0,0012 \text{g} \cdot \text{cm}^{-3}} \cdot \frac{7,7 \text{MeV}}{(7,7 + 3725,68) \text{MeV}} = 6,62 \text{cm}$$

voda:

$$(dE/dx)_{\min} = 2,0 \text{MeV} / (\text{g} \cdot \text{cm}^{-2})$$

$$\rho = 1,0 \text{g} \cdot \text{cm}^{-3}$$

$$R = \frac{T_k}{(dE/dx)_{\min} \cdot \rho} \cdot \frac{T_k}{T_k + m} = \frac{7,7 \text{MeV}}{2,0 \frac{\text{MeV}}{\text{g} \cdot \text{cm}^{-2}} \cdot 1,0 \text{g} \cdot \text{cm}^{-3}} \cdot \frac{7,7 \text{MeV}}{(7,7 + 3725,68) \text{MeV}} = 7,9 \cdot 10^{-3} \text{cm} = 79 \mu\text{m}$$

zlato:

$$(dE/dx)_{\min} = 1,13 \text{MeV} / (\text{g} \cdot \text{cm}^{-2})$$

$$\rho = 20 \text{g} \cdot \text{cm}^{-3}$$

$$R = \frac{T_k}{(dE/dx)_{\min} \cdot \rho} \cdot \frac{T_k}{T_k + m} = \frac{7,7 \text{MeV}}{1,13 \frac{\text{MeV}}{\text{g} \cdot \text{cm}^{-2}} \cdot 20,0 \text{g} \cdot \text{cm}^{-3}} \cdot \frac{7,7 \text{MeV}}{(7,7 + 3725,68) \text{MeV}} = 7,0 \cdot 10^{-4} \text{cm} = 7 \mu\text{m}$$

Interakce elektronů:

Jakož i jiné nabitě částice ionizují,

$$T_{k,\max} = \frac{2m_e\beta^2\gamma^2}{1 + 2\gamma\frac{m_e}{M} + \left(\frac{m_e}{M}\right)^2}$$

$$M = m_e$$

$$T_{k,\max} = \frac{2m_e\beta^2\gamma^2}{1 + 2\gamma + 1} = \frac{2m_e\beta^2\gamma^2}{2(1 + \gamma)} = \frac{m_e(\gamma^2 - 1)}{(1 + \gamma)} = m_e(\gamma - 1) = T_k$$

Při energiích $E > 20 \text{ MeV}$ dominuje brzdné záření – bremsstrahlung

$$-\frac{dE}{dx} = \frac{E}{X_0}$$

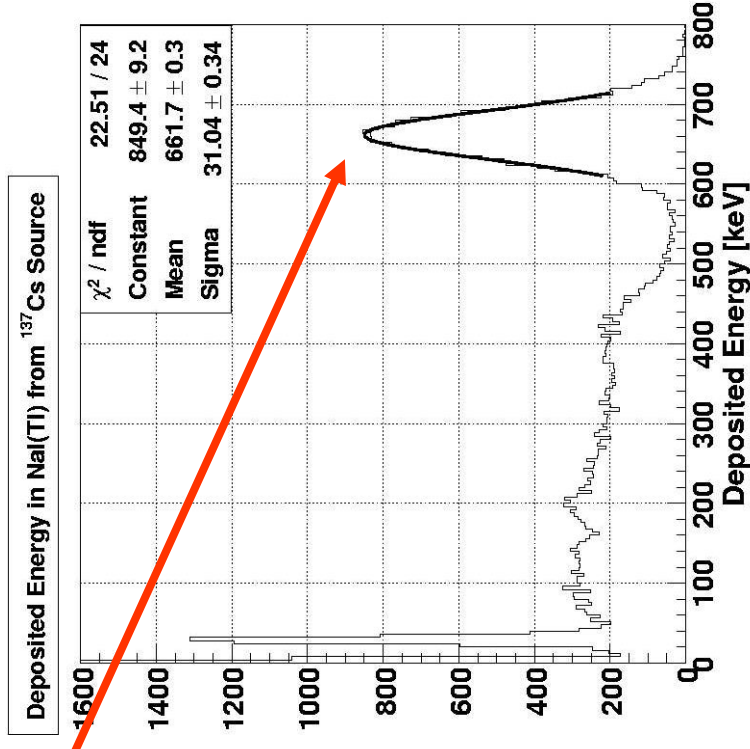
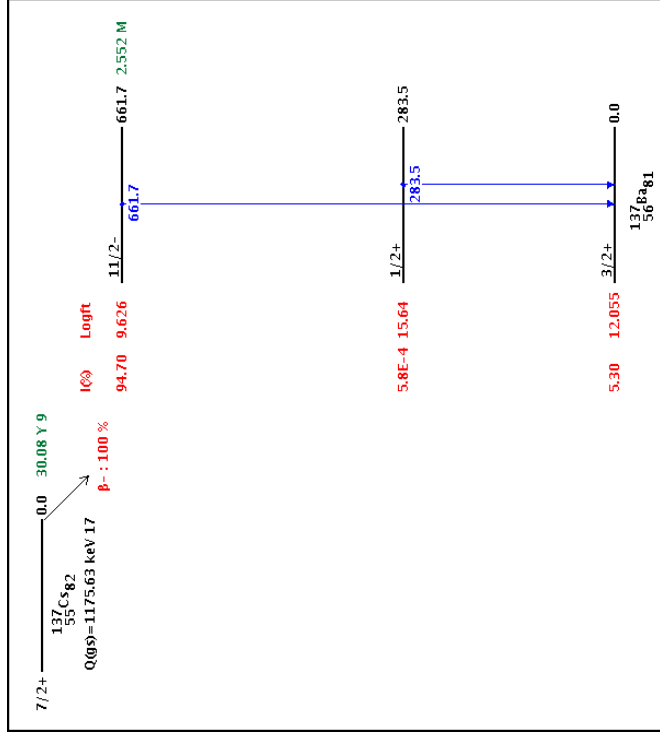
$$E = E_0 e^{-L/X_0}$$

$$X_0(\text{vzduch}) = 304m; X_0(\text{voda}) = 36cm$$

$$X_0(\text{Fe}) = 1,8cm; X_0(\text{Pb}) = 5,6mm$$

Interakce záření gama

Fotoefekt, Comptonův rozptyl, Tvorba páru elektron-positron



Compton :

$$E_\gamma + m_e = E'_\gamma + \sqrt{m_e^2 + p_e^2} \Rightarrow p_e^2 = (E_\gamma - E'_\gamma + m_e)^2 - m_e^2$$

$$E_\gamma = E'_\gamma \cos(\theta) + p_e \cos(\phi)$$

$$E'_\gamma \sin(\theta) = p_e \sin(\phi)$$

$$\Rightarrow p_e^2 = (E_\gamma - E'_\gamma \cos(\theta_\gamma))^2 + (E'_\gamma \sin(\theta_\gamma))^2$$

$$(E_\gamma - E'_\gamma \cos(\theta))^2 + (E'_\gamma \sin(\theta))^2 = (E_\gamma - E'_\gamma + m_e)^2 - m_e^2$$

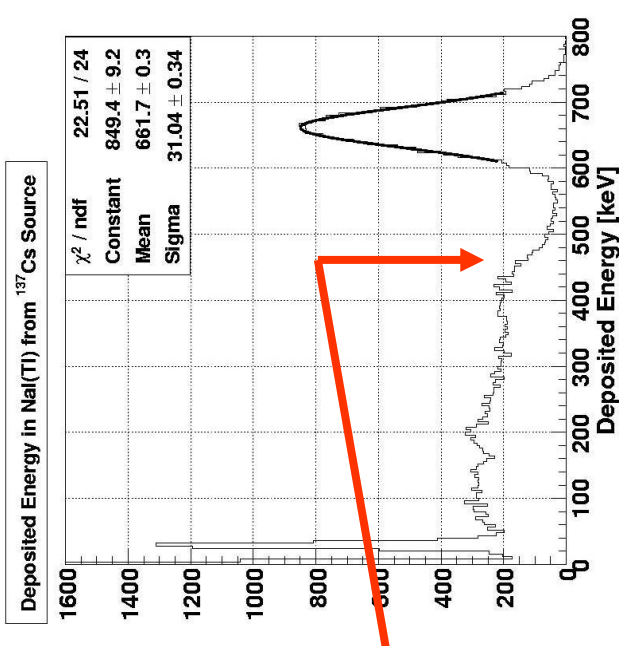
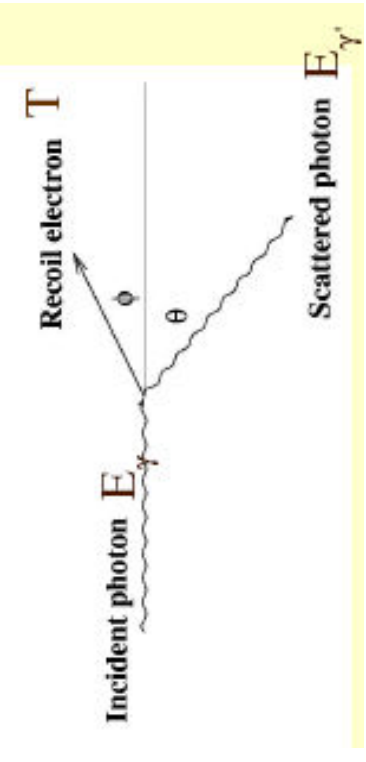
$$E_\gamma^2 - 2E_\gamma E'_\gamma \cos(\theta) + E_\gamma'^2 = E_\gamma^2 + E_\gamma'^2 - 2E_\gamma E'_\gamma + 2m_e E_\gamma - 2E_\gamma m_e$$

$$E'_\gamma (E_\gamma (1 - \cos(\theta)) + m_e) = m_e E_\gamma$$

$$E'_\gamma = \frac{E_\gamma}{1 + \frac{E_\gamma}{m_e} (1 - \cos(\theta))}$$

$$\Delta E_\gamma = E_\gamma - E'_\gamma = E_\gamma - \frac{E_\gamma}{1 + \frac{E_\gamma}{m_e} (1 - \cos(\theta))} = \frac{E_\gamma}{1 + \frac{E_\gamma}{m_e} (1 - \cos(\theta))} \frac{E_\gamma}{m_e} (1 - \cos(\theta))$$

$$\Rightarrow \Delta E_{\gamma_{\max}} = E_\gamma \frac{2 \frac{E_\gamma}{m_e}}{1 + 2 \frac{E_\gamma}{m_e}} = \frac{2E_\gamma^2}{m_e + 2E_\gamma}$$



Tvorba elektron-positronových párů

$$\gamma + M \rightarrow M + e^+ + e^-$$

Prahová energie gama:

$$\left(E_{\gamma}^{thr} + M\right)^2 - E_{\gamma}^{thr\,2} = (M + 2m_e)^2$$

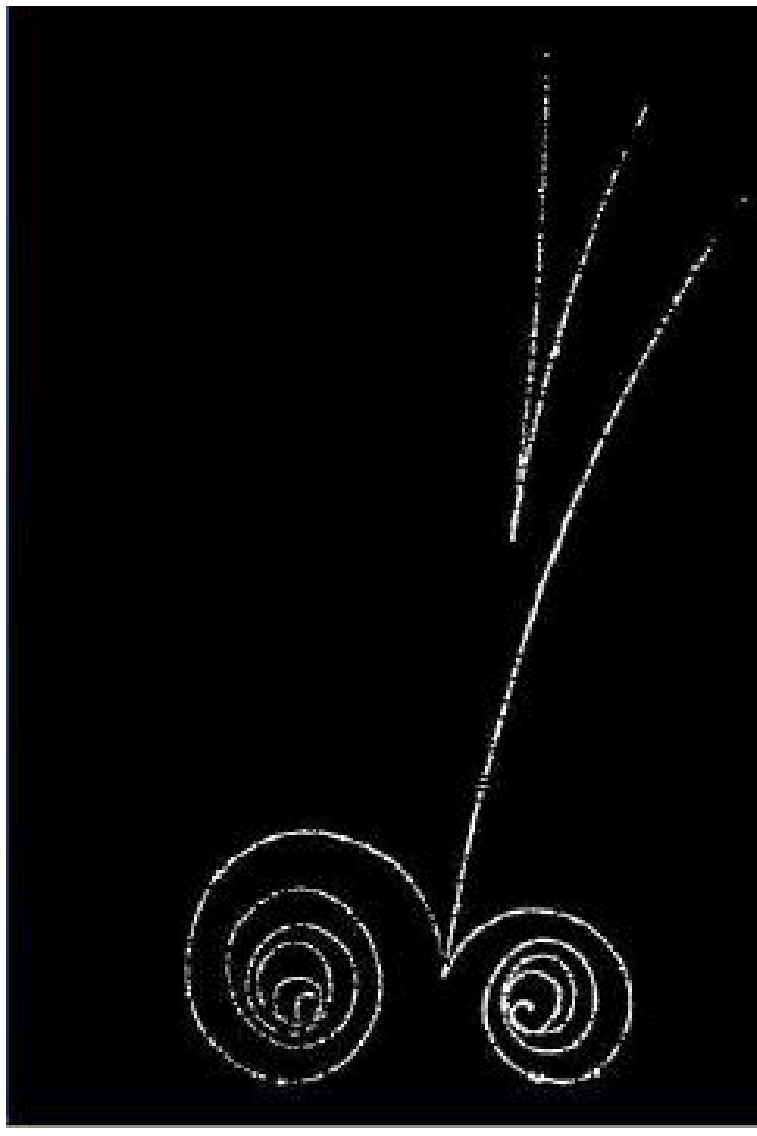
$$2E_{\gamma}^{thr}M = 4m_eM + 4m_e^2$$

$$E_{\gamma}^{thr} = 2m_e \left(1 + \frac{m_e}{M}\right)$$

$$M \gg m_e \Rightarrow E_{\gamma}^{thr} \approx 2m_e$$

$$M = m_e \Rightarrow E_{\gamma}^{thr} = 4m_e$$

$$\gamma + e^- \rightarrow e^- + e^- + e^+$$



$$P(x) = 1 - e^{-\left(\sigma_{\text{fotoefekt}} + \sigma_{\text{Compton}} + \sigma_{\text{pairs}}\right) x \frac{N_A}{A} \rho}$$

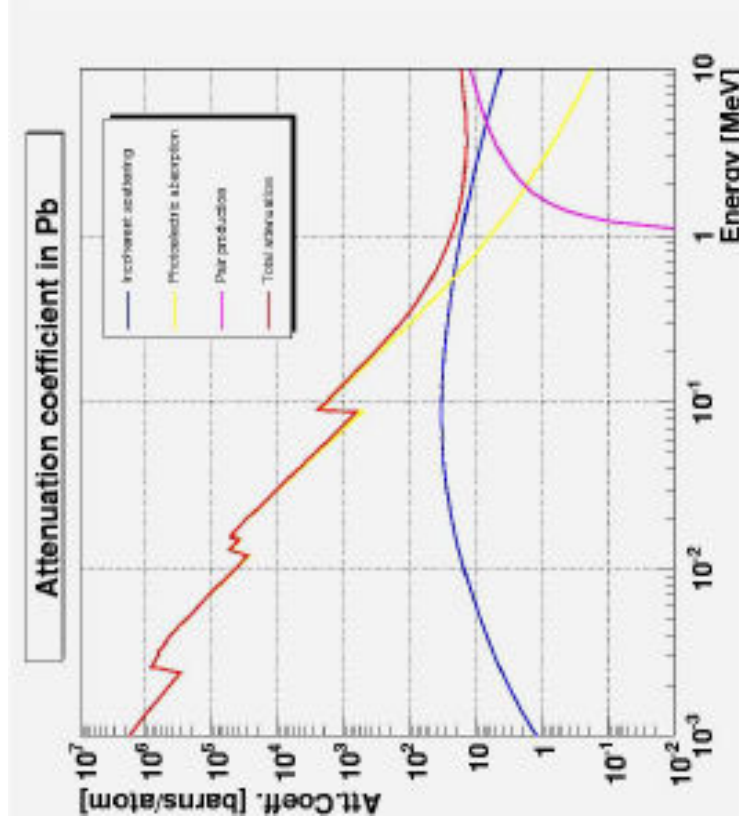
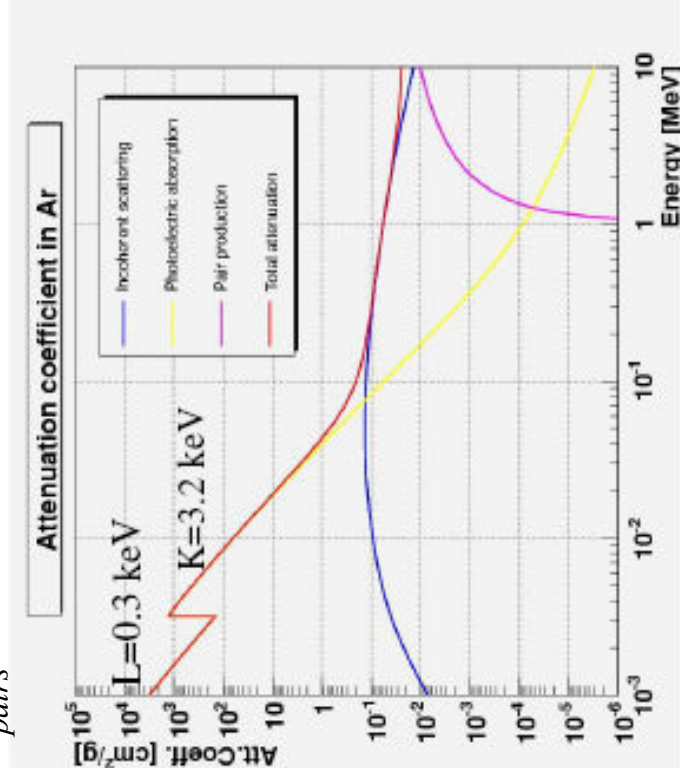
$$I(x) = I_0(1 - P(x)) = I_0 e^{-\left(\sigma_{\text{fotoefekt}} + \sigma_{\text{Compton}} + \sigma_{\text{pairs}}\right) \frac{N_A}{A} \rho x} = I_0 e^{-\mu \rho x}$$

$$\mu = \left(\sigma_{\text{fotoefekt}} + \sigma_{\text{Compton}} + \sigma_{\text{pairs}}\right) \frac{N_A}{A}$$

$$\sigma_{\text{fotoefekt}} \cong Z^{(4-5)}$$

$$\sigma_{\text{Compton}} \cong Z$$

$$\sigma_{\text{pairs}} \cong Z^2$$



Radiace, jednotky

Aktivita se měří v Becquerel Bq = 1 rozpad/s

Dávka Gray G = 1 J/kg

Sievert Sv= Gray x Quality factor – určuje jak je dané záření nebezpečné pro člověka. Nejméně nebezpečné jsou miony, gama, beta (Q=1), nejvíce alfa a ostatní těžké ionty (Q=20) a neutrony s malou energií (Q~5).
Povoleno je 50mSv za rok a 100 mSv za pět let.

1MBq kobaltového gama zářiče:

$$-\Delta N = N \frac{\Delta t}{T_{1/2} / \ln(2)}$$

$$-\Delta N / \Delta t = A = \frac{N}{T_{1/2} / \ln(2)}$$

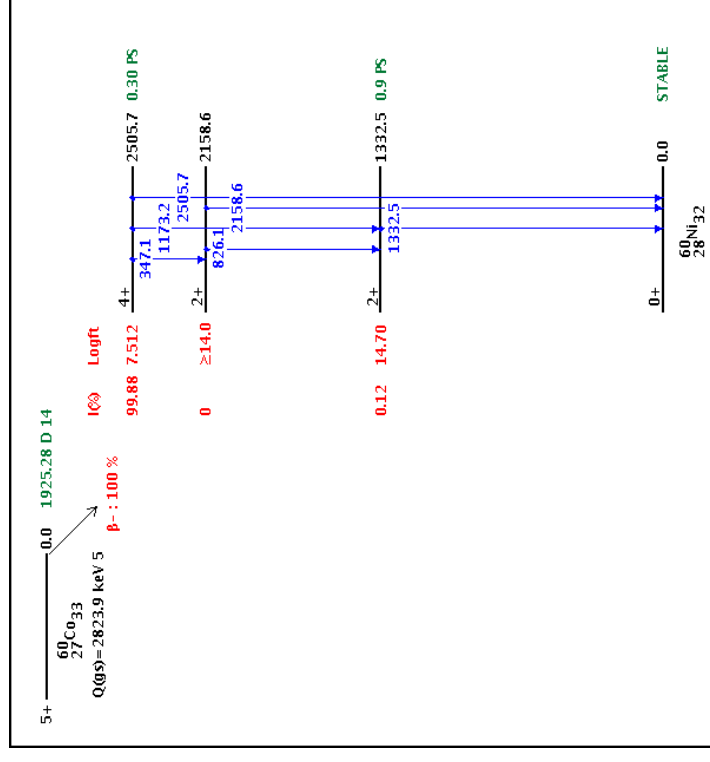
$$N = A [Bq = s^{-1}] \cdot T_{1/2} [s] / \ln(2)$$

$$A = 1MBq; T_{1/2} = 1925days$$

$$N = 10^6 s^{-1} 1925 \cdot 86400s / 0,69 = 0,24 \cdot 10^{15} \text{ } ^{60}_{27}Co$$

$$60gCo = 6,023 \cdot 10^{23} atoms$$

$$0,24 \cdot 10^{15} \text{ } ^{60}_{27}Co \Rightarrow 0,24 \cdot 10^{15} \cdot 60g / 6,023 \cdot 10^{23} = 2,4 \cdot 10^{-8} g = 24ng!$$



Dávky záření:

Miony z kosmického záření:

$$(dE/dx)_{\min} = 2\text{MeV} / \text{g} \cdot \text{cm}^{-2} \Rightarrow \rho = 1\text{g} / \text{cm}^3; (dE/dx)_{\min} = 2\text{MeV} / \text{cm} \\ 50\text{cm}$$

$$1\text{mion} / \text{s} / \text{dm}^2 \Rightarrow 6\text{mion} / \text{s}$$

$$DE = 2\text{MeV} / \text{cm} \cdot 50\text{cm} \cdot 6\text{mion} / \text{s} \cdot 31,4 \cdot 10^7 \text{s} / \text{year} = 1,88 \cdot 10^{11} \text{MeV} / \text{year} = \\ 1,88 \cdot 10^{11} \cdot 10^6 \cdot 1,6 \cdot 10^{-19} \text{J} / \text{year} = 3 \cdot 10^{-2} \text{J} / \text{year}$$

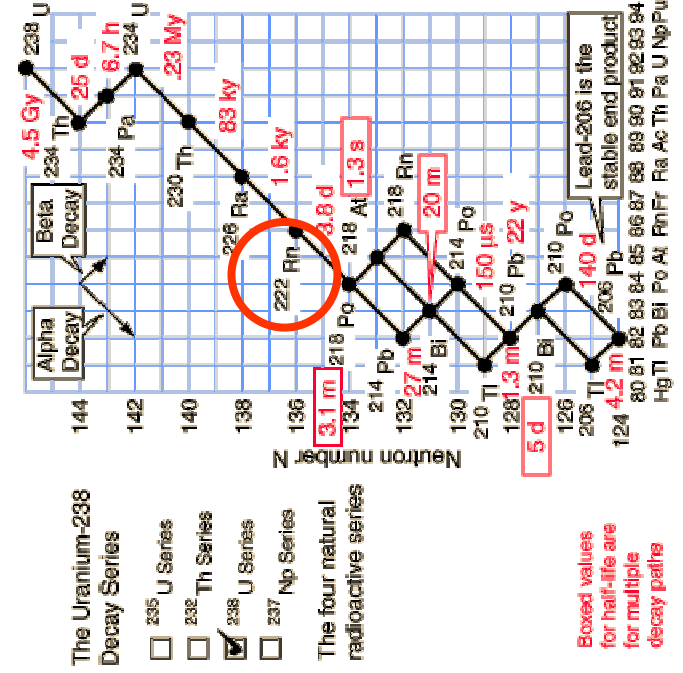
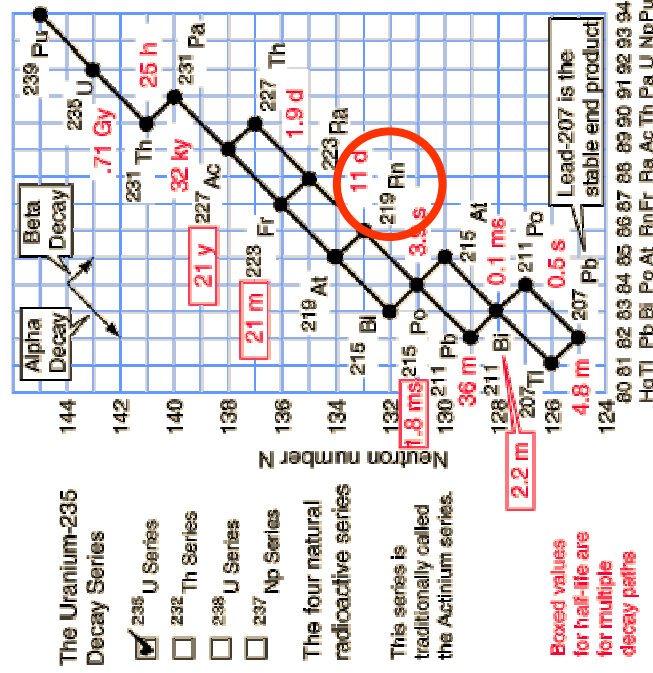
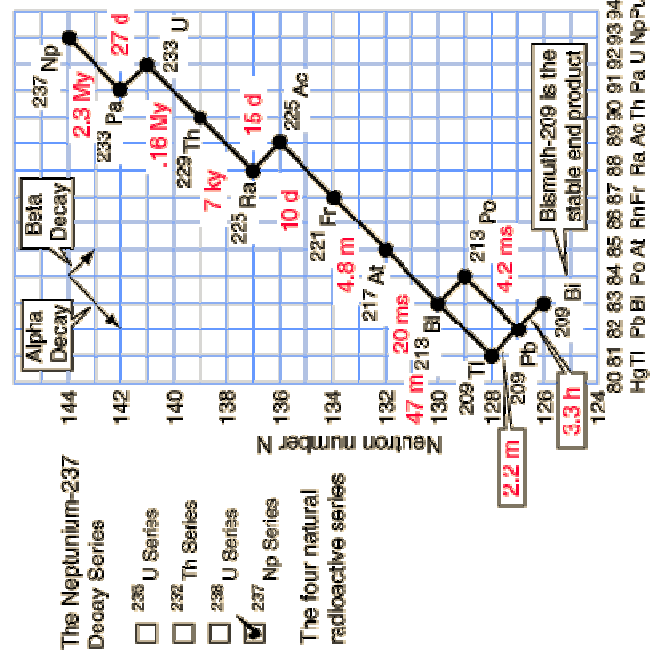
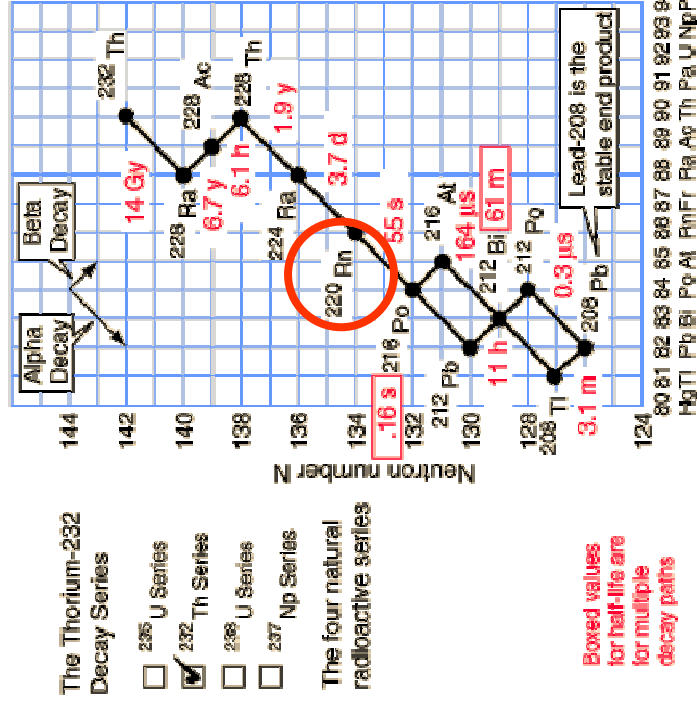
$$D = 3 \cdot 10^{-2} \text{J} / \text{year} / 80\text{kg} = 0,375\text{mJ} / \text{year} = 0,375\text{mSv} / \text{rok}$$

V Troji ve vstupní hale:

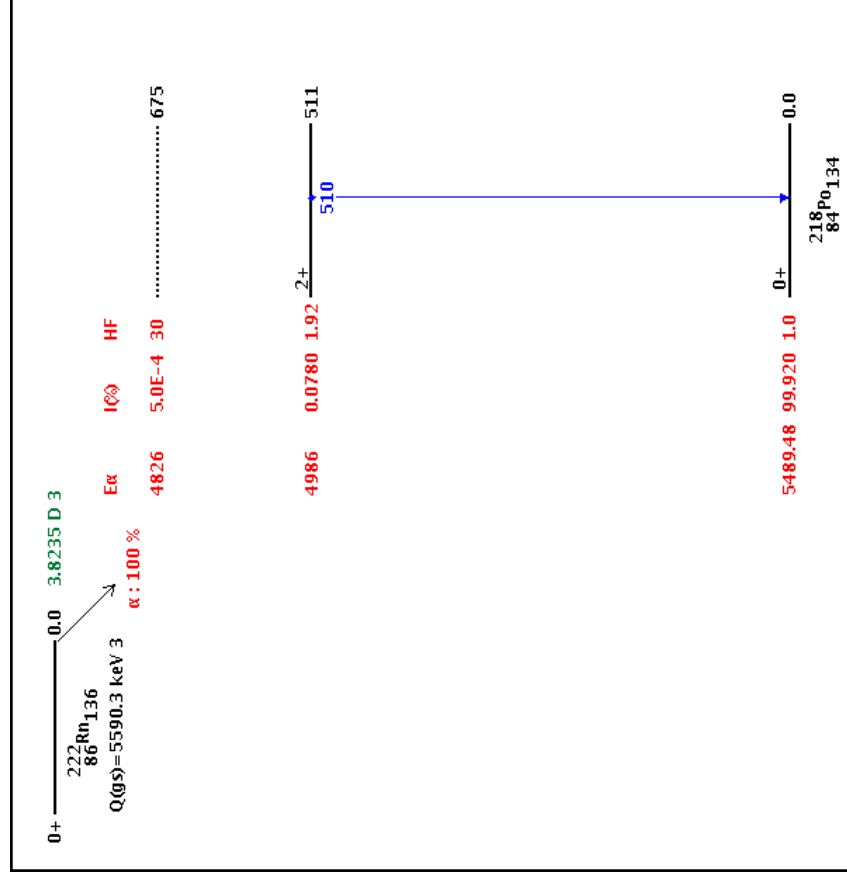
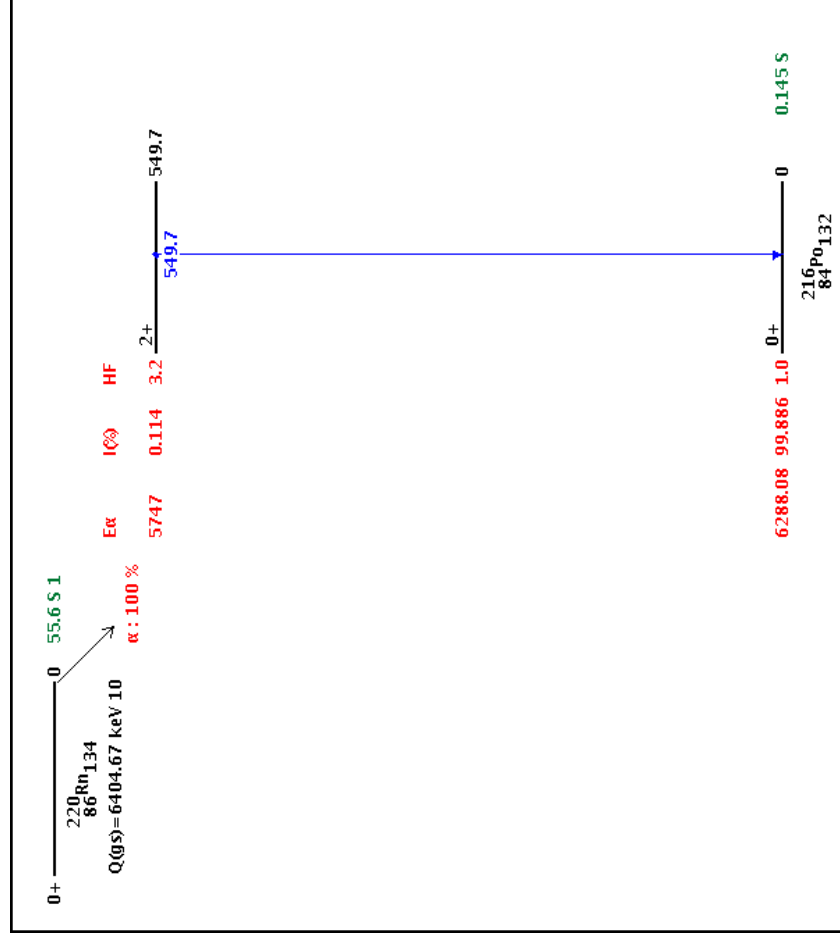
$$D = 0,09\mu\text{Sv} / \text{hod} \Rightarrow 0,09 \cdot 365 \cdot 24\mu\text{Sv} / \text{rok} = 0,788\text{mSv} / \text{rok}$$

Radon 2mSv/rok

Radon



Radon je plyn, vdechujeme ho a alfa částice mohou poškodit plíce



Následky ozáření:

Table of exposure levels and symptoms

Dose-equivalents are presently stated in sieverts:

0.05–0.2 Sv (5–20 REM)

No symptoms.

0.2–0.5 Sv (20–50 REM)

No noticeable symptoms. Red blood cell count decreases temporarily.

0.5–1 Sv (50–100 REM)

Mild radiation sickness with headache and increased risk of infection due to disruption of immunity cells. Temporary male sterility is possible.

1–2 Sv (100–200 REM)

Light radiation poisoning, 10% fatality after 30 days ([LD 10/30](#)). Typical symptoms include mild to moderate nausea (50% probability at 2 Sv), with occasional [vomiting](#), beginning 3 to 6 hours after irradiation and lasting for up to one day. This is followed by a 10 to 14 day latent phase, after which light symptoms like general illness and [fatigue](#) appear (50% probability at 2 Sv). The [immune system](#) is depressed, with convalescence extended and increased risk of infection. Temporary male sterility is common. [Spontaneous abortion](#) or [stillbirth](#) will occur in pregnant women.

2–3 Sv (200–300 REM)

Moderate radiation poisoning, 35% fatality after 30 days ([LD 35/30](#)). Nausea is common (100% at 3 Sv), with 50% risk of vomiting at 2.8 Sv. Symptoms onset at 1 to 6 hours after irradiation and last for 1 to 2 days. After that, there is a 7 to 14 day latent phase, after which the following symptoms appear: loss of hair all over the body (50% probability at 3 Sv), fatigue and general illness. There is a massive loss of [leukocytes](#) (white blood cells), greatly increasing the risk of infection. Permanent female sterility is possible. [Convalescence](#) takes one to several months.

3–4 Sv (300–400 REM)

Severe radiation poisoning, 50% fatality after 30 days ([LD 50/30](#)). Other symptoms are similar to the 2–3 Sv dose, with uncontrollable bleeding in the mouth, under the skin and in the kidneys (50% probability at 4 Sv) after the latent phase. 180 Sv (18,000 REM) to his upper body in an accident at Los Alamos, New Mexico, USA on [30 December 1958](#) survived for 36 hours.

Následky ozáření:

4–6 Sv (400–600 REM)

Acute radiation poisoning, 60% fatality after 30 days ([LD 60/30](#)). Fatality increases from 60% at 4.5 Sv to 90% at 6 Sv (unless there is intense medical care). Symptoms start half an hour to two hours after irradiation and last for up to 2 days. After that, there is a 7 to 14 day latent phase, after which generally the same symptoms appear as with 3–4 Sv irradiation, with increased intensity. Female sterility is common at this point. Convalescence takes several months to a year. The primary causes of death (in general 2 to 12 weeks after irradiation) are infections and [internal bleeding](#).

6–10 Sv (600–1,000 REM)

Acute radiation poisoning, near 100% fatality after 14 days ([LD 100/14](#)). Survival depends on intense medical care. [Bone marrow](#) is nearly or completely destroyed, so a [bone marrow transplant](#) is required. Gastric and intestinal tissue are severely damaged. Symptoms start 15 to 30 minutes after irradiation and last for up to 2 days. Subsequently, there is a 5 to 10 day latent phase, after which the person dies of infection or [internal bleeding](#). Recovery would take several years and probably would never be complete.

Devair Alves Ferreira received a dose of approximately 7.0 Sv (700 REM) during the [Goiânia accident](#) and survived, partially due to his [fractionated exposure](#).

10–50 Sv (1,000–5,000 REM)

Acute radiation poisoning, 100% fatality after 7 days ([LD 100/7](#)). An exposure this high leads to spontaneous symptoms after 5 to 30 minutes. After powerful fatigue and immediate nausea caused by direct activation of chemical receptors in the brain by the irradiation, there is a period of several days of comparative well-being, called the latent (or "[walking ghost](#)") phase[[citation needed](#)]. After that, cell death in the gastric and intestinal tissue, causing massive [diarrhea](#), intestinal bleeding and loss of water, leads to water-electrolyte imbalance. Death sets in with [delirium](#) and coma due to breakdown of circulation. Death is currently inevitable; the only treatment that can be offered is [pain therapy](#).

[Louis Slotin](#) was exposed to approximately 21 Sv in a [criticality accident](#) on 21 May 1946, and died nine days later on 30 May. At this dose the skin can be damaged. Here is a photo of a man who received a 10 to 20 Gy gamma whole body dose as a result of an industrial accident. He died about 10 days after the photo was taken, about 30 days after the event.

More than 50 Sv (>5,000 REM)

A worker receiving 100 Sv (10,000 REM) in an accident at Wood River, Rhode Island, USA on [24 July 1964](#) survived for 49 hours after exposure, and an operator receiving between 60 and 180 Sv (18,000 REM) to his upper body in an accident at Los Alamos, New Mexico, USA on [30 December 1958](#) survived for 36 hours.