Přednáška 5.(29.10.2007)

Měření tvaru jader pomocí mionových atomů.

Štěpení těžkých jader (fission)

Jaderné reakce.

Fúze lehkých jader (fusion)

Objev positronu v kosmickém záření

Atom vodíku - opakování

$$l = 0$$

$$-\frac{(\hbar c)^2}{2m_e}\frac{d^2}{dr^2}u(r) - \alpha\frac{\hbar c}{r}u(r) = Eu(r)$$

$$r_B = \frac{(\hbar c)^2}{2m_e}\frac{2}{\alpha\hbar c} = \frac{\hbar c}{m_e\alpha} = \frac{197 MeVfm}{0{,}511 MeV/137} = 52816\, fm$$

$$E = -\frac{(\hbar c)^2}{2m_e}\frac{1}{r_B^2} = -\frac{(\hbar c)^2}{2m_e}\frac{(m_e\alpha)^2}{(\hbar c)^2} = -\frac{1}{2}m_e\alpha^2 = -\frac{1}{2}0{,}511 MeV\frac{1}{137^2} = -13{,}6 eV$$

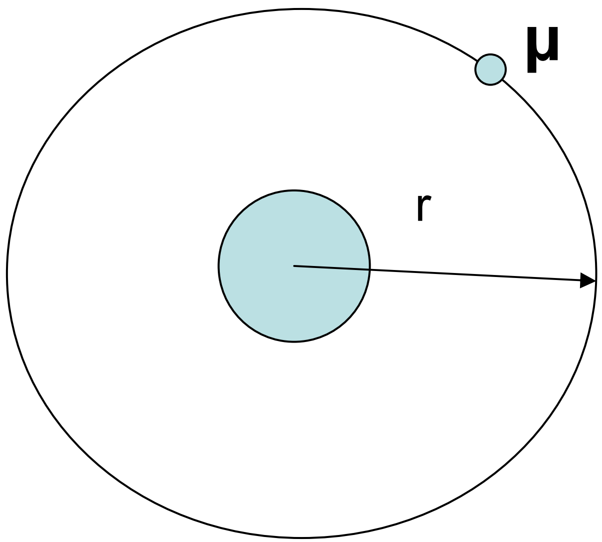
$$E = -\frac{1}{2}m_e\alpha^2 \,;\, E = T + V = \frac{1}{2}m_e\alpha^2 - 2\frac{1}{2}m_e\alpha^2 = 13{,}6 eV - 27{,}2 eV$$

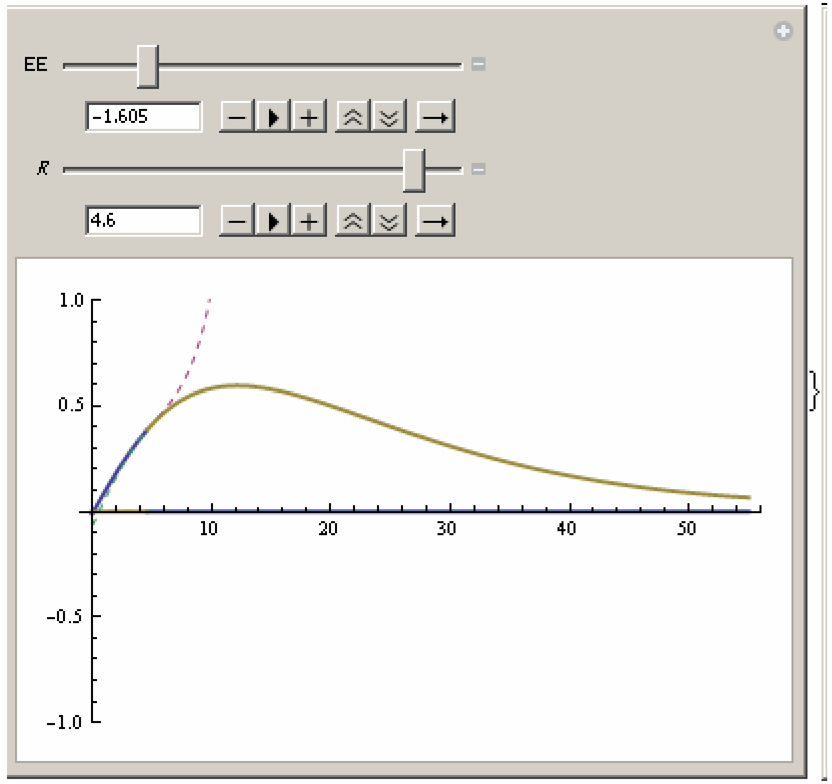
$$T = \frac{1}{2}m_e\left(\frac{v}{c}\right)^2 = \frac{1}{2}m_e\beta^2 \;\; ; \;\; \text{rychlost elektronu } \beta = \frac{1}{137} = \alpha$$

atom s Z protony:

$$E = -\frac{1}{2}m_e(Z\alpha)^2 = Z^2 13{,}6 eV \,;\, E_{Fe} = -26^2 13{,}6 eV = -9{,}19 keV$$

Mionový atom: elektron je nahrazen záporně nabitým mionem





$$r = r_B \frac{m_e}{m_\mu}\frac{1}{Z} \Rightarrow r = 52 \cdot 10^3\, fm \frac{0{,}511 MeV}{105{,}6 MeV}\frac{1}{26} \cong 9{,}7\, fm$$

$$R = 0 \Rightarrow E = -13{,}6 eV \cdot \frac{m_\mu}{m_e} \cdot Z^2{}_{Fe} = -13{,}6 eV \cdot \frac{105{,}6 MeV}{0{,}511 MeV} \cdot 26^2 = -1{,}9 MeV$$

$$R = 1{,}2\, fm \cdot \sqrt[3]{A_{Fe}} = 1{,}2\, fm \cdot \sqrt[3]{56} \cong 4{,}6\, fm \Rightarrow E \cong -1{,}605 MeV$$

The Nobel Prize in Chemistry 1944

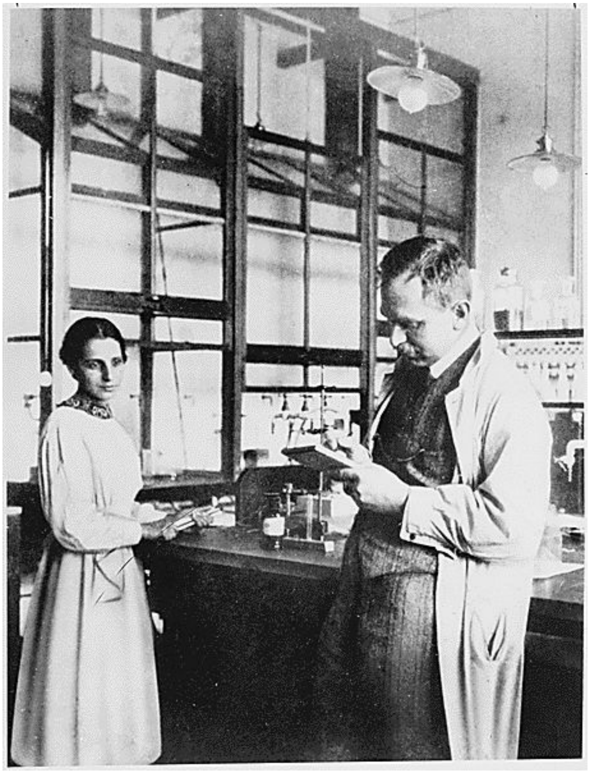
"for his discovery of the fission of heavy nuclei"

Otto Hahn

Germany

Kaiser-Wilhelm-Institut
(now Max-Planck Institut)
für Chemie
Berlin-Dahlem, Germany

b. 1879
d. 1968



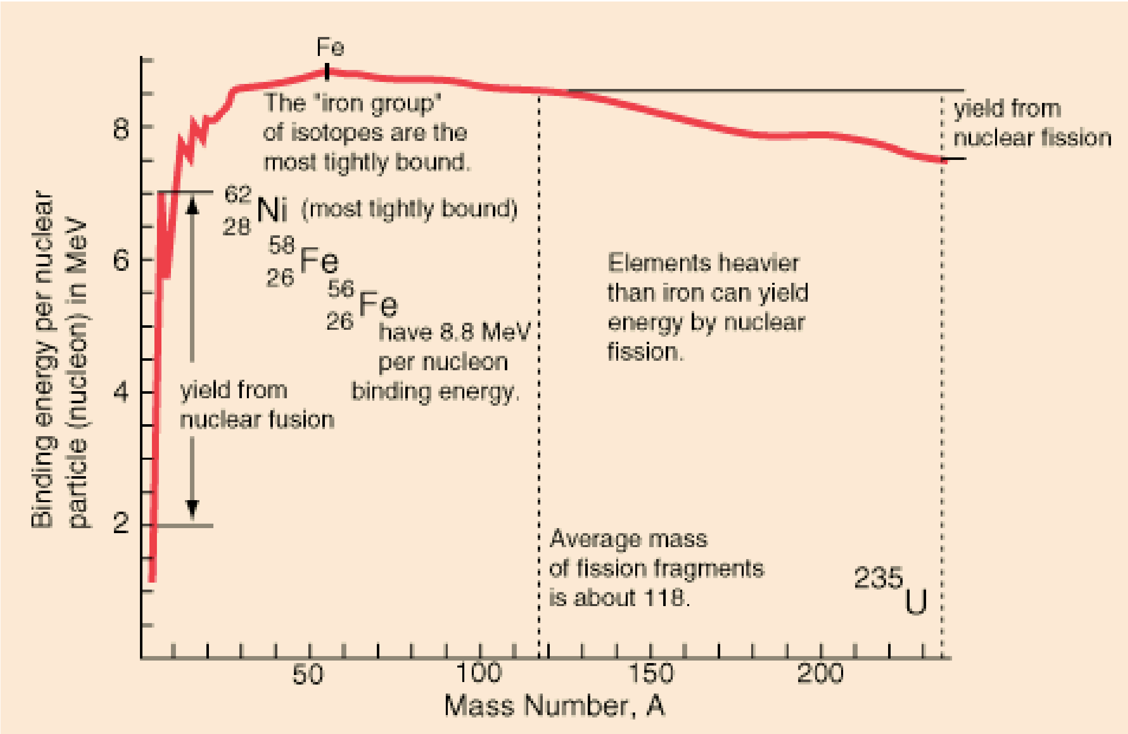
O. Hahn a L. Meitner ozařovali Uran neutrony ve snaze vyrobit prvky těžší než Uran. K velkému překvapení však objevili, že se rodí Barium, tj. že se Uran štěpí na lehčí prvky.

Energie uvolněná při štěpení

Příklad vznik Ba

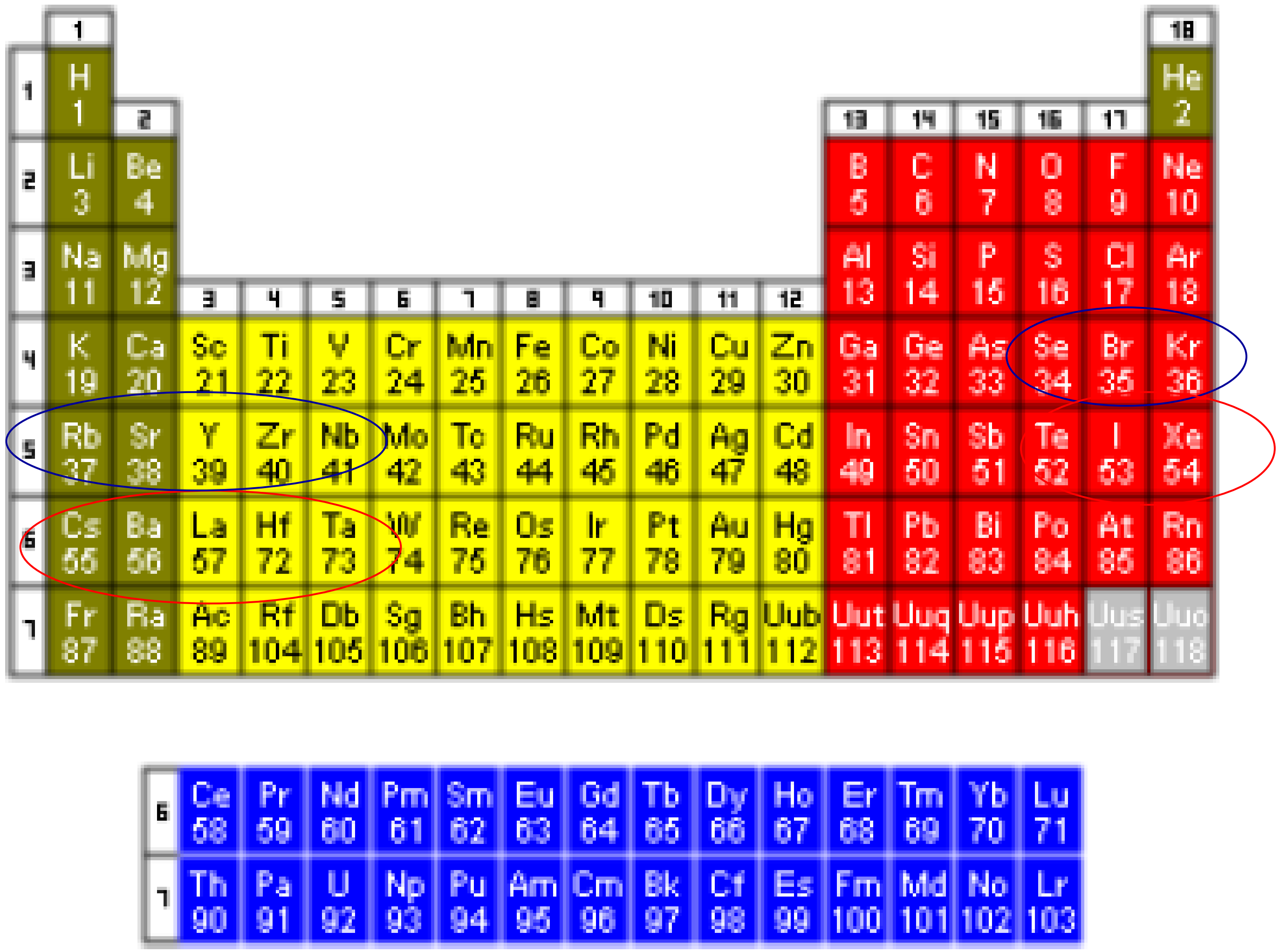
$$ {}_0^1n + {}_{92}^{235}U \rightarrow {}_{56}^{140}Ba + {}_{36}^{93}Kr + 3\,{}_0^1n $$

$$ Q = m_n + 92m_p + 143m_n - B(235,92) - $$
$$ \left(56m_p + 84m_n - B(140,56) + 36m_p + 57m_n - B(93,36) + 3m_n\right) = $$
$$ B(140,56) + B(93,36) - B(235,92) \cong 160MeV $$

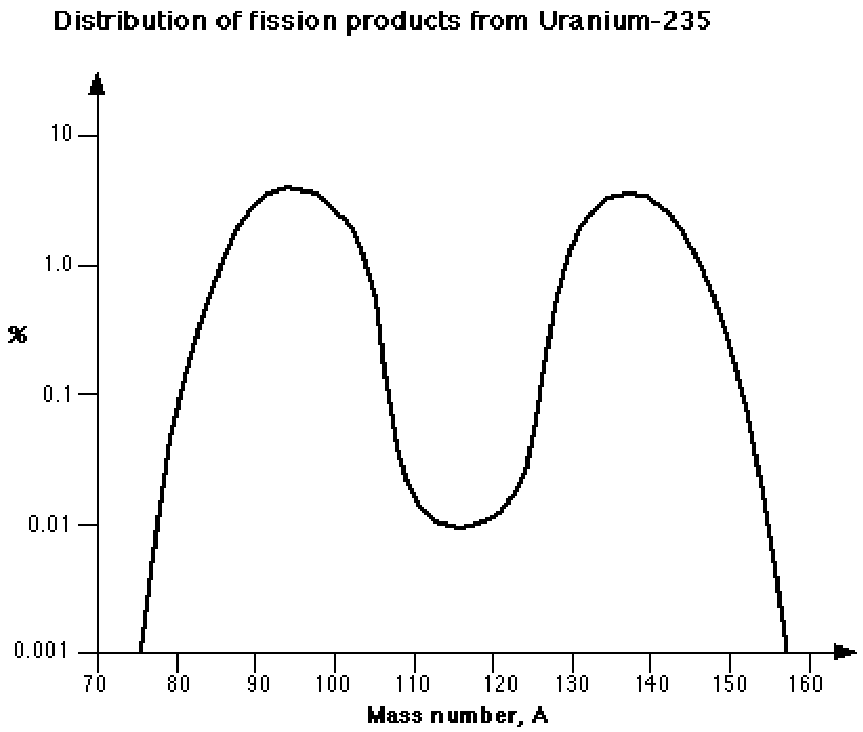


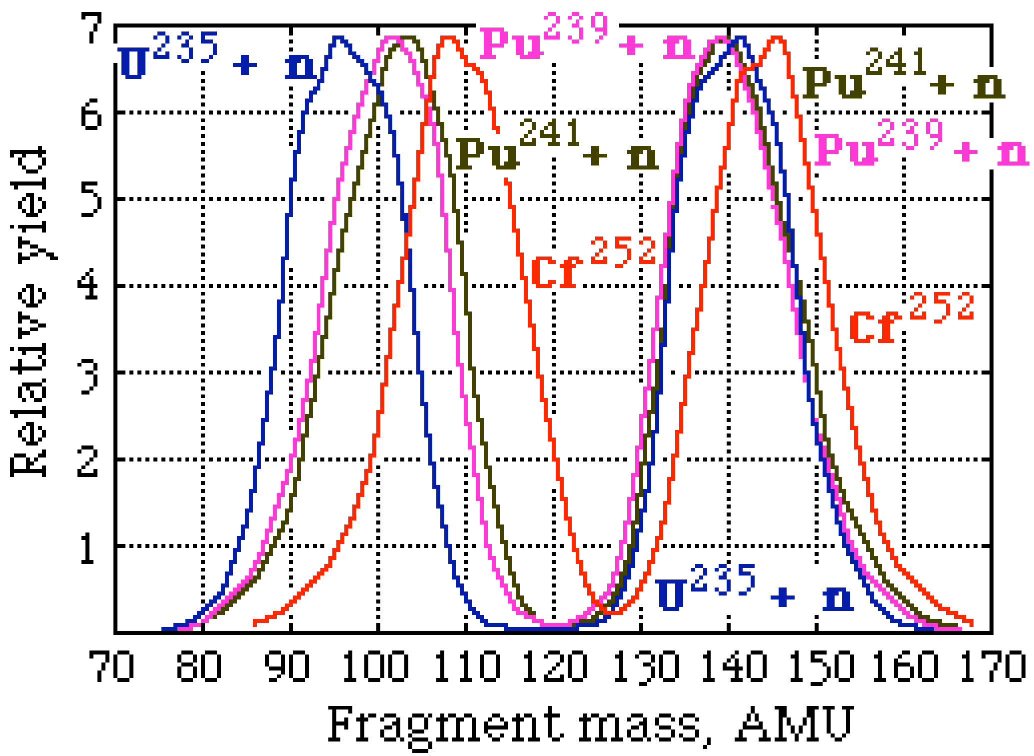
K tomu ještě 5MeV neutrony a 35 MeV radioaktivní rozpady celkem tedy asi 200 MeV

Vzniklá jádra mají příliš mnoho neutronů = nacházejí se mimo údolí stability.
Stabilita je pak dosažena beta-rozpady.

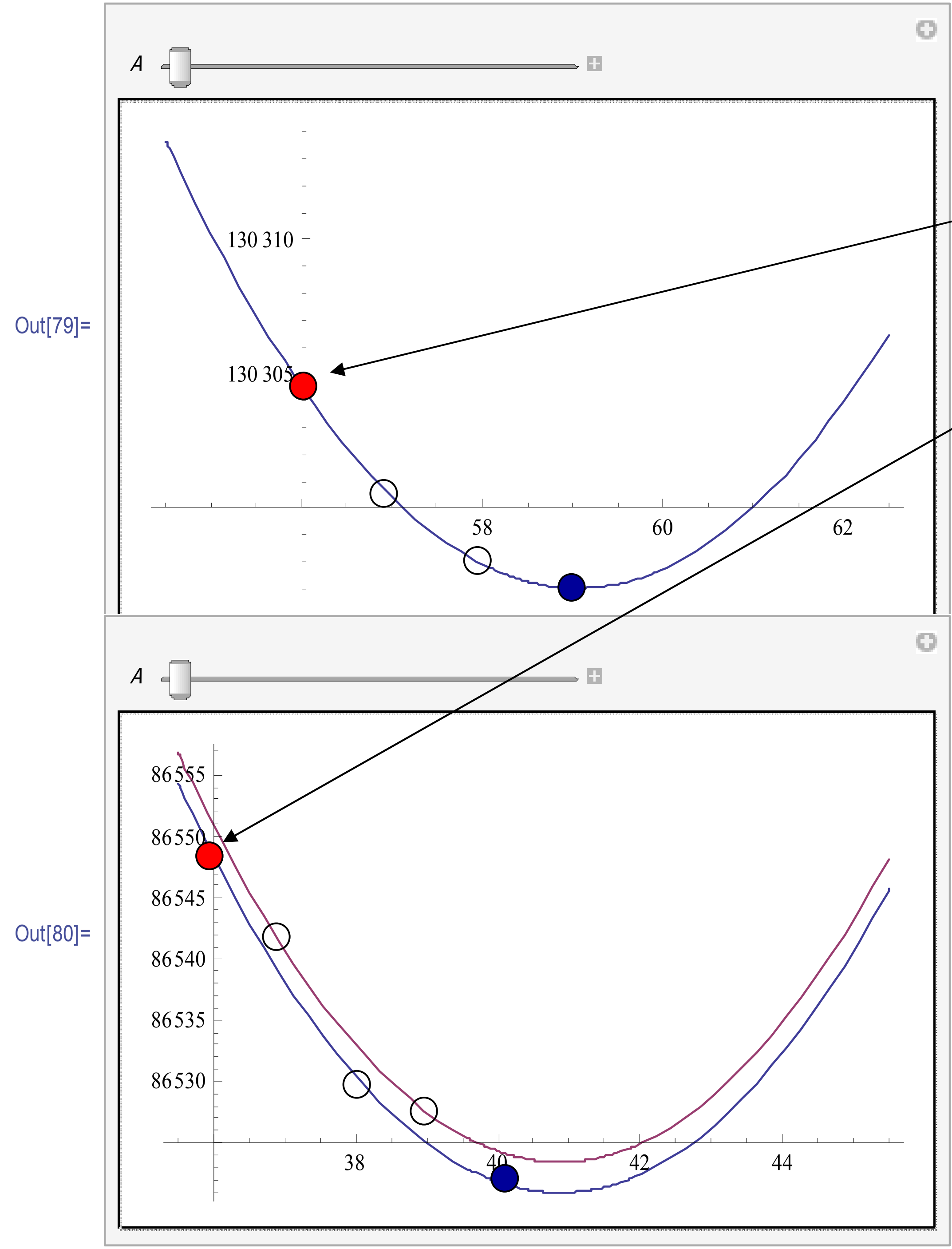

1 2 13 14 15 16 17 18
3 4 5 6 7 8 9 10 11 12
1 H 1 He 2
2 Li 3 Be 4 B 5 C 6 N 7 O 8 F 9 Ne 10
3 Na 11 Mg 12 Al 13 Si 14 P 15 S 16 Cl 17 Ar 18
4 K 19 Ca 20 Sc 21 Ti 22 V 23 Cr 24 Mn 25 Fe 26 Co 27 Ni 28 Cu 29 Zn 30 Ga 31 Ge 32 As 33 Se 34 Br 35 Kr 36
5 Rb 37 Sr 38 Y 39 Zr 40 Nb 41 Mo 42 Tc 43 Ru 44 Rh 45 Pd 46 Ag 47 Cd 48 In 49 Sn 50 Sb 51 Te 52 I 53 Xe 54
6 Cs 55 Ba 56 La 57 Hf 72 Ta 73 W 74 Re 75 Os 76 Ir 77 Pt 78 Au 79 Hg 80 Tl 81 Pb 82 Bi 83 Po 84 At 85 Rn 86
7 Fr 87 Ra 88 Ac 89 Rf 104 Db 105 Sg 106 Bh 107 Hs 108 Mt 109 Ds 110 Rg 111 Uub 112 Uut 113 Uuq 114 Uup 115 Uuh 116 Uus 117 Uuo 118
6 Ce 58 Pr 59 Nd 60 Pm 61 Sm 62 Eu 63 Gd 64 Tb 65 Dy 66 Ho 67 Er 68 Tm 69 Yb 70 Lu 71
7 Th 90 Pa 91 U 92 Np 93 Pu 94 Am 95 Cm 96 Bk 97 Cf 98 Es 99 Fm 100 Md 101 No 102 Lr 103

Štěpení je převážně asymetrické, existuje silná tendence produkovat těžké fragmenty s A okolo 140 a 90. Důvodem jsou magická jádra.





$$ {}_{0}^{1}n + {}_{92}^{235}U \rightarrow {}_{56}^{140}Ba + {}_{36}^{93}Kr + 3\,{}_{0}^{1}n $$



Vzniklá jádra mají příliš mnoho neutronů = nacházejí se mimo údolí stability.

Stabilita je pak dosažena beta-rozpady.

Porovnání chemické a jaderné energie

$$C + O_2 \rightarrow CO_2 + 4{,}1eV$$

$$12g\ C\ \ 6{,}023 \cdot 10^{23} \cdot 4{,}1eV = 6{,}023 \cdot 10^{23} \cdot 6{,}56 \cdot 10^{-19}\,J = 39{,}5 \cdot 10^{4}\,J =$$

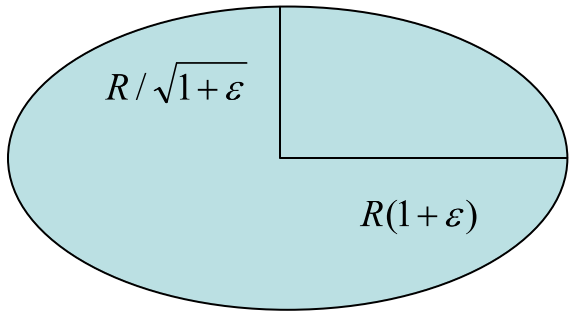
$$395kJ \Rightarrow 1g\ C\ \ 32{,}9kJ$$

$$^{235}_{92}U \rightarrow X_1 + X_2 + 3n + 200MeV$$

$$235g\ \ ^{235}_{92}U\ \ 6{,}023 \cdot 10^{23} \cdot 200MeV = 6{,}023 \cdot 10^{23} \cdot 200 \cdot 10^{6} \cdot 1{,}602 \cdot 10^{-19}\,J =$$

$$1930 \cdot 10^{10}\,J = 19300GJ \Rightarrow 1g\ \ ^{235}_{92}U\ \ 82{,}1GJ$$

Deformace jader



$$V=\frac{4}{3}\pi\left(\frac{R}{\sqrt{1+\varepsilon}}\right)^2 R(1+\varepsilon)=\frac{4}{3}\pi R^3$$

$$\left(\frac{x}{R(1+\varepsilon)}\right)^2+\left(\frac{y}{R/\sqrt{1+\varepsilon}}\right)^2=1\Rightarrow y(x)=\sqrt{\frac{R^2}{1+\varepsilon}-\frac{x^2}{(1+\varepsilon)^3}}\;;$$

```
In[115]:= "ELIPSOID"
          y[x_] := Sqrt[R^2/(1 + ϵ) - x^2/(1 + ϵ)^3]
          "OBJEM"
          Simplify[Series[(y[x])^2, {ϵ, 0, 4}], R > 0 && x > 0 && R > x];
          (π ∫_{-R (1+ϵ)}^{R (1+ϵ)} (%) dx) / (4 π R^3 / 3)
          "POVRCH"
          Simplify[Series[z[x] Sqrt[1 + (∂x y[x])^2], {ϵ, 0, 4}], R > 0 && x > 0 && R > x];
          (2 π ∫_{-R (1+ϵ)}^{R (1+ϵ)} (%) dx) / (4 π R^2)
```

$$S=4\pi R^2\left(1+\frac{2}{5}\varepsilon^2\right)$$

Out[115]= ELIPSOID

Out[117]= OBJEM

Out[119]= $1+O(\epsilon^5)$

Out[120]= POVRCH

Out[122]= $1+\frac{2\epsilon^2}{5}-\frac{52\epsilon^3}{105}+\frac{11\epsilon^4}{21}+O(\epsilon^5)$

$$R^2\rightarrow R^2\left(1+\frac{2}{5}\varepsilon^2\right)$$

$$\frac{1}{R}\rightarrow\frac{1}{R}\left(1-\frac{1}{5}\varepsilon^2\right)$$

$$B(A,Z,\varepsilon) = A\cdot 15{,}6MeV - A^{2/3}\left(1+\frac{2}{5}\varepsilon^2\right)\cdot 17{,}2MeV - \frac{Z^2}{A^{1/3}}\left(1-\frac{1}{5}\varepsilon^2\right)\cdot 0{,}7MeV$$

$$-\frac{(A-2Z)^2}{A}\cdot 23{,}3MeV + \begin{matrix} -\frac{1}{A^{1/2}}12{,}0MeV & \text{.... licho-lichá} \\ 0\text{.... licho-sudá a sudo-lichá} & \\ +\frac{1}{A^{1/2}}12{,}0MeV & \text{.... sudo-sudá} \end{matrix}$$

$B(A,Z,\varepsilon) > B(A,Z,\varepsilon=0) \Rightarrow$ **Spontánní štěpení**

$$-A^{2/3}\left(1+\frac{2}{5}\varepsilon\right)\cdot 17{,}2MeV - \frac{Z^2}{A^{1/3}}\left(1-\frac{1}{5}\varepsilon\right)\cdot 0{,}7MeV > -A^{2/3}\cdot 17{,}2MeV - \frac{Z^2}{A^{1/3}}\cdot 0{,}7MeV$$

$$-A^{2/3}\frac{2}{5}\varepsilon\cdot 17{,}2MeV + \frac{Z^2}{A^{1/3}}\frac{1}{5}\varepsilon\cdot 0{,}7MeV > 0$$

$$\frac{Z^2}{A^{1/3}}\frac{1}{5}\cdot 0{,}7MeV > A^{2/3}\frac{2}{5}\cdot 17{,}2MeV$$

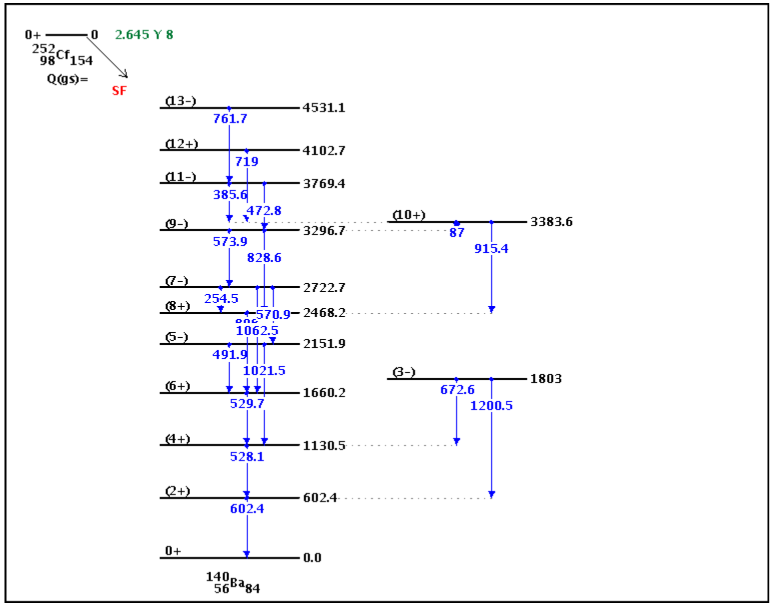
$$\boxed{\frac{Z^2}{A} > \frac{34{,}4}{0{,}7} \cong 49}$$

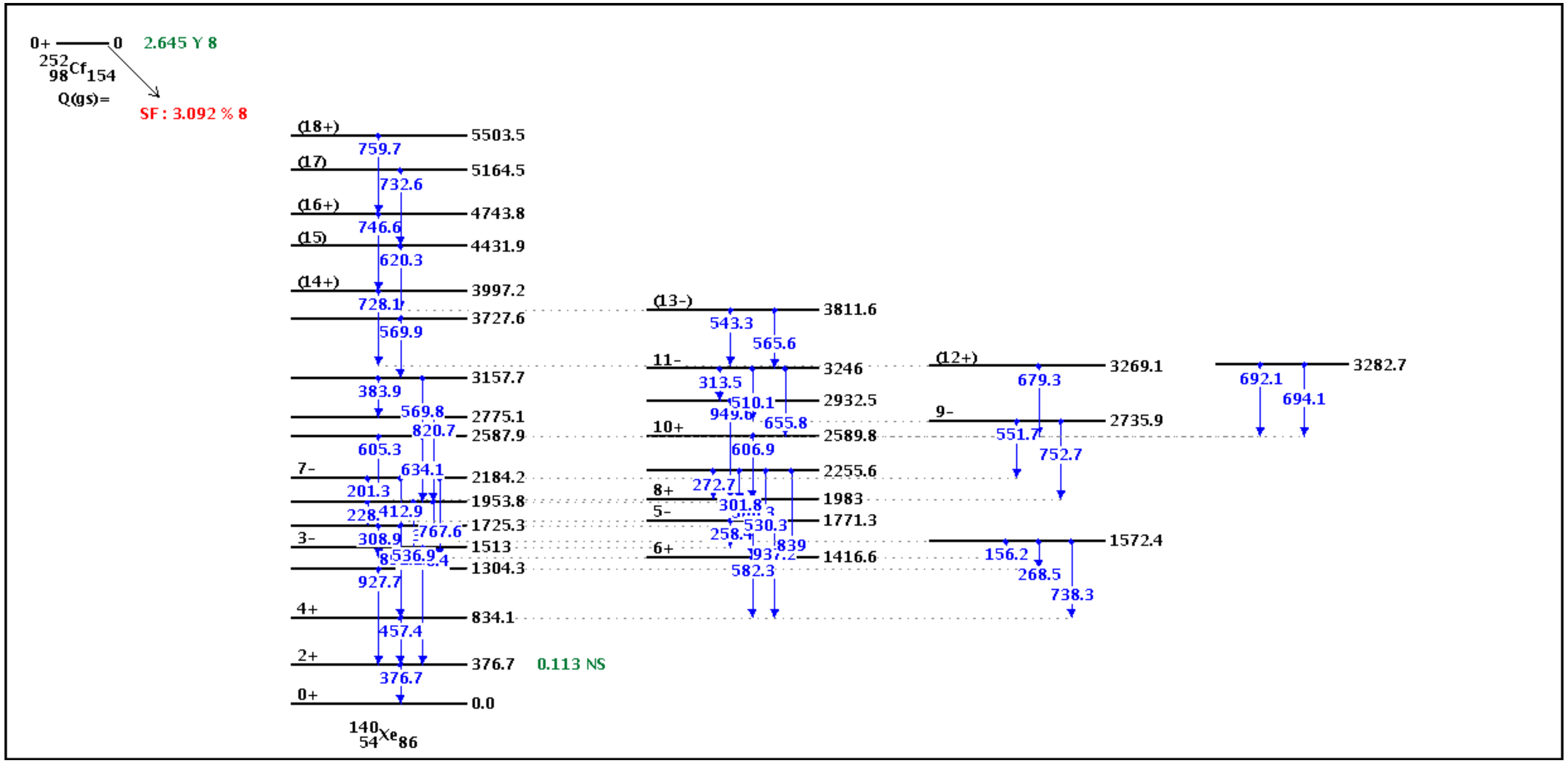
$${}^{235}_{92}U : Z^2 / A = 36$$

$${}^{252}_{98}Cf : Z^2 / A = 38$$

$${}^{252}_{98}Cf \rightarrow {}^{140}_{54}Xe + {}^{112-x}_{44}Ru + x \cdot n$$

$${}^{252}_{98}Cf \rightarrow {}^{140}_{56}Ba + {}^{112-x}_{42}Mo + x \cdot n$$





$$B(A,Z,\varepsilon) = A\cdot 15{,}6MeV - A^{2/3}\left(1+\frac{2}{5}\varepsilon^2\right)\cdot 17{,}2MeV - \frac{Z^2}{A^{1/3}}\left(1-\frac{1}{5}\varepsilon^2\right)\cdot 0{,}7MeV$$

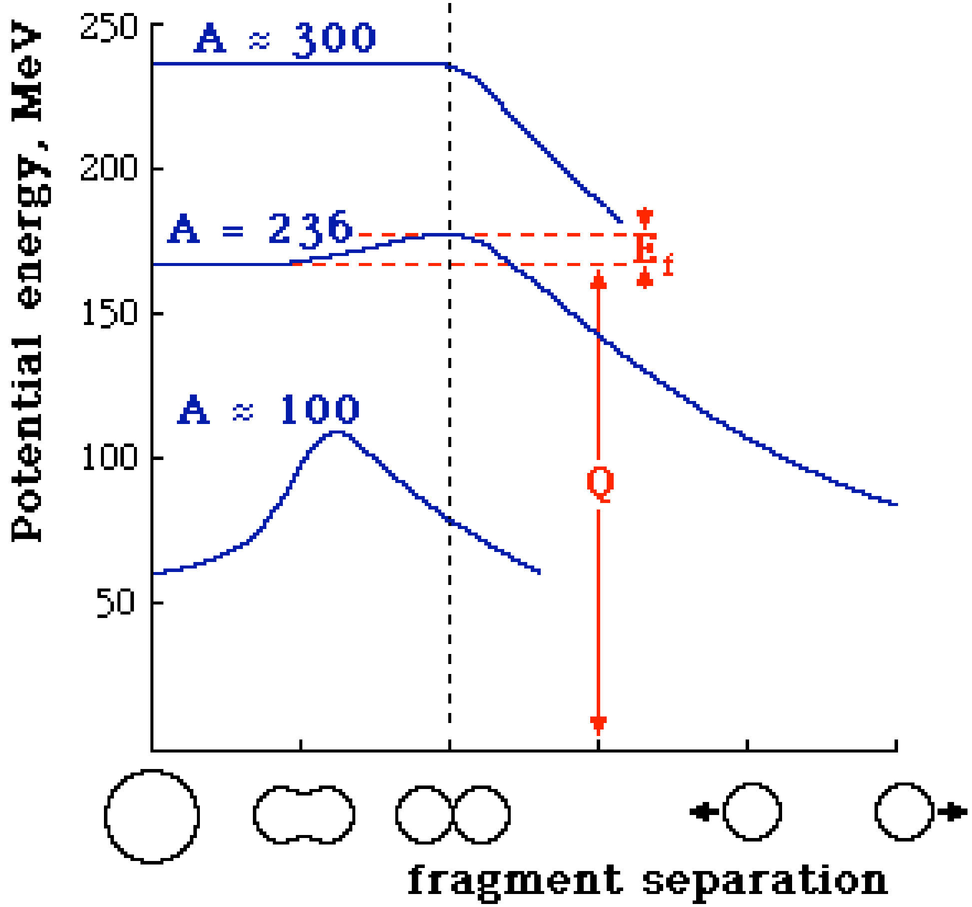
$$-\frac{(A-2Z)^2}{A}\cdot 23{,}3MeV + \begin{array}{l} -\dfrac{1}{A^{1/2}}12{,}0MeV \quad \text{.... licho-lichá} \\ 0\text{.... licho-sudá a sudo-lichá} \\ +\dfrac{1}{A^{1/2}}12{,}0MeV \quad \text{.... sudo-sudá} \end{array}$$

$$B(A,Z,\varepsilon) - B(A,Z,\varepsilon=0) =$$

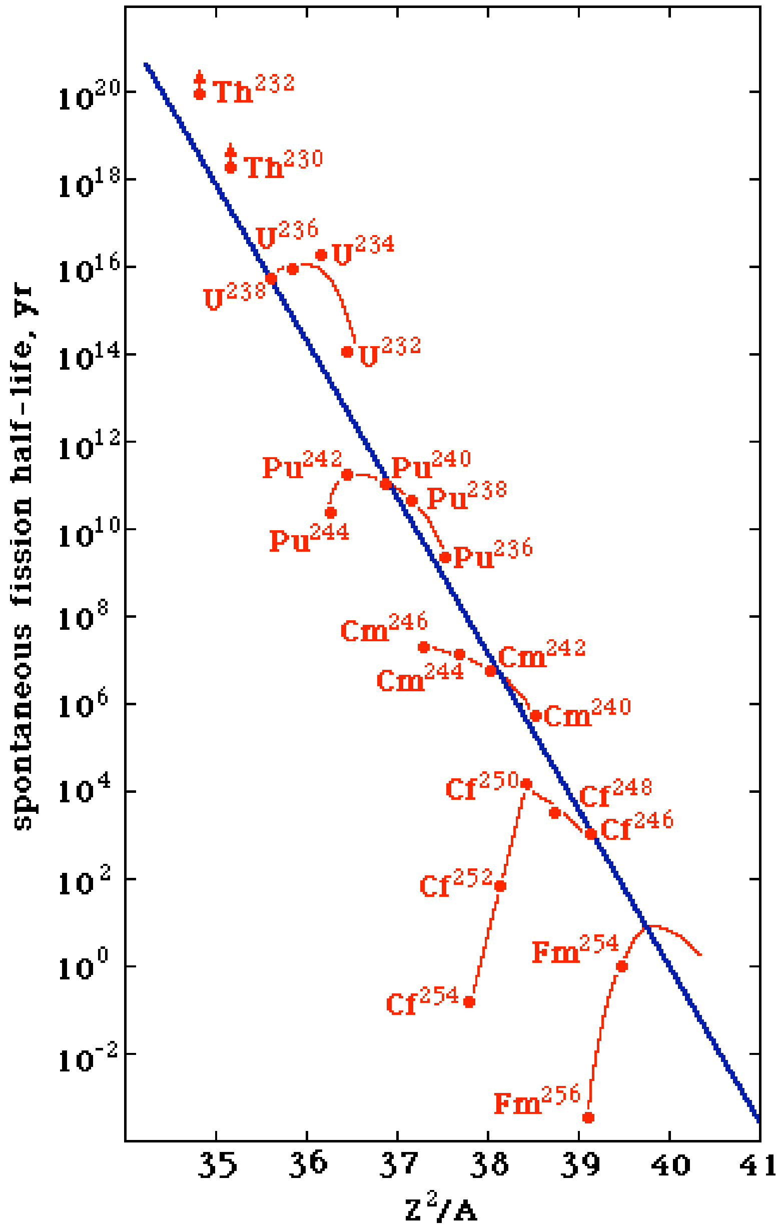
$$-A^{2/3}\left(1+\frac{2}{5}\varepsilon^2\right)\cdot 17{,}2MeV - \frac{Z^2}{A^{1/3}}\left(1-\frac{1}{5}\varepsilon^2\right)\cdot 0{,}7MeV - \left(-A^{2/3}\cdot 17{,}2MeV - \frac{Z^2}{A^{1/3}}\cdot 0{,}7MeV\right) =$$

$$-A^{2/3}\frac{2}{5}\varepsilon^2\cdot 17{,}2MeV + \frac{Z^2}{A^{1/3}}\frac{1}{5}\varepsilon^2\cdot 0{,}7MeV$$

$$-235^{2/3}\frac{2}{5}\varepsilon^2\cdot 17{,}2MeV + \frac{92^2}{235^{1/3}}\frac{1}{5}\varepsilon^2\cdot 0{,}7MeV = \varepsilon^2\left(-262+192\right) = 70\varepsilon^2$$



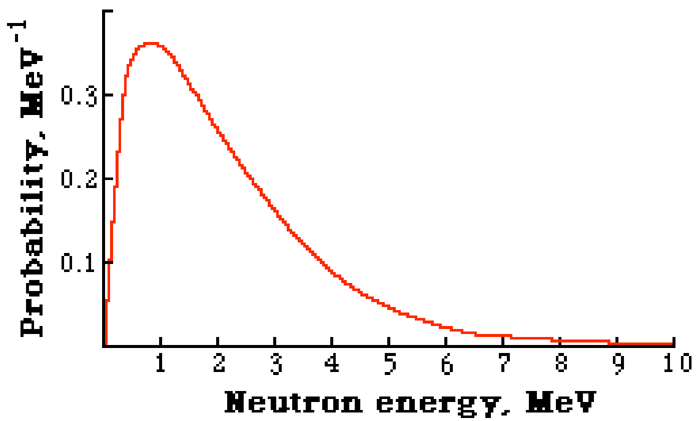
Potential energy, MeV
250
200
150
100
50
A ≈ 300
A = 236
A ≈ 100
E_f
Q
fragment separation

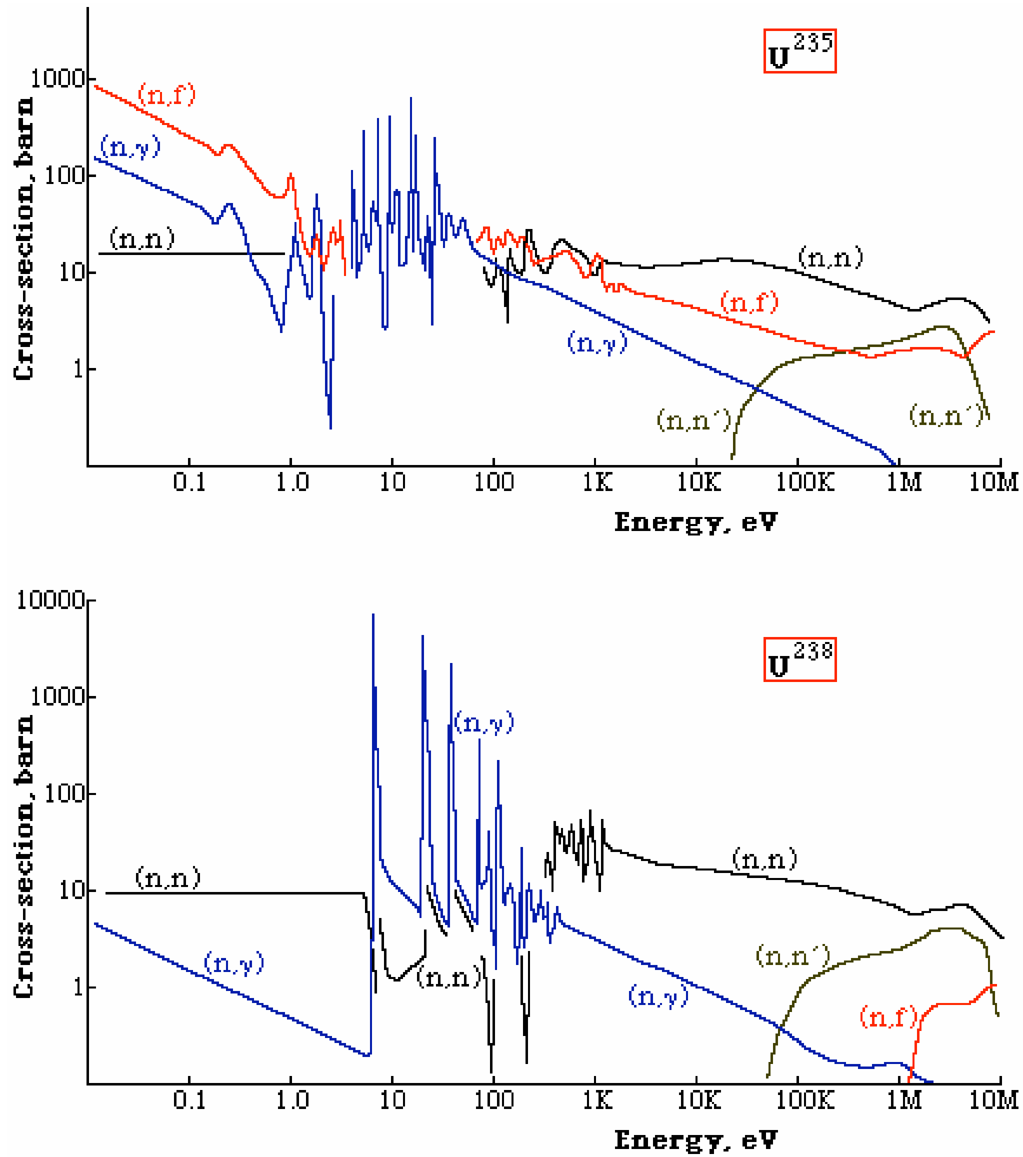


spontaneous fission half-life, yr
10^20
10^18
10^16
10^14
10^12
10^10
10^8
10^6
10^4
10^2
10^0
10^-2
Th^232
Th^230
U^236
U^234
U^238
U^232
Pu^242
Pu^240
Pu^238
Pu^244
Pu^236
Cm^246
Cm^242
Cm^244
Cm^240
Cf^250
Cf^248
Cf^246
Cf^252
Cf^254
Fm^254
Fm^256
35
36
37
38
39
40
41
Z^2/A

Štěpení U235 lze vzvolat záchytem neutronů

Neutrony je třeba zpomalit (moderovat), aby se zvýšila pravděpodobnost štěpné reakce





Chlazení neutronů = zmenšování jejich kinetické energie

$$n + A \rightarrow n + A; \quad p_n = -p_n^{'} + P_A; \quad \frac{p_n^{\,2}}{2m_n} = \frac{p_n^{'\,2}}{2m_n} + \frac{P_A^{\,2}}{2M_A}$$

$$\left(\frac{\Delta T_k}{T_k}\right)_{\max} = \left(\frac{p_n^{\,2}}{2m_n} - \frac{p_n^{'\,2}}{2m_n}\right) \Big/ \left(\frac{p_n^{\,2}}{2m_n}\right) = \left(\frac{P_A^{\,2}}{2M_A}\right) \Big/ \left(\frac{p_n^{\,2}}{2m_n}\right) = \frac{P_A^{\,2}}{p_n^{\,2}} \frac{m_n}{M_A}$$

$$\frac{p_n^{\,2}}{2m_n} = \frac{(P_A - p_n)^2}{2m_n} + \frac{P_A^{\,2}}{2M_A} \Rightarrow P_A^{\,2}\left(\frac{1}{2M_A} + \frac{1}{2m_n}\right) = \frac{P_A p_n}{m_n} \Rightarrow \frac{P_A^{\,2}}{p_n^{\,2}} = \left(\frac{2M_A}{M_A + m_n}\right)^2$$

$$\left(\frac{\Delta T_k}{T_k}\right)_{\max} = \frac{P_A^{\,2}}{p_n^{\,2}} \frac{m_n}{M_A} = \left(\frac{2M_A}{M_A + m_n}\right)^2 \frac{m_n}{M_A} \cong \frac{4A}{(A+1)^2}$$

$$\left\langle \frac{\Delta T_k}{T_k} \right\rangle = \frac{1}{2}\left(\frac{\Delta T_k}{T_k}\right)_{\max} = \frac{2A}{(A+1)^2}$$

$$T_{k,0} \rightarrow T_{k,1} = T_{k,0}\left(1 - \frac{2A}{(A+1)^2}\right) \rightarrow T_{k,2} = T_{k,1}\left(1 - \frac{2A}{(A+1)^2}\right) = T_{k,0}\left(1 - \frac{2A}{(A+1)^2}\right)^2 \rightarrow \ldots T_{k,N} = T_k\left(1 - \frac{2A}{(A+1)^2}\right)^N$$

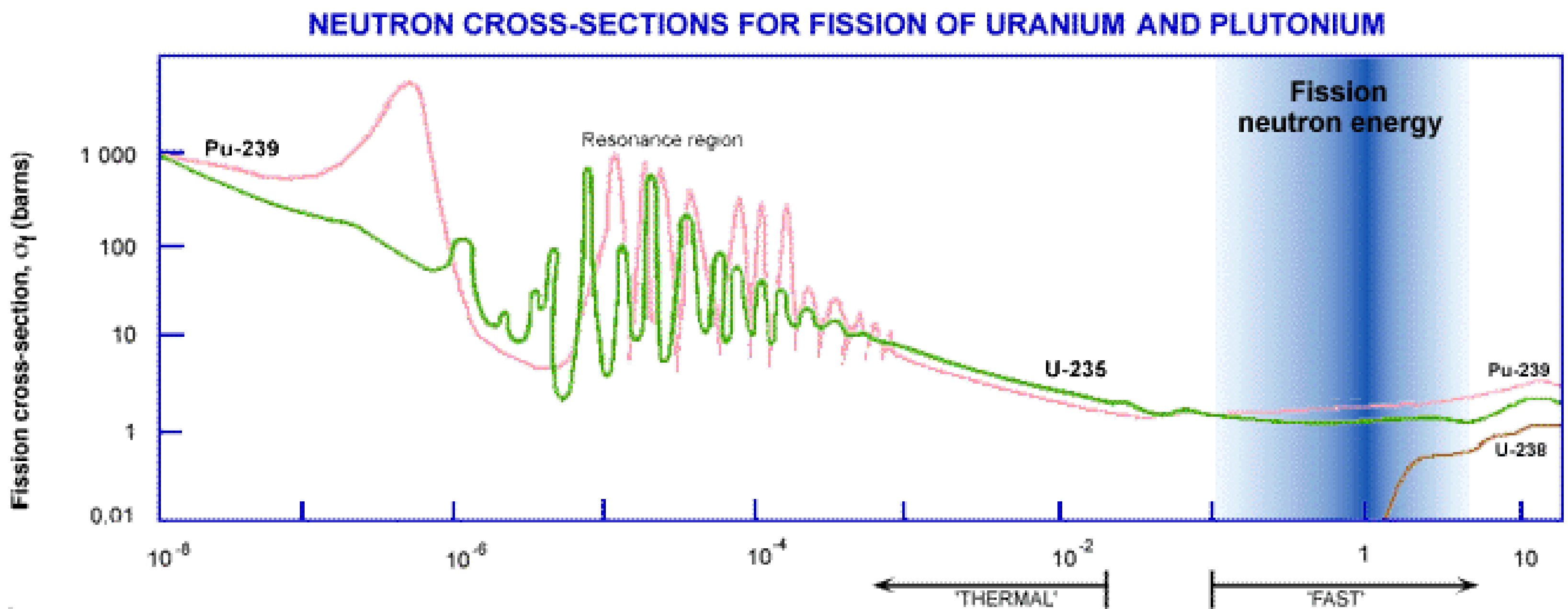
$$1MeV\left(1 - \frac{2 \cdot 12}{(12+1)^2}\right)^N = 100meV \Rightarrow (0{,}858)^N = 10^{-12} \Rightarrow N = \frac{\ln\left(10^{-12}\right)}{\ln(0{,}858)} \cong 180$$

$$1MeV\left(1 - \frac{2 \cdot 1}{(1+1)^2}\right)^N = 100meV \Rightarrow (0{,}5)^N = 10^{-12} \Rightarrow N = \frac{\ln\left(10^{-12}\right)}{\ln(0{,}5)} \cong 40$$

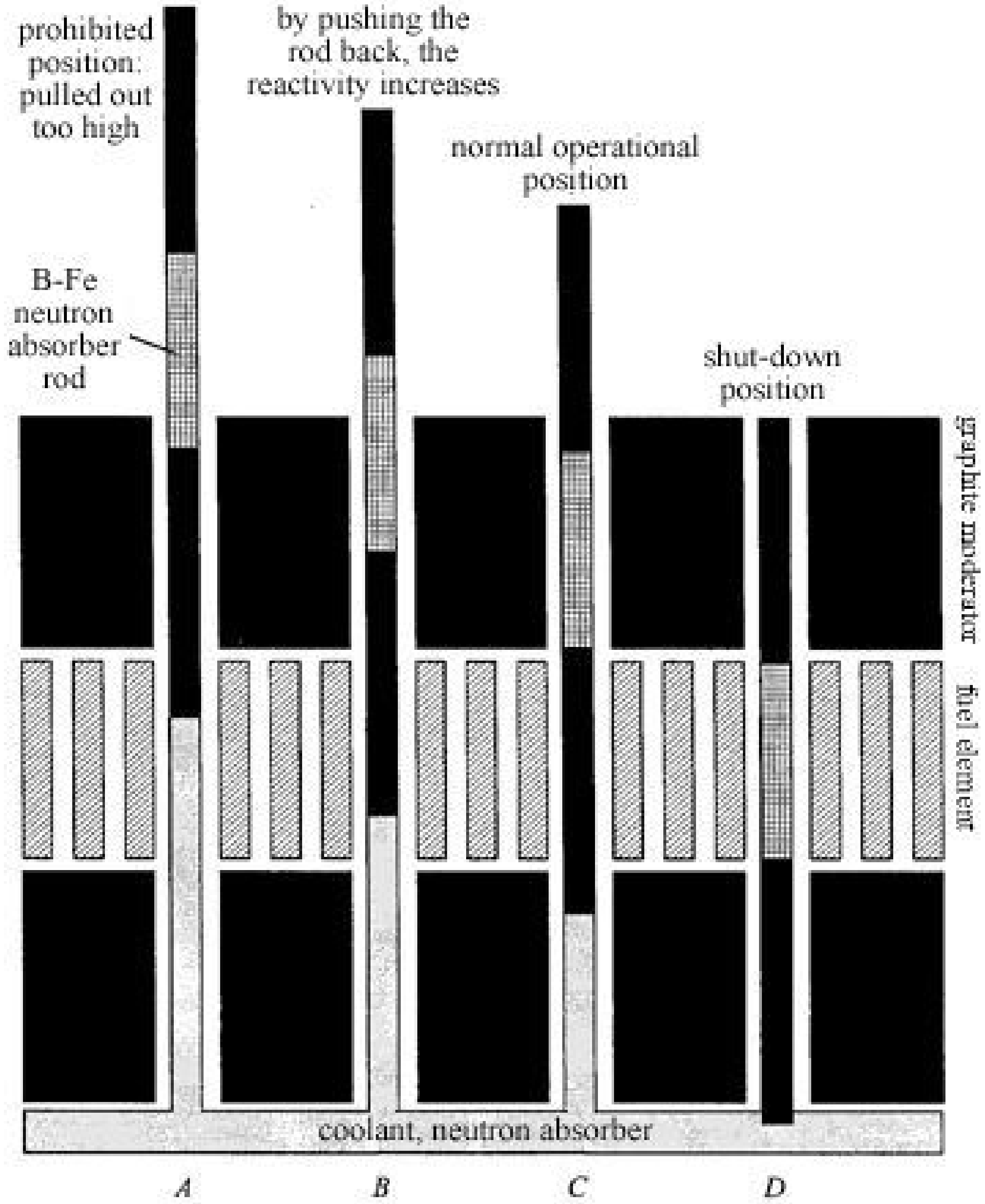
Přírodní uran:
100 neutronů
98 je zachyceno U238 a jen 8 z nich vyvolá štěpení x 3 neutrony = 24 neutronů
2 zbývající štěpí U235 x 3 = 6 neutronů

Tj. ze 100 neutronů bude 30

Přírodní uran je nutné obohatit, tj. zvýšit v něm zastoupení U235, přibližně na 4%. To se dělá pomocí např. pomocí odstředivek.



Sources: OECD / NEA 1989, Plutonium fuel - an assessment.
Taube 1974, Plutonium - a general survey.
1 barn = 10^{-28} m2, 1 MeV = 1.6×10^{-13} J


prohibited position: pulled out too high
by pushing the rod back, the reactivity increases
normal operational position
shut-down position
B-Fe neutron absorber rod
graphite moderator
fuel element
coolant, neutron absorber
A
B
C
D

Jaderné reakce

$$a + A \rightarrow b + B$$

$$A(a,b)B$$

$$Q = M_a + M_A - (M_b + M_B)$$

$Q > 0$ **Exotermické reakce**

$Q < 0$ **Endotermické reakce**

$${}^{4}_{2}\alpha + {}^{14}_{7}N \rightarrow {}^{17}_{8}O + {}^{1}_{1}p$$ **První jaderná reakce objevená Rutherfordem**

$${}^{14}_{7}N(\alpha, p){}^{17}_{8}O$$

$${}^{4}_{2}\alpha + {}^{9}_{4}Be \rightarrow {}^{12}_{6}C + {}^{1}_{0}n$$ **Chadwickův objev neutronu**

$${}^{9}_{4}Be(\alpha, n){}^{12}_{6}C$$

Termojaderná fúze – příkladem je Slunce, kde se vodík mění na helium

$$p + p \rightarrow {}^2_1H + e^+ + \nu_e$$

$$Q = 2m_p - (m_p + m_n - B({}^2_1H) + m_e) = 2 \cdot 938{,}27 - (938{,}27 + 939{,}57 - 2{,}2 + 0{,}511) = 0{,}4 MeV$$

Neexistuje vázaný stav pp ani nn, pouze pn = deuterium.

$${}^2_1H + {}^2_1H \rightarrow {}^4_2He; Q = 23{,}8 MeV$$

Ale uvolněná energie je dostatečná na to, aby se přeměnila na odtržení protonu anebo neutronu, takže častěji nastane:

$${}^2_1H + {}^2_1H \rightarrow {}^3_2He + n; Q = 3{,}3 MeV$$

$${}^2_1H + {}^2_1H \rightarrow {}^3_1H + p; Q = 4{,}0 MeV$$

Termojaderná fúze – příkladem je Slunce, kde se vodík mění na helium

Po fúzi deuteria by helium mohlo vzniknout přímo v reakci:

$$ {}_1^2H + {}_1^2H \rightarrow {}_2^4He; Q = 23{,}8 MeV $$

Ale uvolněná energie je dostatečná na to, aby se přeměnila na odtržení protonu anebo neutronu, takže častěji nastane:

$$ {}_1^2H + {}_1^2H \rightarrow {}_1^3H + p; Q = 4{,}0 MeV $$

$$ {}_1^2H + {}_1^3H \rightarrow {}_2^4He + n; Q = 17{,}6 MeV $$

Další fáze syntézy prvků:

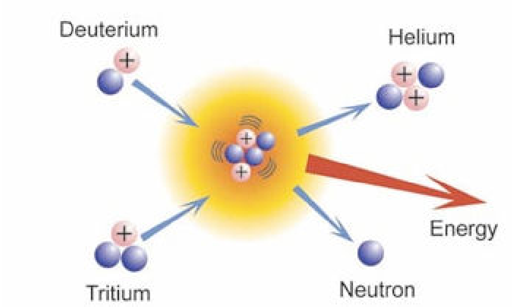
Spalování helia

$$ {}_{2}^{4}He + {}_{2}^{4}He \leftrightarrow {}_{4}^{8}Be; Q = -91{,}9 keV $$

$$ {}_{2}^{4}He + {}_{4}^{8}Be \leftrightarrow {}_{6}^{12}C; Q = -288 keV $$

Další absorbce alfa vedou postupně až ke tvorbě železa

Fúzní reaktory



Využití reakce:

$$ {}_1^2H + {}_1^3H \rightarrow {}_2^4He + n; Q = 17{,}6 MeV $$

$$ P_1(E) \approx e^{-\frac{E}{kT}} $$ **Maxwell-Boltzman**

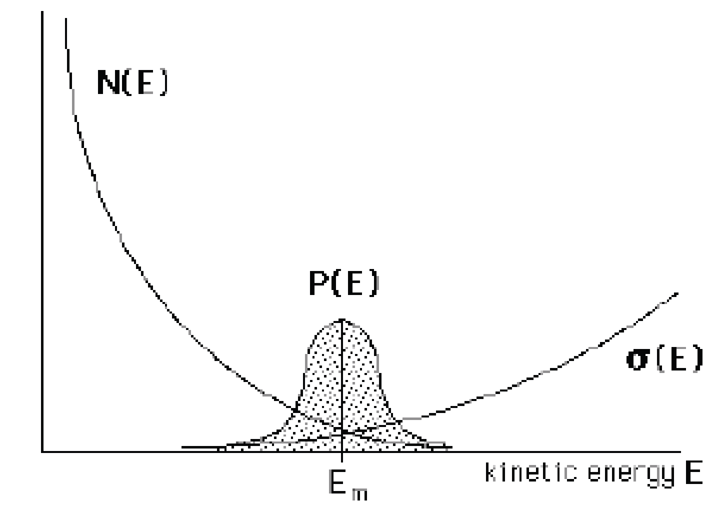
$$ P_2(E) \approx e^{-a\int\sqrt{V(x)-E}dx} $$

$$ P(E) = P_1(E)P_2(E) \approx e^{-(\frac{E}{kT}+a\sqrt{V-E})} $$

$$ \frac{d(\frac{E}{kT}+a\sqrt{V-E})}{dE} = 0 \Rightarrow \frac{1}{kT} - \frac{a}{2\sqrt{V-E}} = 0 $$

Průnik Coulomb. barierou:

$$ P_2(E) = e^{-2\int_R^{R+D}\frac{\sqrt{2M\ (V(x)-E)}}{\hbar c}dx} $$



$$n_D = n_T = n$$

$$N(t) = n \cdot \sigma \cdot v \cdot t \cdot n$$

$$E(t) = N(t) \cdot Q$$

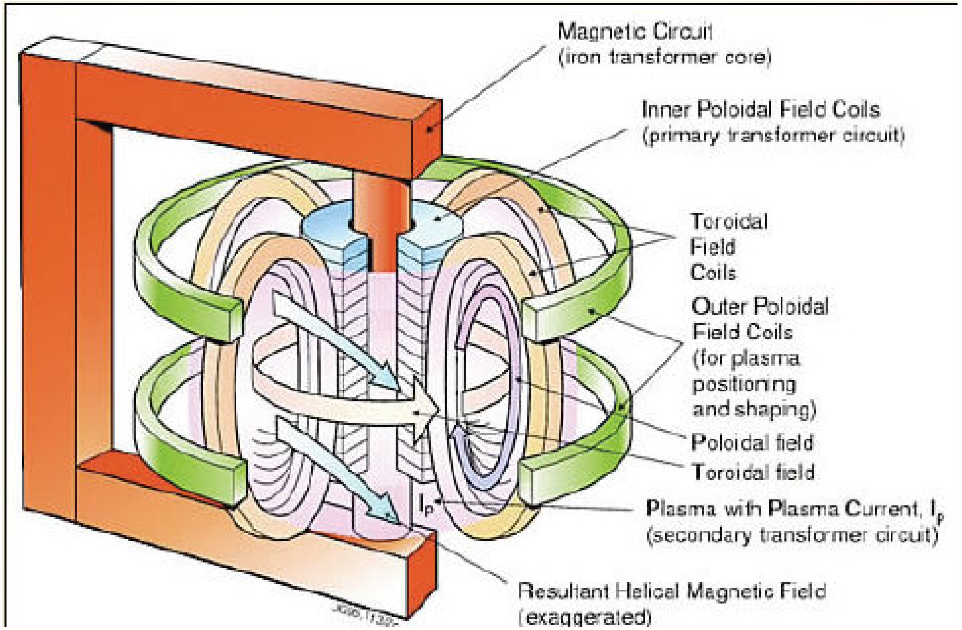
$$E = (n+n) \cdot \frac{3}{2} kT$$

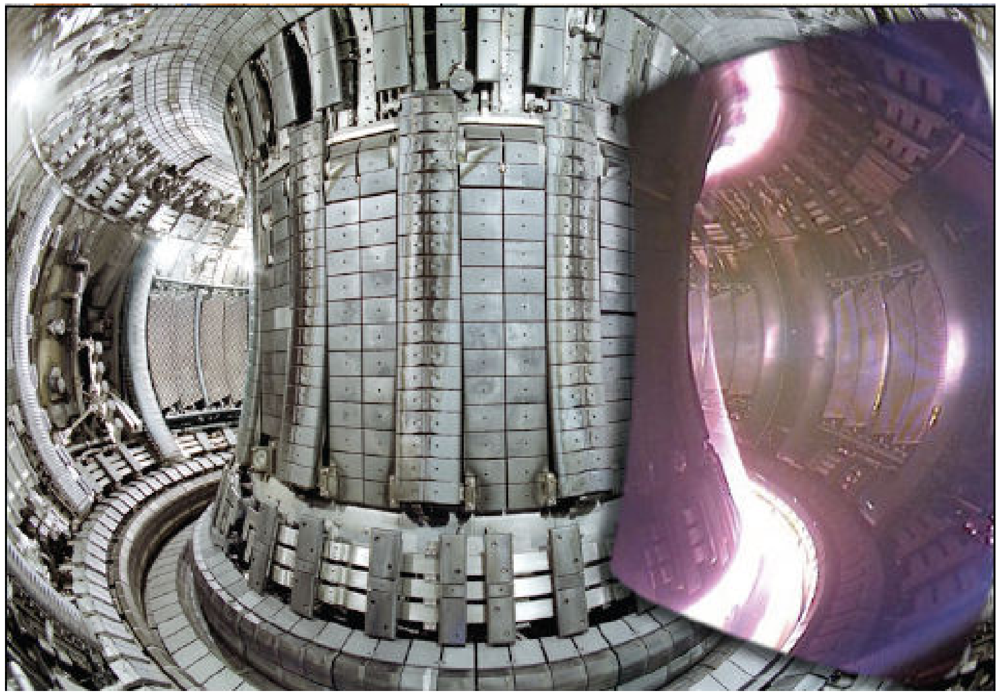
$$E(t) = N(t) \cdot Q > E = (n+n) \cdot \frac{3}{2} kT$$

$$n \cdot \sigma \cdot v \cdot t \cdot n \cdot Q > (n+n) \cdot \frac{3}{2} kT$$

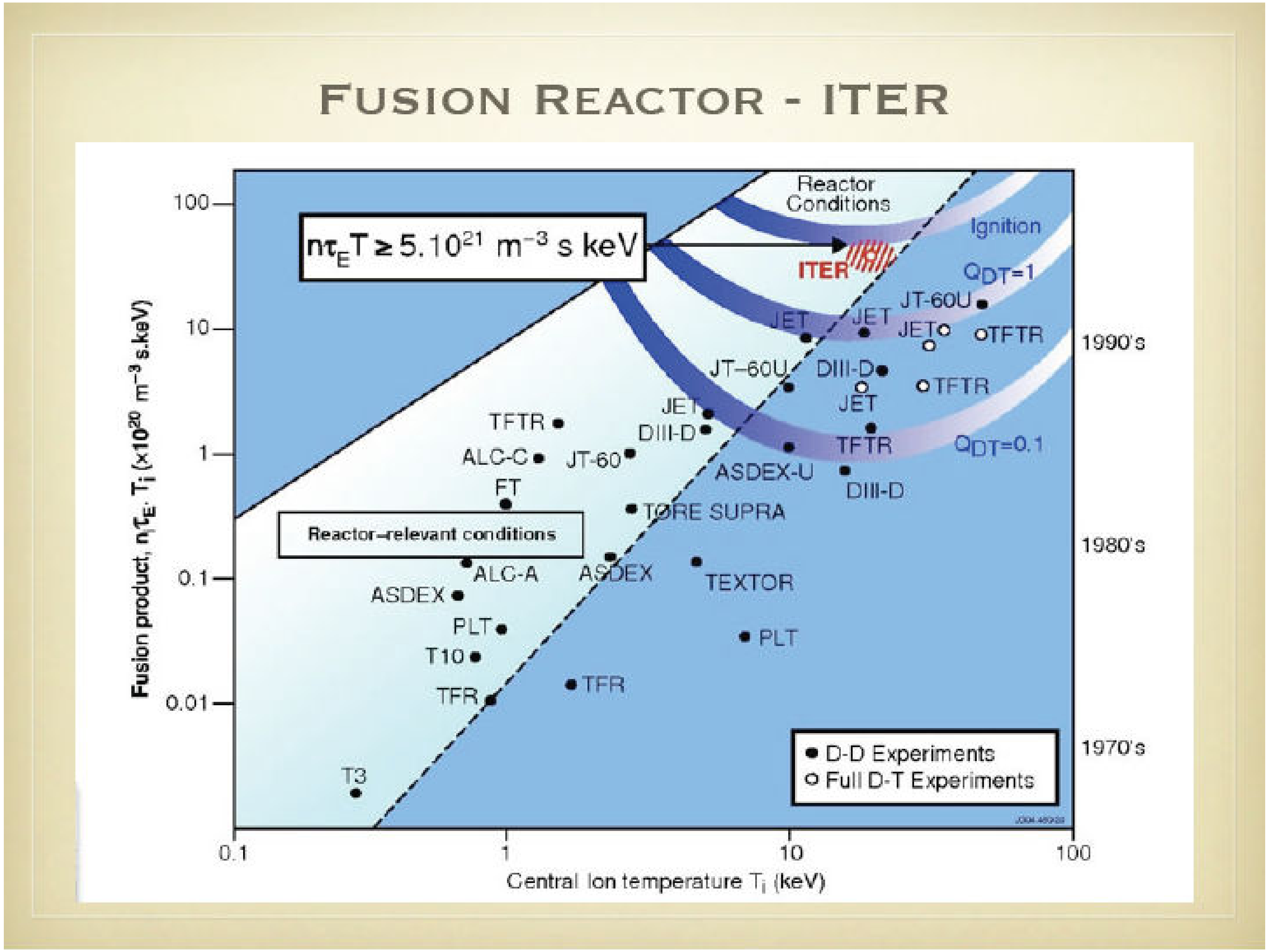
$$n \cdot t > \frac{3kT}{\sigma \cdot v \cdot Q}$$

$$n \cdot t > \frac{3kT}{\sigma \cdot v \cdot Q} = \frac{3 \cdot 25 \cdot 10^{-3} eV \dfrac{1{,}2 \cdot 10^{8} K}{300K}}{10^{-21} m^3 s^{-1} \cdot 17{,}6 MeV} = \frac{3 \cdot 10 keV}{10^{-21} m^3 s^{-1} \cdot 17{,}6 MeV} = 1{,}7 \cdot 10^{20} m^{-3} s^{1}$$


Magnetic Circuit
(iron transformer core)
Inner Poloidal Field Coils
(primary transformer circuit)
Toroidal
Field
Coils
Outer Poloidal
Field Coils
(for plasma
positioning
and shaping)
Poloidal field
Toroidal field
I_p
Plasma with Plasma Current, I_p
(secondary transformer circuit)
Resultant Helical Magnetic Field
(exaggerated)



ITER = nový projekt ve Francii. Bude prvním fúzním reaktorem, kde se bude získávat energie.



Objevy nových částic

V roce 1932:
Proton
Neutron
Elektron
Neutrino – postulováno Paulim

Objev pozitronu = Carl D. Anderson

The Nobel Prize in Physics 1936

"for his discovery of cosmic radiation"	"for his discovery of the positron"
	
Victor Franz Hess	**Carl David Anderson**
1/2 of the prize	1/2 of the prize
Austria	USA
Innsbruck University Innsbruck, Austria	California Institute of Technology (Caltech) Pasadena, CA, USA
b. 1883 d. 1964	b. 1905 d. 1991

Objev kosmického záření Viktorem Hessem

When, in 1912, I was able to demonstrate by means of a series of balloon ascents, that the ionization in a hermetically sealed vessel was reduced with increasing height from the earth (reduction in the effect of radioactive substances in the earth), but that it noticeably increased from 1,000 m onwards, and at 5 km height reached several times the observed value at earth level, I concluded that this ionization might be attributed to the penetration of the earth's atmosphere from outer space by hitherto unknown radiation of exceptionally high penetrating capacity, which was still able to ionize the air at the earth's surface noticeably. Already at that time I sought to clarify the origin of this radiation, for which purpose I undertook a balloon ascent at the time of a nearly complete solar eclipse on the 12th April 1912, and took measurements at heights of two to three kilometres. As I was able to observe no reduction in ionization during the eclipse I decided that, essentially, the sun could not be the source of cosmic rays, at least as far as undeflected rays were concerned.

Objev positronu, Carl D. Anderson

Kladně nabitá částice.

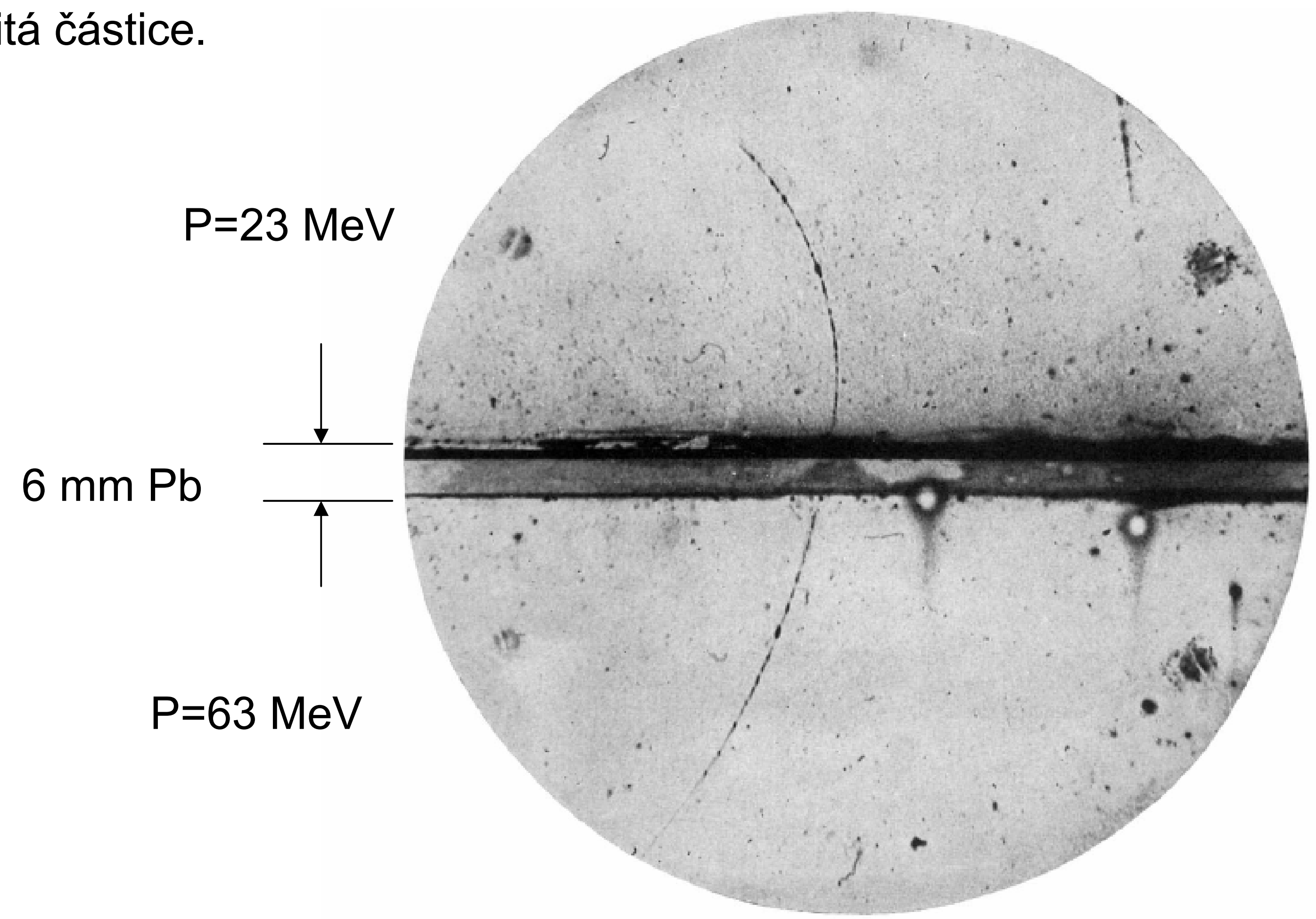


FIG. 1. A 63 million volt positron ($H\rho = 2.1 \times 10^5$ gauss-cm) passing through a 6 mm lead plate and emerging as a 23 million volt positron ($H\rho = 7.5 \times 10^4$ gauss-cm). The length of this latter path is at least ten times greater than the possible length of a proton path of this curvature.

proton: ionizace

$$T_k = \frac{(63MeV)^2}{2\cdot 938{,}27MeV} = 2{,}1MeV;$$

$$R_{Pb} = \frac{1}{\rho_{Pb}(dE/dx)_{\min}}\frac{T^2{}_k}{T_k+m_p} = \frac{1}{11{,}2gcm^{-3}1{,}15MeV/(gcm^{-2})}\frac{2{,}1^2\,MeV^2}{(2{,}1+938{,}27)MeV}$$

$$= 3{,}6\cdot 10^{-4}cm = 3{,}6\mu m$$

$$T_k = \frac{(23MeV)^2}{2\cdot 938{,}27MeV} = 0{,}28MeV$$

$$R_{Air} = \frac{1}{\rho_{Air}(dE/dx)_{\min}}\frac{T^2{}_k}{T_k+m_p} = \frac{1}{1{,}0\cdot 10^{-3}gcm^{-3}2MeV/(gcm^{-2})}\frac{0{,}28^2\,MeV^2}{(0{,}28+938{,}27)MeV}$$

$$= 0{,}042cm = 0{,}42mm$$

Kladný elektron: brzdné záření

$$E(L) = E_0 e^{-\frac{L}{X_0}} = 63e^{-\frac{6mm}{5{,}6mm}} = 21{,}6MeV$$

Částice s neznámou hmotou m - ionizace

$$\frac{1}{\rho_{Pb}(dE/dx)_{\min}}\frac{\left(\sqrt{p_1^2+m^2}-m\right)^2}{\sqrt{p_1^2+m^2}} - \frac{1}{\rho_{Pb}(dE/dx)_{\min}}\frac{\left(\sqrt{p_2^2+m^2}-m\right)^2}{\sqrt{p_2^2+m^2}} = t$$

$$\frac{1}{11{,}2\cdot 1{,}15}\frac{\left(\sqrt{63^2+m^2}-m\right)^2}{\sqrt{63^2+m^2}} - \frac{1}{11{,}2\cdot 1{,}15}\frac{\left(\sqrt{23^2+m^2}-m\right)^2}{\sqrt{23^2+m^2}} = 0{,}6cm$$

$$m = 61{,}5MeV$$

$$\Delta T \approx \frac{1}{\beta^2}$$

$$\Delta T_1/\Delta T_2 = \frac{\beta_2^2}{\beta_1^2} = \frac{\frac{p_2^2}{p_2^2+m^2}}{\frac{p_1^2}{p_1^2+m^2}} = \frac{p_2^2\left(p_1^2+m^2\right)}{p_1^2\left(p_2^2+m^2\right)} = \frac{23^2\left(63^2+61{,}5^2\right)}{63^2\left(23^2+61{,}5^2\right)} \cong 4{,}1$$

```
In[7]:= sol =
  Solve[1/(11.2×1.15) (√(63^2 + m^2) - m)^2/√(63^2 + m^2) - 1/(11.2×1.15) (√(23^2 + m^2) - m)^2/√(23^2 + m^2) ==
    0.6, m]
  ((23^2 + m^2) / 23^2) * (63^2 / (63^2 + m^2)) /. sol
Out[7]= {{m → -61.5412}, {m → 61.5412}}
Out[8]= {4.17526, 4.17526}
```