Přednáška 2. Atomové jádro. Měření rozměrů jader. Jaderné síly.

Rozptyl

Objev neutronu

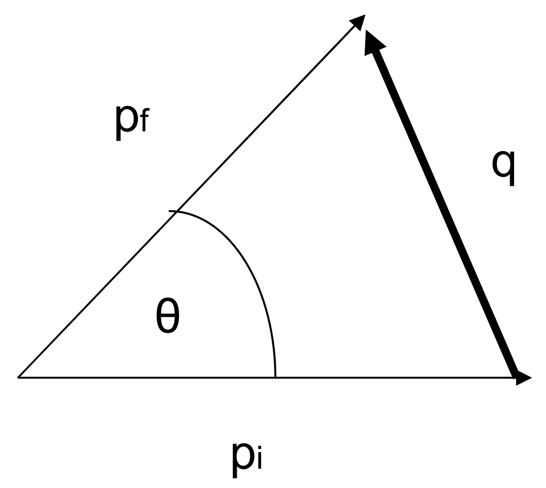
Formfaktory

Měření rozměrů jader

Jaderné síly, potenciál jádra

$$\frac{d\sigma}{d\Omega} = \frac{d\sigma}{d\cos(\theta)d\phi} = \left|f(\theta)\right|^2 = \frac{M^2}{4\pi^2(\hbar c)^4}\left|\int e^{-i\frac{\vec{q}\vec{r}}{\hbar c}} \cdot V(r') \cdot d^3 r'\right|^2$$

$$\left|\vec{q}\right| = \left|\overrightarrow{p_f} - \overrightarrow{p_i}\right| = 2p\sin(\theta/2) \quad ; \quad \left|\overrightarrow{p_f}\right| = \left|\overrightarrow{p_i}\right| = p$$



Fourierův obraz potenciálu:

$$\left|\int e^{-i\frac{\vec{q}\vec{r}'}{\hbar c}}\cdot V(\vec{r}')\cdot d^3\vec{r}'\right|^2 \quad ; \boxed{V(r)=\alpha\frac{\hbar c}{r}e^{-\frac{mr}{\hbar c}}}$$

Pro m=0 dostáváme Coulombický potenciál

$$\vec{q}\vec{r}' = qr'\cos(\theta')$$

$$\left|\int_0^{\infty}\int_{-1}^{+1} e^{-i\frac{qr'\cos(\theta')}{\hbar c}}\, d\cos(\theta')\cdot V(r')\cdot r'^2\, dr'\int_0^{2\pi} d\phi\right|^2 = 4\pi^2\left|\int_0^{\infty}\frac{2\sin(qr'/\hbar c)}{(qr'/\hbar c)}\alpha\frac{\hbar c}{r'}e^{-\frac{mr'}{\hbar c}}r'^2\, dr'\right|^2 =$$

$$16\pi^2\left|\frac{q(\hbar c)^3}{q^3}\alpha\int_0^{\infty}\frac{\sin(qr'/\hbar c)}{(qr'/\hbar c)}\frac{\hbar c}{qr'}e^{-\frac{(m/q)qr'}{\hbar c}}(qr'/\hbar c)^2\, d(qr'/\hbar c)\right|^2 =$$

$$16\pi^2\frac{(\hbar c)^6}{q^4}\alpha\left|\int_0^{\infty}\sin(qr'/\hbar c)e^{-\frac{(m/q)qr'}{\hbar c}}\, d(qr'/\hbar c)\right|^2 = 16\pi^2\frac{(\hbar c)^6}{q^4}\alpha^2\left|\int_0^{\infty}\sin(x)e^{-(m/q)x}\, d(x)\right|^2 =$$

$$16\pi^2\frac{(\hbar c)^6}{q^4}\alpha^2\cdot\left(\frac{1}{1+(m/q)^2}\right)^2 = \boxed{16\pi^2(\hbar c)^6\alpha^2\cdot\left(\frac{1}{q^2+m^2}\right)^2}$$

$m=0:$

$$\left|\int e^{-i\frac{\vec{q}\vec{r}'}{\hbar c}}\cdot V(\vec{r}')\cdot d^3\vec{r}'\right|^2 = 16\pi^2(\hbar c)^6\alpha^2\cdot\frac{1}{q^4} \; ; \; q = 2p\sin(\theta/2)$$

$$|f(q(\theta))|^2 = \frac{M^2}{4\pi^2(\hbar c)^4}16\pi^2(\hbar c)^6\alpha^2\cdot\frac{1}{q^4} = \frac{4M^2(\hbar c)^2\alpha^2}{q^4}$$

Rutherfordův rozptyl kvantově mechanicky

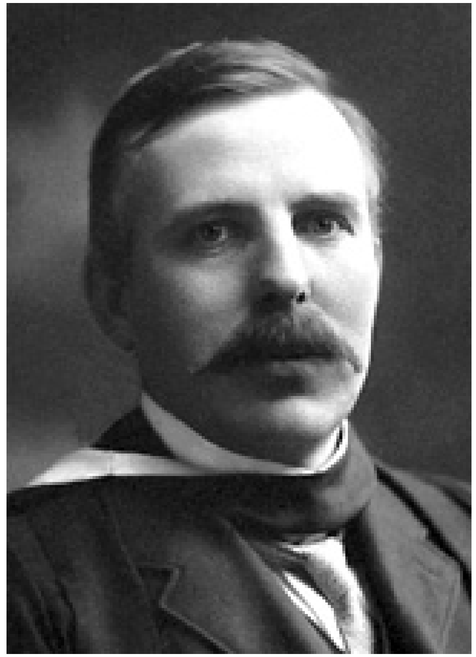
$$\frac{d\sigma}{d\cos(\theta)} = \int_0^{2\pi} \frac{d\sigma}{d\cos(\theta)d\phi} d\phi = 2\pi \cdot \frac{M^2}{4\pi^2(\hbar c)^4} \cdot 16\pi^2(\hbar c)^6 \alpha^2 Z_1^2 Z_2^2 \frac{1}{q^4} =$$

$$= \frac{8\pi\alpha^2 Z_1^2 Z_2^2 (\hbar c)^2}{4\cdot 16\cdot (p^2/2M)^2 \sin^4(\theta/2)} = \frac{\pi\alpha^2 Z_1^2 Z_2^2 (\hbar c)^2}{8T^2 \sin^4(\theta/2)} = \frac{\pi\alpha^2 Z_1^2 Z_2^2 (\hbar c)^2}{2T^2(1-\cos(\theta))^2}$$

$$\sigma(\theta > \theta_0) = \int_{-1}^{\cos(\theta_0)} \frac{d\sigma}{d\cos(\theta)} d\cos(\theta) = \frac{\pi\alpha^2 Z_1^2 Z_2^2 (\hbar c)^2}{2T^2} \int_{-1}^{\cos(\theta_0)} \frac{1}{(1-\cos(\theta))^2} d\cos(\theta) =$$

$$\frac{\pi\alpha^2 Z_1^2 Z_2^2 (\hbar c)^2}{2T^2} \left(\frac{1}{1-\cos(\theta_0)} - \frac{1}{2} \right) = \pi \left(\left(\frac{\alpha Z_1 Z_2 (\hbar c)}{T} \right) / 2 \right)^2 \frac{1+\cos(\theta)}{1-\cos(\theta)} = \frac{\pi (r_{\min}/2)^2}{\tan^2(\theta/2)} = \pi b^2(\theta)$$

Model jádra E. Rutherforda



-Kladný náboj atomu je soustředěn v malé oblasti o rozměru několika fm

-Kladný náboj nesou protony = jádra vodíku

-Hmota jádra je ale přibližně 2xnáboj jádra x hmota protonů

-Model: jádro= 2Z protonů + Z elektronů

Ale: jádro dusíku Z=7,A=14 by mělo být složeno z 14 protonů a 7 elektronů. Protony stejně jako elektrony mají poloviční spin a proto by jádro dusíku složené z lichého počtu fermionů mělo mít také poločíselný spin, ale bylo změřeno, že má spin roven 1.

Lisa Meitner a další zjistili, že elektrony vyletující z jader (beta částice) mají spojité spektrum:

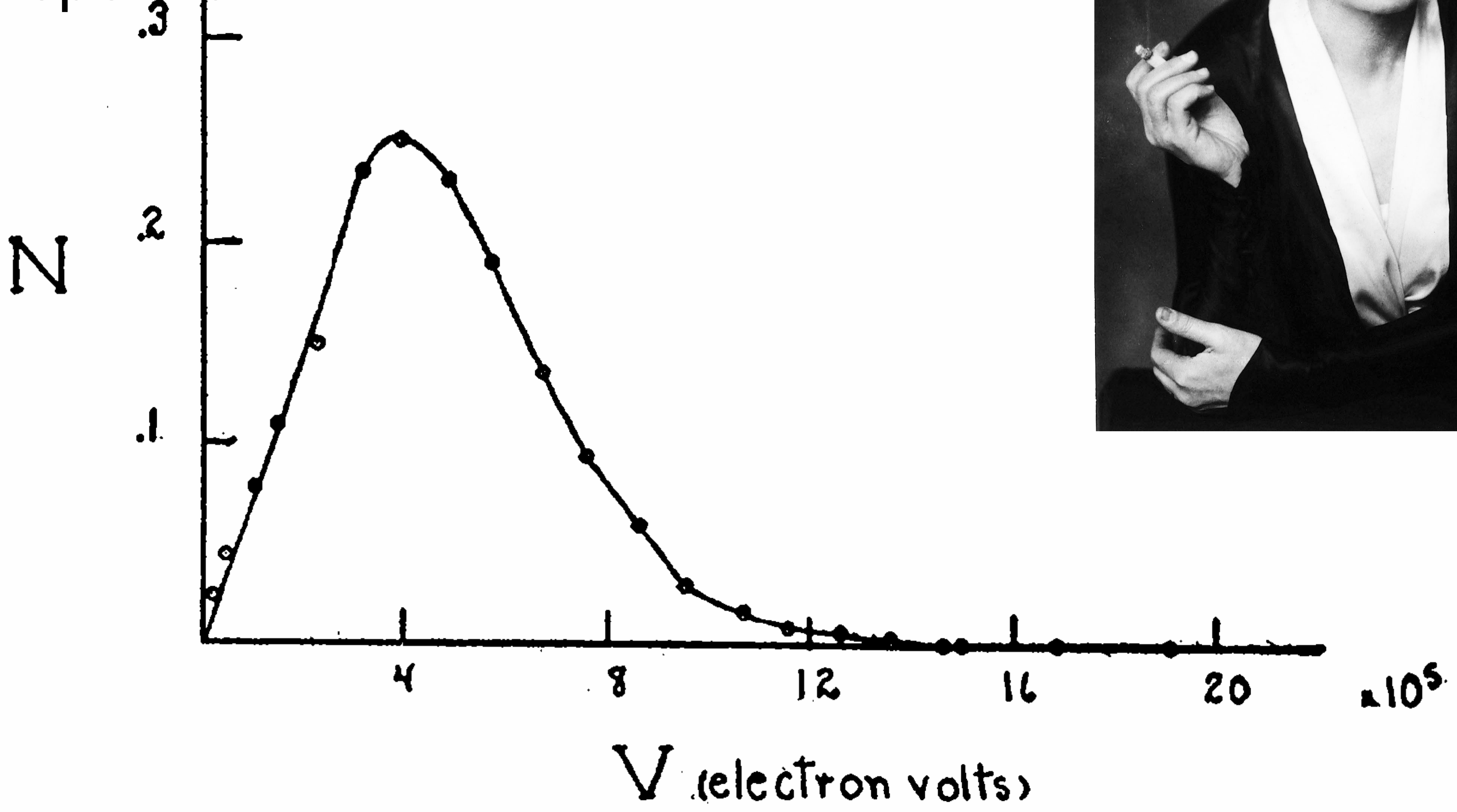


FIG. 5. Energy distribution curve of the beta-rays.

Objev neutronu (1932)

James Chadwick

Při ozařování beryliového terče alfa částicemi vznikalo pronikavé neutrální záření, které dokázalo vyrazit protony a udělit jim až 1/10 rychlosti světla a jádrům dusíku udělit rychlost asi 7,5 krát menší.
Myslelo se, že se jedná o gama záření. Jakou by mělo gama mít energii:

$$E_\gamma + m_p = E'_\gamma + m_p + T_{k,p} (= p^2{}_p / 2m_p)$$

$$p_\gamma (= E_\gamma) + 0 = -p'_\gamma (= E'_\gamma) + p_p$$

$$\beta_p = p_p / E \cong p_p / \sqrt{p^2{}_p + m_p{}^2} \Rightarrow p_p \cong \beta_p m_p$$

$$E_\gamma + m_p = E'_\gamma + m_p + m_p \beta^2{}_p / 2$$

$$E_\gamma = -E'_\gamma + m_p \beta_p$$

$$2E_\gamma = m_p \beta_p + m_p \beta^2{}_p / 2$$

$$E_\gamma = m_p \beta_p / 2 + m_p \beta^2{}_p / 4 \cong 938{,}27 MeV \cdot 0{,}1 / 2 = \underline{47 MeV}$$

Objev neutronu

$$E_\gamma + m_N = E'_\gamma + m_N + T_{k,N} (= p^2{}_N / 2m_N)$$

$$p_\gamma (= E_\gamma) + 0 = -p'_\gamma (= E'_\gamma) + p_N$$

$$\beta_N = p_N / E \cong p_N / \sqrt{p^2{}_N + m_N{}^2} \Rightarrow p_N \cong \beta_N m_N$$

$$E_\gamma + m_N = E'_\gamma + m_N + m_N \beta^2{}_N / 2$$

$$E_\gamma = -E'_\gamma + m_N \beta_N$$

$$2E_\gamma = m_N \beta_N + m_N \beta^2{}_N / 2$$

$$E_\gamma = m_N \beta_N / 2 + m_N \beta^2{}_N / 4 \cong 14 \cdot 938{,}27 MeV \cdot (0{,}1 / 7{,}5) / 2 \cong 91 MeV$$

Ale kdyby záření bylo tvořeno hmotnými neutrálními částicemi (neutrony):

$$\frac{p^2{}_n}{2m_n} + 0 = \frac{p'^2{}_n}{2m_n} + \frac{p^2{}_p}{2m_p} \; ; \; p_n + 0 = p'_n + p_p$$

$$\boxed{{}^4_2He + {}^4_9Be \rightarrow {}^{12}_6C + {}^1_0n}$$

$$p_p = p_n \frac{2m_p}{m_n + m_p}; P_p = p_n \frac{2m_p}{M_N + m_p}; \frac{p_p}{P_p} = \frac{M_N + m_p}{m_n + m_p}$$

$$\frac{M_N + m_p}{m_n + m_p} = \frac{M_N / m_p + 1}{m_n / m_p + 1} \cong \frac{14 + 1}{m_n / m_p + 1} \xrightarrow[7{,}5]{} m_n / m_p \cong 1$$

Postulování neutrina W. Paulim v roce 1930

4th December 1930

Dear Radioactive Ladies and Gentlemen,

As the bearer of these lines, to whom I graciously ask you to listen, will explain to you in more detail, how because of the "wrong" statistics of the N and Li^6 nuclei and the continuous beta spectrum, I have hit upon a desperate remedy to save the "exchange theorem" of statistics and the law of conservation of energy. Namely, the possibility that there could exist in the nuclei electrically neutral particles, that I wish to call neutrons, which have spin 1/2 and obey the exclusion principle and which further differ from light quanta in that they do not travel with the velocity of light. The mass of the neutrons should be of the same order of magnitude as the electron mass and in any event not larger than 0.01 proton masses. The continuous beta spectrum would then become understandable by the assumption that in beta decay a neutron is emitted in addition to the electron such that the sum of the energies of the neutron and the electron is constant...

Neutronem se dnes nazývá Chadwickův neutron, Fermi pojmenoval Pauliho neutron **neutrino**

Jádro je složeno z protonů a neutronů

Jádro dusíku 7 protonů a 7 neutronů, tj. sudý počet fermionů a celočíselný spin

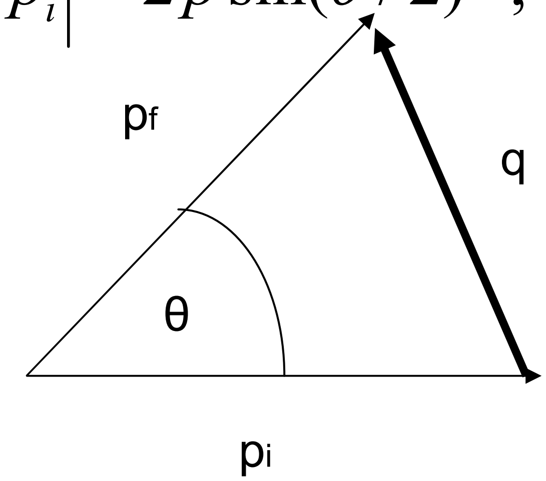
Elektrony v jaderných rozpadech vznikají rozpadem neutronu na proton, elektron a antineutrino. Část energie odnese antineutrino, proto je spektrum beta spojité.

Měření rozměrů jader

Základní myšlenka - úhlové rozdělění závisí na potenciálu a bude zřejmě jiné pro bodový náboj a pro náboj rozdělený např. rovnoměrně uvnitř koule.

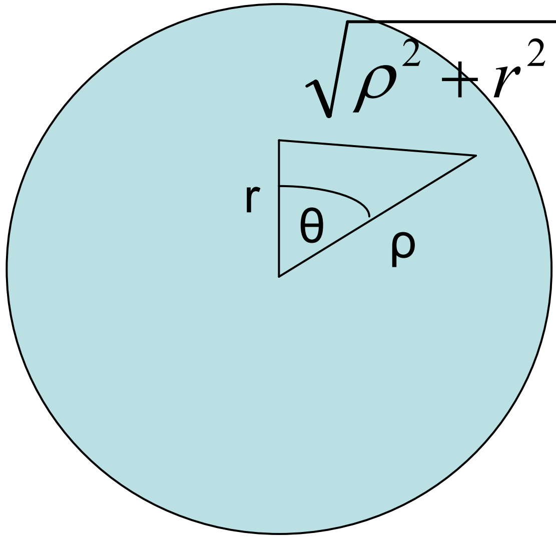
$$\frac{d\sigma}{d\Omega} = \frac{d\sigma}{d\cos(\theta)d\phi} = \left|f(\theta)\right|^2 = \frac{M^2}{4\pi^2(\hbar c)^4}\left|\int e^{-i\frac{\vec{q}\vec{r}}{\hbar c}} \cdot \underline{V(r')} \cdot d^3r'\right|^2$$

$$\left|\vec{q}\right| = \left|\overrightarrow{p_f} - \overrightarrow{p_i}\right| = 2p\sin(\theta/2) \quad ; \quad \left|\overrightarrow{p_f}\right| = \left|\overrightarrow{p_i}\right| = p$$



Rozptyl na homogenní nabité kouli o poloměru R a nábojem Z

Potenciál:



$$\sqrt{\rho^2 + r^2 - 2\rho r\cos(\theta)}$$

$$V(r) = \pm\frac{(Z)}{(4/3)\pi R^3}\alpha\cdot\hbar c\cdot\int_0^{\mathrm{R}}\rho^2 d\rho\int_{-1}^{1} d\cos(\theta)\frac{1}{\sqrt{\rho^2+r^2-2\rho r\cos(\theta)}}\int_0^{2\pi}\mathrm{d}\phi =$$

$$\pm\frac{2\pi(Z)}{(4/3)\pi R^3}\alpha\cdot\hbar c\cdot\int_0^{\mathrm{R}}\rho^2 d\rho\frac{4}{r+\rho+|r-\rho|}$$

$$V(r)=\frac{2\pi(Z)}{(4/3)\pi R^3}\alpha\cdot\hbar c\cdot\int_0^R \rho^2 d\rho\frac{4}{r+\rho+|r-\rho|}$$

pro $r \geq R$ je $|r-\rho| = r-\rho$ a $V(r)=\pm\frac{2\pi(Z)}{(4/3)\pi R^3}\alpha\cdot\hbar c\cdot\frac{2}{r}\int_0^R \rho^2 d\rho =$

$$\frac{2\pi(Z)}{(4/3)\pi R^3}\alpha\cdot\hbar c\cdot\frac{2}{r}\frac{R^3}{3}=(Z)\alpha\cdot\hbar c\cdot\frac{1}{r}$$

pro $r < R$

$\rho \leq r + \rho > r$:

$$V(r)=\frac{2\pi(Z)}{(4/3)\pi R^3}\alpha\cdot\hbar c\cdot\left(\int_0^r \rho^2 d\rho\frac{4}{r+\rho+r-\rho}+\int_r^R \rho^2 d\rho\frac{4}{r+\rho-r+\rho}\right)=$$

$$\frac{2\pi(Z)}{(4/3)\pi R^3}\alpha\cdot\hbar c\cdot\left(\frac{2}{r}\int_0^r \rho^2 d\rho+\int_r^R \rho^2 d\rho\frac{2}{\rho}\right)=$$

$$\pm\frac{2\pi(Z)}{(4/3)\pi R^3}\alpha\cdot\hbar c\cdot\left(\frac{2}{3}r^2+\left(R^2-r^2\right)\right)=\pm\frac{(Z)}{R^3}\alpha\cdot\hbar c\cdot\left(\frac{3}{2}R^2-\frac{1}{2}r^2\right)$$

Potenciál homogenní nabité kouli o poloměru R a nábojem Z :

$$V(r) = (Z)\cdot\alpha\cdot\frac{\hbar c}{r} \qquad \text{pro } r \geq R$$

$$V(r) = (Z)\cdot\alpha\cdot\frac{\hbar c}{R}\cdot\left(1+\frac{1}{2}\left(1-\left(\frac{r}{R}\right)^2\right)\right) \qquad \text{pro } r \leq R$$

Rozptyl na homogenní nabité kouli

$$\frac{d\sigma}{d\Omega} = \frac{d\sigma}{d\cos(\theta)d\phi} = \left| -\frac{2M}{4\pi(\hbar c)^2} \int e^{-i\frac{\vec{q}\vec{r}'}{\hbar c}} \cdot V(r') \cdot d^3 r' \right|^2 = \frac{M^2}{4\pi^2(\hbar c)^4} \left| \int e^{-i\frac{\vec{q}\vec{r}'}{\hbar c}} \cdot V(r') d^3 r' \right|^2$$

$$\vec{q}\vec{r}' = qr'\cos(\theta')$$

$$\frac{d\sigma}{d\cos(\theta)} = \frac{2\pi M^2}{(\hbar c)^4} \left| \int_0^{\infty} \int_{-1}^{+1} e^{-i\frac{qr'\cos(\theta')}{\hbar c}} d\cos(\theta') \cdot V(r') \cdot r'^2\, dr' \right|^2 =$$

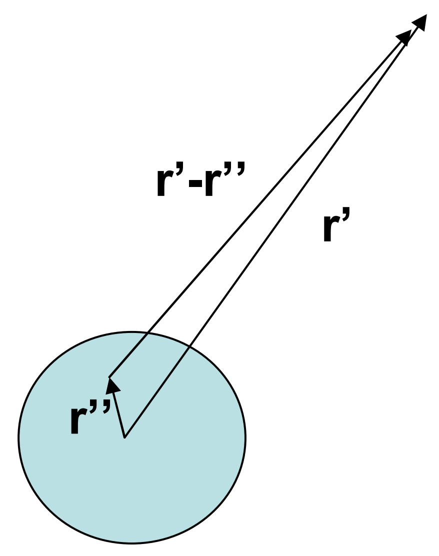
$$\frac{2\pi M^2}{(\hbar c)^4} \left| \int_0^R \frac{2\sin(qr'/\hbar c)}{(qr'/\hbar c)} \alpha Z_1 Z_2 \frac{\hbar c}{R} (1 + 1/2(1-(r'/R)^2)) r'^2\, dr' + \int_R^{\infty} \frac{2\sin(qr'/\hbar c)}{(qr'/\hbar c)} \alpha Z_1 Z_2 \frac{\hbar c}{r'} r'^2\, dr' \right|^2$$

....

$$= \frac{\pi\alpha^2 Z_1^2 Z_2^2 (\hbar c)^2}{2T^2(1-\cos(\theta))^2} \left(3\frac{\sin(qR/\hbar c) - (qR/\hbar c)\cos(qR/\hbar c)}{(qR/\hbar c)^3} \right)^2$$

Poměrně složitý výpočet. Naštěstí existuje jiný způsob.

Formfaktor



$$\frac{d\sigma}{d\Omega} = \frac{d\sigma}{d\cos(\theta)d\phi} = \left| -\frac{2M}{4\pi(\hbar c)^2} \int e^{-i\frac{\vec{q}\vec{r}'}{\hbar c}} \cdot V(\vec{r}') \cdot d^3\vec{r}' \right|^2$$

$$V(\vec{r}') = \int V_0(\vec{r}' - \vec{r}'') \cdot \rho(\vec{r}'') \cdot d^3\vec{r}''$$

$$\frac{d\sigma}{d\Omega} = \frac{d\sigma}{d\cos(\theta)d\phi} = \left| -\frac{2M}{4\pi(\hbar c)^2} \int e^{-i\frac{\vec{q}\vec{r}'}{\hbar c}} \cdot \int V_0(\vec{r}' - \vec{r}'')\rho(\vec{r}'')d^3\vec{r}'' \cdot d^3\vec{r}' \right|^2 =$$

$$\left| -\frac{2M}{4\pi(\hbar c)^2} \int e^{-i\frac{\vec{q}(\vec{r}'-\vec{r}'')}{\hbar c}} e^{-i\frac{\vec{q}\vec{r}''}{\hbar c}} \cdot \int V_0(\vec{r}' - \vec{r}'')\rho(\vec{r}'')d^3\vec{r}'' \cdot d^3\vec{r}' \right|^2 =$$

$$\left| -\frac{2M}{4\pi(\hbar c)^2} \int e^{-i\frac{\vec{q}(\vec{r}'-\vec{r}'')}{\hbar c}} \cdot V_0(\vec{r}' - \vec{r}'') \cdot d^3(\vec{r}' - \vec{r}'') \right|^2 \cdot \left| \int e^{-i\frac{\vec{q}\vec{r}''}{\hbar c}} \rho(\vec{r}'')d^3\vec{r}'' \right|^2 =$$

$$\left| -\frac{2M}{4\pi(\hbar c)^2} \int e^{-i\frac{\vec{q}\vec{r}}{\hbar c}} \cdot V_0(\vec{r}) \cdot d^3\vec{r} \right|^2 \cdot \left| \int e^{-i\frac{\vec{q}\vec{r}''}{\hbar c}} \rho(\vec{r}'')d^3\vec{r}'' \right|^2 = \left(\frac{d\sigma}{d\Omega}\right)_0 \left|F(\vec{q})\right|^2$$

$$\frac{d\sigma}{d\Omega} = \left(\frac{d\sigma}{d\Omega}\right)_0 \left|F(\vec{q})\right|^2$$

$$F(\vec{q}) = \int e^{-i\frac{\vec{q}\vec{r}}{\hbar c}} \rho(\vec{r})d^3\vec{r} \qquad \left|\mathrm{F}(0)\right| = 1$$

Rozptyl na homogenní nabité kouli pomocí formfaktoru

$$\rho(r) = \frac{1}{\frac{4}{3}\pi R^3} \quad r \le R \;\; ; \rho(r) = 0 \quad r > R$$

$$F(q) = \frac{1}{\frac{4}{3}\pi R^3} \int_0^R r^2 dr \int_{-1}^{1} d\cos(\theta) \int_0^{2\pi} d\phi e^{i\frac{qr}{\hbar c}\cos(\theta)} =$$

$$\frac{2\pi}{\frac{4}{3}\pi R^3} \int_0^R r^2 dr \int_{-1}^{1} d\cos(\theta) e^{i\frac{qr}{\hbar c}\cos(\theta)} =$$

$$\frac{4\pi}{\frac{4}{3}\pi R^3} \int_0^R \frac{\sin(qr/\hbar c)}{qr/\hbar c} r^2 dr = 3\frac{\sin(qR/\hbar c) - (qR/\hbar c)\cos(qR/\hbar c)}{(qR/\hbar c)^3}$$

Formfaktor homogenní nabité kulové plochy

$$\rho(r) = \frac{1}{4\pi R^2}\delta(r-R)$$

$$F(q) = \frac{1}{4\pi R^2}\int_0^\infty r^2\delta(r-R)dr\int_{-1}^{1} d\cos(\theta)\int_0^{2\pi} d\phi e^{i\frac{qR}{\hbar c}\cos(\theta)} =$$

$$\frac{R^2 2\pi}{4\pi R^2}\int_{-1}^{1} d\cos(\theta) e^{i\frac{qR}{\hbar c}\cos(\theta)} = \frac{\sin(qR/\hbar c)}{qR/\hbar c}$$

Formfaktor pro exponenciální rozdělení náboje

$$\rho(r) = \frac{1}{8\pi R^3} e^{-r/R}$$

$$F(q) = \frac{1}{8\pi R^3} \int_0^\infty r^2 e^{-r/R} dr \int_{-1}^{1} d\cos(\theta) \int_0^{2\pi} d\phi e^{i\frac{qr}{\hbar c}\cos(\theta)} =$$

$$\frac{2\pi}{4\pi R^3} \int_0^\infty r^2 e^{-r/R} dr \int_{-1}^{1} d\cos(\theta) e^{i\frac{qr}{\hbar c}\cos(\theta)} =$$

$$\frac{1}{2R^3} \int_0^\infty r^2 e^{-r/R} \frac{\sin(qr/\hbar c)}{qr/\hbar c} dr = \frac{1}{2R^3} \cdot \frac{2R^3}{(1+(qR/\hbar c)^2)^2} = \frac{1}{(1+(qR/\hbar c)^2)^2}$$

Formfaktor pro gaussovo rozdělení náboje

$$\rho(r) = \frac{\sqrt{2}}{4\pi\sqrt{\pi}R^3} e^{-r^2/(2R^2)}$$

$$F(q) = \frac{\sqrt{2}}{4\pi\sqrt{\pi}R^3} \int_0^\infty r^2 e^{-r^2/(2R^2)} dr \int_{-1}^{1} e^{i\frac{qR}{\hbar c}\cos(\theta)} d\cos(\theta) \int_0^{2\pi} d\phi =$$

$$\frac{2\pi\sqrt{2}}{4\pi\sqrt{\pi}R^3} \int_0^\infty r^2 e^{-r^2/(2R^2)} dr \int_{-1}^{1} e^{i\frac{qr}{\hbar c}\cos(\theta)} d\cos(\theta) =$$

$$\frac{\sqrt{2}}{\sqrt{\pi}R^3} \int_0^\infty r^2 e^{-r^2/(2R^2)} \frac{\sin(qr/\hbar c)}{qr/\hbar c} dr = e^{-\frac{1}{2}\left(\frac{qR}{\hbar c}\right)^2}$$

Porovnání formfaktorů

formfaktory.nb

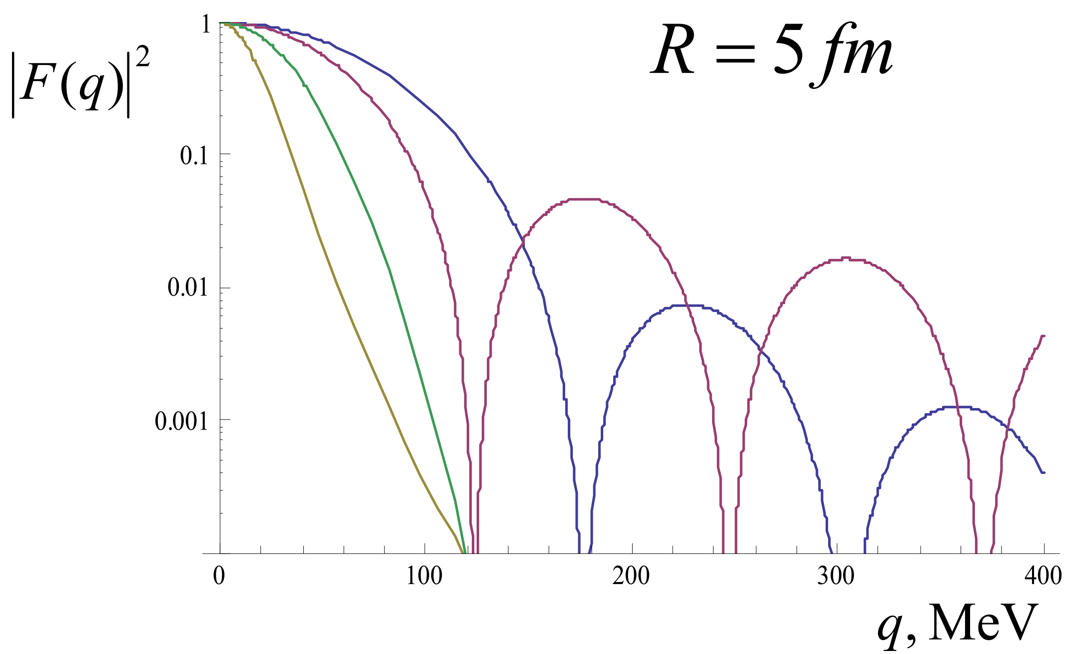
$$\rho(r) = \frac{1}{\frac{4}{3}\pi R^3} \quad r \le R \ ; \rho(r) = 0 \quad r > R;$$

$$F(q) = 3\frac{\sin(qR/\hbar c) - (qR/\hbar c)\cos(qR/\hbar c)}{(qR/\hbar c)^3}$$

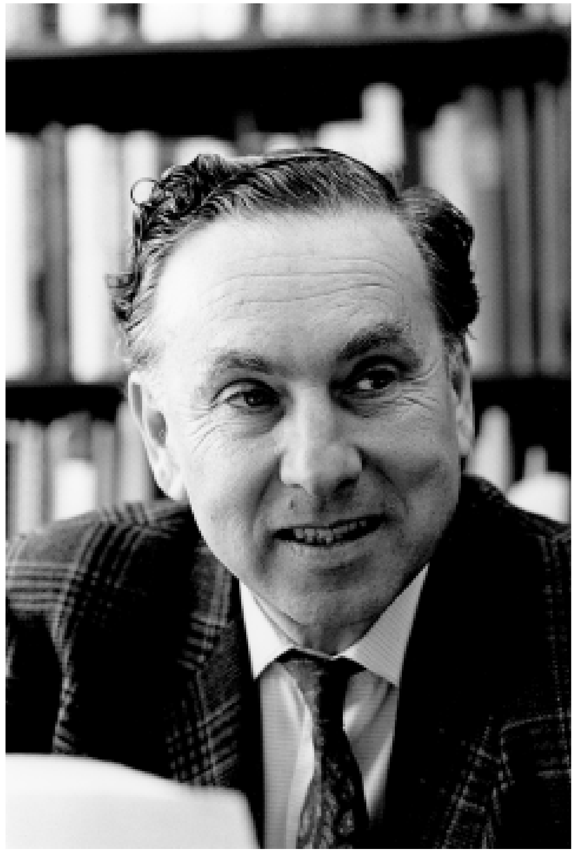
$$\rho(r) = \frac{1}{4\pi R^2}\delta(r-R) \ ; \ F(q) = \frac{\sin(qR/\hbar c)}{qR/\hbar c}$$

$$\rho(r) = \frac{\sqrt{2}}{4\pi\sqrt{\pi}R^3} e^{-r^2/(2R^2)} \quad ; \quad F(q) = e^{-\frac{1}{2}\left(\frac{qR}{\hbar c}\right)^2}$$

$$\rho(r) = \frac{1}{8\pi r R^3} e^{-r/R} \quad ; F(q) = \frac{1}{(1+(qR/\hbar c)^2)^2}$$



R. Hofstadter, Stanford Linear Accelerator Centre (SLAC),



Měřil rozptyl 188 MeV elektronů na jádrech, včetně toho nejlehčího jádra, tj. i na protonech

Electron scattering and nuclear structure.
Robert Hofstadter (Stanford U., Phys. Dept.) **Rev.Mod.Phys.28:214-254,1956**.

Proton form factors from elastic electron-proton scattering.
T. Janssens, R. Hofstadter, E.B. Hughes, M.R. Yearian (Stanford U., HEPL) ,
Phys.Rev.142:922-931,1966.

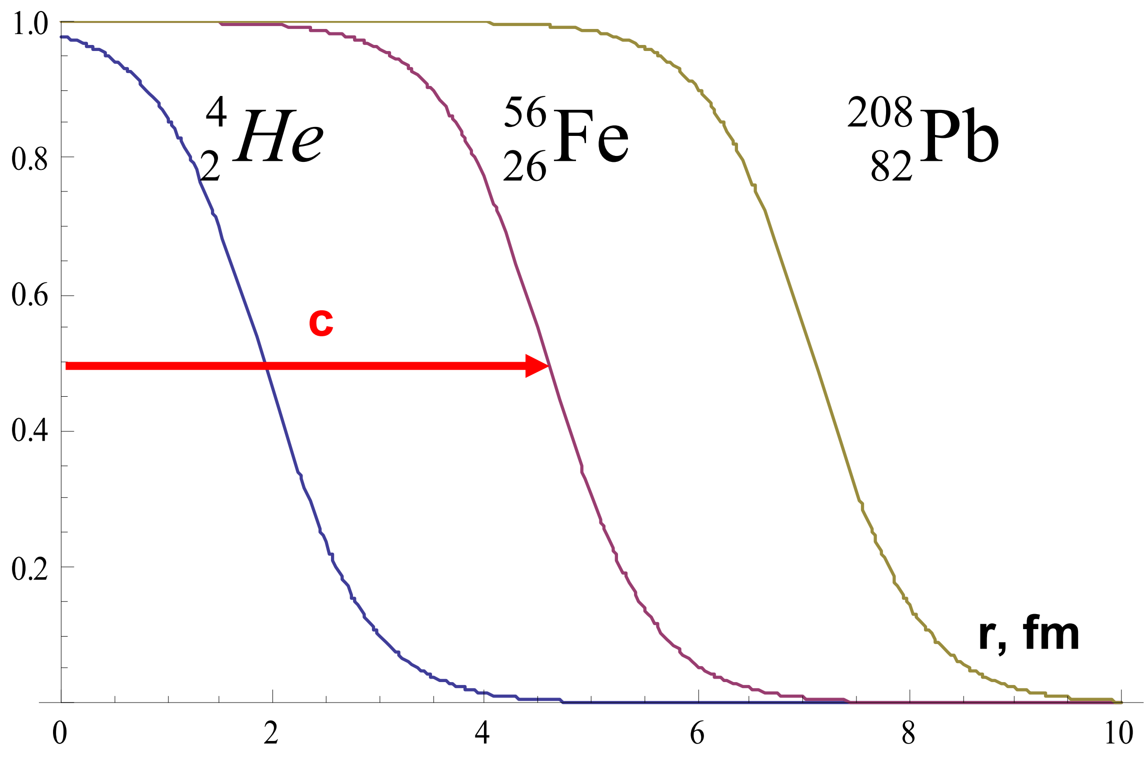
Rozdělení hustoty v jádře je dobře popsáno tzv. Saxon-Woodsovým rozdělením:

Saxon-Woods.nb

$$\rho(r) = \frac{\rho_0}{1 + e^{\frac{r-c}{a}}}$$

$$c \cong 1{,}2 \cdot A^{1/3}\ fm$$

$$a \cong 0{,}5\ \ fm$$



Struktura protonu

$$F(q) = \int e^{i(qr/\hbar c)\cos(\theta)} \rho(r) d^3\vec{r} \cong$$

$$\int \left(1 + i(qr/\hbar c)\cos(\theta) + \frac{1}{2}\left(i(qr/\hbar c)\cos(\theta)\right)^2 + \ldots \right) \rho(r) d^3\vec{r} =$$

$$\int \rho(r) d^3\vec{r} + \int i(qr/\hbar c)\cos(\theta)\rho(r) r^2 dr d\cos(\theta) d\phi -$$

$$\frac{1}{2}\int (qr/\hbar c)^2 \cos^2(\theta)\rho(r) r^2 dr d\cos(\theta) d\phi =$$

$$1 - \frac{q^2}{2(\hbar c)^2}\int r^2 \rho(r) r^2 dr d\cos(\theta) d\phi \frac{\int_{-1}^{+1} \cos^2(\theta) d\cos(\theta)}{\int_{-1}^{+1} d\cos(\theta)} =$$

$$1 - \frac{q^2}{2(\hbar c)^2}\frac{2/3}{2}\int r^2 \rho(r) r^2 dr d\cos(\theta) = 1 - \frac{1}{6}\frac{q^2}{(\hbar c)^2}\int r^2 \rho(r) d^3\vec{r} =$$

$$\underline{1 - \frac{1}{6}\frac{q^2 \langle r^2 \rangle}{(\hbar c)^2}}$$

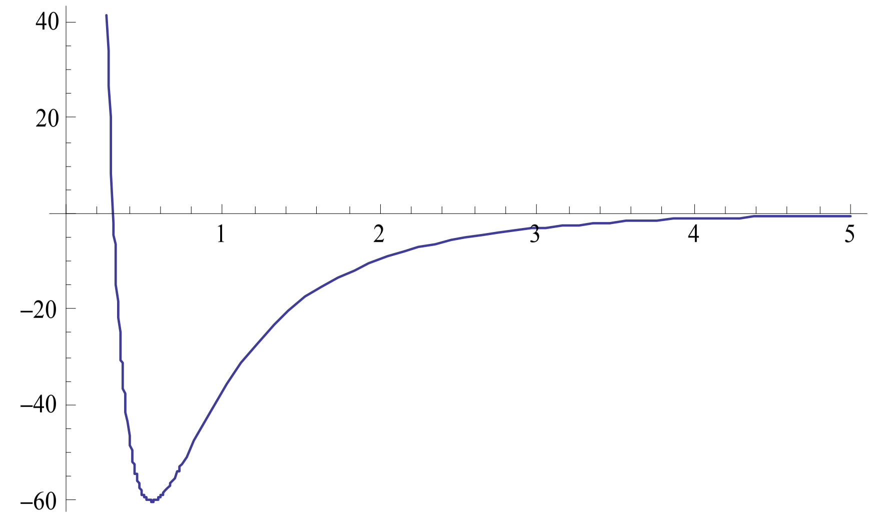
$$\boxed{\langle r^2 \rangle \cong 0{,}8 \;\; fm}$$

Jaderné síly, vazbová energie jader

H. Yukava – jaderné síly jsou výměna nových částic, tzv. mezonů. Tak je možno Zajistit, aby na malých vzdálenostech byly jaderné síly silnější než elektrické a na velkých slabší

$$V_J(r) = -\alpha_J \frac{\hbar c}{r} e^{-\frac{mr}{\hbar c}}$$

$$r \cong fm \quad |V_J(r)| >> V_{em}(r) = \alpha \frac{\hbar c}{r}$$

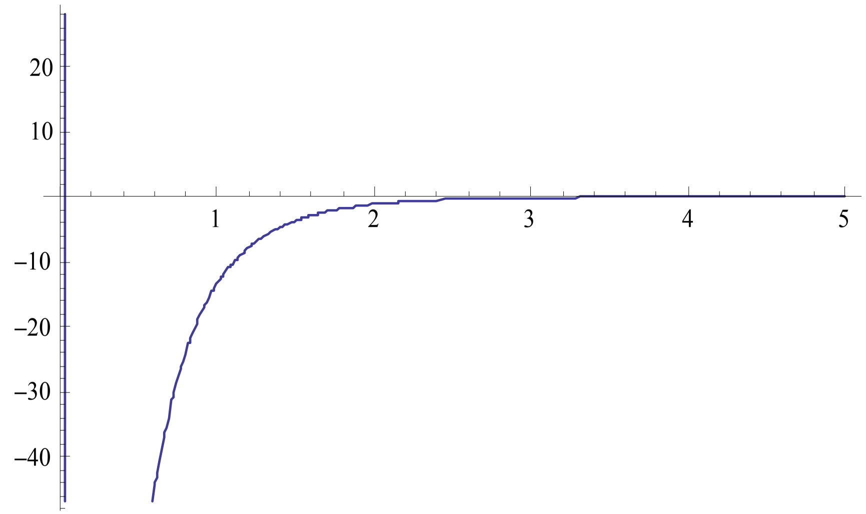


$$\alpha_J \cong 0{,}4 >> \alpha = 1/137$$

$$m = 140 MeV (\pi \text{ meson}),$$

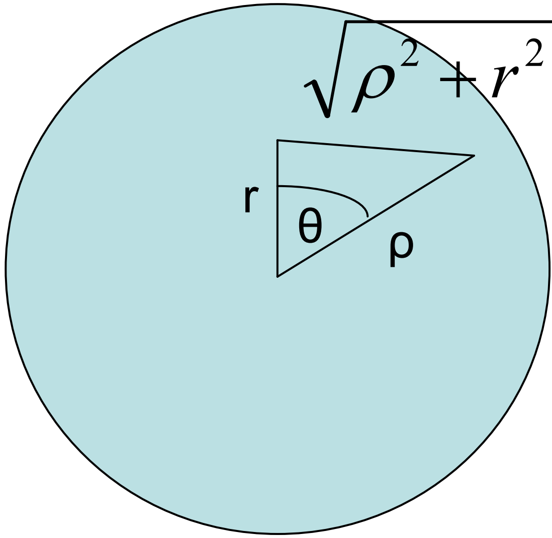
$$560 MeV (\sigma),$$

$$770 MeV (\rho)$$



$$m = \langle m \rangle = 350 MeV, \alpha_J \cong 0{,}4$$

Jak vypočítat energetické hladiny v jádře – na každý nukleon působí zbývajících A-1 nukleonů. Představa - nukleon je v poli zbývajícíh A-1 nukleonů. Ty vyplňují rovnoměrně (jaderně) nabitou kouli



$$\sqrt{\rho^2 + r^2 - 2\rho r\cos(\theta)}$$

$$V(r) = -\frac{(A-1)}{(4/3)\pi R^3}\alpha_J \cdot \hbar c \cdot \int_0^R \rho^2 d\rho \int_{-1}^{1} d\cos(\theta)\frac{e^{-\frac{m\sqrt{\rho^2+r^2-2\rho r\cos(\theta)}}{\hbar c}}}{\sqrt{\rho^2+r^2-2\rho r\cos(\theta)}}\int_0^{2\pi}\mathrm{d}\phi =$$

$$-\frac{(A-1)2\pi}{(4/3)\pi R^3}\alpha_J \cdot \hbar c \cdot \int_0^R \rho^2 \frac{e^{-\frac{m|r-\rho|}{\hbar c}} - e^{-\frac{m(r+\rho)}{\hbar c}}}{\rho} d\rho = -\frac{(A-1)2\pi}{(4/3)\pi R^3}\alpha_J \cdot \hbar c \cdot \left(\int_0^r \rho^2 \frac{e^{\frac{m(r-\rho)}{\hbar c}} - e^{-\frac{m(r+\rho)}{\hbar c}}}{mr\rho} d\rho + \int_r^R \rho^2 \frac{e^{-\frac{m(r-\rho)}{\hbar c}} - e^{-\frac{m(r+\rho)}{\hbar c}}}{mr\rho} d\rho \right) =$$

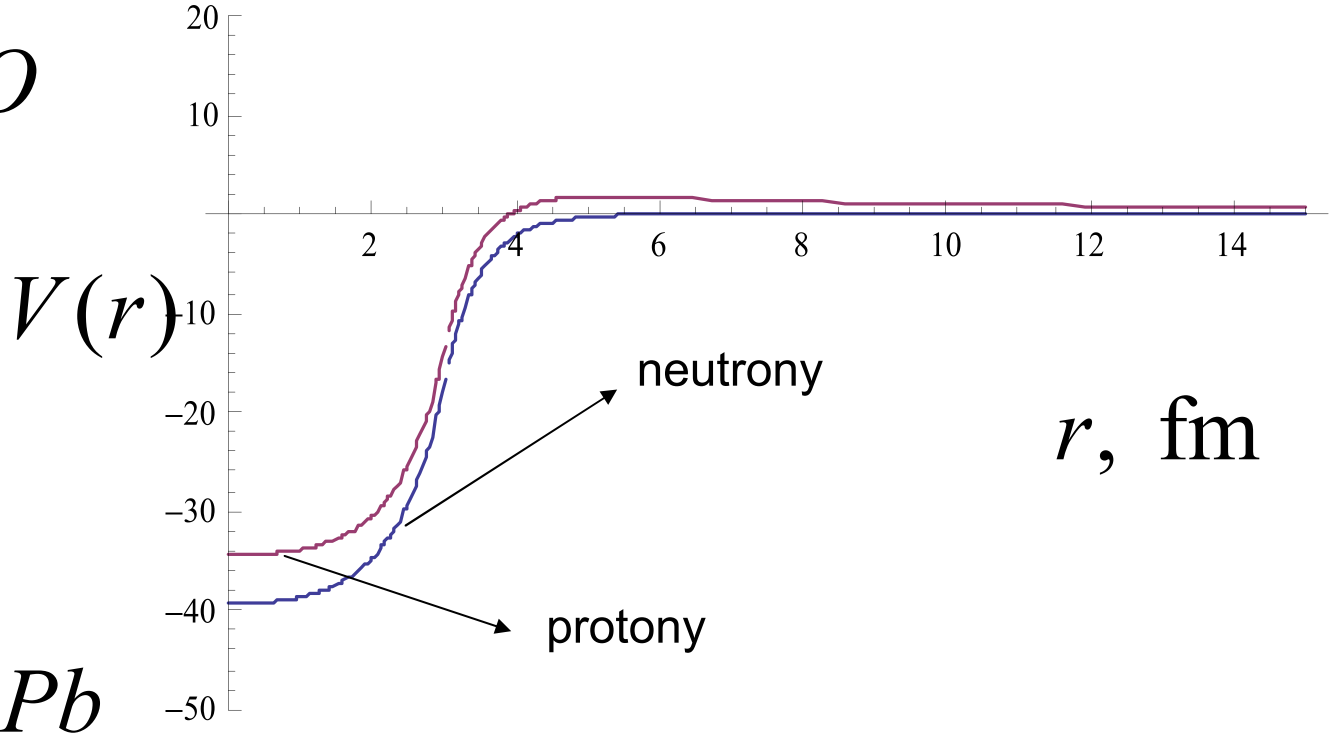
$$V_n(r) = -\frac{(A-1)}{(4/3)\pi R^3}\alpha_J \cdot \hbar c \cdot \int_0^R \rho^2 d\rho \int_{-1}^{1} d\cos(\theta) \frac{e^{-\frac{m\sqrt{\rho^2+r^2-2\rho r\cos(\theta)}}{\hbar c}}}{\sqrt{\rho^2+r^2-2\rho r\cos(\theta)}} \int_0^{2\pi} \mathrm{d}\phi =$$

$$-\frac{(A-1)2\pi}{(4/3)\pi R^3}\alpha_J \cdot \hbar c \cdot \int_0^R \rho^2 \frac{e^{-\frac{m|r-\rho|}{\hbar c}} - e^{-\frac{m(r+\rho)}{\hbar c}}}{mr\rho} d\rho = \quad -\frac{(A-1)2\pi}{(4/3)\pi R^3}\alpha_J \cdot \hbar c \cdot \left(\int_0^r \rho^2 \frac{e^{\frac{m(r-\rho)}{\hbar c}} - e^{-\frac{m(r+\rho)}{\hbar c}}}{mr\rho} d\rho + \int_r^R \rho^2 \frac{e^{-\frac{m(r-\rho)}{\hbar c}} - e^{-\frac{m(r+\rho)}{\hbar c}}}{mr\rho} d\rho \right) =$$

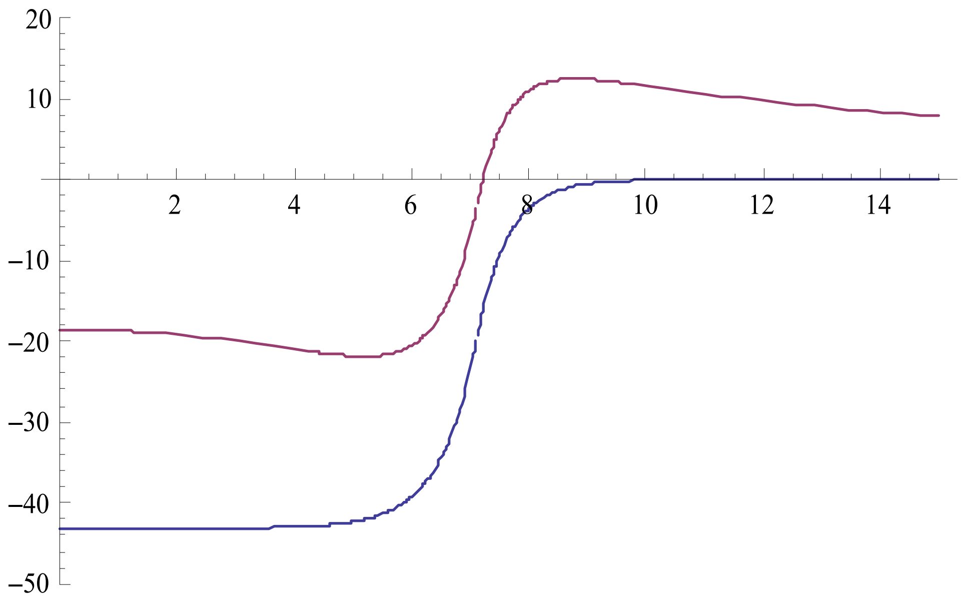
$$V_p(r) = V_n(r) + V_{coul}(r)$$

jaderny_potencial.nb

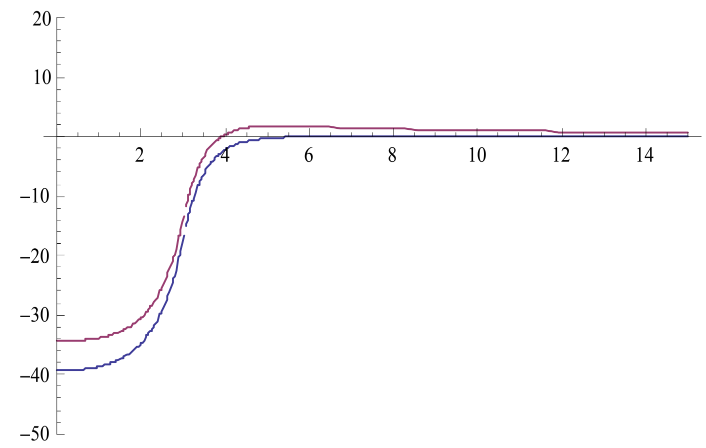
Jádro kyslíku $^{16}_{8}O$



Jádro olova $^{208}_{82}Pb$



Příklad jádro kyslíku



$R = 1{,}2\,fm(16)^{1/3} \cong 3{,}0\,fm$

$V_0 = -38MeV(neutrony)$

$A = 16, Z = 8, N = 8$

Harm. oscilator:

$$V(r) = V_0 + \frac{1}{2}m\omega^2 r^2 = V_0 + \frac{1}{2}mc^2(\hbar\omega)^2 r^2/(\hbar c)^2 =$$

$$V_0 + \frac{1}{2}m(\hbar\omega)^2 r^2/(\hbar c)^2$$

$$V(R) = V_0/2$$

$$V_0 + \frac{1}{2}m(\hbar\omega)^2 R^2/(\hbar c)^2 = V_0/2$$

$$\hbar\omega = \frac{\hbar c}{R}\sqrt{\frac{-V_0}{m}} = \frac{197MeVfm}{3\,fm}\sqrt{\frac{38MeV}{939{,}57MeV}} \cong 13{,}2MeV$$

Hladiny:

neutrony:

$$1p \quad -V_0 + \frac{5}{2}\hbar\omega = -38MeV + \frac{5}{2}13{,}2MeV = -5{,}0MeV\,.....\,6$$

$$1s \quad -V_0 + \frac{3}{2}\hbar\omega = -38MeV + \frac{3}{2}13{,}2MeV = -18{,}2MeV\,.....\,2$$

Vazbova energie:

$$B = 6\cdot 5 + 2\cdot 18{,}2 = 66{,}4MeV$$

$$B/N = 66{,}4MeV/8 = 8{,}3MeV/neutron$$

Harmonický oscilátor

$$-\frac{(\hbar c)^2}{2m}\psi''(x)+\frac{1}{2}m\omega^2x^2\psi(x)=E\psi(x)$$

$$-\frac{(\hbar c)^2}{2m}\psi''(x)+\frac{m(\hbar\omega)^2}{2(\hbar c)^2}x^2\psi(x)=E\psi(x)$$

$$E_{n_x}=(n_x+1/2)\hbar\omega$$

$$-\frac{(\hbar c)^2}{2m}(\partial^2{}_x+\partial^2{}_y+\partial^2{}_y)\varphi(x,y,z)+\frac{1}{2}m\omega^2(x^2+y^2+z^2)\varphi(x,y,z)=(E_{n_x}+E_{n_y}+E_{n_z})\varphi(x,y,z)$$

$$\varphi(x,y,z)=\psi_{n_x}(x)\cdot\psi_{n_y}(x)\cdot\psi_{n_z}(x)$$

$$E=(n_x+1/2+n_y+1/2+n_z+1/2)\hbar\omega=(n_x+n_y+n_z+3/2)\hbar\omega$$

$n_x+n_y+n_z=0.....1...možnost.......3/2\cdot\hbar\omega......1s$

$n_x+n_y+n_z=1.....3...možnosti......5/2\cdot\hbar\omega......1p$

$n_x+n_y+n_z=2.....6...možností......7/2\cdot\hbar\omega......1d.....2s$

$n_x+n_y+n_z=3...10...možností......9/2\cdot\hbar\omega......1f.....2p$

$n_x+n_y+n_z=4...15...možností....11/2\cdot\hbar\omega......1g......2d.....3s$

......

Kulové funkce a parita vlnové funkce

$$Y_{lm}(\theta,\varphi)$$

$$\hat{L}^2\, Y_{lm}(\theta,\varphi) = \hbar^2 l(l+1) Y_{lm}(\theta,\varphi) \quad ; \quad \hat{L}_z\, Y_{lm}(\theta,\varphi) = \hbar m Y_{lm}(\theta,\varphi)$$

$$\int_{-1}^{+1}\left(\int_0^{2\pi} Y^*{}_{lm}(\theta,\varphi) Y_{l'm'}(\theta,\varphi) d\varphi\right) d\cos(\theta) = \delta_{ll'}\delta_{mm'}$$

$$Y_{00}(\theta,\varphi) = \sqrt{\frac{1}{4\pi}}$$

$$Y_{10}(\theta,\varphi) = \sqrt{\frac{3}{4\pi}}\cos(\theta)$$

$$Y_{11}(\theta,\varphi) = -\sqrt{\frac{3}{8\pi}}\sin(\theta)e^{i\varphi}$$

$$Y_{1-1}(\theta,\varphi) = +\sqrt{\frac{3}{8\pi}}\sin(\theta)e^{-i\varphi}$$

$$Y_{20}(\theta,\varphi) = \sqrt{\frac{5}{4\pi}}\left(\frac{3}{2}\cos^2(\theta) - \frac{1}{2}\right)$$

$$Y_{21}(\theta,\varphi) = -\sqrt{\frac{15}{8\pi}}\sin(\theta)\cos(\theta)e^{i\varphi}$$

$$Y_{2-1}(\theta,\varphi) = +\sqrt{\frac{15}{8\pi}}\sin(\theta)\cos(\theta)e^{-i\varphi}$$

$$Y_{22}(\theta,\varphi) = \frac{1}{4}\sqrt{\frac{15}{2\pi}}\sin^2(\theta)e^{i2\varphi}$$

$$Y_{2-2}(\theta,\varphi) = \frac{1}{4}\sqrt{\frac{15}{2\pi}}\sin^2(\theta)e^{-i2\varphi}$$

$$\Psi(\vec{r}) = \Psi(r,\theta,\varphi) = R(r)\cdot Y_{lm}(\theta,\varphi)$$

$$\hat{P}\,\Psi(\vec{r}) = \Psi(-\vec{r}) = \pm\Psi(\vec{r})$$

$$\Psi(\vec{r}) = \Psi(r,\theta,\varphi)$$

$$\Psi(-\vec{r}) = \Psi(r,\pi-\theta,\varphi+\pi)$$

$$\hat{P}\,R(r)\cdot Y_{lm}(\theta,\varphi) = R(r)\cdot\hat{P}\,Y_{lm}(\theta,\varphi)$$

$$\hat{P}\,Y_{lm}(\theta,\varphi) = (-1)^l\, Y_{lm}(\theta,\varphi)$$