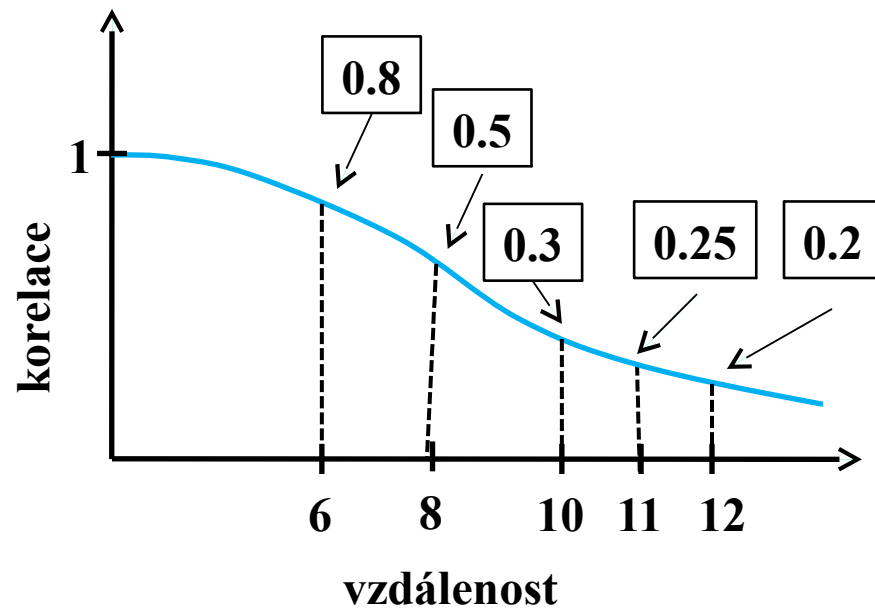
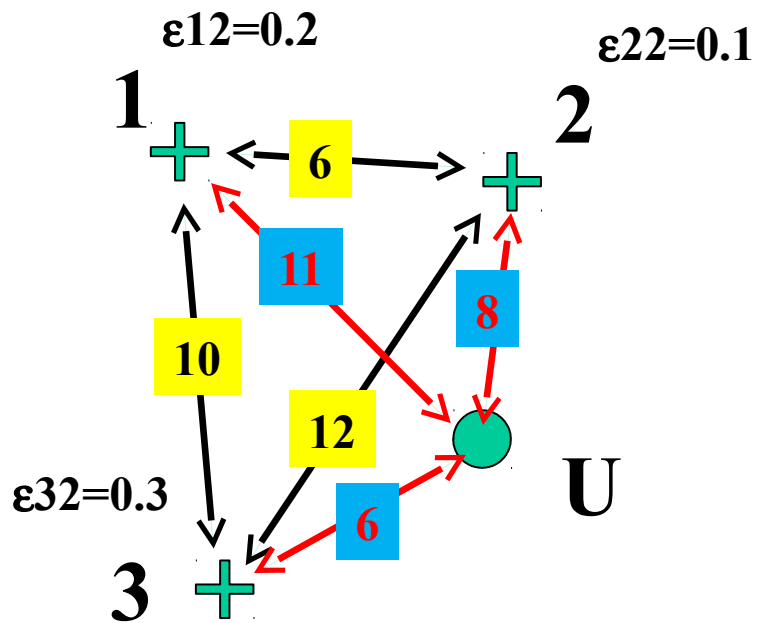


Příklad: Optimální interpolace

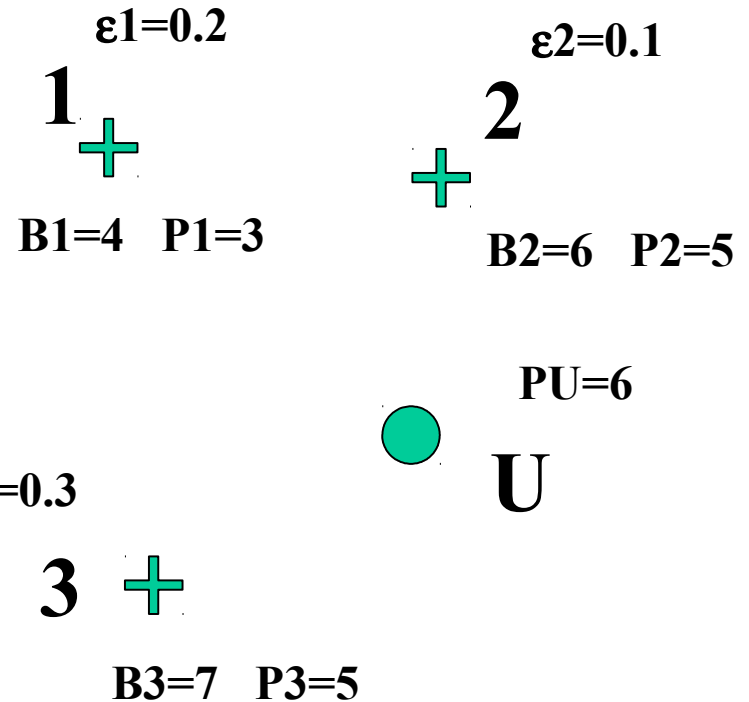


$$\begin{matrix} & \mathbf{M} & & \mathbf{w} & & \mathbf{h} \\ \left[\begin{array}{ccc} 1+0.2 & 0.8 & 0.3 \\ 0.8 & 1+0.1 & 0.2 \\ 0.3 & 0.2 & 1+0.3 \end{array} \right] & \begin{bmatrix} w1 \\ w2 \\ w3 \end{bmatrix} & = & \begin{bmatrix} 0.25 \\ 0.5 \\ 0.8 \end{bmatrix}
 \end{matrix}$$

$$\begin{bmatrix} 1+0.2 & 0.8 & 0.3 \\ 0.8 & 1+0.1 & 0.2 \\ 0.3 & 0.2 & 1+0.3 \end{bmatrix} \begin{bmatrix} w1 \\ w2 \\ w3 \end{bmatrix} = \begin{bmatrix} 0.25 \\ 0.5 \\ 0.8 \end{bmatrix}$$

$$w = [0.33 \quad 0.59 \quad 0.60]$$

$$\begin{aligned} A &= P + \sum_{i=1}^3 w_i (B_i - P_i) \\ &= 6 + 0.33*(4 - 3) + 0.59*(6 - 5) + 0.60*(7 - 5) \\ &= 6 + 0.33 + 0.59 + 1.2 = 8.12 \end{aligned}$$



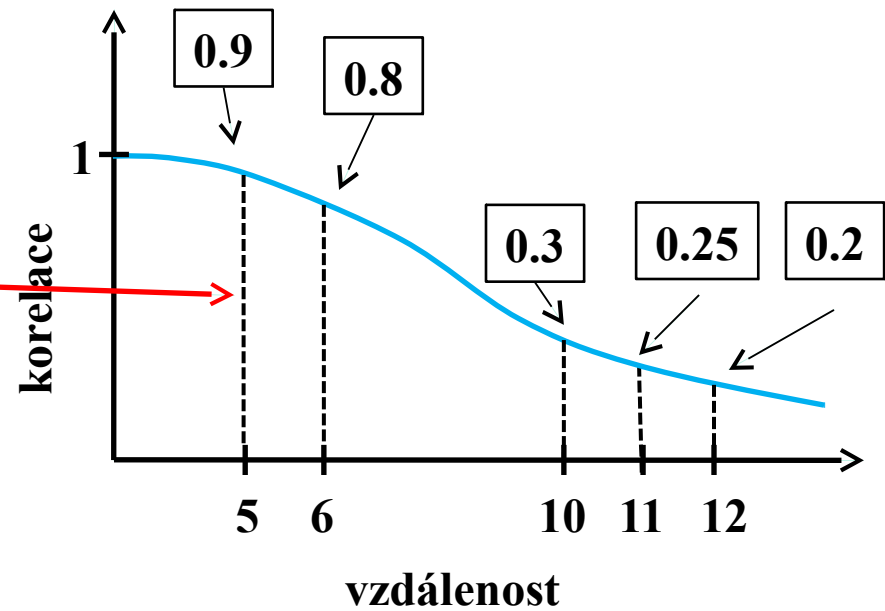
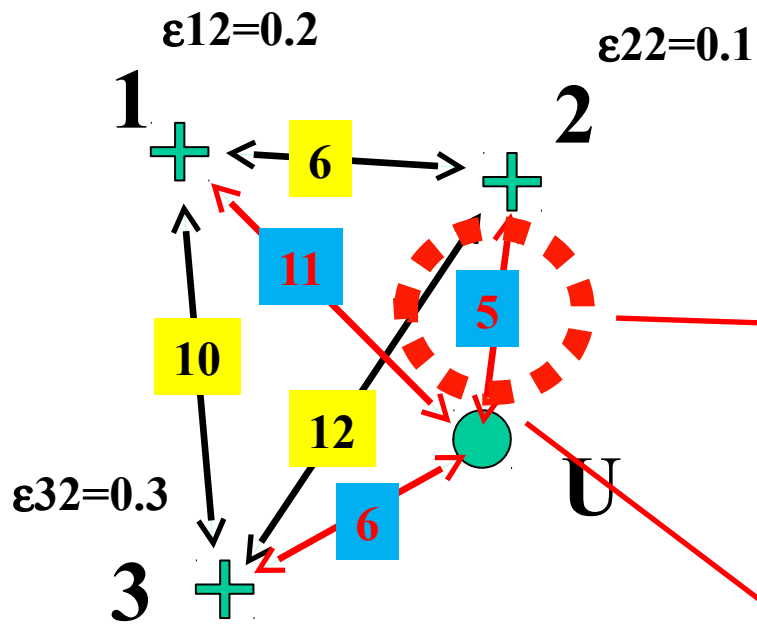
$$\left(\varepsilon^a\right)^2 = 1 - 2\mathbf{w}^T\mathbf{h} + \mathbf{w}^T\mathbf{M}\mathbf{w}$$

$$\begin{aligned}\left(\varepsilon^a\right)^2 &= 1 - \mathbf{h}^T\mathbf{M}^{-1}\mathbf{h} = 1 - \mathbf{h}^T\mathbf{w} \\ &= 1 - (0.25, 0.5, 0.8)^T (0.39, 0.59, 0.60) = 0.31\end{aligned}$$

$$\left(\varepsilon^a\right)^2 = \left(\frac{\mathbf{E}_a}{\mathbf{E}_p}\right)^2 = 0.31$$

$$\mathbf{E}_a = \sqrt{0.31} * \mathbf{E}_p = \sqrt{0.31} * \sqrt{0.4} = 0.35$$

Pozor !!!

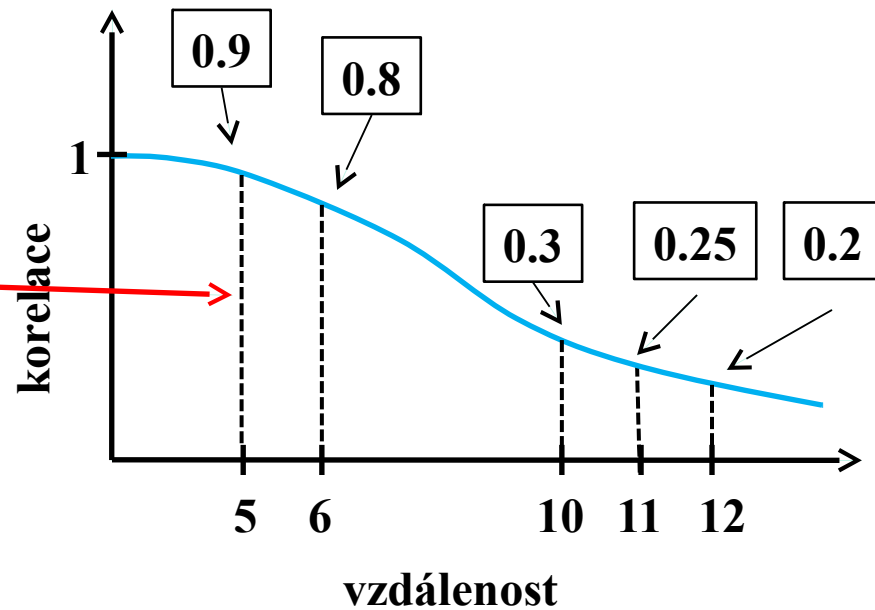
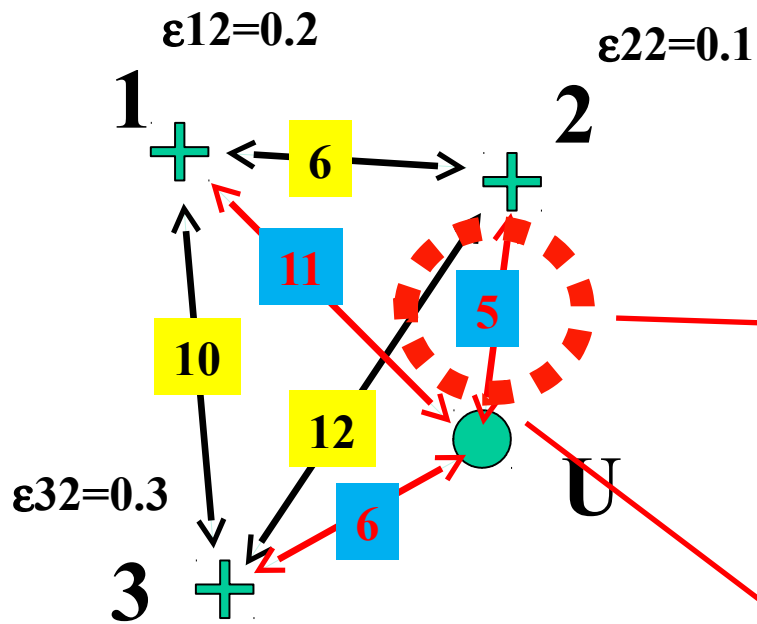


$$\begin{matrix} & \mathbf{M} & & \mathbf{w} & & \mathbf{h} \\ \left[\begin{array}{ccc} 1+0.2 & 0.8 & 0.3 \\ 0.8 & 1+0.1 & 0.2 \\ 0.3 & 0.2 & 1+0.3 \end{array} \right] & & \left[\begin{array}{c} w1 \\ w2 \\ w3 \end{array} \right] & = & \left[\begin{array}{c} 0.25 \\ 0.9 \\ 0.8 \end{array} \right]
 \end{matrix}$$

$$\mathbf{W} = \begin{bmatrix} -1.41 \\ 1.85 \\ 0.78 \end{bmatrix} \quad \mathbf{h} = \begin{bmatrix} 0.25 \\ 0.9 \\ 0.8 \end{bmatrix} \quad \mathbf{M}^{-1} = \begin{bmatrix} 2.5823 & -1.9765 & -0.3481 \\ -1.9765 & 2.5402 & 0.0779 \\ -0.3481 & 0.0779 & 0.9989 \end{bmatrix}$$

$$\mathbf{w} = [-0.80 \quad 1.29 \quad 0.60]$$

$$\left(\varepsilon^a\right)^2 = 1 - \mathbf{h}^T \mathbf{w} = -0.45$$



$$\begin{bmatrix} \mathbf{M} \\ \mathbf{W} \end{bmatrix} = \begin{bmatrix} \mathbf{h} \end{bmatrix}$$

Matrix \mathbf{M} (rows 1, 2, 3):

$$\begin{bmatrix} 1+0.2 & 0.8 & 0.3 \\ 0.8 & 1+0.1 & 0.2 \\ 0.3 & 0.2 & 1+0.3 \end{bmatrix}$$

Matrix \mathbf{W} (rows w1, w2, w3):

$$\begin{bmatrix} w1 \\ w2 \\ w3 \end{bmatrix}$$

Matrix \mathbf{h} (rows 0.25, 0.9, 0.8):

$$\begin{bmatrix} 0.25 \\ 0.9 \\ 0.8 \end{bmatrix}$$

A red arrow points from the value 0.9 in the matrix \mathbf{h} to the value 0.9 in the graph above.

Kriging

Výpočet hodnoty

$$\mathbf{w} = [-0.31 \quad 0.65 \quad 0.66]\mathbf{T}$$

$$\mathbf{B1}=4 \quad \mathbf{B2}=6 \quad \mathbf{B3}=7$$

$$\begin{aligned} \mathbf{A} &= -0.31 * 0.4 + 0.65 * 6 + 0.66 * 7 \\ &= 8.34 \end{aligned}$$

Chyba kriging ve vztahu k OI

$$\left(\varepsilon^a\right)^2 = 1 - 2\mathbf{w}^T\mathbf{h} + \mathbf{w}^T\mathbf{M}\mathbf{w}$$

$$\begin{array}{c} \mathbf{M} \\ \left[\begin{array}{ccc} 1+0.2 & 0.8 & 0.3 \\ 0.8 & 1+0.1 & 0.2 \\ 0.3 & 0.2 & 1+0.3 \end{array} \right] \end{array} \quad \begin{array}{c} \mathbf{h} \\ \left[\begin{array}{c} 0.25 \\ 0.5 \\ 0.8 \end{array} \right] \end{array} \quad \begin{array}{c} \mathbf{w} \\ \left[\begin{array}{c} -0.31 \\ 0.65 \\ 0.66 \end{array} \right] \end{array}$$

Chyba kriging ve vztahu k OI

$$\left(\varepsilon^a\right)^2 = 1 - 2\mathbf{w}^T \mathbf{h} \mathbf{w}^T \mathbf{M} \mathbf{w} = 0.32$$

$$\left(\varepsilon_{OI}^a\right)^2 = 0.31$$

$$\begin{array}{c} \mathbf{M} \\ \left[\begin{array}{ccc} 1+0.2 & 0.8 & 0.3 \\ 0.8 & 1+0.1 & 0.2 \\ 0.3 & 0.2 & 1+0.3 \end{array} \right] \end{array} \quad \begin{array}{c} \mathbf{h} \\ \left[\begin{array}{c} 0.25 \\ 0.5 \\ 0.8 \end{array} \right] \end{array} \quad \begin{array}{c} \mathbf{w} \\ \left[\begin{array}{c} -0.31 \\ 0.65 \\ 0.66 \end{array} \right] \end{array}$$

Iterační metoda OI

$$\begin{aligned}
 \mathbf{M}^{-1} &= \mathbf{Q}^{-1} \{ \mathbf{M} \mathbf{Q}^{-1} \}^{-1} = \\
 &= \mathbf{Q}^{-1} \{ \mathbf{I} - \mathbf{R} \}^{-1} = \\
 &= \mathbf{Q}^{-1} \{ \mathbf{I} - \mathbf{R} \}^{-1}
 \end{aligned}$$

$$\mathbf{M} = \begin{bmatrix} 1+0.2 & 0.8 & 0.3 \\ 0.8 & 1+0.1 & 0.2 \\ 0.3 & 0.2 & 1+0.3 \end{bmatrix}$$

Volba Q:

- Q je diagonální:
- Q je zvoleno tak, aby:

$$\begin{aligned}
 \mathbf{Q} &= \text{diag}\{q_{ii}\} \\
 q_{ii} &= \sum_{j=1}^n |m_{ij}|
 \end{aligned}$$

$$\mathbf{Q} = \begin{bmatrix} 1.2+0.8+0.3 & 0 & 0 \\ 0 & 1.1+0.8+0.2 & 0 \\ 0 & 0 & 1.3+0.3+0.2 \end{bmatrix}$$

$$\mathbf{R} = \mathbf{I} - \mathbf{M}\mathbf{Q}^{-1} = \begin{bmatrix} 0.4783 & -0.3810 & -0.1667 \\ -0.3478 & 0.4762 & -0.1111 \\ -0.1304 & -0.0952 & 0.2778 \end{bmatrix}$$

$$(\mathbf{I} - \mathbf{R})^{-1} = \mathbf{I} + \mathbf{R} + \mathbf{R}^2 + \mathbf{R}^3 + \dots$$

$$\mathbf{M}^{-1} = \mathbf{Q}^{-1} \sum_{j=0}^{\infty} \mathbf{R}^j$$

$$\mathbf{d}_0 = \mathbf{P} - \mathbf{P} = \begin{bmatrix} 4 - 3 \\ 6 - 5 \\ 7 - 5 \end{bmatrix}$$

$$\mathbf{d}_j = \mathbf{d}_{j-1} \mathbf{R} \mathbf{d}_{j-1}$$

$$\mathbf{w} = \mathbf{Q}^{-1} \mathbf{d}_{\infty}$$

$$\mathbf{A}_k = \mathbf{P}_k \mathbf{Q}^{-1} \mathbf{d}_{\infty} \mathbf{h}_k$$

$$\mathbf{A}_k = \mathbf{P}_k + \sum_{i=1}^n \mathbf{w}_{ik} (\mathbf{B}_i - \mathbf{P}_i) = \mathbf{P}_k + \mathbf{w}_k^T (\mathbf{B} - \mathbf{P})$$

$$\mathbf{w}_k = \mathbf{M}^{-1} \mathbf{h}_k$$

$$(\mathbf{I} - \mathbf{R})^{-1} = \mathbf{I} + \mathbf{R} + \mathbf{R}^2 + \mathbf{R}^3 + \dots$$

$$\mathbf{M}^{-1} = \mathbf{Q}^{-1} \sum_{j=0}^{\infty} \mathbf{R}^j$$

$$\begin{aligned} \mathbf{A}_k &= \mathbf{P}_k + \mathbf{w}_k^T (\mathbf{B} - \mathbf{P}) = \mathbf{P}_k + \left(\mathbf{M}^{-1} \mathbf{h}_k \right)^T (\mathbf{B} - \mathbf{P}) = \\ &= \mathbf{P}_k + \left(\mathbf{Q}^{-1} \sum_{j=0}^{\infty} \mathbf{R}^j \mathbf{h}_k \right)^T (\mathbf{B} - \mathbf{P}) = \mathbf{P}_k + (\mathbf{h}_k)^T \mathbf{Q}^{-1} \sum_{j=0}^{\infty} \mathbf{R}^j (\mathbf{B} - \mathbf{P}) \end{aligned}$$

Výpočet

$$\mathbf{d}_0 = \mathbf{B} - \mathbf{P}$$

$$\mathbf{d}_j = \mathbf{R} \mathbf{d}_{j-1}$$

$$\mathbf{A}_k = \mathbf{P}_k + \mathbf{Q}^{-1} \mathbf{d}_{\infty} \mathbf{h}_k$$