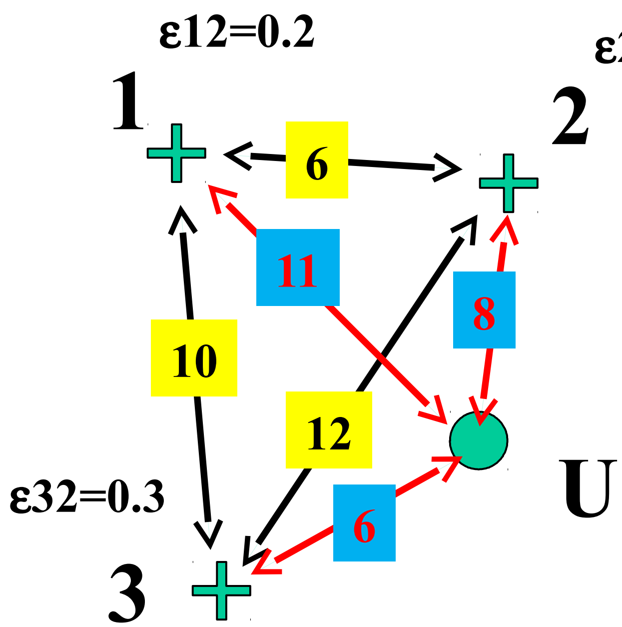
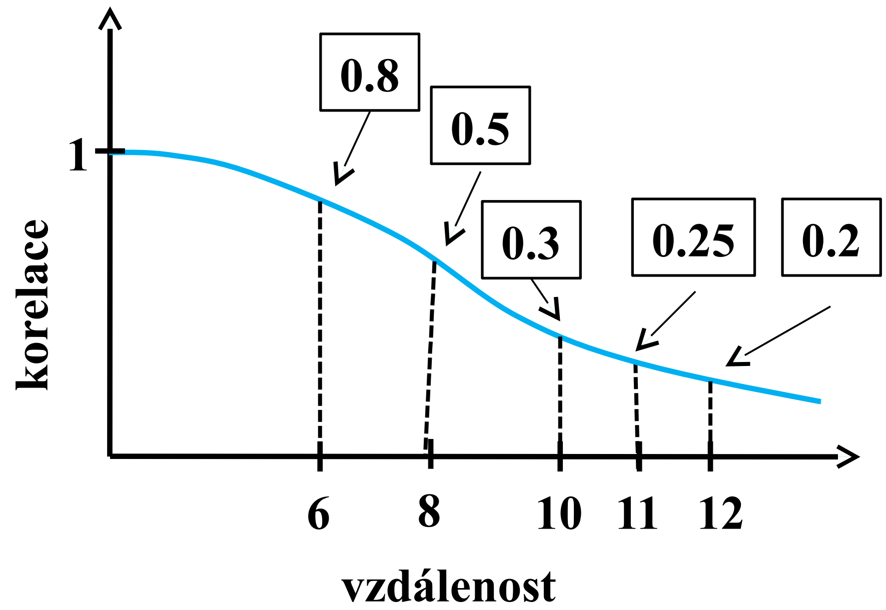
Příklad: Optimální interpolace





$$
\begin{array}{ccc}
\mathbf{M} & \mathbf{w} & & \mathbf{h} \\
\begin{bmatrix} 1+0.2 & 0.8 & 0.3 \\ 0.8 & 1+0.1 & 0.2 \\ 0.3 & 0.2 & 1+0.3 \end{bmatrix} &
\begin{bmatrix} w1 \\ w2 \\ w3 \end{bmatrix} & = &
\begin{bmatrix} 0.25 \\ 0.5 \\ 0.8 \end{bmatrix}
\end{array}
$$

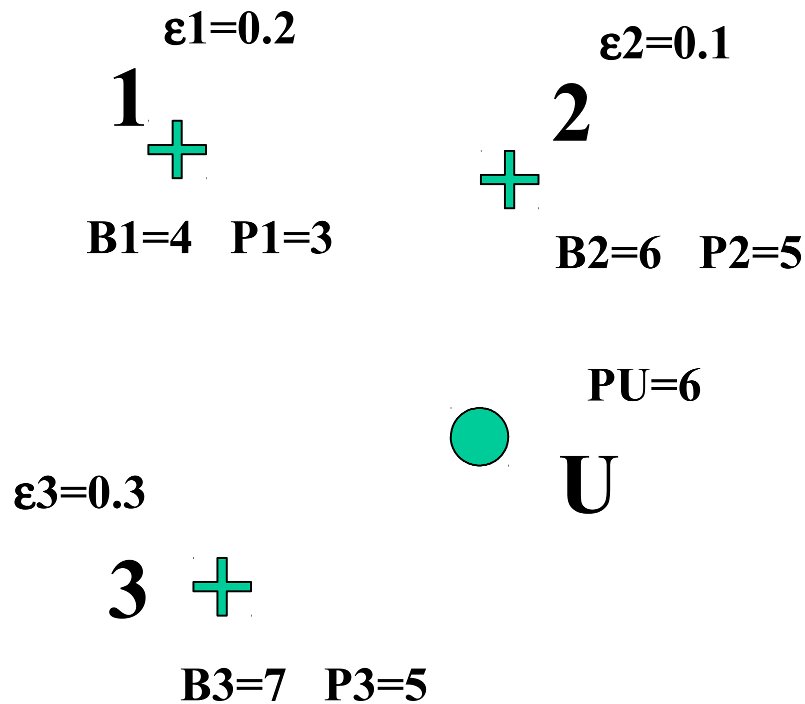
$$
\begin{bmatrix} 1+0.2 & 0.8 & 0.3 \\ 0.8 & 1+0.1 & 0.2 \\ 0.3 & 0.2 & 1+0.3 \end{bmatrix}
\begin{bmatrix} w1 \\ w2 \\ w3 \end{bmatrix}
=
\begin{bmatrix} 0.25 \\ 0.5 \\ 0.8 \end{bmatrix}
$$

w = [0.33   0.59   0.60]

$$
\mathbf{A} = \mathbf{P} + \sum_{\mathbf{i}=1}^{3} \mathbf{w}_{\mathbf{i}}(\mathbf{B}_{\mathbf{i}} - \mathbf{P}_{\mathbf{i}})
$$

$$
= 6 + 0.33*(4-3) + 0.59*(6-5) + 0.60*(7-5)
$$

$$
= 6 + 0.33 + 0.59 + 1.2 = 8.12
$$



$$\left(\varepsilon^{\mathbf{a}}\right)^2 = 1 - 2\mathbf{w}^T\mathbf{h} + \mathbf{w}^T\mathbf{M}\mathbf{w}$$

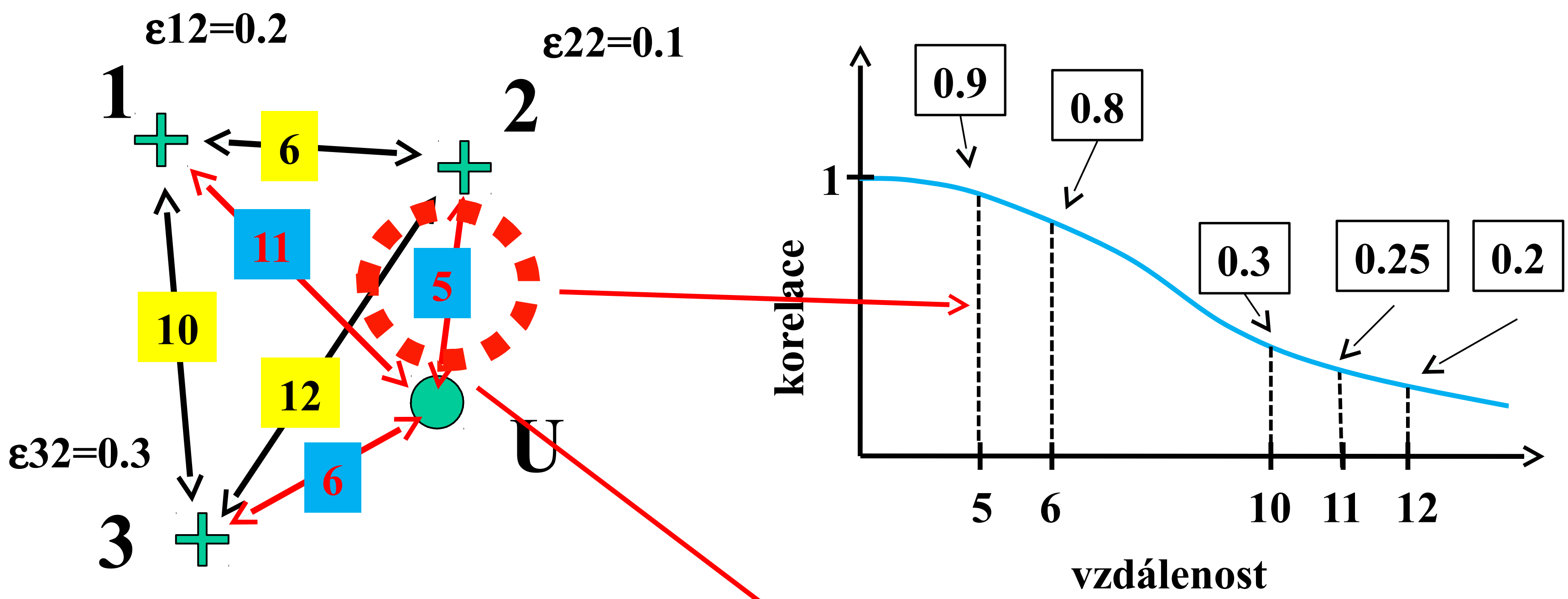
$$\left(\varepsilon^{\mathbf{a}}\right)^2 = 1 - \mathbf{h}^T\mathbf{M}^{-1}\mathbf{h} = 1 - \mathbf{h}^T\mathbf{w}$$

$$= 1 - (0.25,0.5,0.8)^T(0.39,0.59,0.60) = 0.31$$

$$\left(\varepsilon^{\mathbf{a}}\right)^2 = \left(\frac{\mathbf{E_a}}{\mathbf{E_p}}\right)^2 = 0.31$$

$$\mathbf{E_a} = \sqrt{0.31} * \mathbf{E_p} = \sqrt{0.31} * \sqrt{0.4} = 0.35$$

Pozor !!!

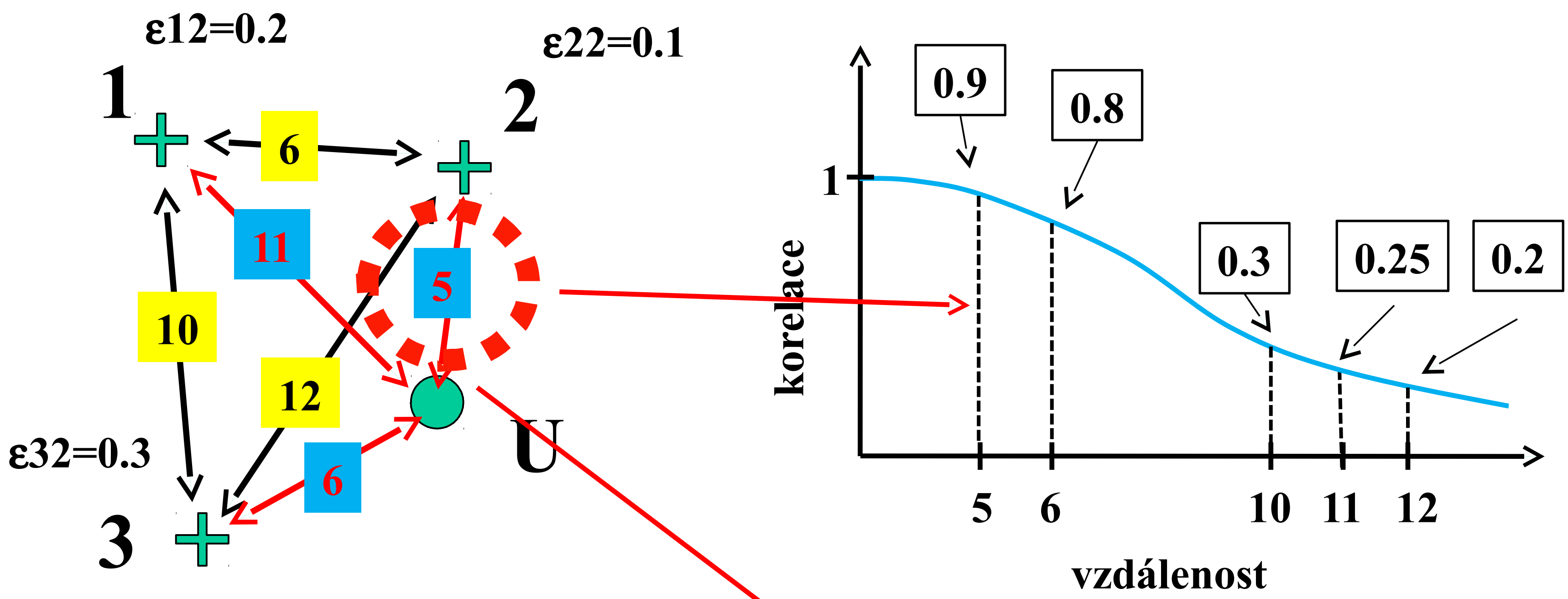


$$\overset{\mathbf{M}}{\begin{bmatrix} 1+0.2 & 0.8 & 0.3 \\ 0.8 & 1+0.1 & 0.2 \\ 0.3 & 0.2 & 1+0.3 \end{bmatrix}} \overset{\mathbf{w}}{\begin{bmatrix} w1 \\ w2 \\ w3 \end{bmatrix}} = \overset{\mathbf{h}}{\begin{bmatrix} 0.25 \\ 0.9 \\ 0.8 \end{bmatrix}}$$

$$\mathbf{w} = \begin{bmatrix} -1.41 \\ 1.85 \\ 0.78 \end{bmatrix} \quad \mathbf{h} = \begin{bmatrix} 0.25 \\ 0.9 \\ 0.8 \end{bmatrix} \quad \mathbf{M\text{-}1} = \begin{bmatrix} 2.5823 & -1.9765 & -0.3481 \\ -1.9765 & 2.5402 & 0.0779 \\ -0.3481 & 0.0779 & 0.9989 \end{bmatrix}$$

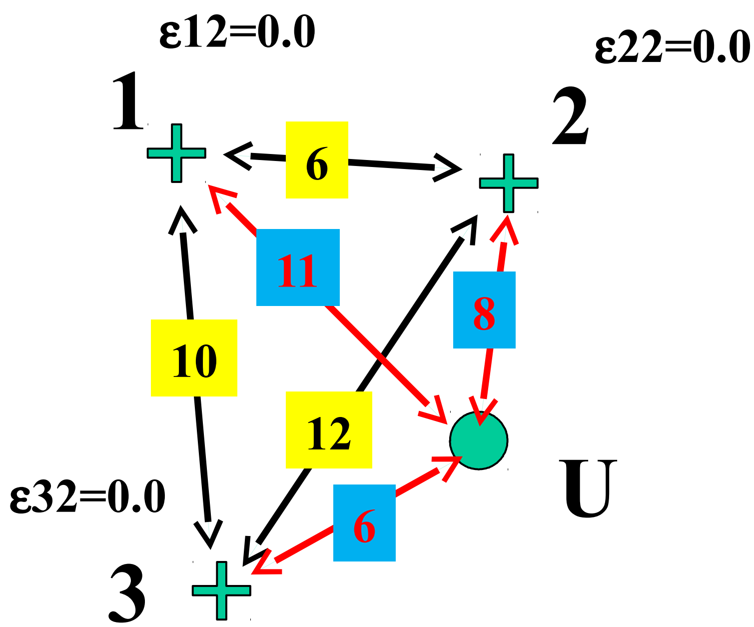
w = [-0.80 1.29 0.60]

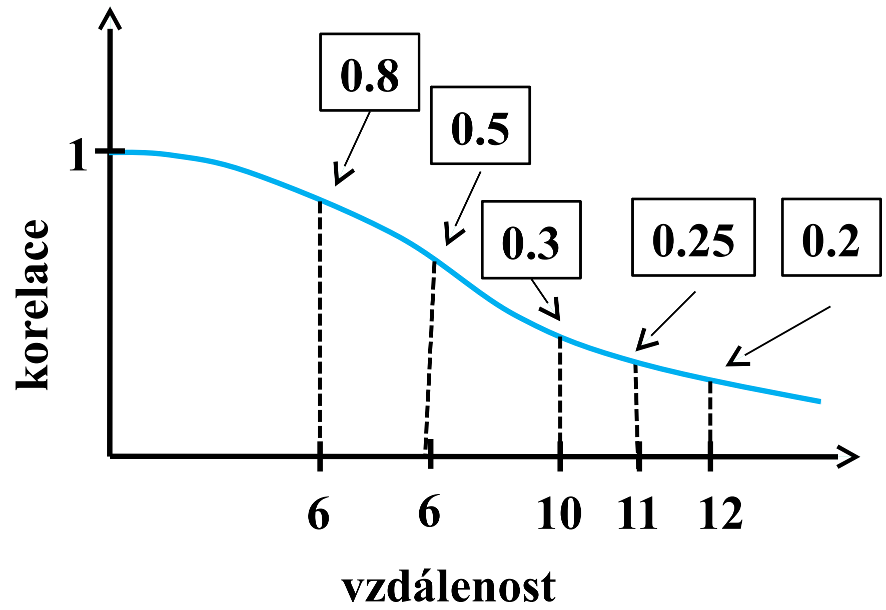
$$\left(\varepsilon^{\mathbf{a}}\right)^2 = 1 - \mathbf{h}^{\mathbf{T}}\mathbf{w} = -\ 0.45$$



$$\underset{\mathbf{M}}{\begin{bmatrix} 1+0.2 & 0.8 & 0.3 \\ 0.8 & 1+0.1 & 0.2 \\ 0.3 & 0.2 & 1+0.3 \end{bmatrix}} \underset{\mathbf{w}}{\begin{bmatrix} w1 \\ w2 \\ w3 \end{bmatrix}} = \underset{\mathbf{h}}{\begin{bmatrix} 0.25 \\ 0.9 \\ 0.8 \end{bmatrix}}$$

Kriging





$$\overset{\mathbf{M}}{\begin{bmatrix} 1 & 0.8 & 0.3 & 1 \\ 0.8 & 1 & 0.2 & 1 \\ 0.3 & 0.2 & 1 & 1 \\ 1 & 1 & 1 & 0 \end{bmatrix}} \overset{\mathbf{w}}{\begin{bmatrix} w1 \\ w2 \\ w3 \\ \lambda \end{bmatrix}} = \overset{\mathbf{h}}{\begin{bmatrix} 0.25 \\ 0.5 \\ 0.8 \\ 1 \end{bmatrix}}$$

Výpočet hodnoty

w = [-0.31   0.65   0.66]T

B1=4   B2=6   B3=7

A = -0.31* 0.4 + 0.65*6 + 0.66*7
= 8.34

Chyba kriging ve vztahu k OI

$$\left(\varepsilon^{\mathbf{a}}\right)^2 = 1 - 2\mathbf{w}^T\mathbf{h} + \mathbf{w}^T\mathbf{M}\mathbf{w}$$

$$\mathbf{M} = \begin{bmatrix} 1+0.2 & 0.8 & 0.3 \\ 0.8 & 1+0.1 & 0.2 \\ 0.3 & 0.2 & 1+0.3 \end{bmatrix} \qquad \mathbf{h} = \begin{bmatrix} 0.25 \\ 0.5 \\ 0.8 \end{bmatrix} \qquad \mathbf{w} = \begin{bmatrix} -0.31 \\ 0.65 \\ 0.66 \end{bmatrix}$$

Chyba kriging ve vztahu k OI

$$\left(\varepsilon^{\mathbf{a}}\right)^2 = 1 - 2\mathbf{w}^T\mathbf{h} + \mathbf{w}^T\mathbf{M}\mathbf{w} = 0.32$$

$$\left(\varepsilon^{\mathbf{a}}_{\mathbf{OI}}\right)^2 = 0.31$$

$$\mathbf{M} = \begin{bmatrix} 1+0.2 & 0.8 & 0.3 \\ 0.8 & 1+0.1 & 0.2 \\ 0.3 & 0.2 & 1+0.3 \end{bmatrix} \qquad \mathbf{h} = \begin{bmatrix} 0.25 \\ 0.5 \\ 0.8 \end{bmatrix} \qquad \mathbf{w} = \begin{bmatrix} -0.31 \\ 0.65 \\ 0.66 \end{bmatrix}$$

Iterační metoda OI

$$
\begin{aligned}
\mathbf{M}^{-1} &= \mathbf{Q}^{-1}\left\{\mathbf{MQ}^{-1}\right\}^{-1} = \\
&= \mathbf{Q}^{-1}\left\{\mathbf{I} - \left[\mathbf{I} - \mathbf{MQ}^{-1}\right]\right\}^{-1} = \\
&= \mathbf{Q}^{-1}\left\{\mathbf{I} - \mathbf{R}\right\}^{-1}
\end{aligned}
$$

$$
\mathbf{M} = \begin{bmatrix} 1+0.2 & 0.8 & 0.3 \\ 0.8 & 1+0.1 & 0.2 \\ 0.3 & 0.2 & 1+0.3 \end{bmatrix}
$$

Volba Q:

- **Q je diagonální:** $\mathbf{Q} = \left\{q_{ii}\right\}$
- **Q je zvoleno tak, aby:** $q_{ii} = \sum_{j=1}^{n} \left| m_{ij} \right|$

$$
\mathbf{Q} = \begin{bmatrix} 1.2+0.8+0.3 & 0 & 0 \\ 0 & 1.1+0.8+0.2 & 0 \\ 0 & 0 & 1.3+0.3+0.2 \end{bmatrix}
$$

$$\mathbf{R} = \mathbf{I} - \mathbf{M}\mathbf{Q}^{-1} = \begin{bmatrix} 0.4783 & -0.3810 & -0.1667 \\ -0.3478 & 0.4762 & -0.1111 \\ -0.1304 & -0.0952 & 0.2778 \end{bmatrix}$$

$$(\mathbf{I} - \mathbf{R})^{-1} = \mathbf{I} + \mathbf{R} + \mathbf{R}^2 + \mathbf{R}^3 + \ldots$$

$$\mathbf{M}^{-1} = \mathbf{Q}^{-1} \sum_{j=0}^{\infty} \mathbf{R}^j$$

$$\mathbf{d}_0 = \mathbf{B} - \mathbf{P} = \begin{bmatrix} 4-3 \\ 6-5 \\ 7-5 \end{bmatrix}$$

$$\mathbf{d}_j = \mathbf{d}_{j-1} + \mathbf{R}\,\mathbf{d}_{j-1}$$

$$\mathbf{w} = \mathbf{Q}^{-1}\mathbf{d}_\infty$$

$$\mathbf{A}_k = \mathbf{P}_k + \mathbf{Q}^{-1}\mathbf{d}_\infty \mathbf{h}_k$$

$$\mathbf{A}_k = \mathbf{P}_k + \sum_{i=1}^{n} \mathbf{w}_{ik}(\mathbf{B}_i - \mathbf{P}_i) = \mathbf{P}_k + \mathbf{w}_k^T(\mathbf{B} - \mathbf{P})$$

$$\mathbf{w}_k = \mathbf{M}^{-1}\mathbf{h}_k$$

$$(\mathbf{I} - \mathbf{R})^{-1} = \mathbf{I} + \mathbf{R} + \mathbf{R}^2 + \mathbf{R}^3 + \ldots$$

$$\mathbf{M}^{-1} = \mathbf{Q}^{-1}\sum_{j=0}^{\infty} \mathbf{R}^j$$

$$\mathbf{A}_k = \mathbf{P}_k + \mathbf{w}_k^T(\mathbf{B} - \mathbf{P}) = \mathbf{P}_k + \left(\mathbf{M}^{-1}\mathbf{h}_k\right)^T(\mathbf{B} - \mathbf{P}) =$$

$$= \mathbf{P}_k + \left(\mathbf{Q}^{-1}\sum_{j=0}^{\infty} \mathbf{R}^j\mathbf{h}_k\right)^T(\mathbf{B} - \mathbf{P}) = \mathbf{P}_k + (\mathbf{h}_k)^T\mathbf{Q}^{-1}\sum_{j=0}^{\infty} \mathbf{R}^j(\mathbf{B} - \mathbf{P})$$

Výpočet

$$\mathbf{d}_0 = \mathbf{B} - \mathbf{P}$$

$$\mathbf{d}_j = \mathbf{d}_{j-1} + \mathbf{R}\,\mathbf{d}_{j-1}$$

$$\mathbf{A}_k = \mathbf{P}_k + \mathbf{Q}^{-1}\mathbf{d}_\infty \mathbf{h}_k$$