Stellar populations and star clusters as galactic building blocks

Lecture 4

The stellar binary population: deriving the birth distribution functions
Binary dynamical population synthesis: the stellar populations of galaxies

Selected Chapters on Astrophysics
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Lecture 1 :

The stellar IMF : solar neighbourhood as average IMF
theoretical expectations : a *variable IMF*

Lecture 2 :

The stellar IMF : constraints from star-forming events **:**
a non-varying IMF ?

Lecture 3 :

The integrated galactic initial mass function (IGIMF) : a new theory
How to calculate the stellar population of a galaxy.

Lecture 4 :

The stellar binary population**:** deriving the birth distribution functions
Binary dynamical population synthesis**:** the stellar populations of galaxies

The Initial Binary Population

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We saw that deriving (or rather constraining) the IMF was not a trivial task.

Similarly, deriving (or rather constraining) the initial or birth distribution functions of binary systems is not trivial. It is, however, essential because most and perhaps all stars form in binary systems.

It has already become apparent that corrections for *unresolved multiple systems* are of much importance in deriving the IMF.

For clusters, multiple systems are likewise of importance because they add additional *energy-exchange channels*, and they may absorb a significant amount of binding energy.

Binaries have internal degrees of freedom that single stars do not.

Dynamical Issues

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The *most-important dynamical quantities* describing a mutliple system are

- the period P (always in days) or semi-major axis a (in AU)
- the system mass $m_{\rm sys} = m_1 + m_2$
- the mass ratio $q \equiv \frac{m_2}{m_1} \le 1$, where $m_1 =$ primary, $m_2 =$ secondary mass
- the eccentricity $e = \frac{R_{\rm apo} - R_{\rm peri}}{R_{\rm aop} + R_{\rm peri}}$

Given a *snap-shot*, these orbital parameters can be calculated as follows, where it is assumed that the *relative position* and *velocity vectors* and the masses of the two companion stars are known, $\vec{R}_{\rm rel}$, $\vec{V}_{\rm rel}$, m_1 , m_2 (in consistent units).

The binding energy of the binary (virial theorem) :

$$E_{\rm b} = \frac{1}{2}\,\mu\,V_{\rm rel}^2 - \frac{G\,m_1\,m_2}{R_{\rm rel}} = -\frac{G\,m_1\,m_2}{2a} \quad \Longrightarrow \quad a$$

$$\left(\mu = \frac{m_1\,m_2}{m_1 + m_2}\right)$$

Kepler's third law : $\quad m_{\rm sys} = \frac{a_{\rm AU}^3}{P_{\rm yr}^2} \quad \Longrightarrow \quad P = P_{\rm yr} \times 365.25 \text{ days}$

Eccentricity $\quad e = \left[\left(1 - \frac{R_{\rm rel}}{a}\right)^2 + \frac{(\vec{R}_{\rm rel}\cdot\vec{V}_{\rm rel})^2}{a\,G\,m_{\rm sys}}\right]^{\frac{1}{2}}$

The relative equation of motion $\quad \frac{d^2\vec{R}_{\rm rel}}{dt^2} = -G\,\frac{m_1 + m_2}{|R_{\rm rel}|^3}\,\vec{R}_{\rm rel} + \vec{a}_{\rm pert}$

where $\vec{a}_{\rm pert}$ is the time-dependent perturbation from other cluster members.

$P = P(t), \quad a = a(t)$

$e = e(t)$

$q = q(t)$ during strong encounters when partners exchange

Since most stars form in embedded clusters it follows that the binary-star properties of a given population cannot be taken to represent the initial or primordial values.

The comparison of two populations with different ***dynamical properties*** constrains the dynamical history of these.

Simple overview of binary processes in a cluster :

Binaries can absorp energy and thus cool a cluster.
Binaries can emit energy an thus heat a cluster.

$$\text{cluster} \underset{\text{energy exchange}}{\longleftrightarrow} \text{binaries}$$

There are two extreme regimes that can be understood with a simple *gedanken experiment* **:**

$$E_{\rm bin} \equiv -E_{\rm b} > 0 \qquad\qquad E_{\rm k} \equiv \frac{1}{2}\,\overline{m}\,\sigma^2 \approx \frac{1}{N}\,\text{kinetic energy of cluster}$$

soft binaries $E_{\rm bin} \ll E_{\rm k}$

hard binaries $E_{\rm bin} \gg E_{\rm k}$

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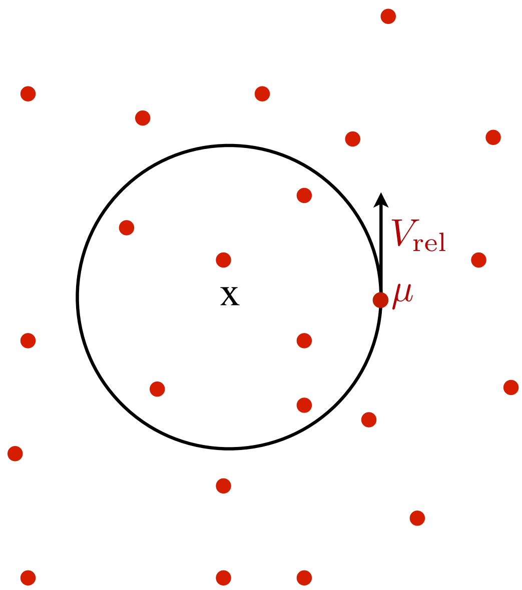
The binary consists of a central (fixed) potential about which a reduced particle μ orbits on a circular trajectory.
It is immersed in a bath of thermal particles with velocity dispersion σ . (Note: $lP[\text{days}] = 6.986 + lm_{\rm sys}[M_\odot] - 3\,lV_{\rm orb}[\text{km/s}]$ $\quad(\log_{10}() \equiv l()\,)$)

An encounter

soft binary $\longleftrightarrow$ thermal particle
$(V_{\rm rel} \ll \sigma)$

- $V_{\rm rel} \uparrow$ by energy equipartition
- $|E_{\rm b}| \downarrow \quad \Longrightarrow \quad a \uparrow,\ P \uparrow$
- *softer*



hard binary $\longleftrightarrow$ thermal particle
$(V_{\rm rel} \gg \sigma)$

- $V_{\rm rel} \downarrow$ by energy equipartition
- $|E_{\rm b}| \uparrow \quad \Longrightarrow \quad a \downarrow,\ P \downarrow$
- *harder*

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***Heggie-Hills law*:** (Heggie 1975; Hills 1975)

Hard binaries harden,
soft binaries soften.

Thus, *hard* binaries *heat* the thermal bath, they are an *energy source*, mostly near the centre of a cluster, *cf.* to thermonuclear reactions at a star's centre.

Soft binaries *cool* a cluster.
(Kroupa, Petr & McCaughrean 1999, NewA)

Note: The energy range $10^{-2}\,E_{\rm k} \lesssim |E_{\rm b}| \lesssim 10^{2}\,E_{\rm k}$

$$33^{-1}\,\sigma \lesssim V_{\rm orb} \lesssim 33\,\sigma$$

cannot be dealt with analytically because of the complex resonances.

It is these binaries which may be highly important for the early evolution of a cluster, depending on its $\sigma = \sigma(M_{\rm cl}, R)$.

This can only be studied numerically.

The Binary Population

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In order to set-up a
realistic stellar population
in a cluster
we need to have
distribution functions
of the
multiple-star properties,
such as the periods, eccentricities
and mass-ratios.

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$$N_{\rm mult} \equiv (N_{\rm sing} : N_{\rm bin} : N_{\rm trip} : N_{\rm quad} : \ldots)$$
$$= (\mathcal{S} : \mathcal{B} : \mathcal{T} : \mathcal{Q} : \ldots)$$

$$f_{\rm mult} = \frac{N_{\rm mult}}{N_{\rm sys}} = \frac{\mathcal{B}+\mathcal{T}+\mathcal{Q}+\ldots}{\mathcal{S}+\mathcal{B}+\mathcal{T}+\mathcal{Q}+\ldots}$$
$$f_{\rm bin} = \frac{\mathcal{B}}{N_{\rm sys}}$$

In the Galactic field :

for G-dwarfs : $N_{\rm mult} = (\,57 : 38 : 4 : 1\,)$ (Duquennoy & Mayor 1991)

for M-dwarfs : $N_{\rm mult} = (\,58 : 33 : 7 : 1\,)$ (Fischer & Marcy 1992)

$$^{\rm G}f_{\rm mult} = 0.43\,; \quad ^{\rm G}f_{\rm bin} = 0.38$$
$$^{\rm M}f_{\rm mult} = 0.41\,; \quad ^{\rm M}f_{\rm bin} = 0.33$$

→ most multiples in the Galactic field are binaries

→ higher-order multiples can be neglected to 1st order.

After correcting for incompleteness :

$$^{\rm G}f_{\rm bin} = 0.53 \pm 0.08 \equiv f_{\rm G}$$ (Duquennoy & Mayor 1991)

$$^{\rm K}f_{\rm bin} = 0.45 \pm 0.07 \equiv f_{\rm K}$$ (Mayor et al. 1992)

$$^{\rm M}f_{\rm bin} = 0.42 \pm 0.09 \equiv f_{\rm M}$$ (Fischer & Marcy 1992)

→ $f_{\rm G} \approx f_{\rm K} \approx f_{\rm M} \approx 0.5 = f_{\rm tot}$ in the Galactic field

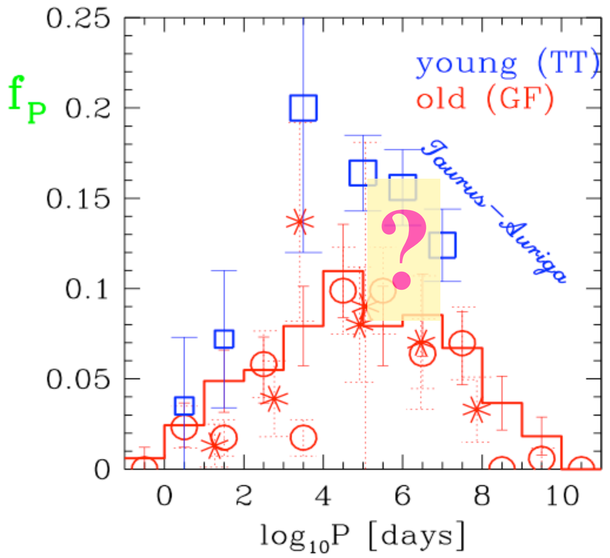
But : about 1 Myr old stars have $f_{\rm TTauri} \approx 1$ (e.g. Duchene 1999)

For BDs, $f_{\rm BD} \approx 0.15$, independently of age. (Close et al. 2003; Buoy et al. 2003; Burgasser et al. 2003; Mertin et al. 2003)

The distribution of periods :

G- and M-dwarfs: $f_P(lP) = f_{\mathrm{tot}} \left(\frac{1}{\sigma_{lP}\sqrt{2\,\pi}} \right) e^{\left[-\frac{1}{2} \frac{(lP - \overline{lP})^2}{\sigma^2_{lP}} \right]}$ (Duquennoy & Mayor 1991; Fischer & Marcy 1992)

Initial/primordial $f_P(lP) = 2.5 \frac{lP - 1}{45 + (lP - 1)^2}$ (Kroupa 1995; Kroupa et al. 2003; Goodwin & Kroupa 2006)



where the normalisation

$$\int_{lP_{\mathrm{min}}=1}^{lP_{\mathrm{max}}} f_P(lP)\, dlP = f_{\mathrm{tot}} \; (= 1 \text{ TTauri})$$

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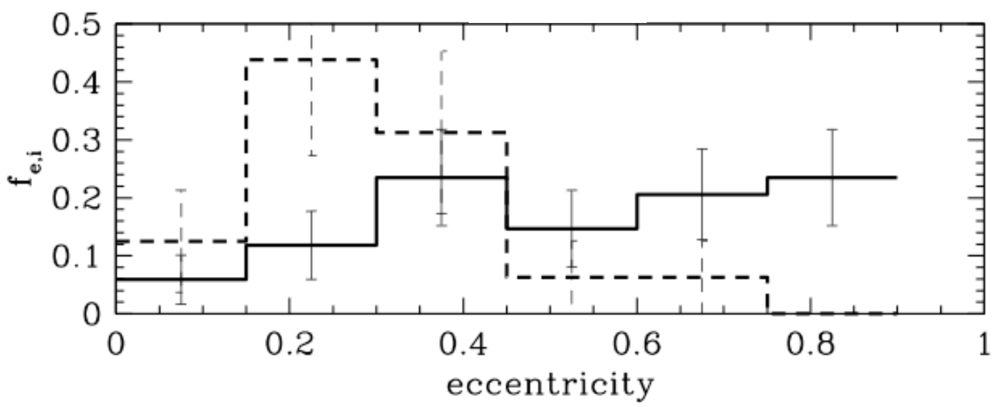
The distribution of eccentricities :



$f_e(e) = $ bell shaped; $\quad lP \lesssim 3$

$f_e(e) = 2\,e; \quad lP \gtrsim 3$

(Duquennoy & Mayor 1991)

The thermal distribution :

Orbital angular momentum $L^2 = \frac{G}{m_{\mathrm{sys}}} \frac{G\, m_1\, m_2}{2\, E_{\mathrm{bin}}} (1 - e^2)\, (m_1\, m_2)^2$

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$$e = \left(1 - 2\,E_{\rm bin}\,L^2\,\frac{m_{\rm sys}}{G^2\,(m_1\,m_2)^2}\right)^{\frac{1}{2}}$$

$$\frac{de}{dE_{\rm bin}} = \left[-L^2\,\frac{m_{\rm sys}}{G^2\,(m_1\,m_2)^2}\right](\ )^{-\frac{1}{2}} = [\ ]\ e^{-1}$$

The number of binaries with eccentricity in the range $e, e+de$ is the same number with binding-energy in the range $E_{\rm bin}, E_{\rm bin}+dE_{\rm bin}$,

$$f(e)\,de = f(E_{\rm bin})\,dE_{\rm bin} = f(E_{\rm bin})\ \ [\ ]^{-1}\ e\,de$$

But $\displaystyle\int_0^1 f(e)\,de = 1$

$$= f(E_{\rm bin})\ \ [\ ]^{-1}\int_0^1 e\,de = f(E_{\rm bin})\ \ [\ ]^{-1}\ \frac{1}{2}\,e^2\,|_0^1$$

so $f(E_{\rm bin})\ \ [\ ]^{-1} = 2\ = \text{const}$ → $\boxed{f(e)\,de = 2\,e\,de}$

Thus, $f(e) = 2\,e$ is a *thermalised distribution*; all binding energy levels are equally occupied ($f(E_{\rm bin}) = \text{const}$).

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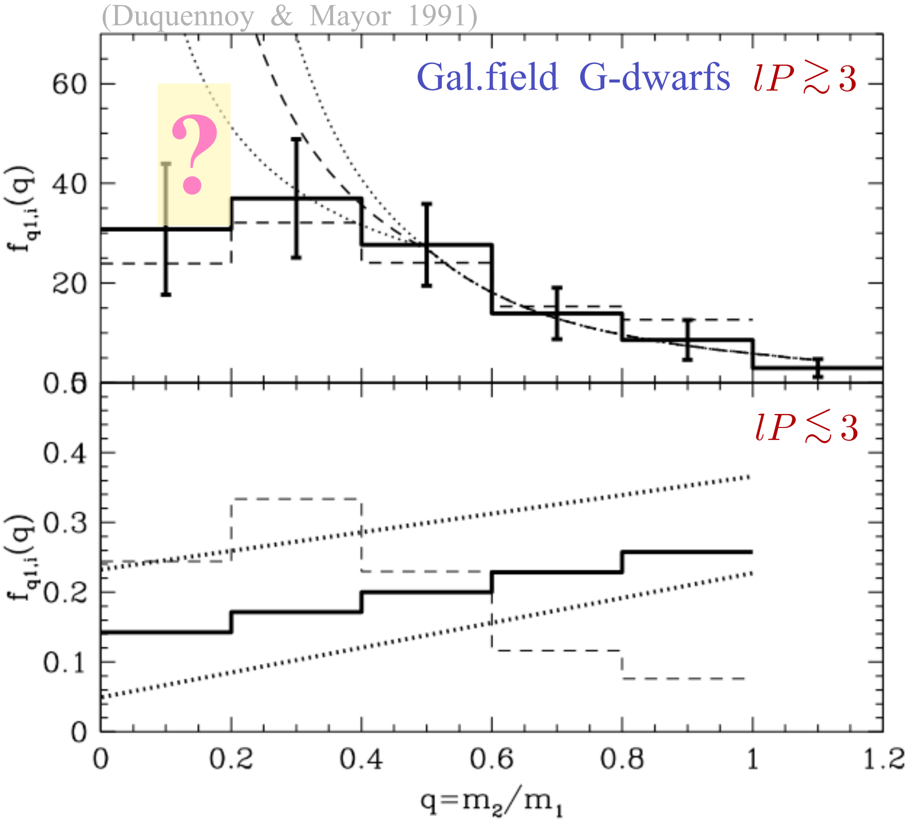
The mass-ratio distribution :

(Duquennoy & Mayor 1991)



For pre-main sequence binaries, the q-distribution is flat being consistent with random pairing for $q \gtrsim 0.2$

↓

supports *fragmentation* rather than common-accretion as origin of companion masses but is *inconsistent* with Galactic-field stars

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Summary :

- In order to do realistic star-cluster evolution models we need an exact description of the initial stellar population **:**
- IMF: $dN = \xi(m)\,dm$
- Initial binary-star population:

$$df = f_P(lP)\,dlP \quad \text{or} \quad df = f(E_{\rm bin})\,dE_{\rm bin}$$

$$df = f_e(e)\,de$$

$$df = f_q(q)\,dq$$

These are the four distribution functions describing a stellar population.

Some questions to ponder :

Can the discrepant pre-main-sequence and main-sequence binary-star properties be explained by clustered star formation **?**

How universal are the IPF, IEF, IMRF **?**
(f_P f_e f_q)

Assuming there are unique and universal IPF, IEF, IMRF :
which dynamical structure can unify the above discrepancy **?**

→ *inverse dynamical population synthesis* (IDPS) **:**

Ansatz : Assume the initial orbital-parameter distribution function can be separated : $D_{\rm in}(lP, e, q : m_1) = f_P \; f_e \; f_q$ for primary-star masses m_1 . (Kroupa 2002: habilitation thesis; Marks, Kroupa & Oh 2011)

Introduce : $D_{\rm fin}(lP, e, q : m_1) = \boxed{\Omega^{N, R_{0.5}}} \; [D_{\rm in}(lP, e, q : m_1)]$

e.g. the Galactic-field population

the "stellar-dynamical operator"

the initial TTauri population

Aim : Solve for $\boxed{\Omega^{N, R_{0.5}}}$:

Questions :

- Is there a solution *such that*
  $f_{P,\rm in} \longrightarrow f_{P,\rm fin}$
  $f_{e,\rm in} \longrightarrow f_{e,\rm fin}$
  $f_{q,\rm in} \longrightarrow f_{q,\rm fin}$
  simultaneously ?

- Is the solution *unique* ?

Preview

(Marks, Kroupa & Oh 2011)

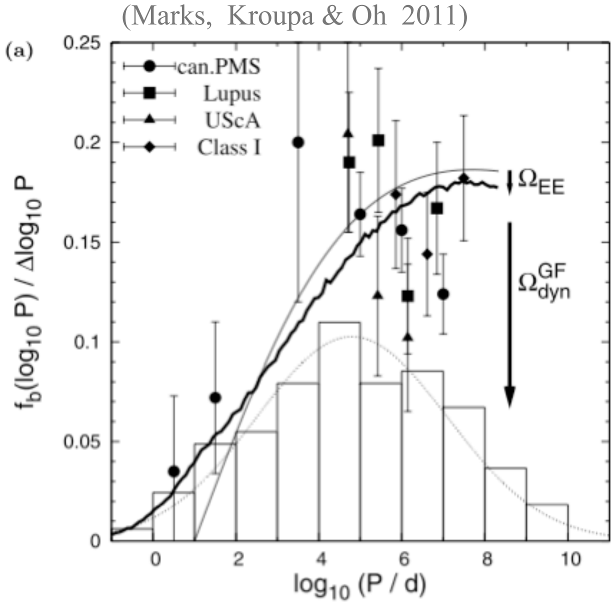
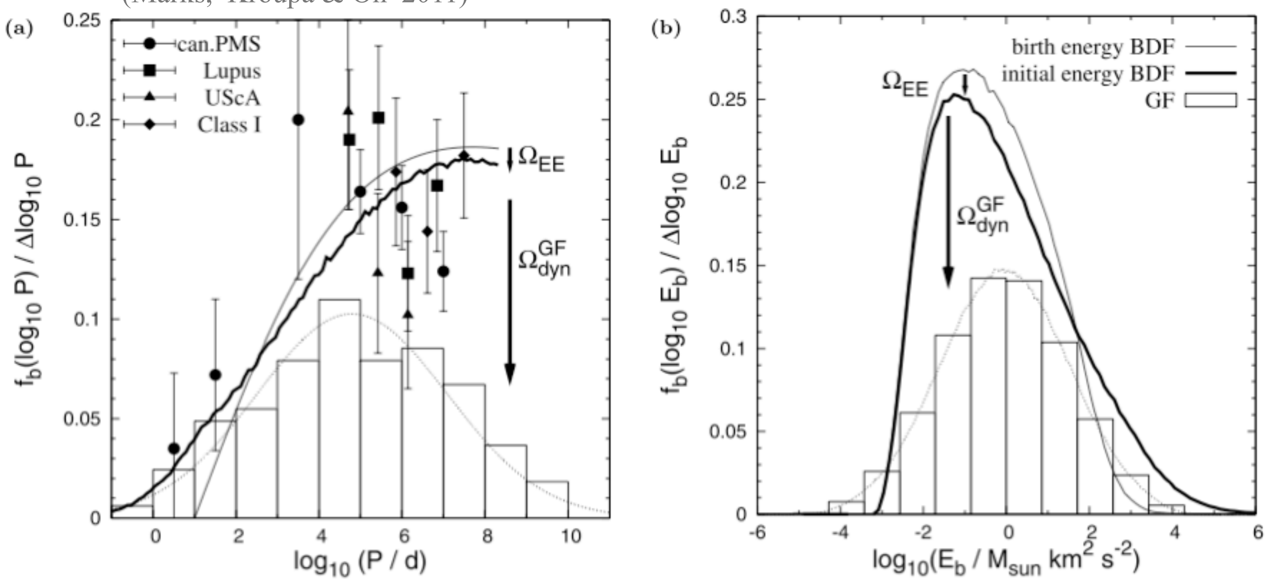


Figure 1. The left- and right-hand panels show the adopted period and energy BDFs, respectively. Both panels depict the same birth (weak solid lines, equation 7), initial (thick solid lines, equation (7) + pre-main-sequence EE) and G-dwarf GF (solid histograms and dotted lines, Duquennoy & Mayor 1991) distributions. The energy BDFs in panel (b) follow from the period BDFs in panel (a) by applying a Monte Carlo method by sampling binaries from analytical distribution functions (Section 2.2; Küpper, Kroupa & Baumgardt 2008). For comparison with the birth and initial period BDFs, the symbols with the error bars in panel (a) show results from pre-main-sequence observation of Taurus-Auriga (labelled can.PMS, from Leinert et al. 1993; Mathieu 1994; Richichi et al. 1994; Kohler & Leinert 1998), Lupus (Koehler, private communication), Upper Sco A (UScA, Brandner & Koehler 1998; Köhler et al. 2000) and Class I protostellar objects (Connelley et al. 2008). The GF BDF originates from the birth distribution, $\mathcal{D}_{\rm birth}$, after pre-main-sequence EE and stimulated evolution in the *dominant-mode* cluster (Section 2.2). The EE operator, $\Omega_{\rm EE}$ (equation 10), transforms $\mathcal{D}_{\rm birth}$ into the initial, i.e. eigenevolved birth distribution, $\mathcal{D}_{\rm in}$ (Section 2.3). The GF distribution, $\mathcal{D}^{M_{\rm ecl}, r_{\rm h}}(t) \equiv \mathcal{D}_{\rm GF}$, results from $\mathcal{D}_{\rm in}$ after applying the stellar-dynamical operator, $\Omega^{M_{\rm ecl}, r_{\rm h}}_{\rm dyn}(t)$ (equation 9), for the dominant-mode cluster ($M_{\rm ecl}/M_\odot = 128$, $r_{\rm h}/{\rm pc} = 0.8$, $t = 1$ Gyr, Section 2.3).

Preview

(Marks, Kroupa & Oh 2011)



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Numerical experiments to test the existence hypothesis :

$R_{0.5} = 0.08 \quad 0.25 \quad 0.77 \quad 2.53$ pc

→ $\rho(\text{stars/pc}^3)$ changes by factor $27 (= 3^3)$

$N_{\rm bin} = 200$ binaries

Standard IMF ($\alpha_1 = 1.3 : 0.08 - 0.5\, M_\odot; \quad \alpha_2 = 2.3 : 0.5 - 1\, M_\odot$)
between $0.1 - 1.0\, M_\odot$ to avoid stellar-evolution issues.

$f_a(la) =$ const. between $1.69 - 1690$ AU; $10^{2.9}{\rm d} \le P \le 10^{7.4}{\rm d}$

$$f_a(la) = [la_{\rm max} - la_{\rm min}]^{-1} \quad (\, la \equiv \log_{10} a\,)$$
$$a(X) = a_{\rm min}\, 10^{X \log_{10}(a_{\rm max}/a_{\rm min})}$$

$f_e(e) = 2\,e$ (thermal distribution)

$$dn(e) = N_{\rm bin}\, f(e)\, de$$
$$e(X) = \sqrt{X}$$

$f_q(q) =$ random pairing from the IMF

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Disruption of binaries :

Least bound binaries should get disrupted :

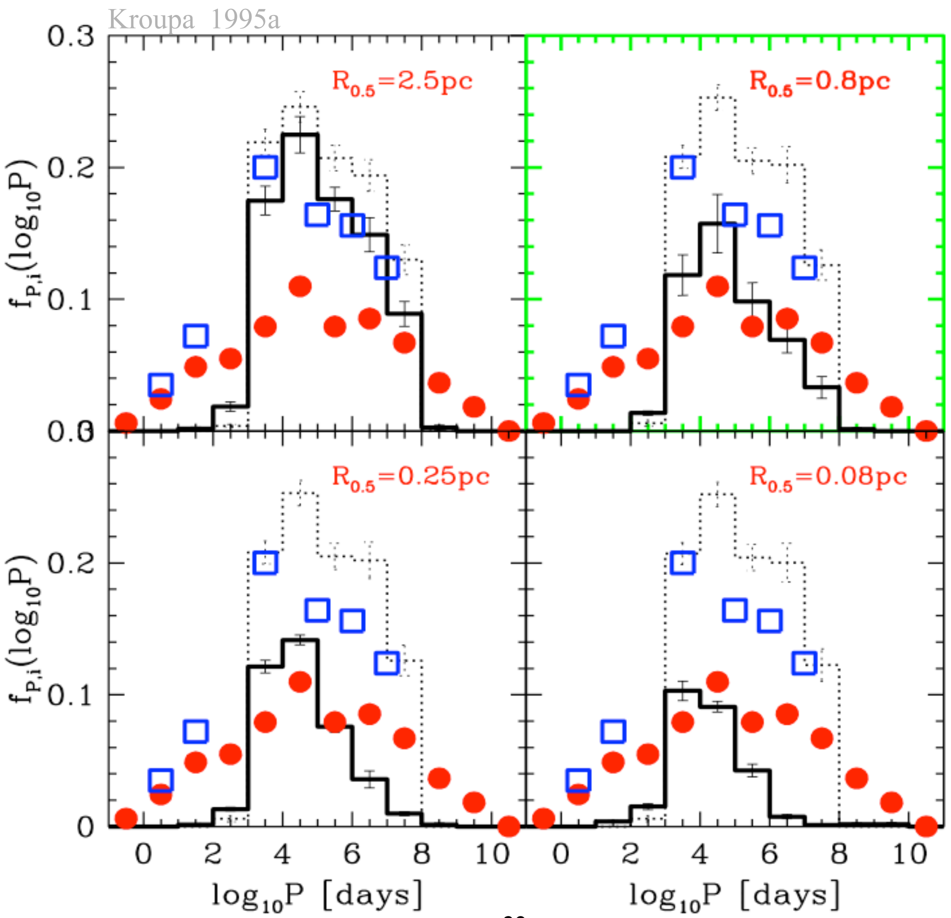
$$E_{\rm b} = -G\,\frac{m_1^2}{2\,a}\,q$$
$$= -\frac{1}{2}\mu\,v_{\rm orb}^2 \quad \text{for a circular orbit}$$

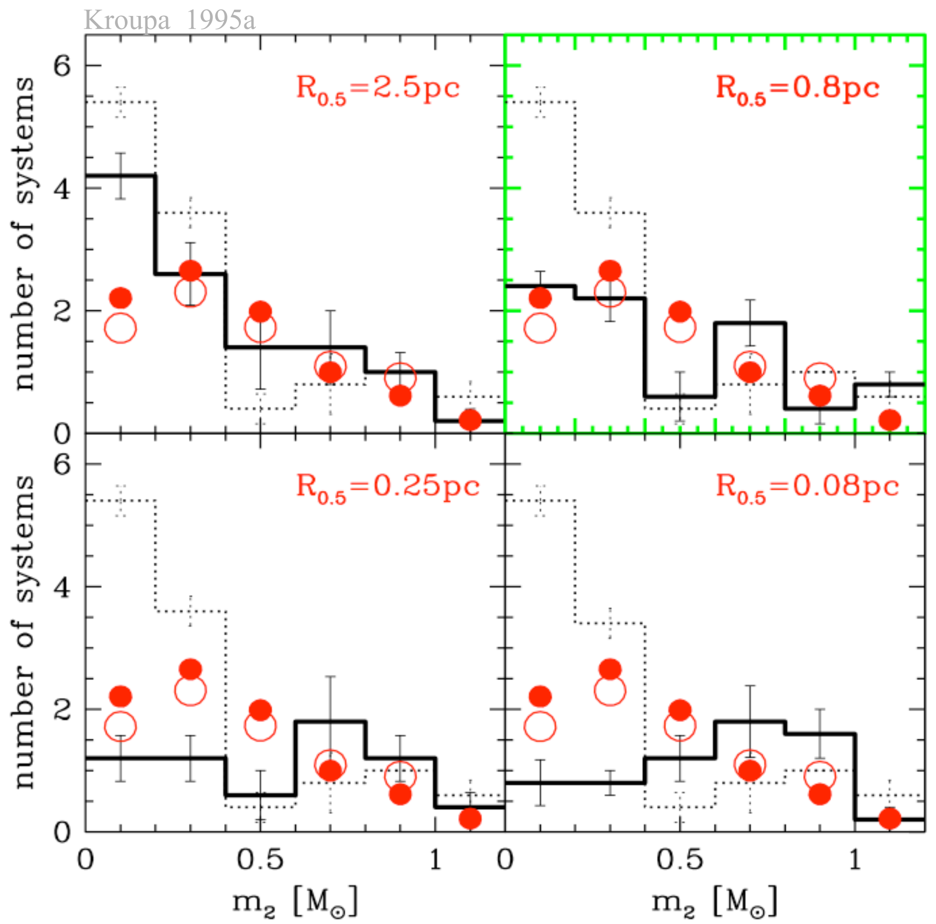
Star-cluster vel.disp. $\sigma^2 = G\,\frac{M_{\rm cl}}{2\,R_{0.5}}\,s^2$ $(s = 0.88$ for Plummer model)

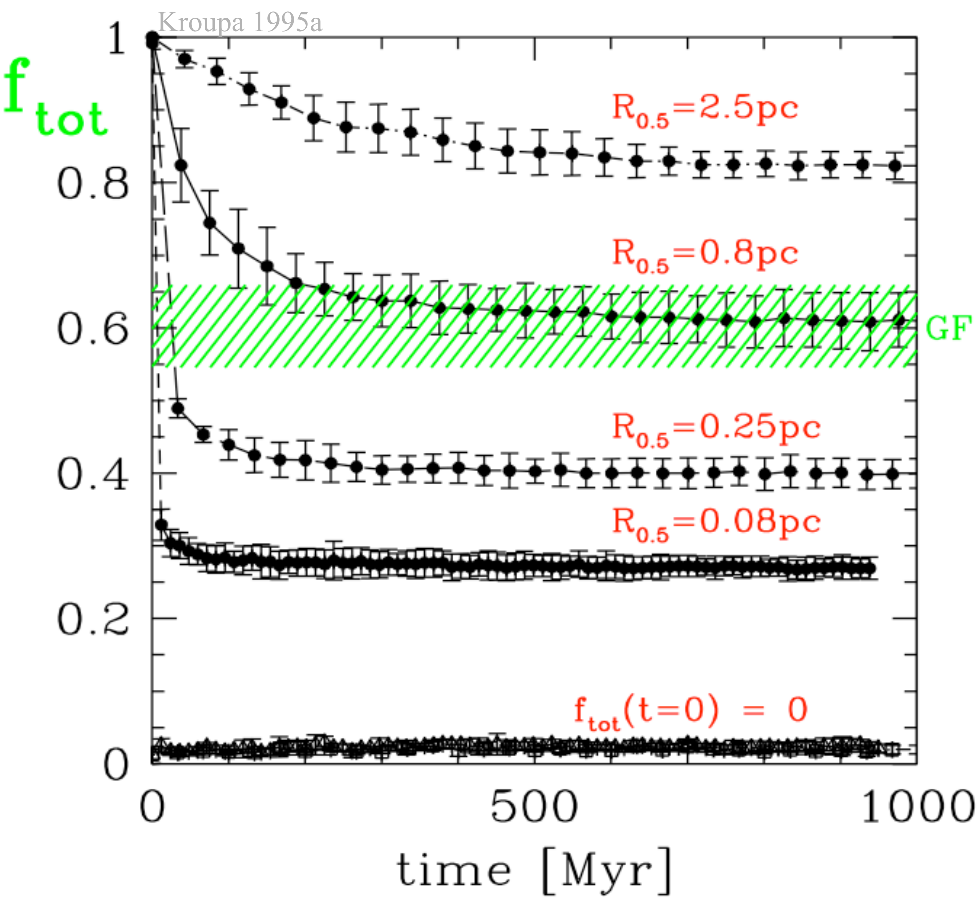
$$v_{\rm orb}^2 = \sigma^2$$

With $lP = 6.986 + lm_{\rm sys} - 3\,lv_{\rm orb}$ derive!

one can compute the "thermal" or "disruption" period above which binaries are likely to be disrupted in a given cluster.




Kroupa 1995a
$R_{0.5}$=2.5pc
$R_{0.5}$=0.8pc
$R_{0.5}$=0.25pc
$R_{0.5}$=0.08pc
number of systems
number of systems
m_2 [$M_\odot$]
m_2 [$M_\odot$]


Kroupa 1995a
f_{tot}
$R_{0.5}$=2.5pc
$R_{0.5}$=0.8pc
GF
$R_{0.5}$=0.25pc
$R_{0.5}$=0.08pc
$f_{tot}(t=0) = 0$
time [Myr]

Results :

There exists a solution with the required property **!**

The "*stellar dynamical operator*" $\Omega^{N,R_{0.5}}$ is given by the class of star clusters *dynamically equivalent* to the solution

$$\Omega^{400,0.8} \cong (N_{\rm bin}, R_{0.5}) = (200, 0.8\,{\rm pc})$$

$\longleftrightarrow$

Most stars form in clusters dynamically equivalent to 200 binaries in 0.8 pc.

Deriving the Initial Binary Distribution Functions

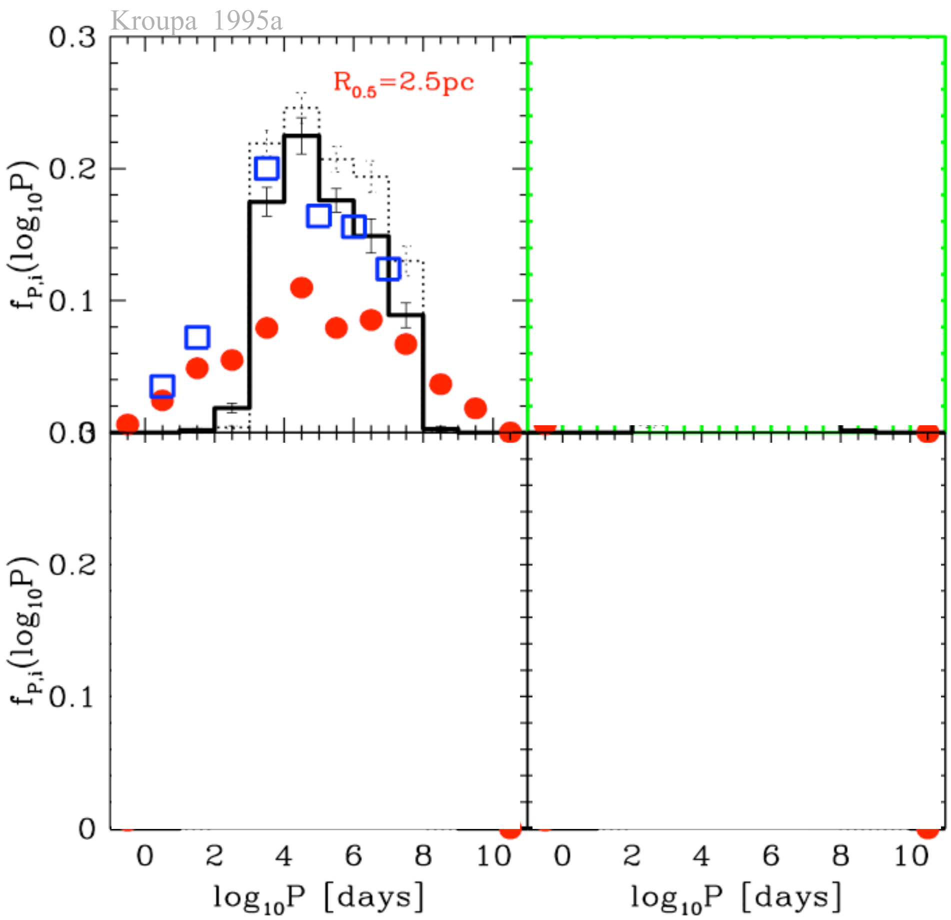
The initial period distribution function

We now understand how a zeroth-order approximation to $f_P(lP)$ evolves in star-clusters.

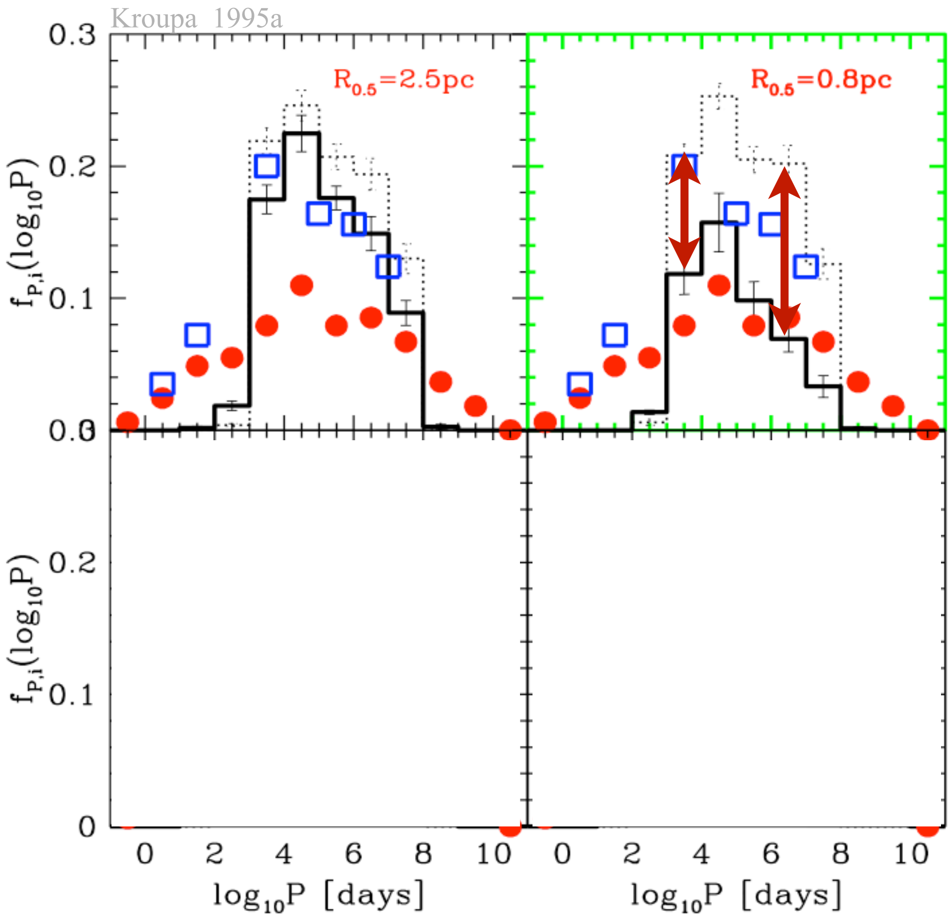
In particular, it has become evident that an *observed* period- or semi-major-axis distribution does *not* represent the *initial distribution*, unless the stellar population is in a *dynamically unevolved* structure (a cluster with age $\tau < t_{\rm cross}$).

Question :
What is the form of the initial $f_P(lP)$ **?**

Define : The "*orbit depletion function*" $c_{P,i}(t) = \dfrac{f_{P,i}(t)}{f_{P,i}(0)}$



$$c_{P,i}(t) = \frac{f_{P,i}(t)}{f_{P,i}(0)}$$



$$c_{P,i}(t) = \frac{f_{P,i}(t)}{f_{P,i}(0)}$$

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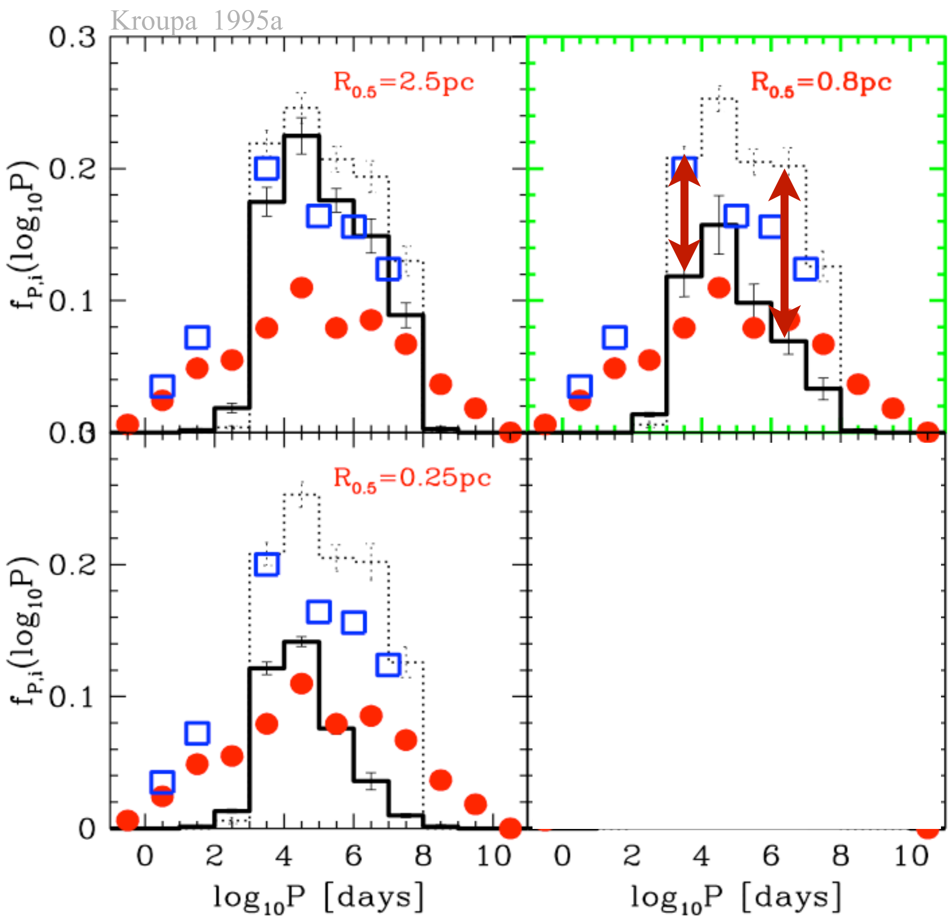

$$c_{P,i}(t) = \frac{f_{P,i}(t)}{f_{P,i}(0)}$$

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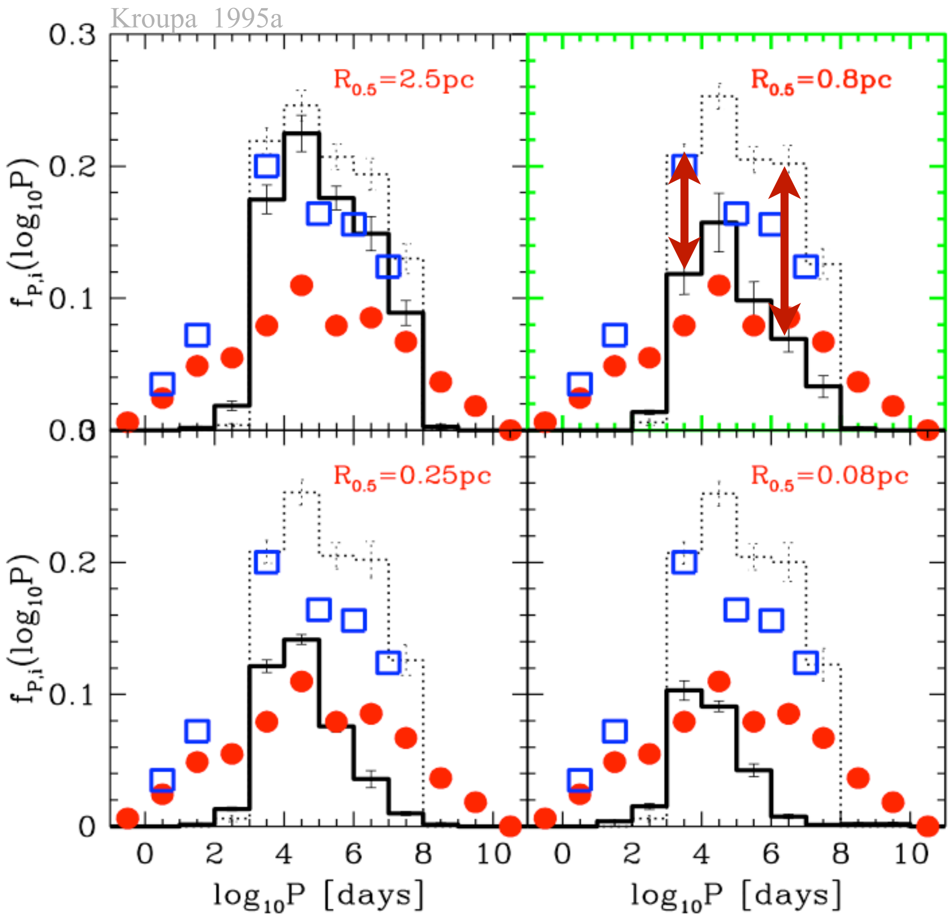
Kroupa 1995a



$$c_{P,i}(t) = \frac{f_{P,i}(t)}{f_{P,i}(0)}$$

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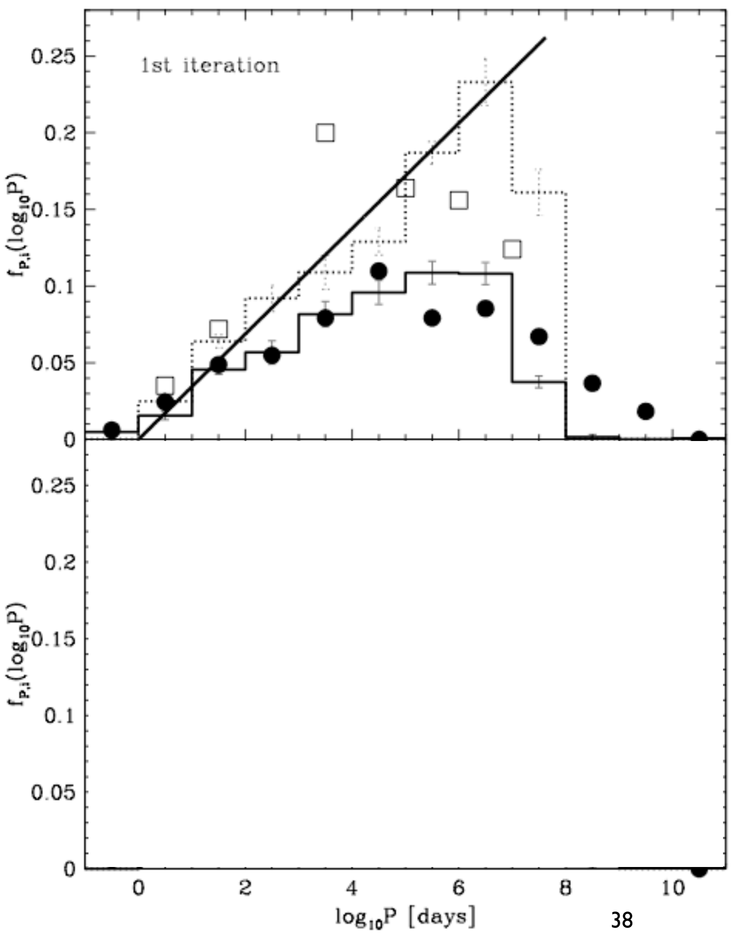

Kroupa 1995a

1st iteration :

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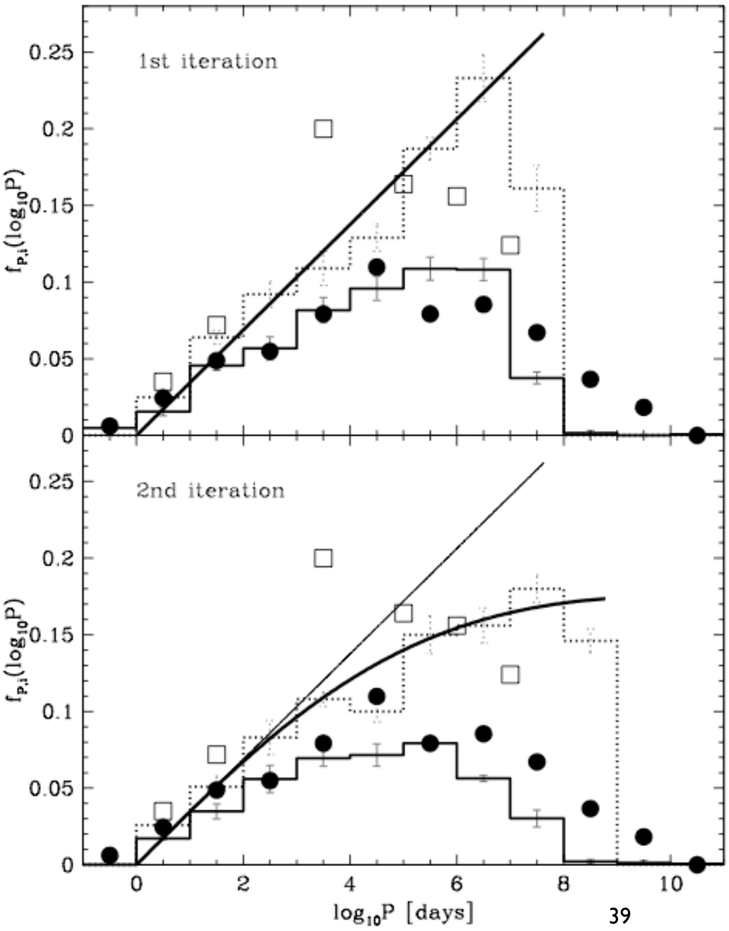

Kroupa 1995a

1st iteration :

2nd iteration :

$$f^{(2)}_{P,\mathrm{in}} = \eta \, \frac{lP - lP_{\mathrm{min}}}{\delta + (lP - lP_{\mathrm{min}})^2}$$

$$lP(X) = \left[\delta \left(e^{\frac{2X}{\eta}} - 1\right)\right]^{\frac{1}{2}} + lP_{\mathrm{min}}$$

$$\eta = 3.50, \quad \delta = 100, \quad lP_{\mathrm{min}} = 0$$

$$\Longrightarrow \quad lP_{\mathrm{max}} = 8.78$$

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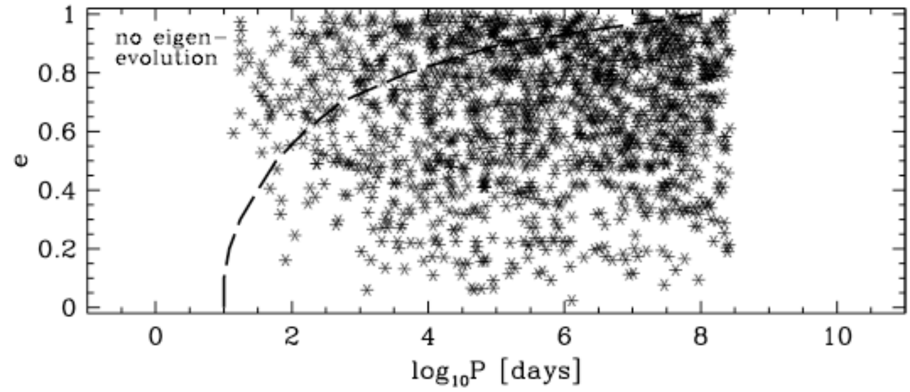
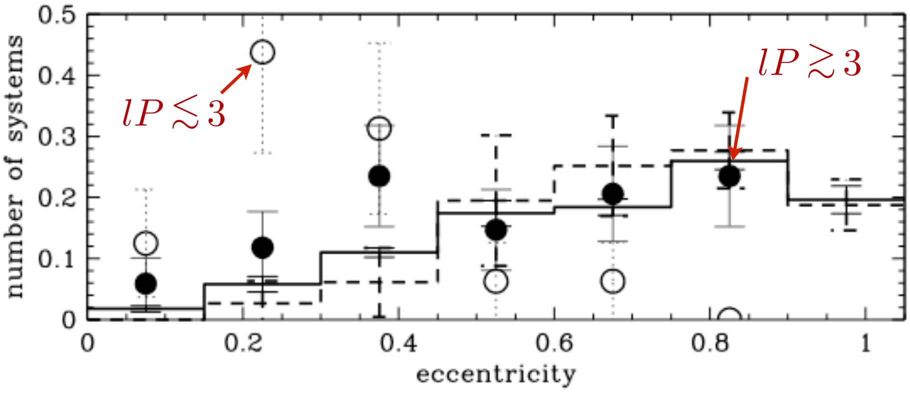
Implications for e, lP diagram:





There are orbits in the *forbidden region* *i.e.* no bell-shaped $f_e(e)$ for short-period binaries.

Need 3rd iteration for $f_{P,\mathrm{in}}$

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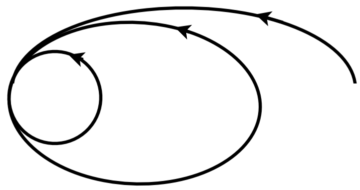
3rd (final) iteration :

Thus, to quantify a realistic initial (t=0) binary population we need to correctly account for the observed correlations between P, e, q for $lP \lesssim 3$.

Physics :

The correlations are most probably due to dissipative processes in the short-period pre-main-sequence binaries **:**

- tidal circularisation
- dissipation in circumstellar disks
- accretion onto the two stars → change of angular momentum



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We need to *develop an algorithm* that transforms our initial (actually ***birth***) distribution

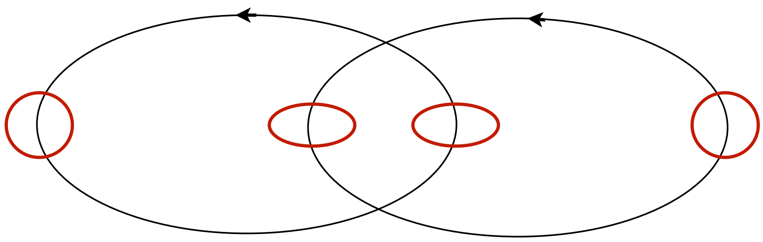
$$D_{\rm birth} = f_P \, f_e \, f_q \quad \Big| \quad dN_{\rm bin\ at\ birth} = N_{\rm sys} \, D_{\rm birth} \, dlP \, de \, dq$$

to an ***initial*** distribution which matches the observational data. The evolution of the birth orbital parameters to the initial ones progresses within 0.1 Myr during the early pre-main-sequence phase, and the *initial binaries* then enter the N-body computations.

Note : 0.1Myr is also about the time after which a proto star has acquired about 95% of its final mass (Wuchterl & Tscharnuter 2003).

This algorithm needs to be derived from tidal circularisation theory.

Tidal circularisation theory is difficult because the tidally-distorted star needs to be calculated during an orbit integration. 3D stellar modelling is not possible at present though.



Resort to semi-analytical estimates :

Eigenevolution : (= sum of all binary-internal dissipational processes) occurs when the accreting proto-stars are at peri-astron (Kroupa 1995b)

$$R_{\rm peri} = (1-e)\, P_{\rm yr}^{\frac{2}{3}}\, (m_1+m_2)^{\frac{1}{3}} \qquad (P_{\rm yr} = P/365.25)$$

Let the binary be born with eccentricity $e_{\rm birth}$, then the system evolves approximately according to (Goldman &Mazeh 1994)

$$\frac{1}{e}\frac{de}{dt} = -\rho' \quad \Longrightarrow \quad \log_{10} e_{\rm in} = -\rho + \log_{10} e_{\rm birth}$$

$\frac{1}{\rho'} =$ tidal-circularisation time-scale

$$\text{where} \quad \rho = \int_0^{\Delta t} \rho'\, dt = \left(\frac{\lambda R_\odot}{R_{\rm peri}}\right)^{\chi}$$

$R_\odot = 4.6523 \times 10^{-3}$ AU

$R_{\rm peri} =$ const. because dissipative force acts at periastron and thus tangentially only

$\Delta t \lesssim 10^5$ yr is the time-scale within which pre-main-sequence eigenevolution completes.

References: Tidal-circularisation theory: Goldman &Mazeh (1994); Zahn & Boucher (1989) .
Eigenevolution: Kroupa (1995b)

The "initial" period becomes (from $R_{\rm peri} = (1-e)\, P_{\rm yr}^{\frac{2}{3}}\, (m_1+m_2)^{\frac{1}{3}}$)

$$P_{\rm in} = P_{\rm birth} \left(\frac{m_{\rm tot,birth}}{m_{\rm tot,in}}\right)^{\frac{1}{2}} \left(\frac{1-e_{\rm birth}}{1-e_{\rm in}}\right)^{\frac{3}{2}}$$

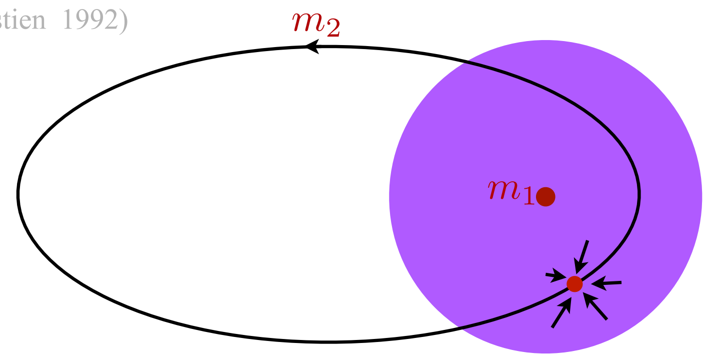
Assume the stars merge if $a_{\rm in} \le 10\, R_\odot$: $m_1 + m_2 \longrightarrow m$

We now need to find $\mathcal{X}$ and λ such that $(e,P)_{\rm birth} \longrightarrow (e,P)_{\rm in}$ so that $(e,P)_{\rm in}$ matches observational constraints.

$$\left[\rho = \left(\frac{\lambda\, R_\odot}{R_{\rm peri}}\right)^{\mathcal{X}}\right]$$

Mass ratios :

$lP \lesssim 3$ binaries have f_q biased towards 1. This may be a result of accretion by the secondary from the circum-stellar disk of the primary at $R_{\rm peri}$: (Bonnell & Bastien 1992)



Assume :

$$q_{\rm in} = q_{\rm birth} + (1 - q_{\rm birth})\, \rho^*$$

$$\rho^* = \begin{cases} \rho & \text{if} \quad \rho \le 1 \\ 1 & \text{if} \quad \rho > 1 \end{cases}$$

$$\left[\rho = \left(\frac{\lambda\, R_\odot}{R_{\rm peri}}\right)^{\mathcal{X}}\right]$$

"Initial" masses then become $m_{1,\rm in} = m_{1,\rm birth} = m_1$

$$m_{2,\rm in} = q_{\rm in}\, m_1$$

Note : $m_{{\rm sys},in} > m_{{\rm sys},birth}$

We now have the following parameters that need to be determined :

$$lP_{\min},\ \eta,\ \delta \quad \text{in} \quad f_{\mathrm{P,birth}}(lP)$$
$$\lambda,\ \mathcal{X} \quad \text{in} \quad \rho$$

Choose $lP_{\min} = 1$ since binaries with $lP \approx 0$ are observed but eigenevolution shrinks orbits.

Then $$\boxed{f_{P,\mathrm{birth}} = 2.5\,\frac{lP - 1}{45 + (lP-1)^2}}$$

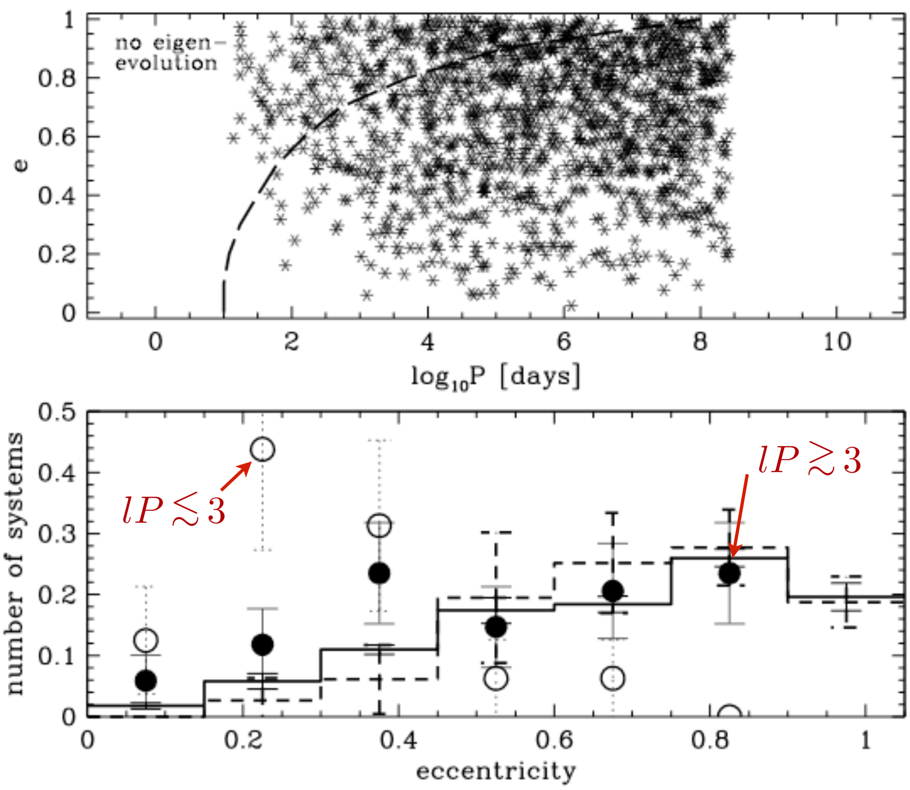
$\eta = 2.5$

$\delta = 45$

and $lP_{\max} = 8.43$.

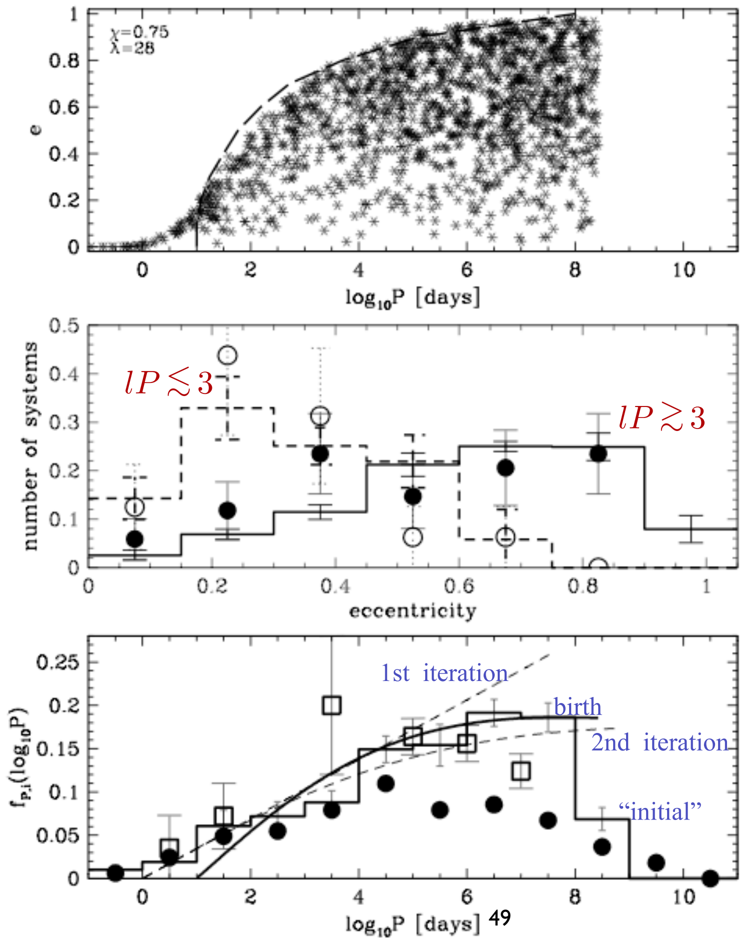
Furthermore, numerical experiments show that $\lambda = 28$ and $\mathcal{X} = 0.75$ lead to good results :

Remember: Implications for e, lP diagram:



There are orbits in the *forbidden region* *i.e.* no bell-shaped $f_e(e)$ for short-period binaries.

Need 3rd iteration for $f_{P,\mathrm{in}}$



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Test of the algorithm :

Set-up the cluster $N_{\text{bin}} = 200$ binaries, $R_{0.5} = 0.85$ pc (Plummer model).

1st step : Eigenevolve the pre-main sequence population

→ birth population ⟶ initial population

2nd step : Load this initial population into the N-Body integrator (e.g. Aarseth's Nbody4/5/6).

→ evolve the cluster for 1 Gyr when it is completely dissolved. Do this 20 times (ensemble averaging).

3rd step : At the end of the Nbody calculations there exist "*forbidden binaries*". These are unphysical (they would tidally circularise) but give an insight into their frequency of occurence. Evolve these using "*main-sequence eigenevolution*" :

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$$\left[\rho = \left(\frac{\lambda R_\odot}{R_{\rm peri}} \right)^{\mathcal{X}} \right]$$

Note here that in order to match the upper envelope of the *e, P* diagram, $\lambda_{\rm ms} = 24.7$, (Duquennoy & Mayor 1991),

and to match the observed circularisation period of 12 d for G dwarfs $\mathcal{X}_{\rm ms} = 8$ (*c.f.* Zahn 1977).

For pre-main sequence eigenevolution we needed

$\mathcal{X} = 0.75 < \mathcal{X}_{\rm ms}$

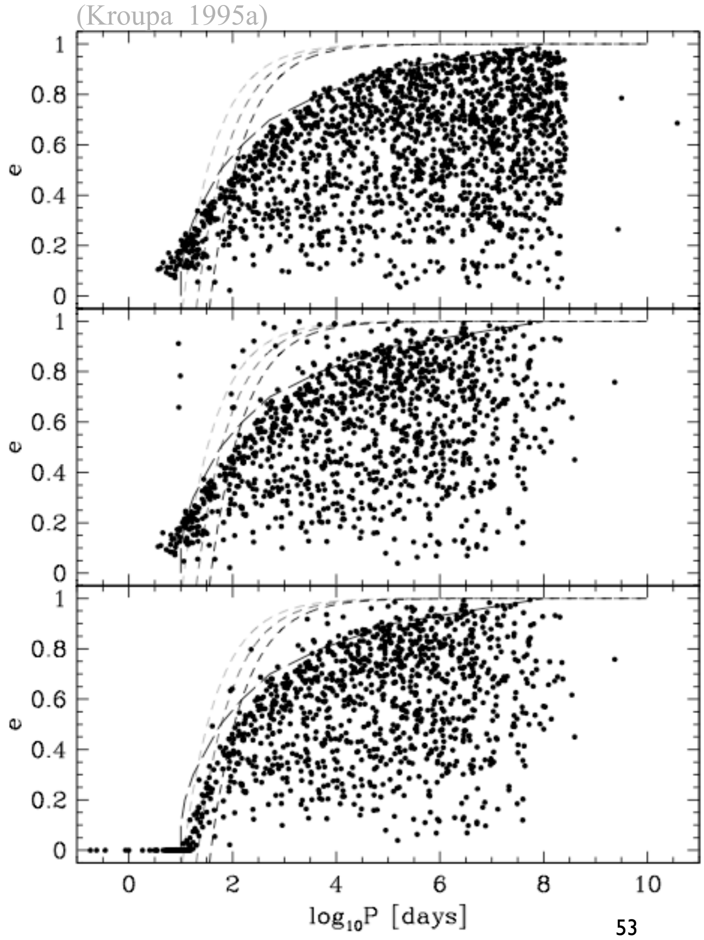
$\lambda = 28 > \lambda_{\rm ms}$

which can be understood nicely in terms of main-sequence stars being smaller and more compact.

4th step : Average the 20 runs to obtain statistically averaged

$f_P, \quad f_e, \quad f_q$

and other data on cluster evolution.



20 runs, 4000 binaries at birth

$t = 0$ after pre-main sequence eigenevolution

$t = 1\,\mathrm{Gyr}$ after cluster disruption

$t = 1\,\mathrm{Gyr}$ after main-sequence eigenevolution

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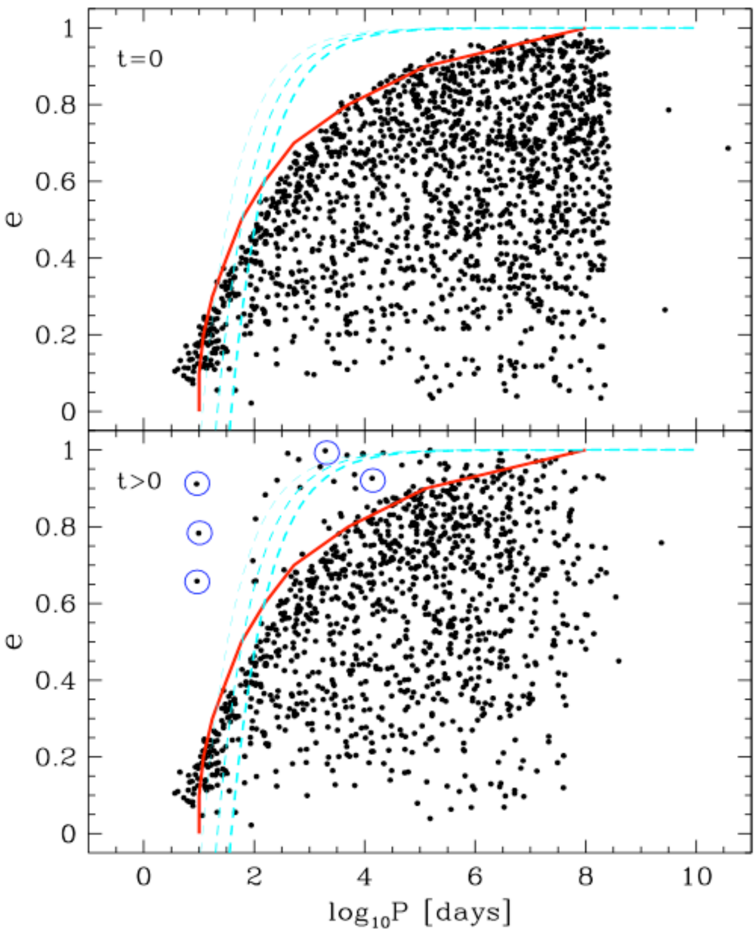

forbidden binaries

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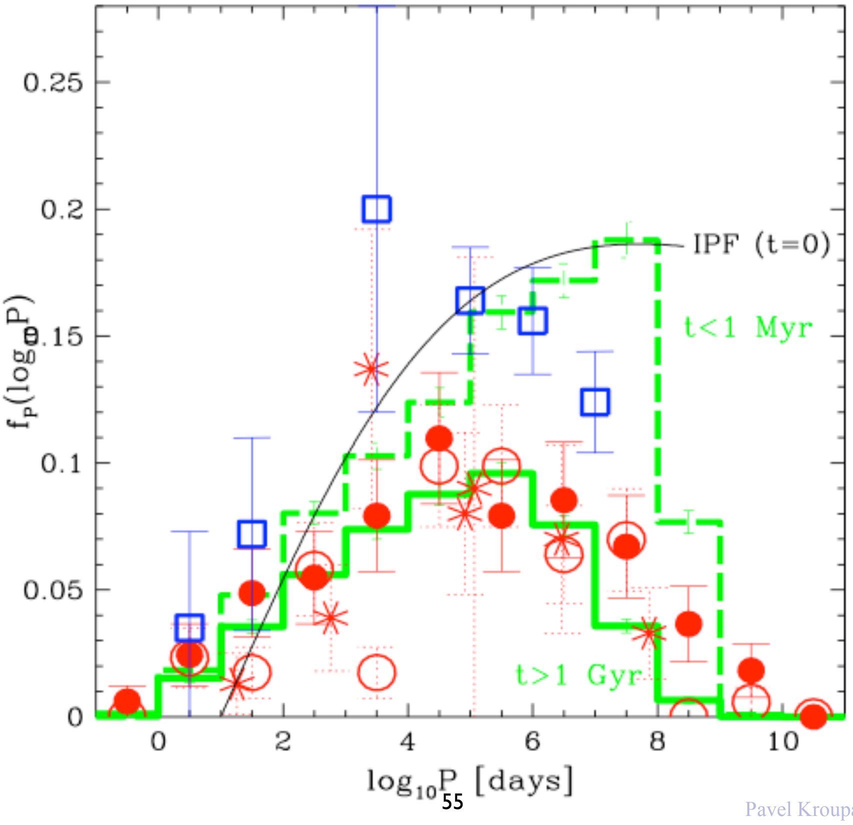
The final period-distribution function



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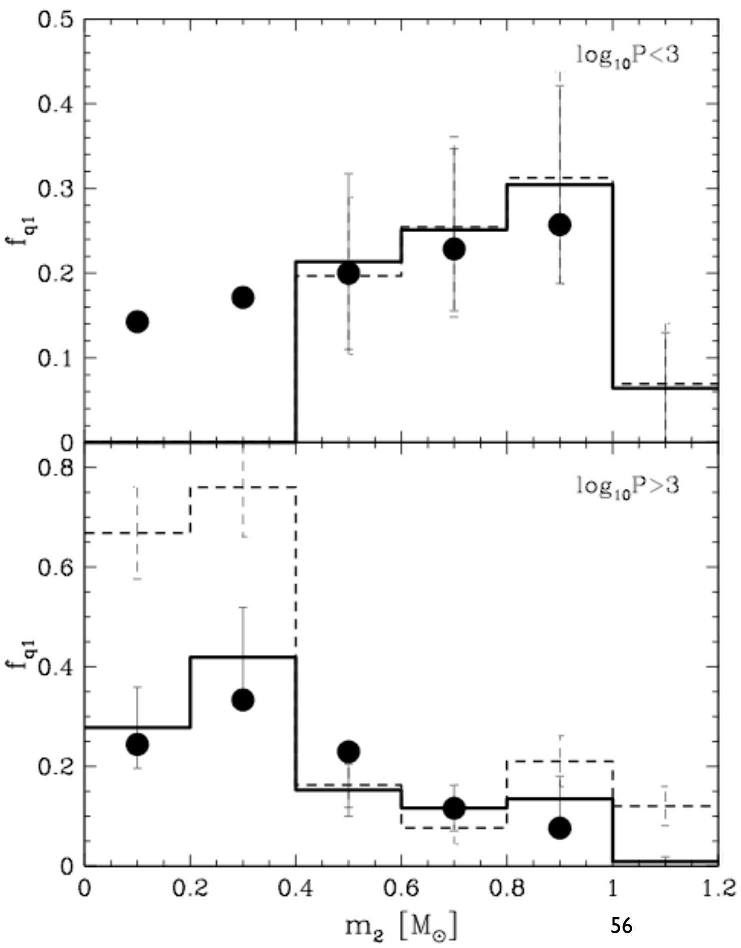

The final mass-ratio distribution for G-dwarfs

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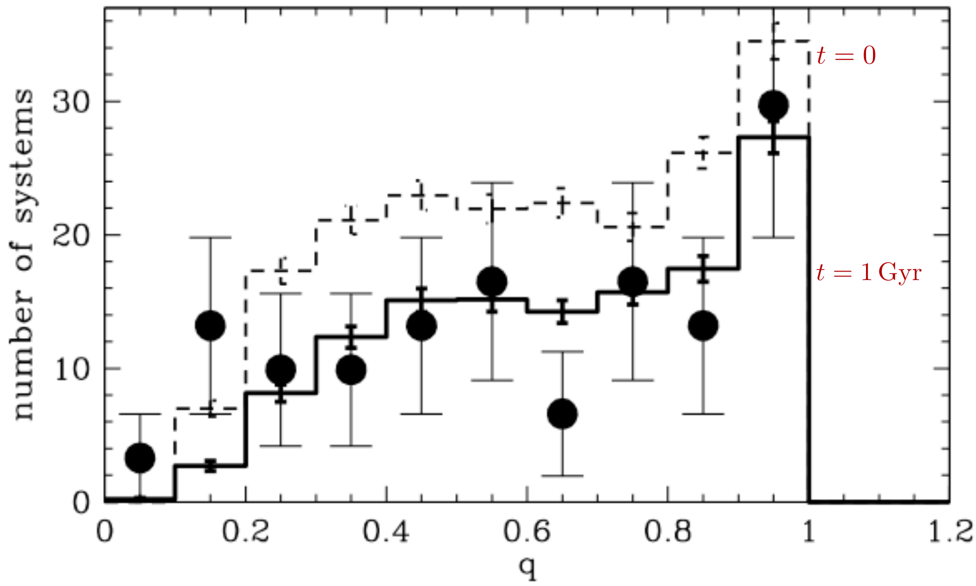
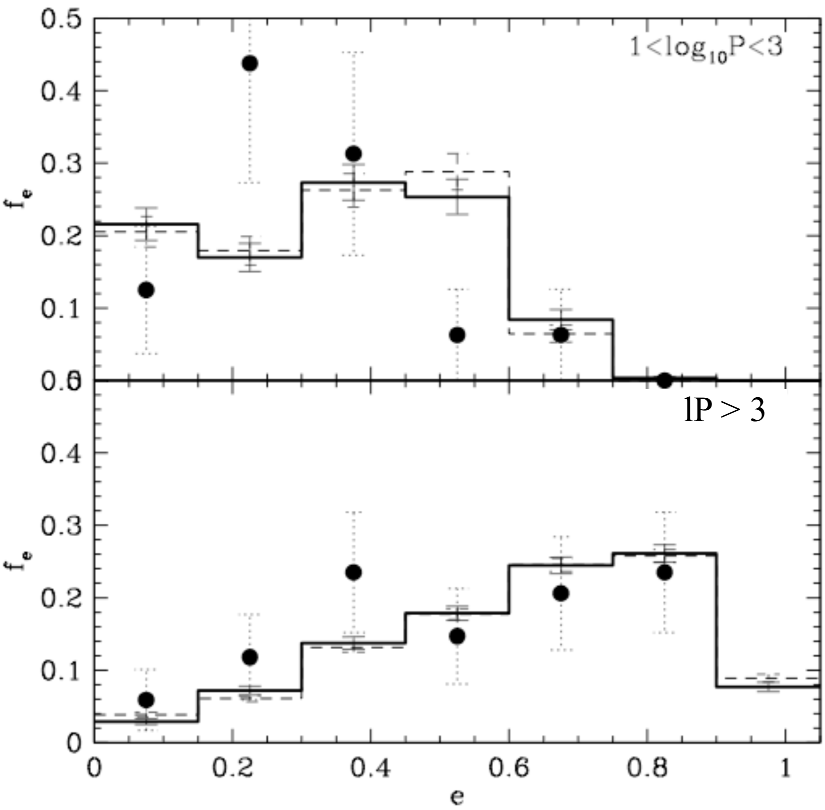
The final mass-ratio distribution for late-type dwarfs



● Reid & Gizis (1997)

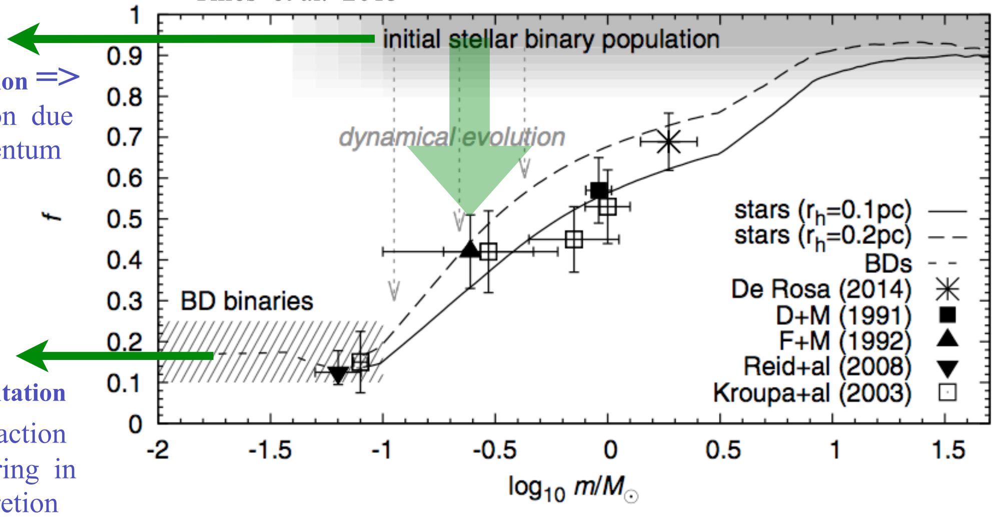
The final eccentricity distribution



The binary fraction as a function of primary mass

Thies et al. 2015

Primary fragmentation => high binary fraction due to angular momentum conservation



Peripheral fragmentation => low binary fraction due to chance pairing in circumstellar accretion discs

==> it is not correct to deduce from f=fn(m_prim) that BDs form a continuous extension of the stellar population

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Preview

(Marks, Kroupa & Oh 2011)

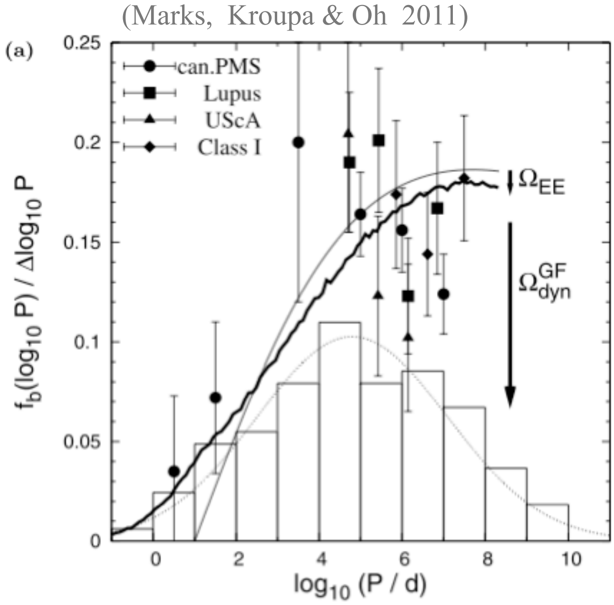

Figure 1. The left- and right-hand panels show the adopted period and energy BDFs, respectively. Both panels depict the same birth (weak solid lines, equation 7), initial (thick solid lines, equation (7) + pre-main-sequence EE) and G-dwarf GF (solid histograms and dotted lines, Duquennoy & Mayor 1991) distributions. The energy BDFs in panel (b) follow from the period BDFs in panel (a) by applying a Monte Carlo method by sampling binaries from analytical distribution functions (Section 2.2; Küpper, Kroupa & Baumgardt 2008). For comparison with the birth and initial period BDFs, the symbols with the error bars in panel (a) show results from pre-main-sequence observation of Taurus-Auriga (labelled can.PMS, from Leinert et al. 1993; Mathieu 1994; Richichi et al. 1994; Kohler & Leinert 1998), Lupus (Koehler, private communication), Upper Sco A (UScA, Brandner & Koehler 1998; Köhler et al. 2000) and Class I protostellar objects (Connelley et al. 2008). The GF BDF originates from the birth distribution, $\mathcal{D}_{\rm birth}$, after pre-main-sequence EE and stimulated evolution in the *dominant-mode* cluster (Section 2.2). The EE operator, $\Omega_{\rm EE}$ (equation 10), transforms $\mathcal{D}_{\rm birth}$ into the initial, i.e. eigenevolved birth distribution, $\mathcal{D}_{\rm in}$ (Section 2.3). The GF distribution, $\mathcal{D}^{M_{\rm ecl},r_h}(t) \equiv \mathcal{D}_{\rm GF}$, results from $\mathcal{D}_{\rm in}$ after applying the stellar-dynamical operator, $\Omega^{M_{\rm ecl},r_h}_{\rm dyn}(t)$ (equation 9), for the dominant-mode cluster ($M_{\rm ecl}/M_\odot = 128, r_h/{\rm pc} = 0.8, t = 1$ Gyr, Section 2.3).60

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Preview

(Marks, Kroupa & Oh 2011)

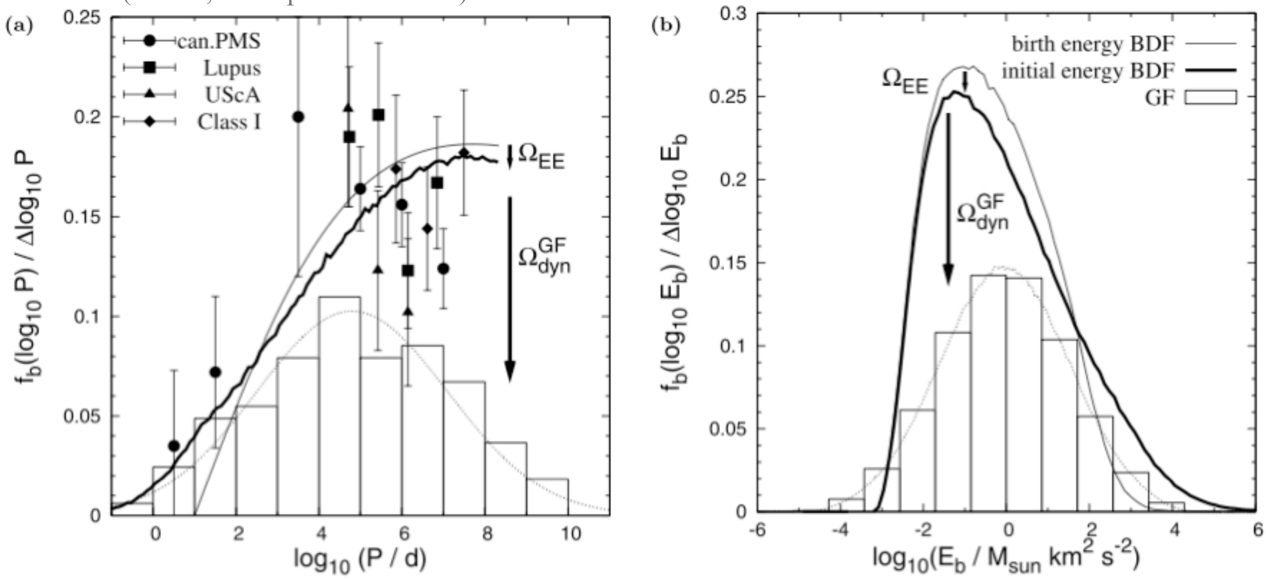


Figure 1. The left- and right-hand panels show the adopted period and energy BDFs, respectively. Both panels depict the same birth (weak solid lines, equation 7), initial (thick solid lines, equation (7) + pre-main-sequence EE) and G-dwarf GF (solid histograms and dotted lines, Duquennoy & Mayor 1991) distributions. The energy BDFs in panel (b) follow from the period BDFs in panel (a) by applying a Monte Carlo method by sampling binaries from analytical distribution functions (Section 2.2; Küpper, Kroupa & Baumgardt 2008). For comparison with the birth and initial period BDFs, the symbols with the error bars in panel (a) show results from pre-main-sequence observation of Taurus-Auriga (labelled can.PMS, from Leinert et al. 1993; Mathieu 1994; Richichi et al. 1994; Kohler & Leinert 1998), Lupus (Koehler, private communication), Upper Sco A (UScA, Brandner & Koehler 1998; Köhler et al. 2000) and Class I protostellar objects (Connelley et al. 2008). The GF BDF originates from the birth distribution, $\mathcal{D}_{\rm birth}$, after pre-main-sequence EE and stimulated evolution in the *dominant-mode* cluster (Section 2.2). The EE operator, $\Omega_{\rm EE}$ (equation 10), transforms $\mathcal{D}_{\rm birth}$ into the initial, i.e. eigenevolved birth distribution, $\mathcal{D}_{\rm in}$ (Section 2.3). The GF distribution, $\mathcal{D}^{M_{\rm ecl},r_{\rm h}}(t) \equiv \mathcal{D}_{\rm GF}$, results from $\mathcal{D}_{\rm in}$ after applying the stellar-dynamical operator, $\Omega^{M_{\rm ecl},r_{\rm h}}_{\rm dyn}(t)$ (equation 9), for the dominant-mode cluster ($M_{\rm ecl}/M_\odot = 128, r_{\rm h}/{\rm pc} = 0.8, t = 1$ Gyr, Section 2.3).61

$$f^{\rm obs}_{\rm mult} = 0.43 \Longleftrightarrow f^{\rm model}_{\rm mult} = 0.48 \quad ✓$$

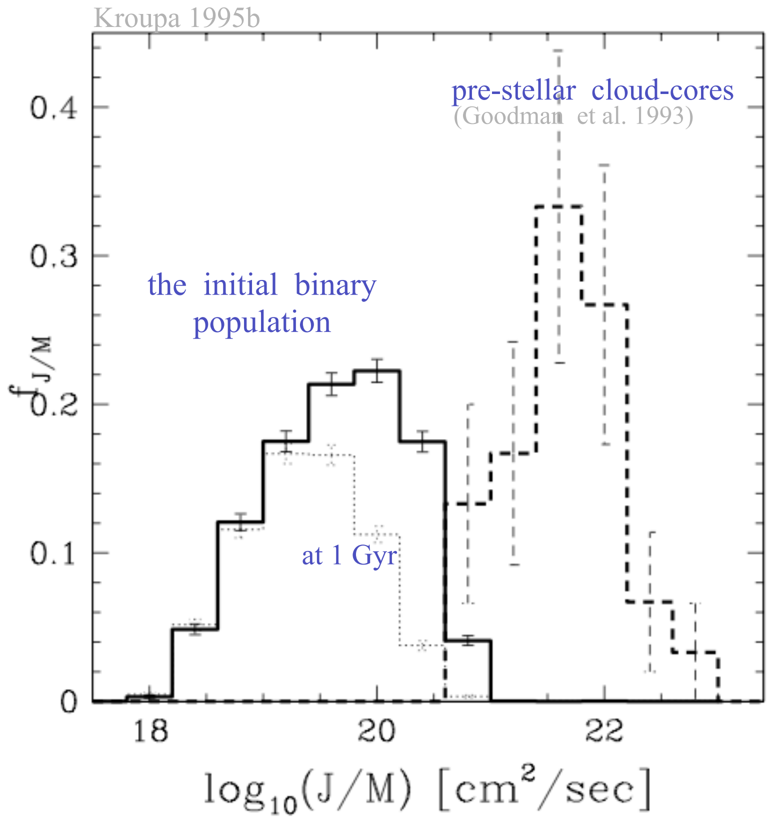
but

$$f^{\rm obs}_{\mathcal{T},\mathcal{Q}} = 0.050 \Longleftrightarrow f^{\rm model}_{\mathcal{T},\mathcal{Q}} = 0.0070 \quad ✗$$

➡ Most triple and quadruple systems cannot have formed dynamically in clusters from binaries.

They are either *primordial*, or perhaps the *cores of dead clusters* (Goodwin & Kroupa 2005).

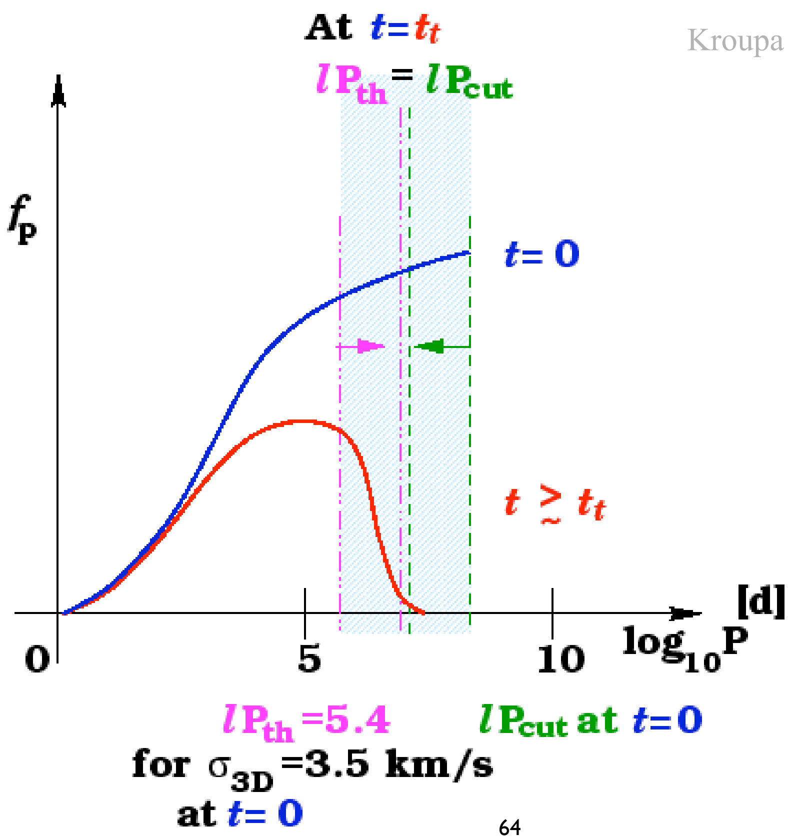
The distribution of specific angular momenta



. . . the origin of the binary-star specific angular momenta ?

The binary - star cluster interaction

Kroupa (2000, aph/0001202)



From

CSFEs

to

galaxies :

binary populations

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What do we know about
Correlated Star Formation Events (CSFEs) ?

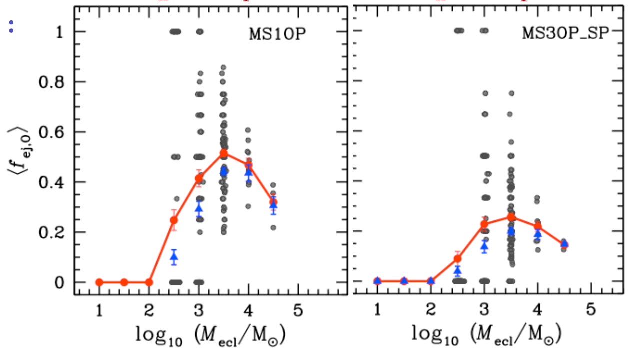
Stellar-mass distribution : $\xi_{\rm ECMF}(M_{\rm ecl}) = k\, M_{\rm ecl}^{-\beta}; \quad \beta \approx 2$ (Lada & Lada 2003)

Radius - mass relation : $\frac{r_{\rm h}}{\rm pc} = 0.1^{+0.07}_{-0.04} \times \left(\frac{M_{\rm ecl}}{M_\odot}\right)^{0.13\pm0.04}$ (Marks & Kroupa 2012)

c.f. simulation results from **Matthew Bate**
& observational results for Carina SF region by **Mauricio Tapia**

Efficient ejections of O stars :
(within 3Myr)

$r_{\rm h} = 0.1$ pc $\qquad r_{\rm h} = 0.3$ pc



(Oh, Kroupa & Pflamm-Altenburg 2015)

100 % initial binarity

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Angular momentum problem ==> most (all?) stars form as binaries

$\mathcal{D}_{\rm in} = \mathcal{D}_{\rm in}(P,e,q)$ from observational constraints

Kroupa 1995; Duchene 1999

In a CSFE an initial binary population evolves with time through disruption and hardening events

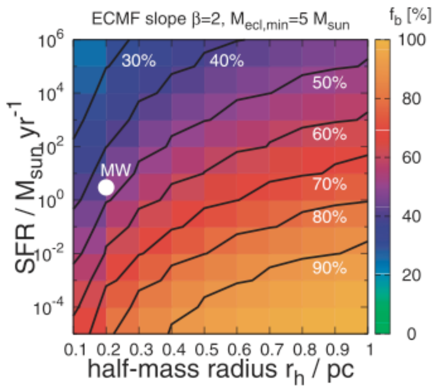
$\mathcal{D}_{\rm in}(P,e,q) \longrightarrow \mathcal{D}_{\rm evolved}(t)$

$$\mathcal{D}^{M_{\rm ecl},r_{\rm h}}(t) = \Omega_{\rm dyn}^{M_{\rm ecl},r_{\rm h}}(t) \times \mathcal{D}_{\rm in}.$$

Superposition principle (as for IGIMF) :

Marks & Kroupa 2011

$$\mathcal{D}_{\rm GF}^{r_{\rm h}} = \int_{M_{\rm ecl,min}}^{M_{\rm ecl,max}} \Omega_{\rm dyn}^{M_{\rm ecl},r_{\rm h}}(t_{\rm freeze})\, \mathcal{D}_{\rm in}\, \xi_{\rm ecl}(M_{\rm ecl})\, {\rm d}M_{\rm ecl},$$



Dwarf galaxies should have high field binary fractions $f_{\rm bin} \approx 80\%$

Massive galaxies should have low field binary fractions $f_{\rm bin} \approx 30-40\%$

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Summary of the Binary-Star Birth Distribution Functions

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Summary of birth/initial binary-star population :

Late-type stars :

- random pairing from the IMF : $0.08 - 1\, M_\odot$ approximately
- $f_{P,\mathrm{birth}} = \eta \, \dfrac{lP - lP_{\min}}{\delta + (lP - lP_{\min})^2}$ $\qquad lP(X) = \left[\delta \left(e^{\frac{2X}{\eta}} - 1\right)\right]^{\frac{1}{2}} + lP_{\min}$
- $f(e) = 2\, e$ $\qquad e(X) = \sqrt{X}$
- pre-main sequence eigenevolution to impose observed correlations for $lP < 3$ systems

Early-type stars :

q between 0.1 and 1

Purely random pairing from whole stellar mass range excluded. Period distribution biased towards shorter Ps (Oh et al. 2015).

BDs :

BD--BD binaries : narrow P distribution; BDs & stars don't mix (Thies et al. 2015)

The End of 4th Lecture.