Stellar winds of hot stars

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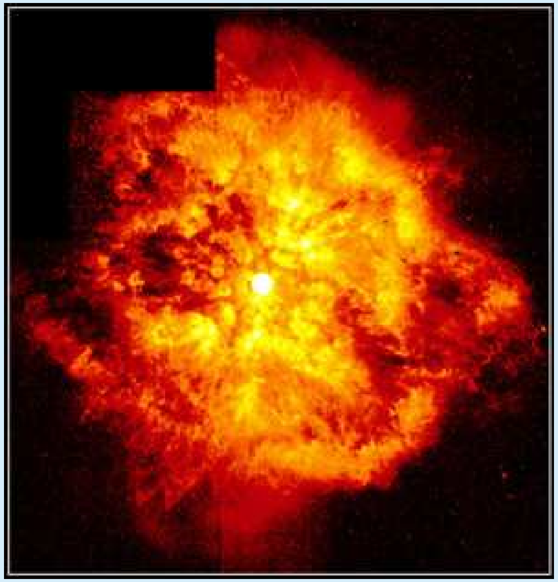
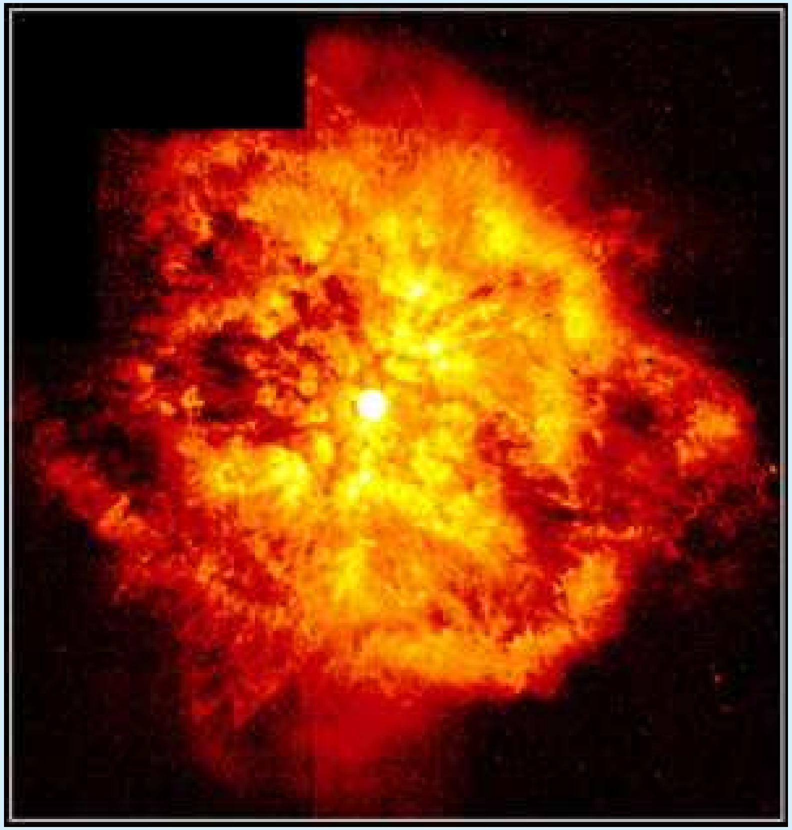
Masaryk University, Brno, Czech Republic



Observation of hot stars

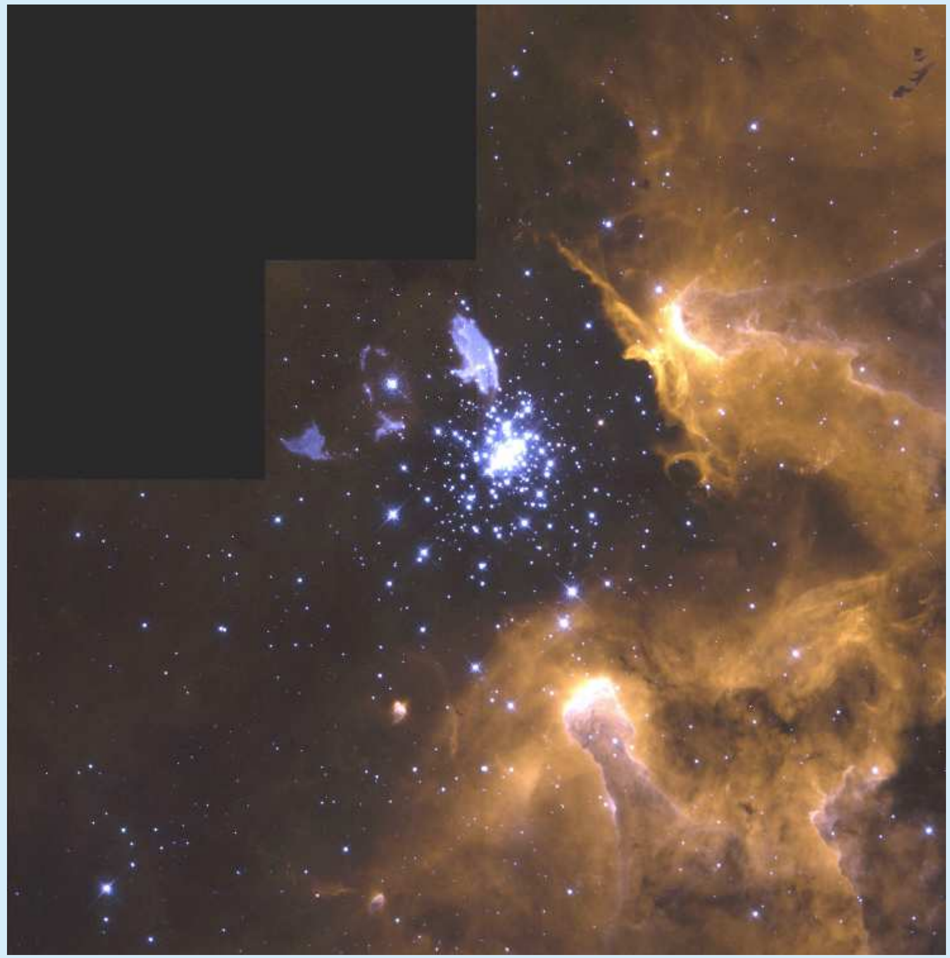
- shells in the surroundings of hot stars



nebula close to the star WR 124 (HST)

Observation of hot stars

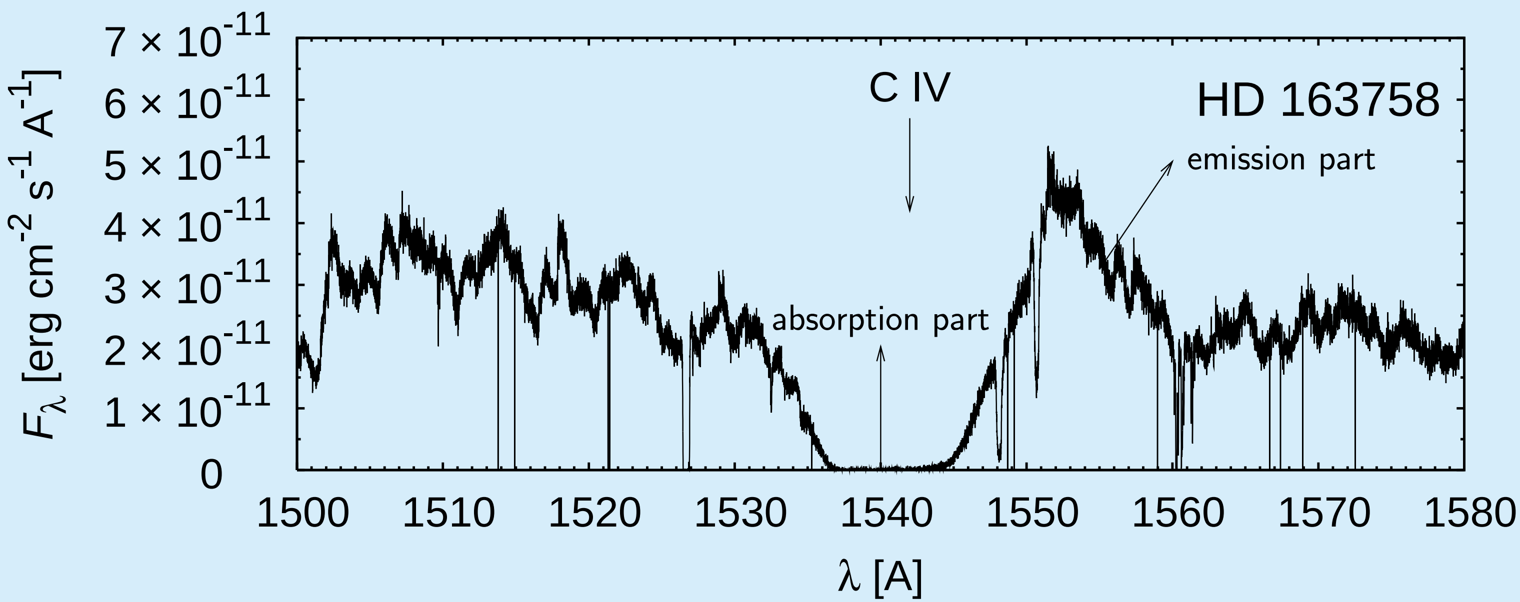
- the interstellar medium around hot stars



open cluster NGC 3603 (HST)

Observation of hot stars

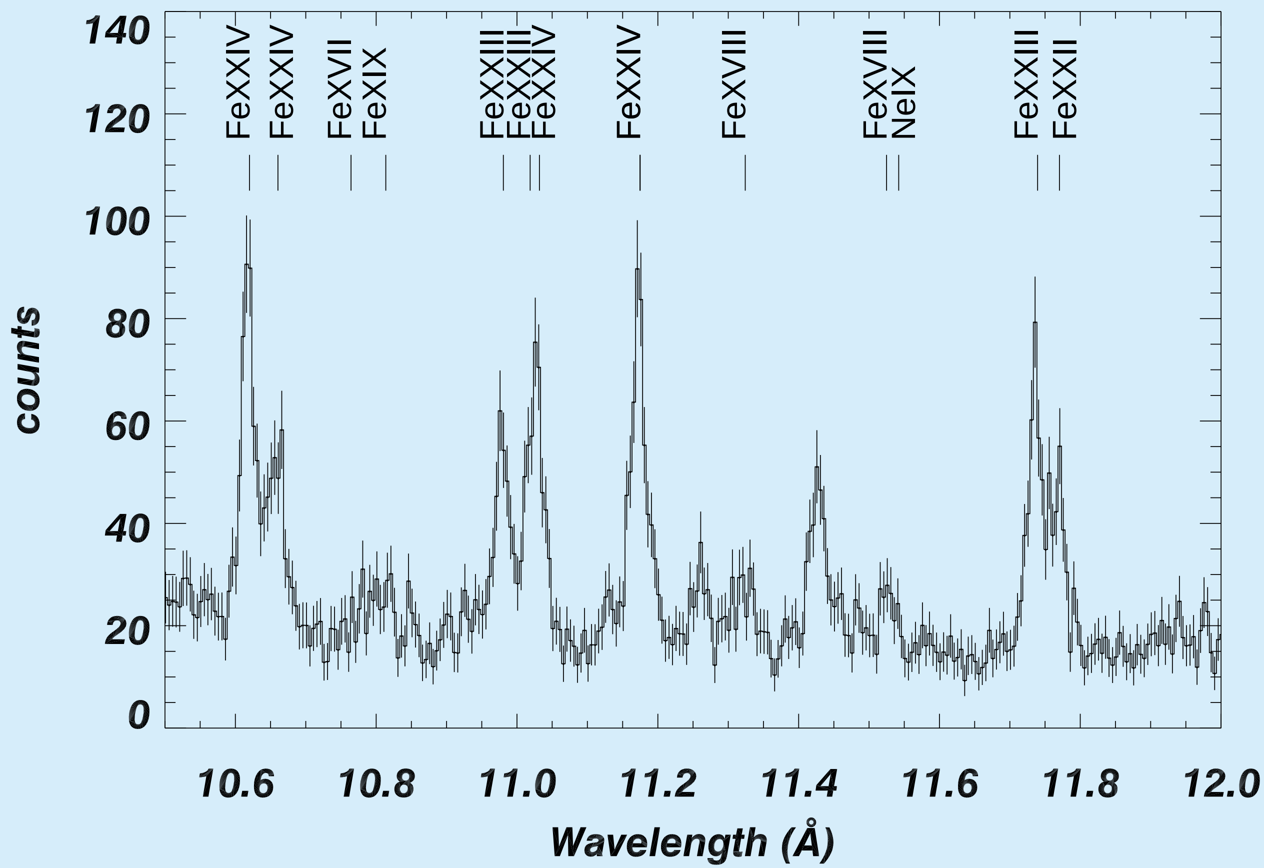
- P Cyg line profiles in UV



HD 163758 (HST)

Observation of hot stars

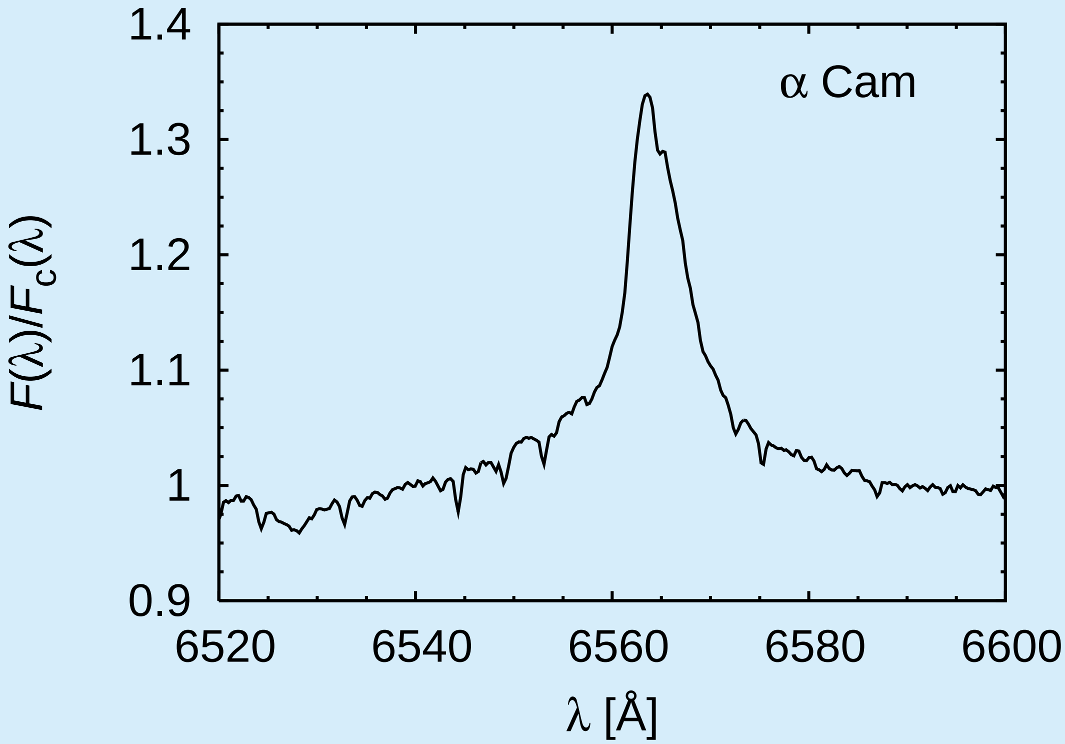
- X-ray emission



X-ray spectrum θ^1 Ori C
(CHANDRA, Schulz et al. 2003)

Observation of hot stars

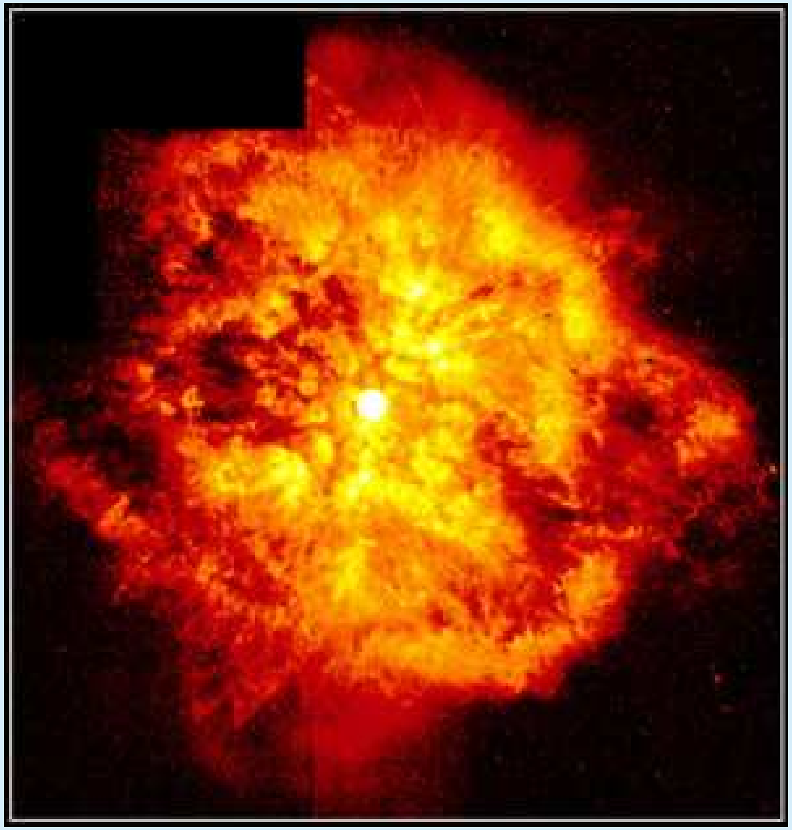
- Hα emission line



α Cam, 2m telescope in Ondřejov (Kubát 2003)

How to explain the observations?

- nebulae



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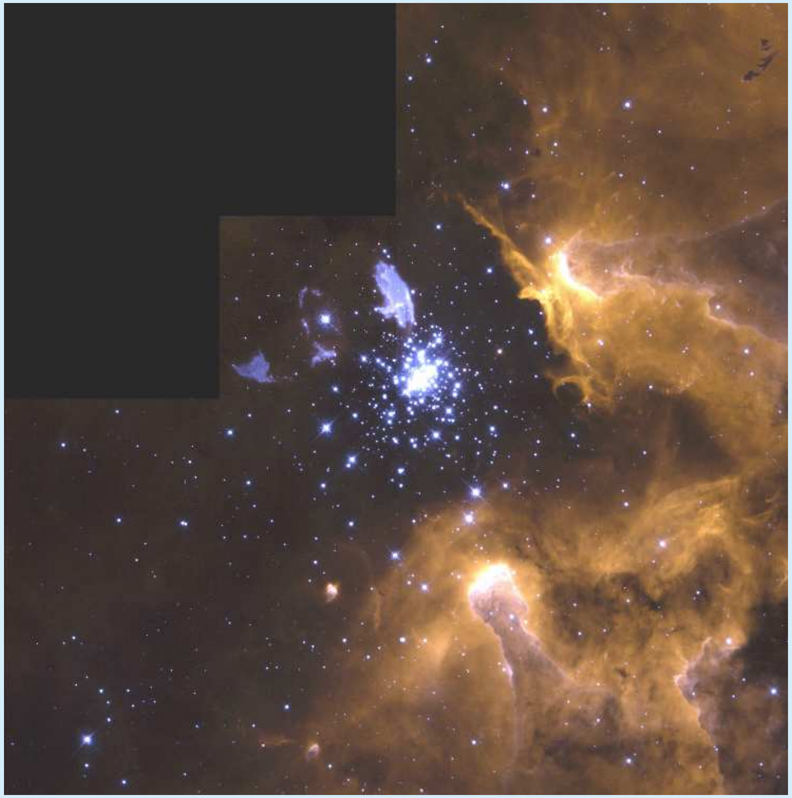

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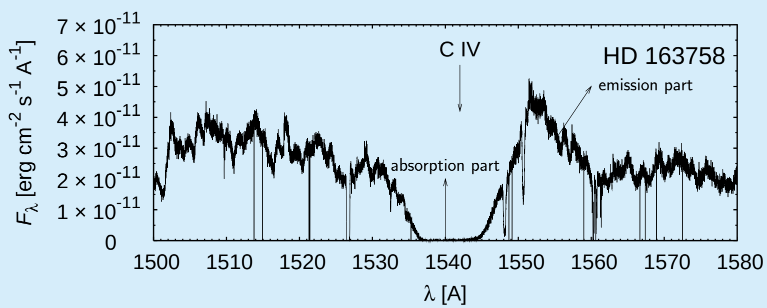

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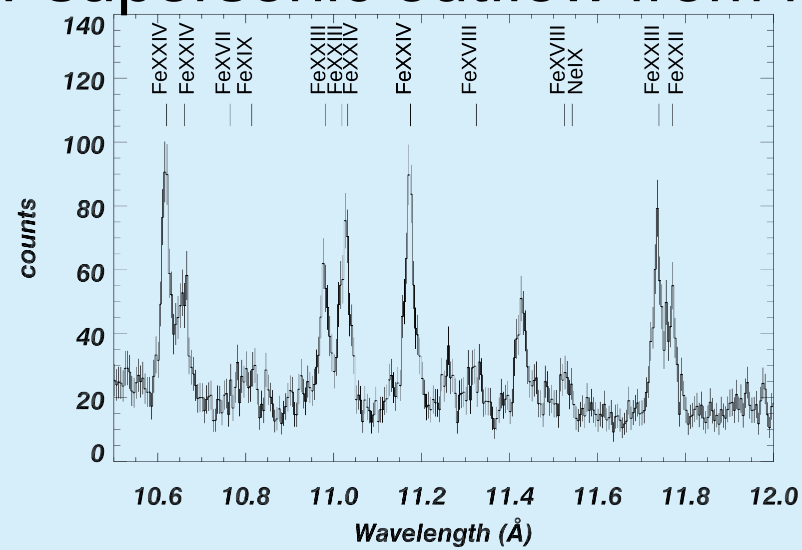

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$\Rightarrow$ quantitative study of the wind

Hot star wind theory

- why is the wind blowing from hot stars?
- what are the main wind parameters (mass-loss rate, velocity)?
- how to predict the wind line profiles?
- how the wind influences the stellar evolution and the circumstellar environment?

Why is the wind blowing?

- some force accelerates the material from the stellar atmosphere to the circumstellar environment

Why is the wind blowing?

- hot stars are luminous: radiative force?

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- hot stars are luminous: radiative force?

$$f_{\mathrm{rad}} = \frac{1}{c}\int_0^\infty \chi(r,\nu)F(r,\nu)\,\mathrm{d}\nu$$

  - spherically symmetric case
  - $\chi(r,\nu)$ absorption coefficient
  - $F(r,\nu)$ radiative flux

Why is the wind blowing?

- hot stars are luminous: radiative force?

$$f_{\mathrm{rad}} = \frac{1}{c}\int_0^\infty \chi(r,\nu) F(r,\nu)\,\mathrm{d}\nu$$

  - radiative force due to the light scattering on free electrons

$$\chi(r,\nu) = \sigma_{\mathrm{Th}} n_{\mathrm{e}}(r)$$

    - σ_{Th} Thomson scattering cross-section
    - $n_{\mathrm{e}}(r)$ electron density

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$$f_{\mathrm{rad}} = \frac{\sigma_{\mathrm{Th}} n_{\mathrm{e}}(r) L}{4\pi r^2 c}$$

where $L = 4\pi r^2 \int_0^\infty F(r,\nu)\,\mathrm{d}\nu$

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$$f_{\mathrm{rad}} = \frac{\sigma_{\mathrm{Th}} n_{\mathrm{e}}(r) L}{4\pi r^2 c}$$

  - comparison with the gravity force

$$f_{\mathrm{grav}} = \frac{\rho(r) G M}{r^2}$$

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$$\Gamma \equiv \frac{f_{\mathrm{rad}}}{f_{\mathrm{grav}}} = \frac{\sigma_{\mathrm{T}} \frac{n_{\mathrm{e}}(r)}{\rho(r)} L}{4\pi c G M}$$

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$$\Gamma \approx 10^{-5}\left(\frac{L}{1\,\mathrm{L}_\odot}\right)\left(\frac{M}{1\,\mathrm{M}_\odot}\right)^{-1}$$

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  - comparison with the gravity force
  - example: α Cam, $L = 6.2 \times 10^5 \mathrm{L}_\odot$, $M = 43\,\mathrm{M}_\odot$, $\Gamma \approx 0.1$

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$$f_{\mathrm{rad}} = \frac{\sigma_{\mathrm{Th}} n_{\mathrm{e}}(r) L}{4\pi r^2 c}$$

  - comparison with the gravity force

$\Rightarrow$ radiative force due to the light scattering on free electrons is important, but it never (?) exceeds the gravity force

Why is the wind blowing?

- hot stars are luminous: radiative force?

$$f_{\mathrm{rad}} = \frac{1}{c}\int_0^\infty \chi(r,\nu)F(r,\nu)\,\mathrm{d}\nu$$

  - radiative force due to the line transitions

$$\chi(r,\nu) = \frac{\pi e^2}{m_{\mathrm{e}}c}\sum_{\mathrm{lines}}\varphi_{ij}(\nu)g_i f_{ij}\left(\frac{n_i(r)}{g_i}-\frac{n_j(r)}{g_j}\right)$$

    - $\varphi_{ij}(\nu)$ line profile, $\int_0^\infty \varphi_{ij}(\nu) = 1$
    - f_{ij} oscillator strength
    - $n_i(r)$, $n_j(r)$ level occupation number, g_i, g_j statistical weights

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$$f_{\mathrm{line}} = \frac{\pi e^2}{m_{\mathrm{e}}c^2}\int_0^\infty \sum_{\mathrm{line}} g_i f_{ij}\left(\frac{n_i(r)}{g_i} - \frac{n_j(r)}{g_j}\right)\varphi_{ij}(\nu)F(r,\nu)\,\mathrm{d}\nu$$

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- problem: influence of lines on $F(r,\nu)$?
- crude solution: $F(r,\nu)$ constant for frequencies corresponding to a given line, $\nu \approx \nu_{ij}$

Why is the wind blowing?

- hot stars are luminous: radiative force?

$$f_{\text{rad}} = \frac{1}{c}\int_0^\infty \chi(r,\nu)F(r,\nu)\,\mathrm{d}\nu$$

- radiative force due to the line transitions
  - maximum force

$$f_{\text{lines}}^{\text{max}} = \frac{\pi e^2}{m_{\text{e}} c^2}\sum_{\text{lines}} g_i f_{ij}\left(\frac{n_i(r)}{g_i} - \frac{n_j(r)}{g_j}\right)F(r,\nu_{ij})$$

  - ν_{ij} is the line center frequency

Why is the wind blowing?

- hot stars are luminous: radiative force?

$$f_{\text{rad}} = \frac{1}{c}\int_0^\infty \chi(r,\nu) F(r,\nu)\,\mathrm{d}\nu$$

- radiative force due to the line transitions
  - maximum force: comparison with gravity

$$\frac{f_{\text{line}}^{\text{max}}}{f_{\text{grav}}} = \frac{Le^2}{4m_{\text{e}}\rho GMc^2}\sum_{\text{line}} f_{ij} n_i(r) \frac{L_\nu(\nu_{ij})}{L}$$

  - neglect of $n_j(r) \ll n_i(r)$
  - $L_\nu(\nu_{ij}) = 4\pi r^2 F(r,\nu_{ij})$

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$$\frac{f_{\text{lines}}^{\text{max}}}{f_{\text{grav}}} = \Gamma \sum_{\text{lines}} \frac{\sigma_{ij}}{\sigma_{\text{Th}}} \frac{n_i}{n_{\text{e}}} \frac{\nu_{ij} L_\nu(\nu_{ij})}{L}$$

$$\sigma_{ij} = \frac{\pi e^2 f_{ij}}{\nu_{ij} m_{\text{e}} c}$$

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    - hydrogen: mostly ionised in the stellar envelopes $\Rightarrow$ n_i/n_{e} very small $\Rightarrow$ negligible contribution to radiative force

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  - neutral helium: n_i/n_{e} very small $\Rightarrow$ negligible contribution to radiative force

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    - ionised helium: nonnegligible contribution to the radiative force

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    - heavier elements (iron, carbon, nitrogen, oxygen, ...): large number of lines, $\sigma_{ij}/\sigma_{\mathrm{Th}} \approx 10^7 \Rightarrow f_{\mathrm{line}}^{\mathrm{max}}/f_{\mathrm{grav}}$ up to 10^3

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$\Rightarrow$ radiative force may be larger than gravity (for many O stars $f_{\mathrm{lines}}^{\mathrm{max}}/f_{\mathrm{grav}} \approx 2000$, Abbott 1982, Gayley 1995)

$\Rightarrow$ stellar wind

Radiative force?

- speculations of Kepler, Newton

Radiative force?

- predicted by James Clerk Maxwell (1873) in the book *A Treatise on Electricity and Magnetism*

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$\Rightarrow$ for $E_\mathrm{p} = E_\nu$ the momentum ratio is

$$\frac{p_\nu}{p_\mathrm{p}} \approx \frac{v}{c}$$

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  - particle with thermal energy $E_\mathrm{p} \approx kT$

$$\frac{p_\nu}{p_\mathrm{p}} \approx \frac{h\nu}{c\sqrt{mkT}} \approx 0.001 \left(\frac{\nu}{10^{15}\,\mathrm{s}^{-1}}\right) \left(\frac{T}{100\,\mathrm{K}}\right)^{-1/2}$$

  - two possibilities:

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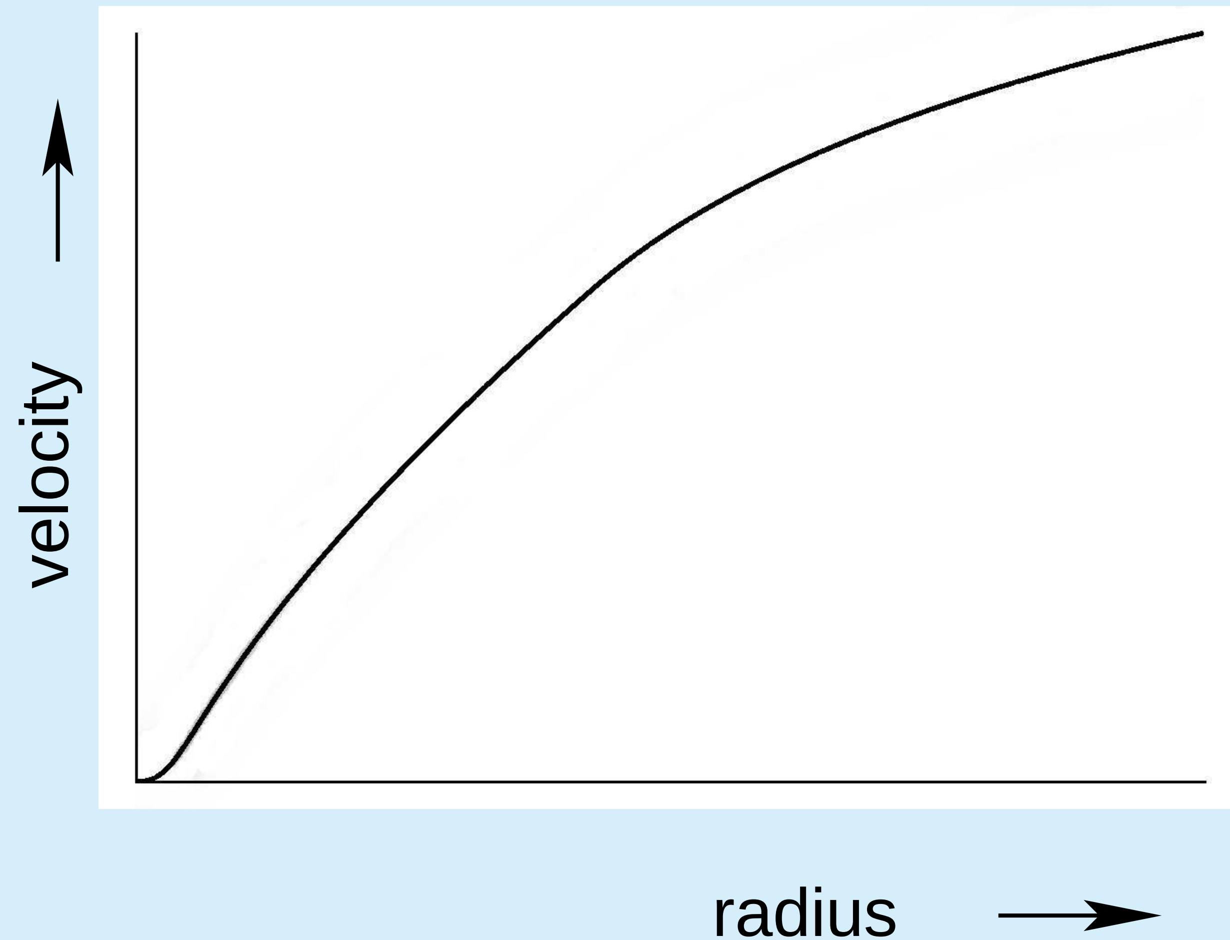
    - line absorption followed by emission
    - Thomson scattering
    - both processes important in hot star winds

The Sobolev approximation

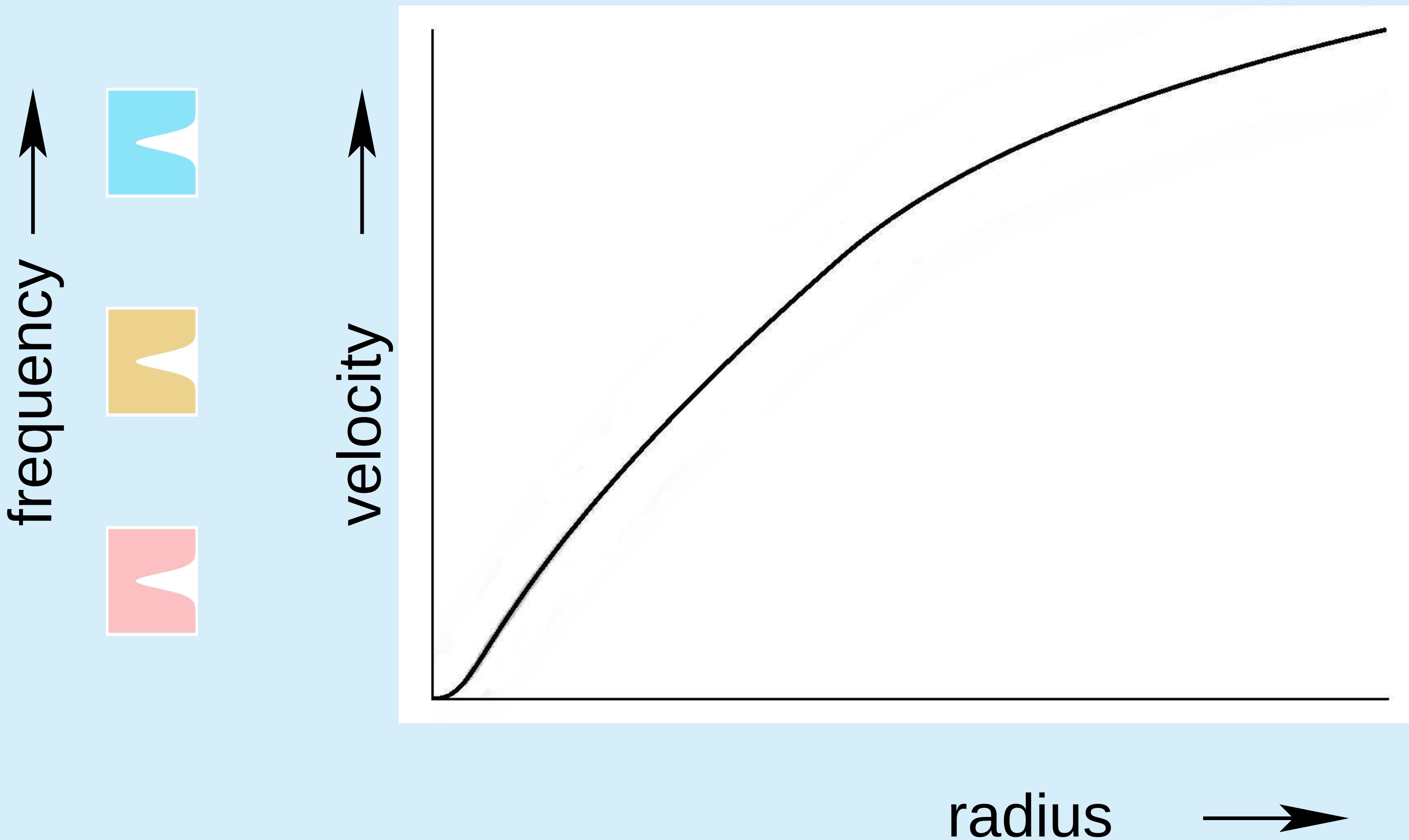
- the main problem: the line opacity (lines may be optically thick)

$\Rightarrow$ necessary to solve the radiative transfer equation

The Sobolev approximation

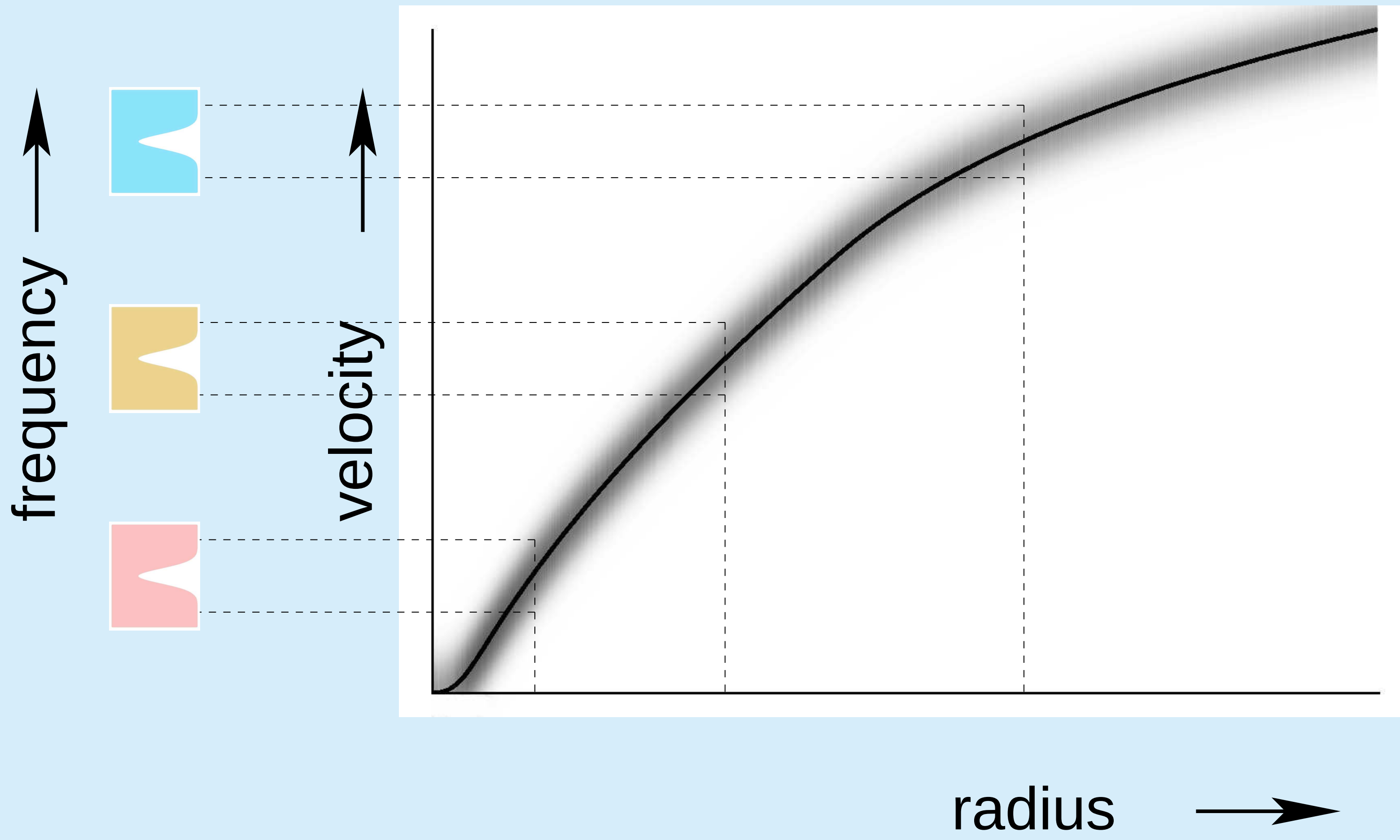


The Sobolev approximation



the Doppler effect in the wind

The Sobolev approximation



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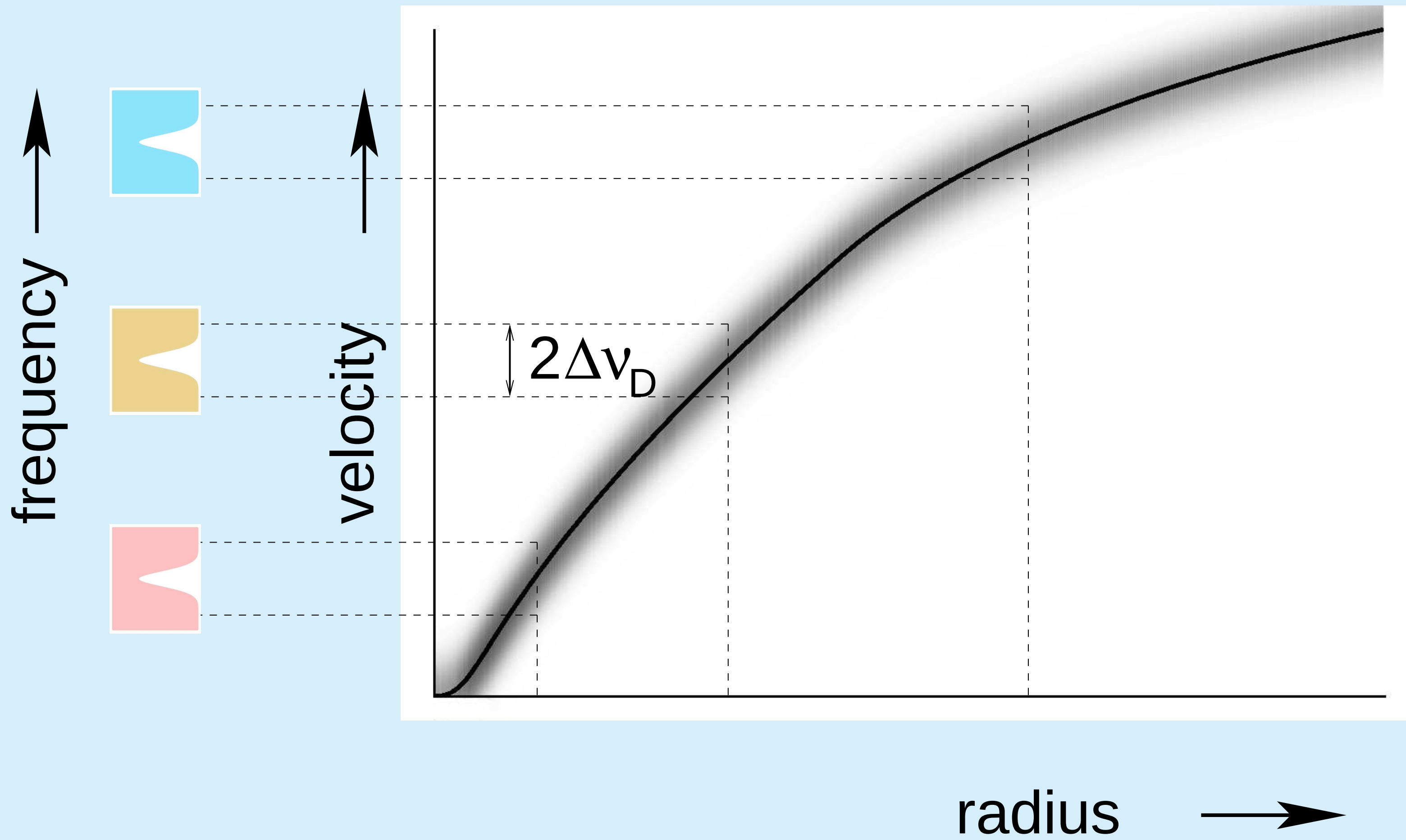
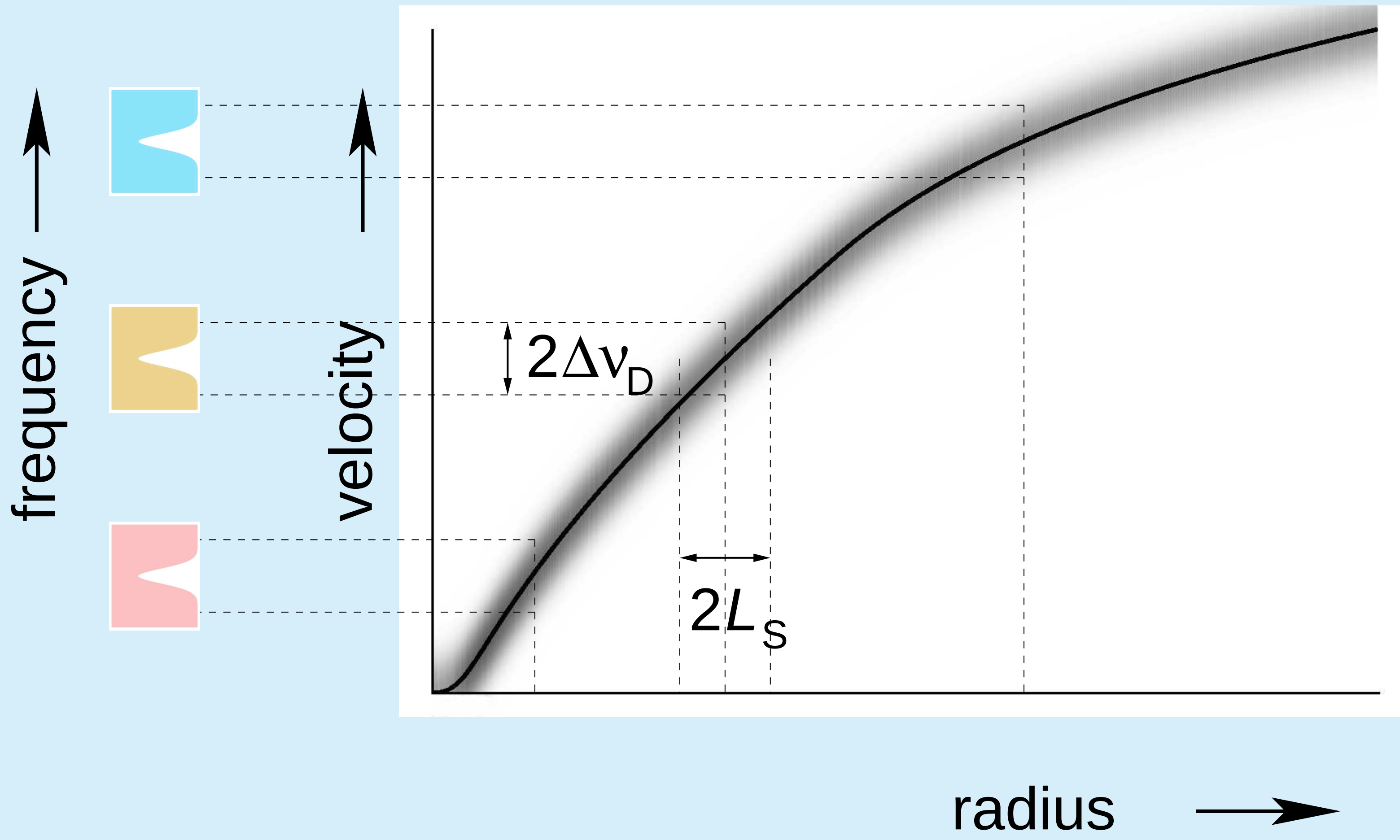


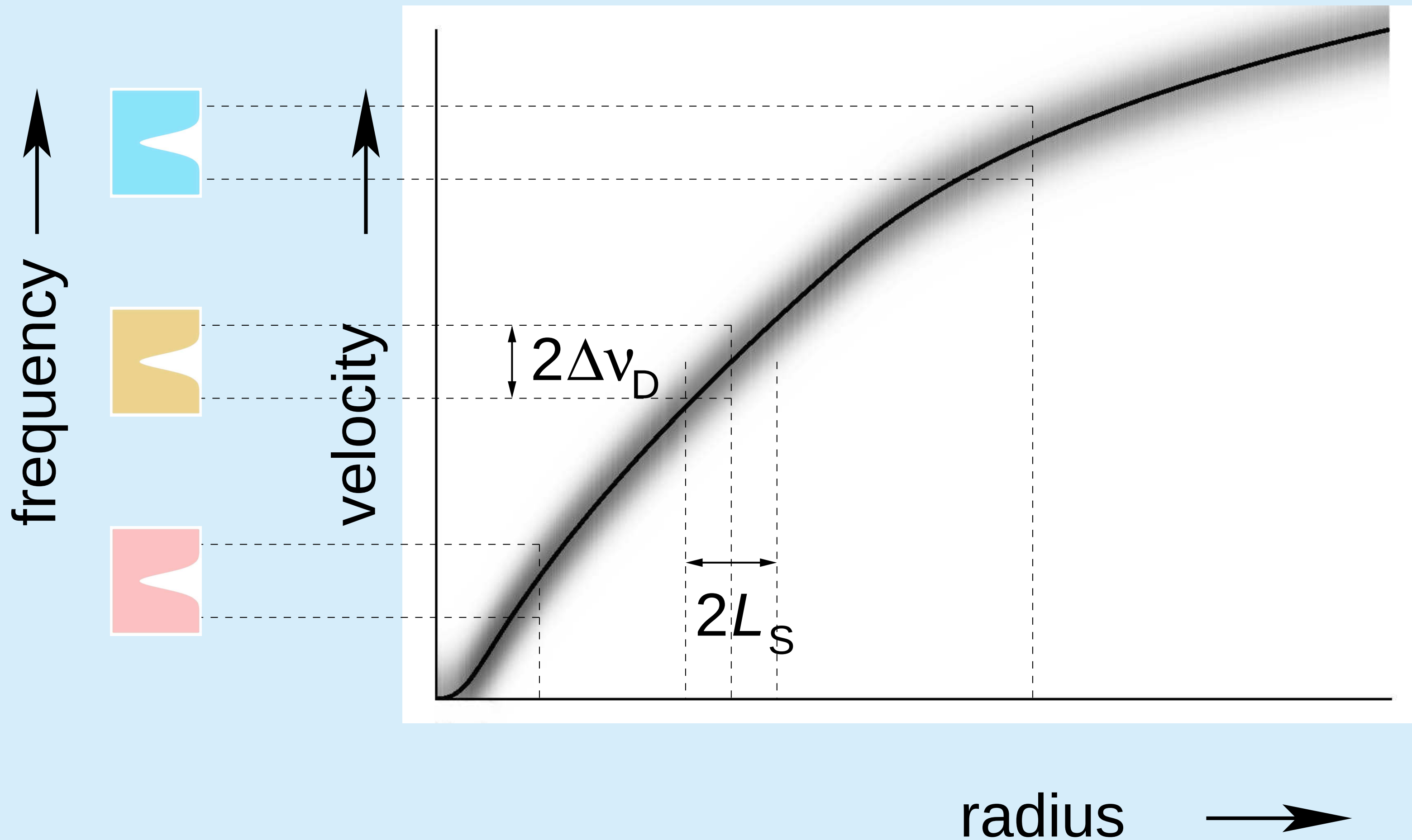
- $\Delta\nu_D$ is the Doppler width of the line

The Sobolev approximation



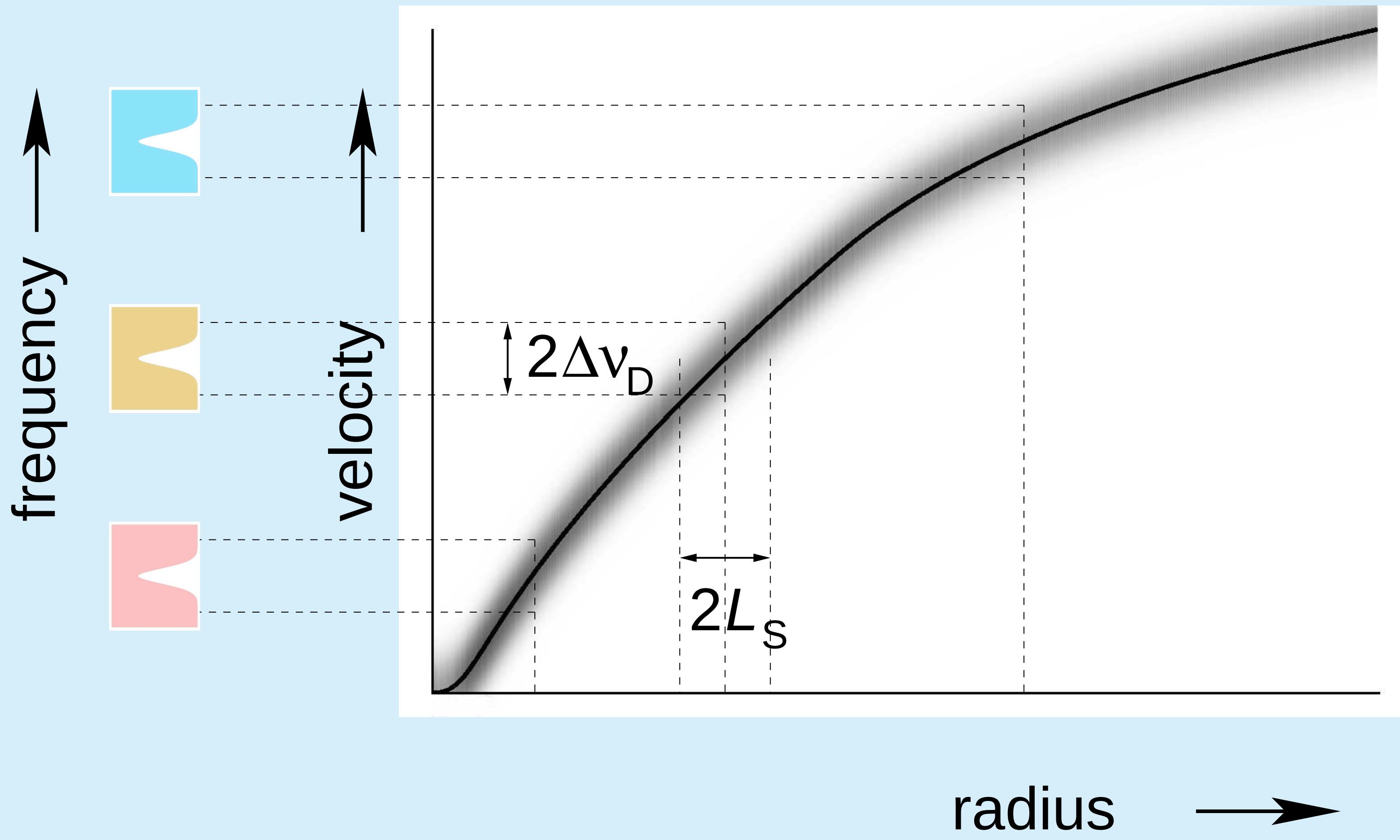
- $L_S \equiv \dfrac{v_{th}}{\frac{dv}{dr}} = c\dfrac{\Delta\nu_D}{\nu_{ij}}\dfrac{1}{\frac{dv}{dr}}$ is the Sobolev length

The Sobolev approximation



- structure does not significantly vary over L_S $\Rightarrow$ simplification of the calculation of f^rad possible

The Sobolev approximation



- opacity nonnegligible only over $L_S \Rightarrow$ solution of RTE in the „gray“ zone only

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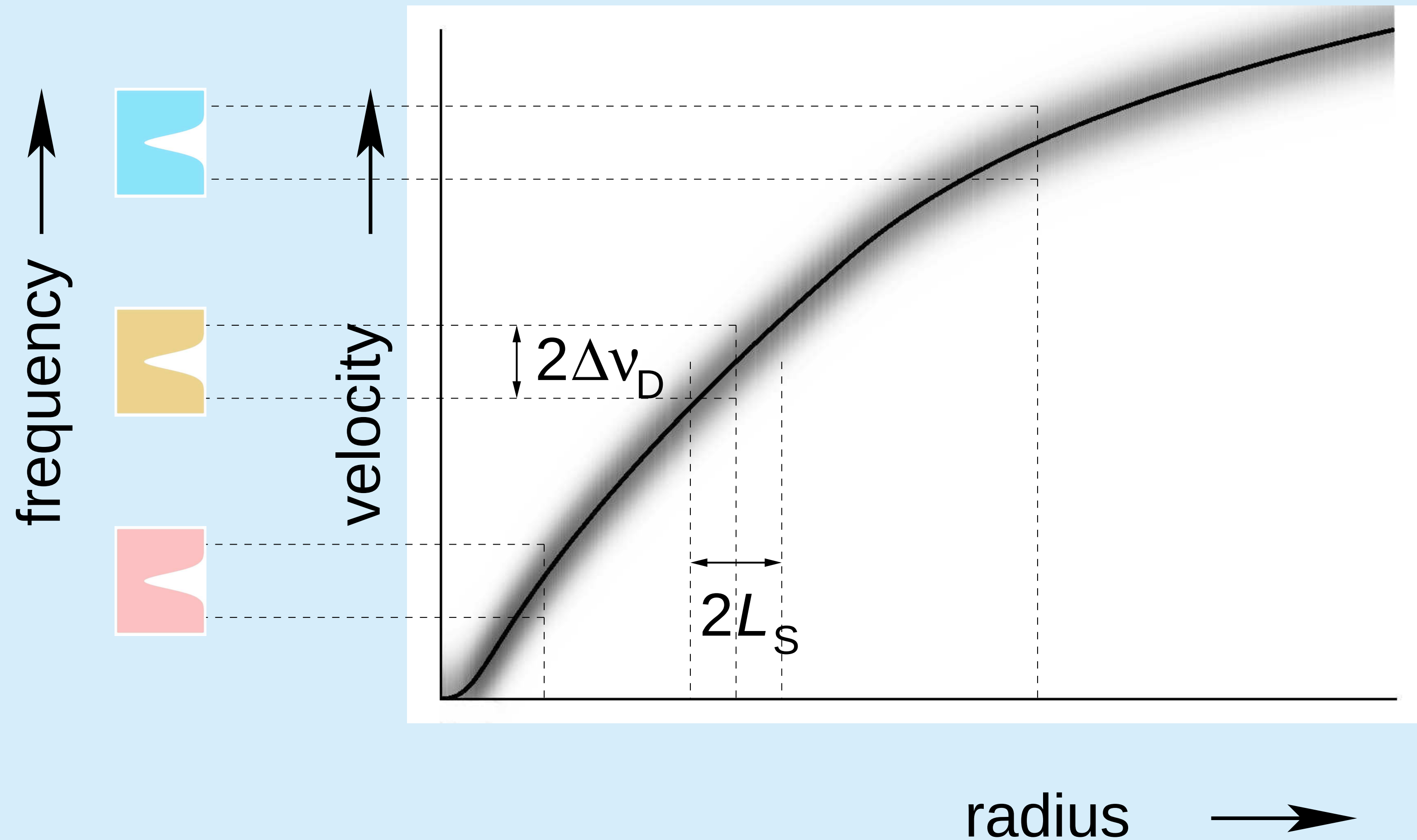

- $H \equiv \dfrac{\rho}{\left(\frac{\mathrm{d}\rho}{\mathrm{d}r}\right)} \approx \dfrac{v}{\left(\frac{\mathrm{d}v}{\mathrm{d}r}\right)} \gg \dfrac{v_{\mathrm{th}}}{\left(\frac{\mathrm{d}v}{\mathrm{d}r}\right)} \equiv L_{\mathrm{S}}\ (v \gg v_{\mathrm{th}})$

Our assumptions

- spherical symmetry

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- spherical symmetry
- stationary (time-independent) flow

The Sobolev line force I.

- the radiative transfer equation

$$\mu \frac{\partial}{\partial r} I(r,\mu,\nu) + \frac{1-\mu^2}{r}\frac{\partial}{\partial \mu} I(r,\mu,\nu) =$$
$$= \eta(r,\mu,\nu) - \chi(r,\mu,\nu) I(r,\mu,\nu)$$

- frame of static observer
- stationarity, spherical symmetry
- μ is frequency, $\mu = \cos\theta$
- $I(r,\mu,\nu)$ is specific intensity
- $\chi(r,\mu,\nu)$ is absorption (extinction) coefficient
- $\eta(r,\mu,\nu)$ is emissivity (emission coefficient)

The Sobolev line force I.

- the radiative transfer equation

$$\mu \frac{\partial}{\partial r} I(r,\mu,\nu) + \frac{1-\mu^2}{r} \frac{\partial}{\partial \mu} I(r,\mu,\nu) = \\ = \eta(r,\mu,\nu) - \chi(r,\mu,\nu) I(r,\mu,\nu)$$

- problem: $\chi(r,\mu,\nu)$ and $\eta(r,\mu,\nu)$ depend on μ due to the Doppler effect

The Sobolev line force I.

- the radiative transfer equation

$$\mu \frac{\partial}{\partial r} I(r,\mu,\nu) + \frac{1-\mu^2}{r} \frac{\partial}{\partial \mu} I(r,\mu,\nu) =$$
$$= \eta(r,\mu,\nu) - \chi(r,\mu,\nu) I(r,\mu,\nu)$$

- problem: $\chi(r,\mu,\nu)$ and $\eta(r,\mu,\nu)$ depend on μ due to the Doppler effect
- solution: use comoving frame!

The Sobolev line force I.

- CMF radiative transfer equation

$$\mu \frac{\partial}{\partial r} I(r,\mu,\nu) + \frac{1-\mu^2}{r} \frac{\partial}{\partial \mu} I(r,\mu,\nu) -$$
$$\frac{\nu v(r)}{cr} \left(1 - \mu^2 + \frac{\mu^2 r}{v(r)} \frac{\mathrm{d}v(r)}{\mathrm{d}r}\right) \frac{\partial}{\partial \nu} I(r,\mu,\nu) =$$
$$= \eta(r,\nu) - \chi(r,\nu) I(r,\mu,\nu)$$

- comoving frame (CMF) equation
- $v(r)$ is the fluid velocity
- $\chi(r,\nu)$ and $\eta(r,\nu)$ do depend on μ

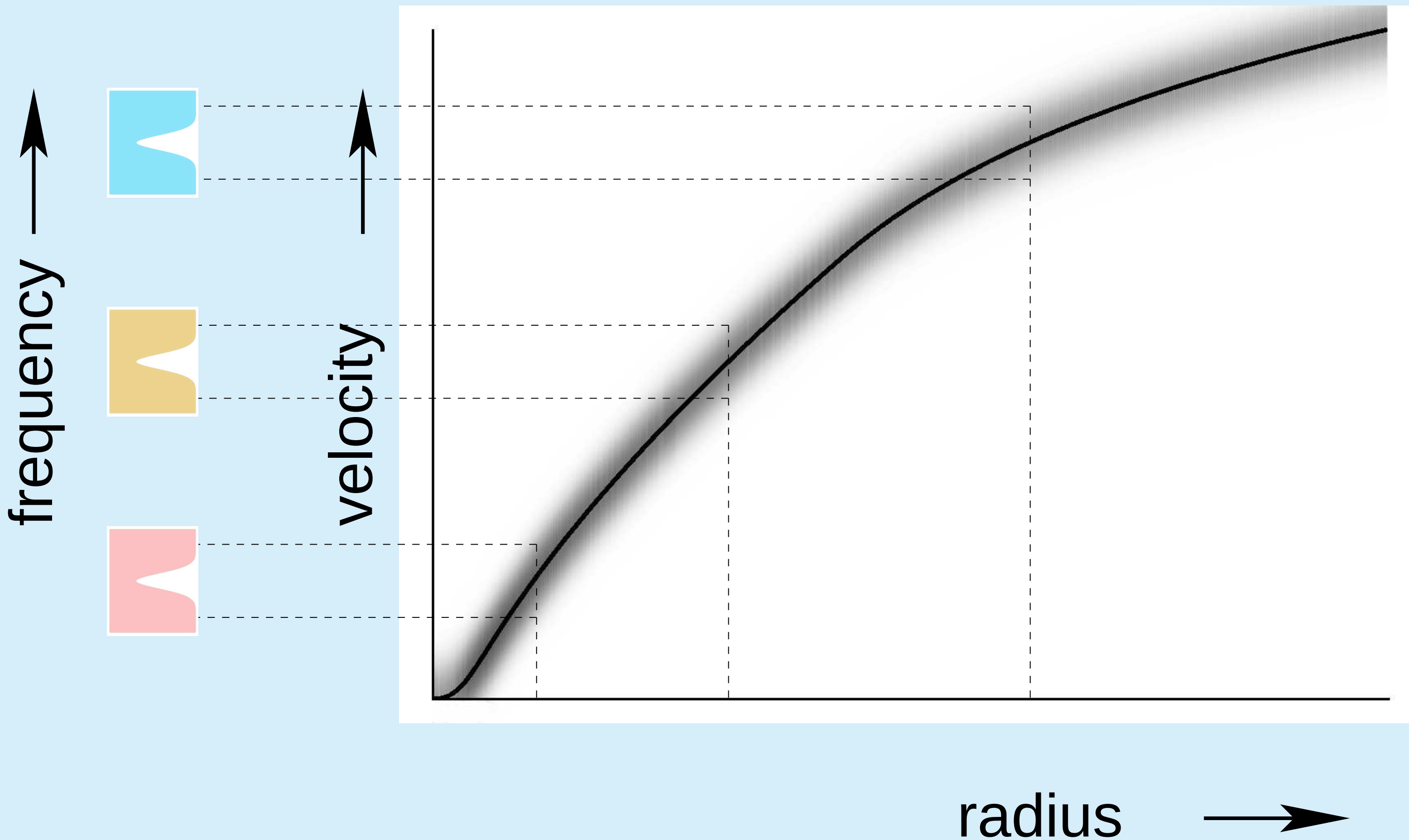
The Sobolev line force I.

- CMF radiative transfer equation

$$\mu \frac{\partial}{\partial r} I(r,\mu,\nu) + \frac{1-\mu^2}{r}\frac{\partial}{\partial \mu} I(r,\mu,\nu) - \frac{\nu v(r)}{cr}\left(1-\mu^2+\frac{\mu^2 r}{v(r)}\frac{\mathrm{d}v(r)}{\mathrm{d}r}\right)\frac{\partial}{\partial \nu} I(r,\mu,\nu) = \\ = \eta(r,\nu) - \chi(r,\nu) I(r,\mu,\nu)$$

- neglected aberration, advection (unimportant for $v \ll c$, e.g., Korčáková & Kubát 2003)
- neglect of the transformation of $I(r,\mu,\nu)$ between individual inertial frames

Intermezzo: the interpretation



- in CMF: continuous redshift of a given photon

The Sobolev line force II.

- the Sobolev transfer equation (Castor 2004)

$$\cancel{\mu\frac{\partial}{\partial r}I(r,\mu,\nu)+\frac{1-\mu^2}{r}\frac{\partial}{\partial\mu}I(r,\mu,\nu)}-$$
$$-\frac{\nu v(r)}{cr}\left(1-\mu^2+\frac{\mu^2 r}{v(r)}\frac{\mathrm{d}v(r)}{\mathrm{d}r}\right)\frac{\partial}{\partial\nu}I(r,\mu,\nu)=$$
$$=\eta(r,\nu)-\chi(r,\nu)I(r,\mu,\nu)$$

The Sobolev line force II.

- the Sobolev transfer equation (Castor 2004)

$$\cancel{\mu \frac{\partial}{\partial r} I(r,\mu,\nu) + \frac{1-\mu^2}{r} \frac{\partial}{\partial \mu} I(r,\mu,\nu)} -$$

$$- \frac{\nu v(r)}{cr} \left(1 - \mu^2 + \frac{\mu^2 r}{v(r)} \frac{\mathrm{d}v(r)}{\mathrm{d}r} \right) \frac{\partial}{\partial \nu} I(r,\mu,\nu) =$$

$$= \eta(r,\nu) - \chi(r,\nu) I(r,\mu,\nu)$$

- possible when $\frac{\nu v(r)}{cr} \frac{\partial}{\partial \nu} I(r,\mu,\nu) \gg \frac{\partial}{\partial r} I(r,\mu,\nu)$
- dimensional arguments:
  - $\frac{\partial}{\partial r} I(r,\mu,\nu) \sim \frac{I(r,\mu,\nu)}{r}$,
  - $\frac{\partial}{\partial \nu} I(r,\mu,\nu) \sim \frac{I(r,\mu,\nu)}{\Delta\nu}$,
    $\Delta\nu = \nu \frac{v_{\mathrm{th}}}{c}$ is the line Doppler width

The Sobolev line force II.

- the Sobolev transfer equation (Castor 2004)

$$\cancel{\mu\frac{\partial}{\partial r}I(r,\mu,\nu)+\frac{1-\mu^2}{r}\frac{\partial}{\partial \mu}I(r,\mu,\nu)}-$$

$$-\frac{\nu v(r)}{cr}\left(1-\mu^2+\frac{\mu^2 r}{v(r)}\frac{\mathrm{d}v(r)}{\mathrm{d}r}\right)\frac{\partial}{\partial \nu}I(r,\mu,\nu)=$$

$$=\eta(r,\nu)-\chi(r,\nu)I(r,\mu,\nu)$$

- possible when $v(r)\gg v_{\mathrm{th}}$

The Sobolev line force III.

- solution of the transfer equation for one line

$$-\frac{\nu v(r)}{cr}\left(1-\mu^2+\frac{\mu^2 r}{v(r)}\frac{\mathrm{d}v(r)}{\mathrm{d}r}\right)\frac{\partial}{\partial\nu}I(r,\mu,\nu)=$$
$$=\eta(r,\nu)-\chi(r,\nu)I(r,\mu,\nu)$$

The Sobolev line force III.

- solution of the transfer equation for one line

$$-\frac{\nu v(r)}{cr}\left(1-\mu^2+\frac{\mu^2 r}{v(r)}\frac{\mathrm{d}v(r)}{\mathrm{d}r}\right)\frac{\partial}{\partial\nu}I(r,\mu,\nu)=$$
$$=\eta(r,\nu)-\chi(r,\nu)I(r,\mu,\nu)$$

- line absorption and emission coefficients are

$$\chi(r,\nu)=\frac{\pi e^2}{m_\mathrm{e}c}\varphi_{ij}(\nu)g_i f_{ij}\left(\frac{n_i(r)}{g_i}-\frac{n_j(r)}{g_j}\right)$$
$$\eta(r,\nu)=\frac{2h\nu^3}{c^2}\frac{\pi e^2}{m_\mathrm{e}c}\varphi_{ij}(\nu)g_i f_{ij}\frac{n_j(r)}{g_j}$$

The Sobolev line force III.

- solution of the transfer equation for one line

$$-\frac{\nu v(r)}{cr}\left(1-\mu^2+\frac{\mu^2 r}{v(r)}\frac{\mathrm{d}v(r)}{\mathrm{d}r}\right)\frac{\partial}{\partial\nu}I(r,\mu,\nu)=$$
$$=\eta(r,\nu)-\chi(r,\nu)I(r,\mu,\nu)$$

- the line opacity and emissivity are

$$\chi(r,\nu)=\chi_{\mathrm{L}}(r)\varphi_{ij}(\nu)$$

$$\eta(r,\nu)=\chi_{\mathrm{L}}(r)S_{\mathrm{L}}(r)\varphi_{ij}(\nu)$$

where $\chi_{\mathrm{L}}(r)=\frac{\pi e^2}{m_{\mathrm{e}}c}g_i f_{ij}\left(\frac{n_i(r)}{g_i}-\frac{n_j(r)}{g_j}\right)$

The Sobolev line force III.

- solution of the transfer equation for one line

$$-\frac{\nu v(r)}{cr}\left(1-\mu^2+\frac{\mu^2 r}{v(r)}\frac{\mathrm{d}v(r)}{\mathrm{d}r}\right)\frac{\partial}{\partial\nu}I(r,\mu,\nu)=$$
$$=\chi_{\mathrm{L}}(r)\varphi_{ij}(\nu)\left(S_{\mathrm{L}}(r)-I(r,\mu,\nu)\right)$$

The Sobolev line force III.

- solution of the transfer equation for one line

$$-\frac{\nu v(r)}{cr}\left(1-\mu^2+\frac{\mu^2 r}{v(r)}\frac{\mathrm{d}v(r)}{\mathrm{d}r}\right)\frac{\partial}{\partial\nu}I(r,\mu,\nu)=$$
$$=\chi_\mathrm{L}(r)\varphi_{ij}(\nu)\left(S_\mathrm{L}(r)-I(r,\mu,\nu)\right)$$

- introduce a new variable

$$y=\int_\nu^\infty \mathrm{d}\nu'\varphi_{ij}(\nu')$$

- where
  - $y=0$: the incoming side of the line
  - $y=1$: the outgoing side of the line

The Sobolev line force III.

- solution of the transfer equation for one line

$$\frac{\nu v(r)}{cr}\left(1-\mu^2+\frac{\mu^2 r}{v(r)}\frac{\mathrm{d}v(r)}{\mathrm{d}r}\right)\frac{\partial}{\partial y}I(r,\mu,y)=$$

$$=\chi_{\mathrm{L}}(r)\left(S_{\mathrm{L}}(r)-I(r,\mu,y)\right)$$

The Sobolev line force III.

- solution of the transfer equation for one line

$$\frac{\nu v(r)}{cr}\left(1-\mu^2+\frac{\mu^2 r}{v(r)}\frac{\mathrm{d}v(r)}{\mathrm{d}r}\right)\frac{\partial}{\partial y}I(r,\mu,y)=$$
$$=\chi_{\mathrm{L}}(r)\left(S_{\mathrm{L}}(r)-I(r,\mu,y)\right)$$

- assumptions:
  - variables do not significantly vary with r within the „resonance zone“

  $\Rightarrow$ fixed r, $\frac{\partial}{\partial y}\rightarrow\frac{\mathrm{d}}{\mathrm{d}y}$
  - $\nu\rightarrow\nu_0$

$\Rightarrow$ integration possible

The Sobolev line force III.

- solution of the transfer equation for one line

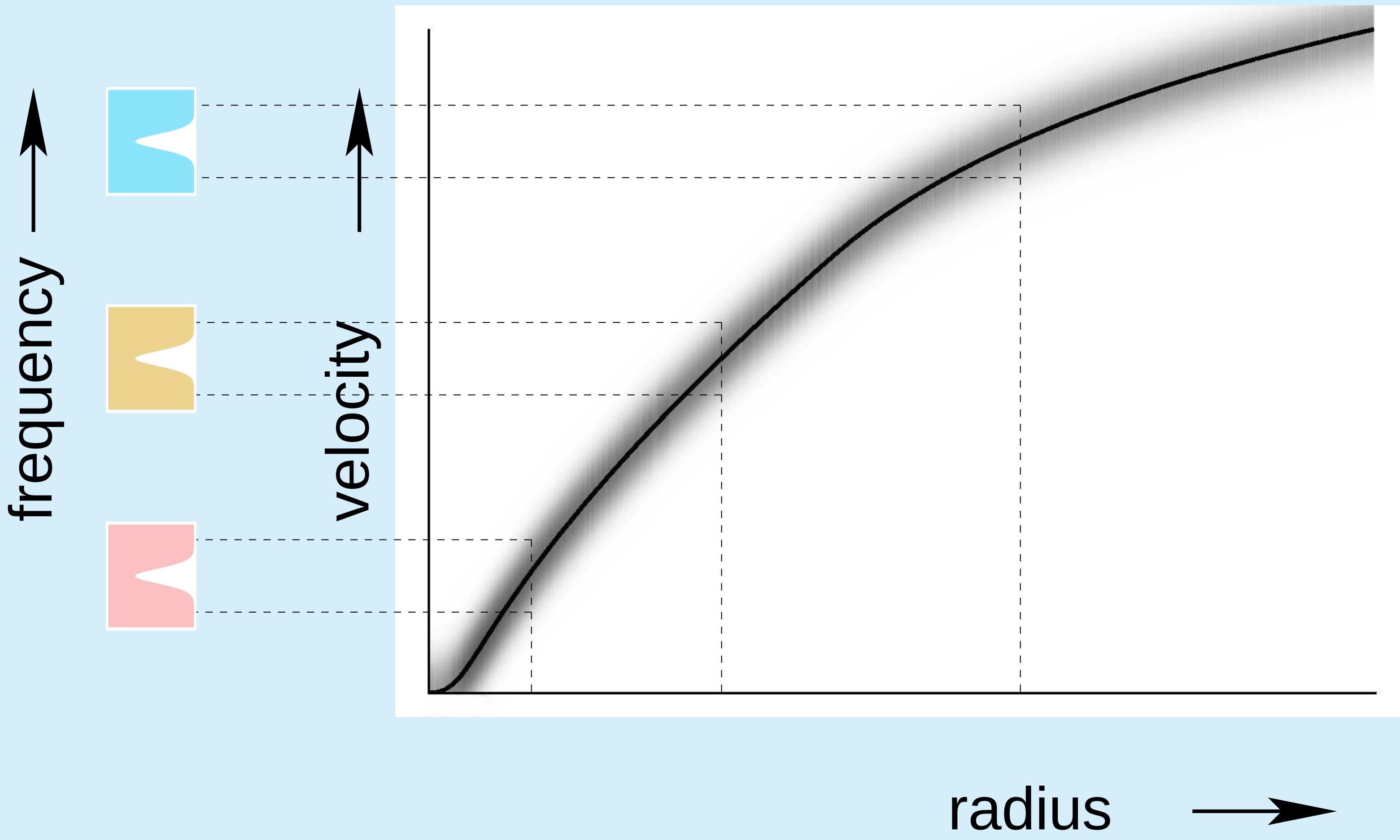
$$I(y) = I_{\mathrm{c}}(\mu)\exp\left[-\tau(\mu)y\right] + S_{\mathrm{L}}\left\{1 - \exp\left[-\tau(\mu)y\right]\right\}$$

- where
  - the Sobolev optical depth is

$$\tau(\mu) = \frac{\chi_{\mathrm{L}}(r)cr}{\nu_0 v(r)\left(1 - \mu^2 + \frac{\mu^2 r}{v(r)}\frac{\mathrm{d}v(r)}{\mathrm{d}r}\right)}$$

  - the boundary condition is $I(y=0) = I_{\mathrm{c}}(\mu)$

Intermezzo: the interpretation



- τ is given by the slope $\Rightarrow \tau \sim \left(\frac{\mathrm{d}v}{\mathrm{d}r}\right)^{-1}$

The Sobolev line force IV.

- the radiative force (the radial component; force per unit of volume)

$$f_{\mathrm{rad}} = \frac{1}{c}\int_0^\infty \chi(r,\nu)F(r,\nu)\,\mathrm{d}\nu$$

The Sobolev line force IV.

- the radiative force (the radial component; force per unit of volume)

$$f_{\mathrm{rad}} = \frac{1}{c} \int_0^{\infty} \mathrm{d}\nu \, \chi(r,\nu) \oint \mathrm{d}\Omega \, \mu I(r,\mu,\nu)$$

The Sobolev line force IV.

- the radiative force (the radial component; force per unit of volume)

$$f_{\mathrm{rad}} = \frac{2\pi}{c} \int_0^\infty \mathrm{d}\nu\, \chi_{\mathrm{L}}(r) \varphi_{ij}(\nu) \int_{-1}^{1} \mathrm{d}\mu\, \mu I(r,\mu,\nu)$$

The Sobolev line force IV.

- the radiative force (the radial component; force per unit of volume)

$$f_{\mathrm{rad}} = \frac{2\pi\chi_{\mathrm{L}}(r)}{c}\int_0^1 \mathrm{d}y \int_{-1}^1 \mathrm{d}\mu\, \mu I(r,\mu,y)$$

The Sobolev line force IV.

- the radiative force (the radial component; force per unit of volume)

$$f_{\mathrm{rad}} = \frac{2\pi\chi_{\mathrm{L}}(r)}{c}\int_0^1 \mathrm{d}y \times$$

$$\int_{-1}^{1} \mathrm{d}\mu\, \mu \left\{ I_{\mathrm{c}}(\mu) \exp\left[-\tau(\mu) y\right] + S_{\mathrm{L}} \left\{1 - \exp\left[-\tau(\mu) y\right]\right\}\right\}$$

- where the Sobolev optical depth is

$$\tau(\mu) = \frac{\chi_{\mathrm{L}}(r) c r}{\nu_0 v(r) \left(1 - \mu^2 + \frac{\mu^2 r}{v(r)} \frac{\mathrm{d}v(r)}{\mathrm{d}r}\right)}$$

- $\tau(\mu)$ is an even function of μ

The Sobolev line force IV.

- the radiative force (the radial component; force per unit of volume)

$$f_{\mathrm{rad}} = \frac{2\pi\chi_{\mathrm{L}}(r)}{c} \int_0^1 \mathrm{d}y \int_{-1}^{1} \mathrm{d}\mu\, \mu I_{\mathrm{c}}(\mu) \exp\left[-\tau(\mu) y\right]$$

- no net contribution of the emission to the radiative force (S_{L} is isotropic in the CMF)

The Sobolev line force IV.

- the radiative force (the radial component; force per unit of volume)

$$f_{\mathrm{rad}} = \frac{2\pi\chi_{\mathrm{L}}(r)}{c}\int_{-1}^{1} \mathrm{d}\mu\, \mu I_{\mathrm{c}}(\mu)\frac{1-\exp\left[-\tau(\mu)\right]}{\tau(\mu)}$$

- inserting

$$\tau(\mu) = \frac{\chi_{\mathrm{L}}(r)cr}{\nu_0 v(r)\left(1-\mu^2+\frac{\mu^2 r}{v(r)}\frac{\mathrm{d}v(r)}{\mathrm{d}r}\right)}$$

The Sobolev line force IV.

- the radiative force (the radial component; force per unit of volume)

$$f_{\mathrm{rad}} = \frac{2\pi\nu_0 v(r)}{rc^2}\int_{-1}^{1}\mathrm{d}\mu\,\mu I_{\mathrm{c}}(\mu)\left[1+\mu^2\sigma(r)\right]\times$$
$$\times\left\{1-\exp\left[-\frac{\chi_{\mathrm{L}}(r)cr}{\nu_0 v(r)\left(1+\mu^2\sigma(r)\right)}\right]\right\}$$

- where $\sigma(r) = \dfrac{r}{v(r)}\dfrac{\mathrm{d}v(r)}{\mathrm{d}r} - 1$
- Sobolev (1957), Castor (1974), Rybicki & Hummer (1978)

Optically thin lines

- optically thin line:

$$\frac{\chi_{\mathsf{L}}(r)cr}{\nu_0 v(r)\left(1+\mu^2\sigma(r)\right)} \ll 1$$

Optically thin lines

- optically thin line:

$$\frac{\chi_{\mathrm{L}}(r)cr}{\nu_0 v(r)\left(1+\mu^2\sigma(r)\right)} \ll 1$$

- the radiative force proportional to

$$f_{\mathrm{rad}} \sim 1 - \exp\left[-\frac{\chi_{\mathrm{L}}(r)cr}{\nu_0 v(r)\left(1+\mu^2\sigma(r)\right)}\right]$$

Optically thin lines

- optically thin line:

$$\frac{\chi_{\mathrm{L}}(r) c r}{\nu_0 v(r)\left(1+\mu^2 \sigma(r)\right)} \ll 1$$

- the radiative force proportional to

$$f_{\mathrm{rad}} \sim 1-\exp\left[-\frac{\chi_{\mathrm{L}}(r) c r}{\nu_0 v(r)\left(1+\mu^2 \sigma(r)\right)}\right] \approx \frac{\chi_{\mathrm{L}}(r) c r}{\nu_0 v(r)\left(1+\mu^2 \sigma(r)\right)}$$

Optically thin lines

$$f_{\mathsf{rad}} = \frac{2\pi}{c} \int_{-1}^{1} \mathsf{d}\mu \, \mu I_{\mathsf{c}}(\mu) \chi_{\mathsf{L}}(r)$$

Optically thin lines

$$f_{\mathrm{rad}} = \frac{1}{c}\chi_{\mathrm{L}}(r)F(r)$$

Optically thin lines

$$f_{\text{rad}} = \frac{1}{c}\chi_{\text{L}}(r)F(r)$$

- optically thin radiative force proportional to the radiative flux $F(r)$
- optically thin radiative force proportional to the normalised line opacity $\chi_{\text{L}}(r)$ (or to the density)
- the same result as for the static medium

Optically thick lines

- optically thick line:

$$\frac{\chi_{\mathrm{L}}(r) c r}{\nu_0 v(r)\left(1+\mu^2 \sigma(r)\right)} \gg 1$$

Optically thick lines

- optically thick line:

$$\frac{\chi_{\mathrm{L}}(r)cr}{\nu_0 v(r)\left(1+\mu^2\sigma(r)\right)} \gg 1$$

- the radiative force proportional to

$$f_{\mathrm{rad}} \sim 1 - \exp\left[-\frac{\chi_{\mathrm{L}}(r)cr}{\nu_0 v(r)\left(1+\mu^2\sigma(r)\right)}\right]$$

Optically thick lines

- optically thick line:

$$\frac{\chi_{\mathrm{L}}(r)cr}{\nu_0 v(r)\left(1+\mu^2\sigma(r)\right)} \gg 1$$

- the radiative force proportional to

$$\begin{aligned} f_{\mathrm{rad}} &\sim 1-\exp\left[-\frac{\chi_{\mathrm{L}}(r)cr}{\nu_0 v(r)\left(1+\mu^2\sigma(r)\right)}\right] \\ &\approx 1 \end{aligned}$$

Optically thick lines

$$f_{\mathrm{rad}} = \frac{2\pi\nu_0 v(r)}{r c^2} \int_{-1}^{1} \mathrm{d}\mu \, \mu I_{\mathrm{c}}(\mu) \left[1 + \mu^2 \sigma(r)\right]$$

Optically thick lines

$$f_{\text{rad}} = \frac{2\pi\nu_0 v(r)}{r c^2} \int_{-1}^{1} \mathrm{d}\mu\, \mu I_{\text{c}}(\mu) \left[1 + \mu^2 \sigma(r)\right]$$

- neglect of the limb darkening:

$$I_{\text{c}}(\mu) = \begin{cases} I_{\text{c}} = \text{const.}, & \mu \geq \mu_*, \\ 0, & \mu < \mu_* \end{cases},$$

where $\mu_* = \sqrt{1 - \frac{R_*^2}{r^2}}$

Optically thick lines

$$f_{\mathrm{rad}} = \frac{2\pi \nu_0 v(r)}{r c^2} \int_{\mu_*}^{1} \mathrm{d}\mu \, \mu I_{\mathrm{c}} \left[1 + \mu^2 \sigma(r)\right]$$

Optically thick lines

$$f_{\mathrm{rad}} = \frac{\nu_0 v(r) F(r)}{r c^2} \left[1 + \sigma(r) \left(1 - \frac{1}{2} \frac{R_*^2}{r^2} \right) \right]$$

where $F = 2\pi \int_{\mu_*}^{1} \mathrm{d}\mu \, \mu I_{\mathrm{c}} = \pi \frac{R_*^2}{r^2} I_{\mathrm{c}}$

Optically thick lines

$$f_{\text{rad}} = \frac{\nu_0 v(r) F(r)}{r c^2} \left[1 + \sigma(r) \left(1 - \frac{1}{2} \frac{R_*^2}{r^2} \right) \right]$$

- large distance from the star: $r \gg R_*$

Optically thick lines

$$f_{\mathrm{rad}} = \frac{\nu_0 v(r) F(r)}{r c^2} \left[1 + \sigma(r) \left(1 - \frac{1}{2} \frac{R_*^2}{r^2} \right) \right]$$

- large distance from the star: $r \gg R_*$

$$f_{\mathrm{rad}} \approx \frac{\nu_0 F(r)}{c^2} \frac{\mathrm{d} v(r)}{\mathrm{d} r}$$

Optically thick lines

$$f_{\mathrm{rad}} = \frac{\nu_0 v(r) F(r)}{r c^2} \left[1 + \sigma(r) \left(1 - \frac{1}{2} \frac{R_*^2}{r^2}\right)\right]$$

- large distance from the star: $r \gg R_*$

$$f_{\mathrm{rad}} \approx \frac{\nu_0 F(r)}{c^2} \frac{\mathrm{d}v(r)}{\mathrm{d}r}$$

- optically thick radiative force proportional to the radiative flux $F(r)$
- optically thick radiative force proportional to $\frac{\mathrm{d}v}{\mathrm{d}r}$
- optically thick radiative force does not depend on the level populations or the density

Wind driven by thick lines

- continuity and momentum equation of isothermal spherically symmetric wind

$$\frac{\partial \rho}{\partial t} + \frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2 \rho v\right) = 0$$

$$\frac{\partial v}{\partial t} + \rho v \frac{\partial v}{\partial r} = -a^2 \frac{\partial \rho}{\partial r} + f_{\mathrm{rad}} - \frac{\rho G M (1-\Gamma)}{r^2}$$

- ρ, v are the wind density and velocity
- a is the sound speed

Wind driven by thick lines

- continuity and momentum equation of isothermal spherically symmetric wind

$$\frac{1}{r^2}\frac{\mathrm{d}}{\mathrm{d}r}\left(r^2\rho v\right) = 0$$

$$\rho v\frac{\mathrm{d}v}{\mathrm{d}r} = -a^2\frac{\mathrm{d}\rho}{\mathrm{d}r} + f_{\mathrm{rad}} - \frac{\rho G M(1-\Gamma)}{r^2}$$

- assumption: stationary flow

Wind driven by thick lines

- continuity equation

$$\frac{1}{r^2}\frac{\mathrm{d}}{\mathrm{d}r}\left(r^2\rho v\right) = 0 \Rightarrow \dot{M} \equiv 4\pi r^2\rho v = \text{const.}$$

- $\dot{M}$ is the wind mass-loss rate

Wind driven by thick lines

- momentum equation

$$v\frac{\mathrm{d}v}{\mathrm{d}r} = \frac{f_{\mathrm{rad}}}{\rho} - \frac{GM(1-\Gamma)}{r^2}$$

- neglect of the gas-pressure term $a^2\frac{\mathrm{d}\rho}{\mathrm{d}r} \ll f_{\mathrm{rad}}$ (possible in the supersonic part of the wind)

Wind driven by thick lines

- momentum equation

$$v\frac{\mathrm{d}v}{\mathrm{d}r} = \frac{\nu_0 v(r) F(r)}{\rho r c^2}\left[1+\sigma(r)\left(1-\frac{1}{2}\frac{R_*^2}{r^2}\right)\right] - \frac{GM(1-\Gamma)}{r^2}$$

- inclusion of the expression for the optically thick line force
- $F(r) = \frac{L_\nu}{4\pi r^2}$, where L_ν is the monochromatic stellar luminosity (constant)
- $\sigma(r) = \frac{r}{v}\frac{\mathrm{d}v}{\mathrm{d}r} - 1$

Wind driven by thick lines

- momentum equation

$$\left[v - \frac{\nu_0 L_\nu}{4\pi r^2 \rho c^2}\left(1 - \frac{1}{2}\frac{R_*^2}{r^2}\right)\right]\frac{\mathrm{d}v}{\mathrm{d}r} = \frac{\nu_0 v(r) L_\nu}{8\pi\rho c^2 r^3} - \frac{GM(1-\Gamma)}{r^2}$$

Wind driven by thick lines

- momentum equation

$$\left[v - \frac{\nu_0 L_\nu}{4\pi r^2 \rho c^2}\left(1 - \frac{1}{2}\frac{R_*^2}{r^2}\right)\right]\frac{\mathrm{d}v}{\mathrm{d}r} = \frac{\nu_0 v(r) L_\nu}{8\pi\rho c^2 r^3} - \frac{GM(1-\Gamma)}{r^2}$$

- has a critical point

$$v - \frac{\nu_0 L_\nu}{4\pi r^2 \rho c^2}\left(1 - \frac{1}{2}\frac{R_*^2}{r^2}\right) = 0$$

Wind driven by thick lines

- momentum equation

$$\left[v - \frac{\nu_0 L_\nu}{4\pi r^2 \rho c^2}\left(1 - \frac{1}{2}\frac{R_*^2}{r^2}\right)\right]\frac{\mathrm{d}v}{\mathrm{d}r} = \frac{\nu_0 v(r) L_\nu}{8\pi\rho c^2 r^3} - \frac{GM(1-\Gamma)}{r^2}$$

- has a critical point

$$v - \frac{\nu_0 L_\nu}{4\pi r^2 \rho c^2}\left(1 - \frac{1}{2}\frac{R_*^2}{r^2}\right) = 0$$

- neglect of $\frac{R_*}{r}$ term:

$$\dot{M} \equiv 4\pi r^2 \rho v(r) = \frac{\nu_0 L_\nu}{c^2}$$

Wind driven by thick lines

- momentum equation

$$\left[v - \frac{\nu_0 L_\nu}{4\pi r^2 \rho c^2}\left(1 - \frac{1}{2}\frac{R_*^2}{r^2}\right)\right]\frac{\mathrm{d}v}{\mathrm{d}r} = \frac{\nu_0 v(r) L_\nu}{8\pi\rho c^2 r^3} - \frac{GM(1-\Gamma)}{r^2}$$

- has a critical point

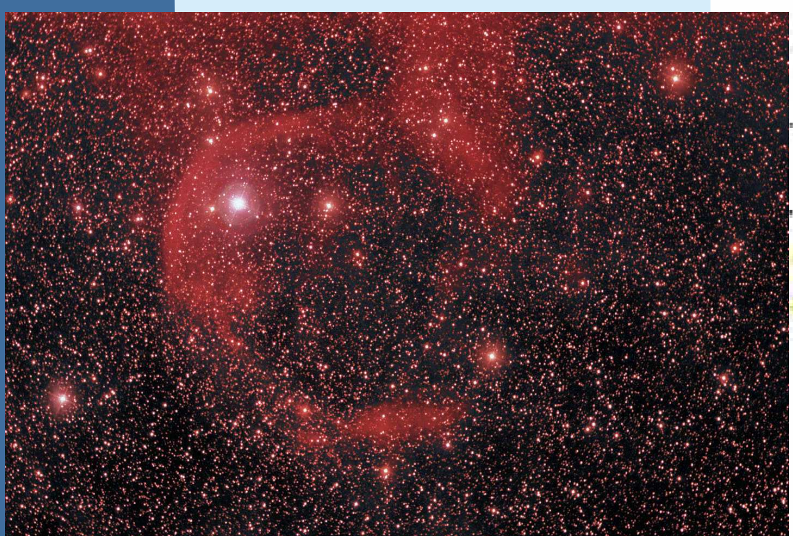
$$v - \frac{\nu_0 L_\nu}{4\pi r^2 \rho c^2}\left(1 - \frac{1}{2}\frac{R_*^2}{r^2}\right) = 0$$

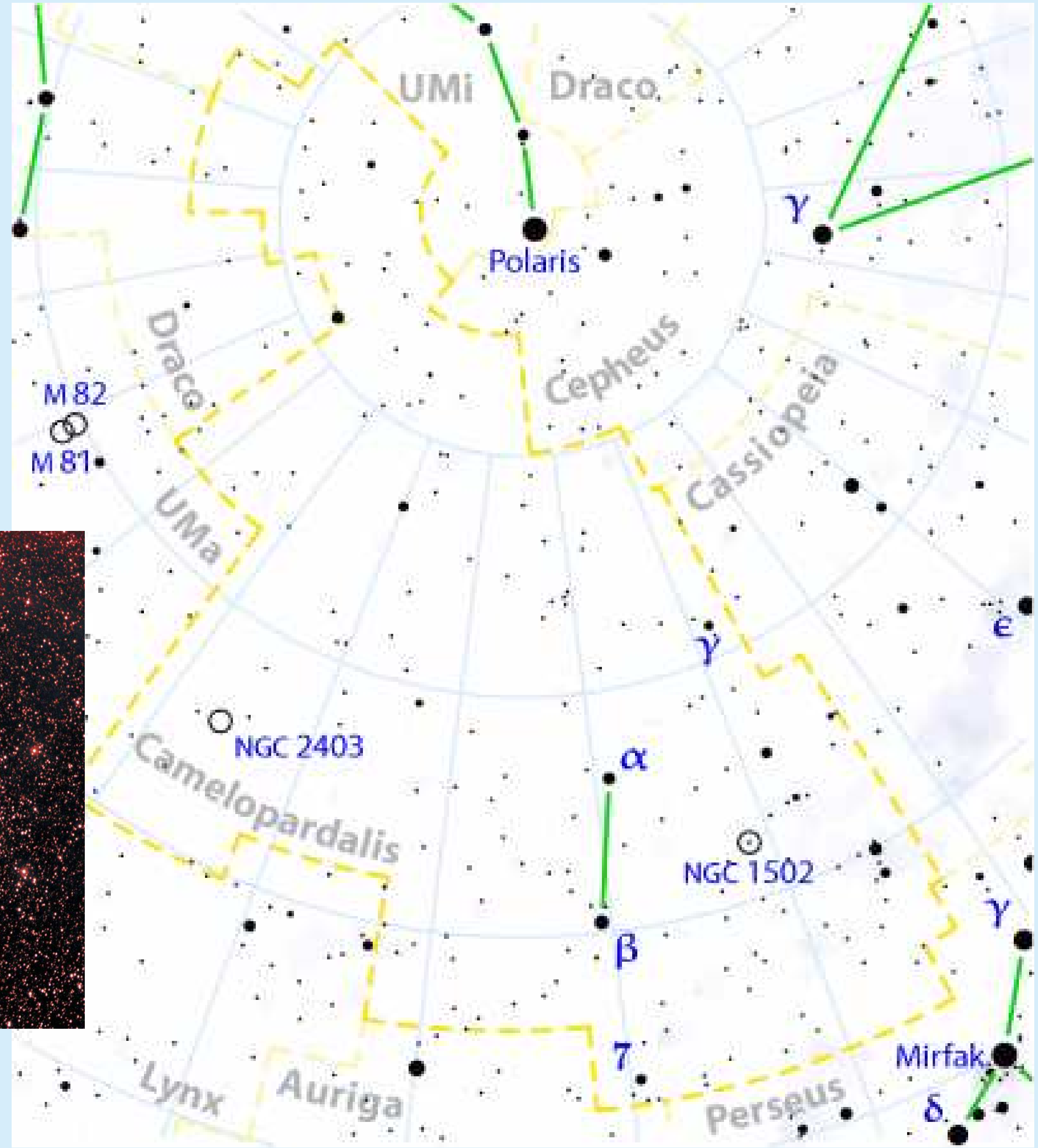
- neglect of $\frac{R_*}{r}$ term:

$$\dot{M} \equiv 4\pi r^2 \rho v(r) = \frac{\nu_0 L_\nu}{c^2} \approx \frac{L}{c^2}$$

$\Rightarrow$ mass-loss rate due to one optically thick line approximatively equal to the „photon mass-loss rate“ (L is stellar luminosity)

Example: α Cam





Example: α Cam

temperature $T_{\rm eff}$	$30\,900$ K
radius R_*	$27.6\,\mathrm{R}_\odot$
mass M	$43\,\mathrm{M}_\odot$

(Lamers et al. 1995)

Example: α Cam

temperature T_{eff}	$30\,900$ K
radius R_*	$27.6\,\text{R}_\odot$
mass M	$43\,\text{M}_\odot$

- mass-loss rate due to one optically thick line
  $\dot{M} \approx L/c^2$

Example: α Cam

temperature T_{eff}	$30\,900$ K
radius R_*	$27.6\,\mathrm{R}_\odot$
mass M	$43\,\mathrm{M}_\odot$

- mass-loss rate due to one optically thick line
  $\dot{M} \approx L/c^2$
- mass-loss rate due to N_{thick} optically thick lines
  $\dot{M} \approx N_{\text{thick}} L/c^2$

Example: α Cam

temperature T_{eff}	$30\,900$ K
radius R_*	$27.6\,\mathrm{R}_\odot$
mass M	$43\,\mathrm{M}_\odot$

- mass-loss rate due to one optically thick line
  $\dot{M} \approx L/c^2$
- mass-loss rate due to N_{thick} optically thick lines
  $\dot{M} \approx N_{\text{thick}} L/c^2$
- NLTE calculations: $N_{\text{thick}} \approx 1000$

Example: α Cam

temperature T_{eff}	$30\,900$ K
radius R_*	$27.6\,\mathrm{R}_\odot$
mass M	$43\,\mathrm{M}_\odot$

- mass-loss rate due to one optically thick line $\dot{M} \approx L/c^2$
- mass-loss rate due to N_{thick} optically thick lines $\dot{M} \approx N_{\text{thick}} L/c^2$
- NLTE calculations: $N_{\text{thick}} \approx 1000$
- $L = 4\pi\sigma R_*^2 T_{\text{eff}}^4$, $L = 620\,000\,\mathrm{L}_\odot$

Example: α Cam

temperature T_{eff}	$30\,900$ K
radius R_*	$27.6\,\mathrm{R}_\odot$
mass M	$43\,\mathrm{M}_\odot$

- mass-loss rate due to one optically thick line $\dot{M} \approx L/c^2$
- mass-loss rate due to N_{thick} optically thick lines $\dot{M} \approx N_{\mathrm{thick}} L/c^2$
- NLTE calculations: $N_{\mathrm{thick}} \approx 1000$
- $L = 4\pi\sigma R_*^2 T_{\mathrm{eff}}^4$, $L = 620\,000\,\mathrm{L}_\odot$
- $\dot{M} \approx 4 \times 10^{-5}\,\mathrm{M}_\odot\,\mathrm{yr}^{-1}$, more precise estimate: $1.5 \times 10^{-6}\,\mathrm{M}_\odot\,\mathrm{yr}^{-1}$ (Krtička & Kubát 2008)

CAK theory

- in reality the wind is driven by a mixture of optically thick and thin lines
  - optically thin line force

$$f_{\mathrm{rad}} = \frac{1}{c}\chi_{\mathrm{L}}(r)F(r)$$

  - optically thick line force

$$f_{\mathrm{rad}} = \frac{\nu_0 F(r)}{c^2}\frac{\mathrm{d}v}{\mathrm{d}r}$$

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- Sobolev optical depth $\tau_{\mathsf{S}} = \frac{\chi_{\mathsf{L}}(r)c}{\nu_0\frac{\mathrm{d}v}{\mathrm{d}r}}$

$$f_{\mathsf{rad}} = \frac{1}{c}\chi_{\mathsf{L}}(r)F(r)\left(\tau_{\mathsf{S}}^{-1}\right)^{\alpha}$$

where $\alpha = 0$ (thin) or $\alpha = 1$ (thick)

CAK theory

- in reality the wind is driven by a mixture of optically thick and thin lines

$$\Rightarrow \quad 0 < \alpha < 1$$

CAK theory

- in reality the wind is driven by a mixture of optically thick and thin lines
- the radiative force in the CAK approximation (Castor, Abbott & Klein 1975)

$$f_{\mathrm{rad}} = k \frac{\sigma_{\mathrm{Th}} n_{\mathrm{e}} L}{4\pi r^2 c} \left(\frac{1}{\sigma_{\mathrm{Th}} n_{\mathrm{e}} v_{\mathrm{th}}} \frac{\mathrm{d}v}{\mathrm{d}r} \right)^{\alpha}$$

- where
  - k, α are constants (force multipliers)
  - σ_{Th} is the Thomson scattering cross-section
  - n_{e} is the electron number density
  - v_{th} is hydrogen thermal speed (for $T = T_{\mathrm{eff}}$)

(Abbott 1982)

CAK theory

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- the radiative force in the CAK approximation (Castor, Abbott & Klein 1975)

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- nondimensional parameters k and α describe the line-strength distribution function (CAK, Puls et al. 2000)
- in general NLTE calculations necessary to obtain k and α (Abbott 1982)

CAK theory

- momentum equation with CAK line force (neglecting the gas pressure term)

$$\rho v \frac{\mathrm{d}v}{\mathrm{d}r} = f_{\mathrm{rad}} - \frac{\rho G M (1 - \Gamma)}{r^2}$$

CAK theory

- momentum equation with CAK line force (neglecting the gas pressure term)

$$\rho v \frac{\mathrm{d}v}{\mathrm{d}r} = k \frac{\sigma_{\mathrm{Th}} n_{\mathrm{e}} L}{4\pi r^2 c} \left(\frac{1}{\sigma_{\mathrm{Th}} n_{\mathrm{e}} v_{\mathrm{th}}} \frac{\mathrm{d}v}{\mathrm{d}r} \right)^{\alpha} - \frac{\rho G M (1 - \Gamma)}{r^2}$$

CAK theory

- momentum equation with CAK line force (neglecting the gas pressure term)

$$r^2 v \frac{\mathrm{d}v}{\mathrm{d}r} = k \frac{\sigma_{\mathrm{Th}} L}{4\pi c} \frac{n_{\mathrm{e}}}{\rho} \left(\frac{\rho}{n_{\mathrm{e}}} \frac{4\pi r^2 v}{\sigma_{\mathrm{Th}} \dot{M} v_{\mathrm{th}}} \frac{\mathrm{d}v}{\mathrm{d}r} \right)^{\alpha} - GM(1-\Gamma)$$

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- velocity in terms of the escape speed

$$w \equiv \frac{v^2}{v_{\mathrm{esc}}^2}, \text{ where } v_{\mathrm{esc}}^2 = \frac{2GM(1-\Gamma)}{R_*}$$

- new radial variable

$$x \equiv 1 - \frac{R_*}{r}$$

(Owocki 2004)

CAK theory

- momentum equation with CAK line force (neglecting the gas pressure term)

$$1 + w' = C\left(w'\right)^{\alpha}$$

- where
  - $w' \equiv \frac{\mathrm{d}w}{\mathrm{d}x}$
  - $C \equiv \dfrac{k\sigma_{\mathrm{Th}}L}{4\pi c G M(1-\Gamma)}\dfrac{n_{\mathrm{e}}}{\rho}\left(\dfrac{\rho}{n_{\mathrm{e}}}\dfrac{4\pi G M(1-\Gamma)}{\sigma_{\mathrm{Th}}\dot{M}v_{\mathrm{th}}}\right)^{\alpha}$
  - $\dfrac{\rho}{n_{\mathrm{e}}} \approx m_{\mathrm{H}}$
  - algebraic equation

CAK theory

- momentum equation with CAK line force (neglecting the gas pressure term)

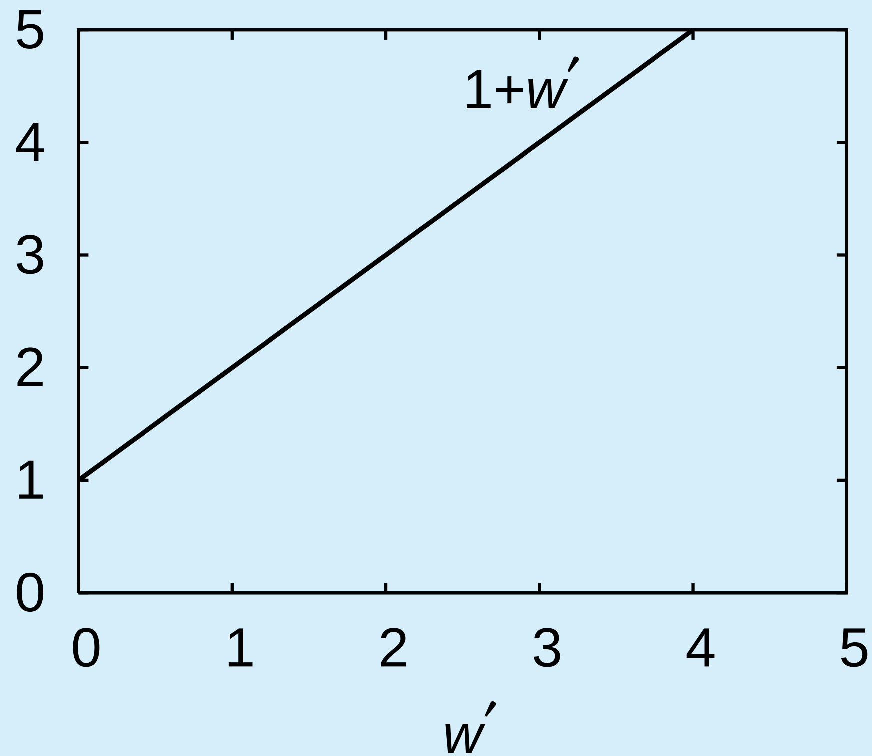
$$1 + w' = C\left(w'\right)^{\alpha}$$

- different solutions for different values of C (or mass-loss rate $\dot{M}$)

CAK theory

- momentum equation with CAK line force (neglecting the gas pressure term)

$$1 + w' = C\left(w'\right)^{\alpha}$$



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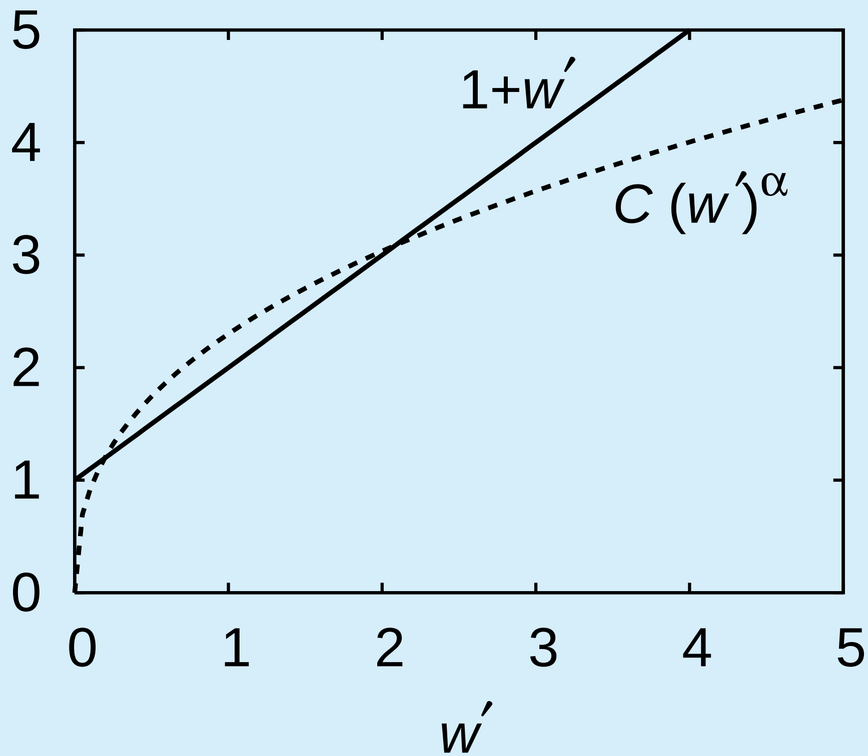
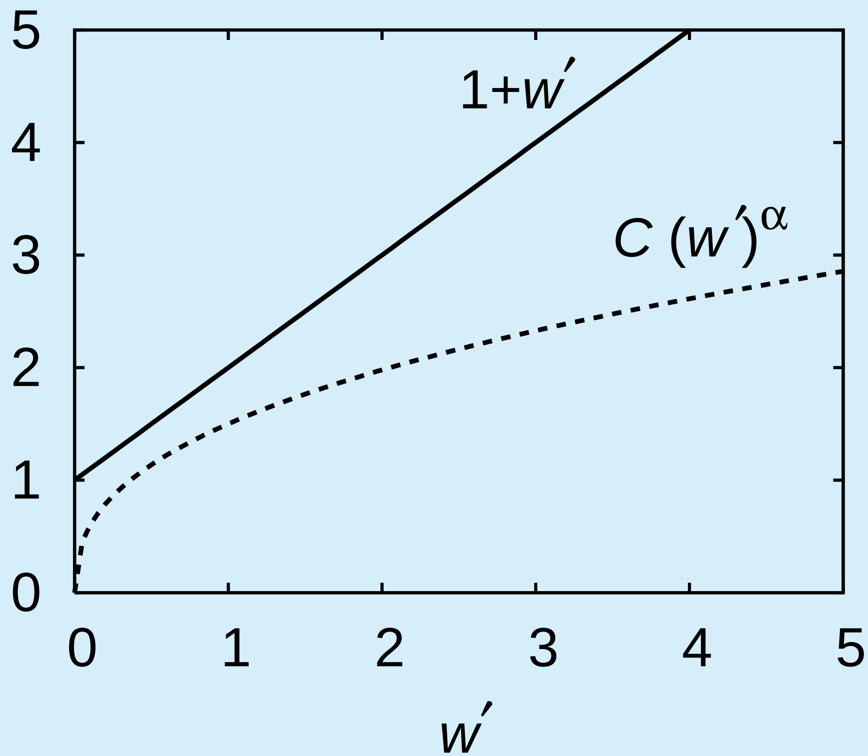


- large C (small $\dot{M}$): two solutions

CAK theory

- momentum equation with CAK line force (neglecting the gas pressure term)

$$1 + w' = C\left(w'\right)^{\alpha}$$



- small C (large $\dot{M}$): no solution

CAK theory

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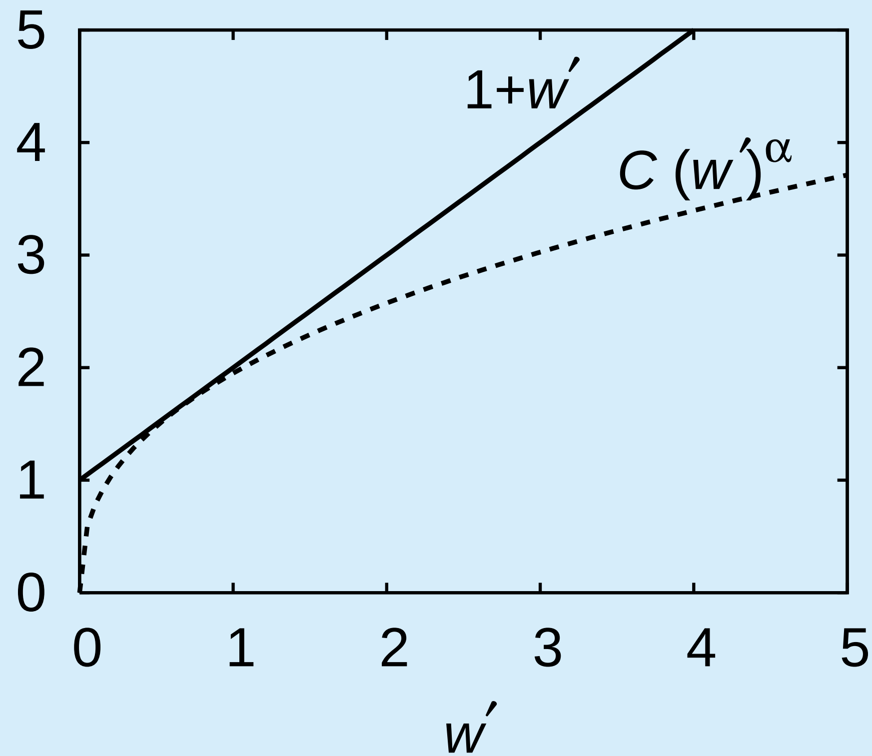
$$1 + w' = C \left(w'\right)^{\alpha}$$



- critical value of C ($\dot{M}$): one solution

CAK theory

- momentum equation with CAK line force (neglecting the gas pressure term)

$$1 + w' = C\left(w'\right)^{\alpha}$$

- critical (CAK) solution for a specific value of $\dot{M}$: the only smooth solution of detailed momentum equation from the stellar surface to infinity
- CAK solution: the largest $\dot{M}$ possible

CAK theory

- momentum equation with CAK line force (neglecting the gas pressure term)

$$1 + w' = C \left(w'\right)^{\alpha}$$

- critical (CAK) solution for a specific value of $\dot{M}$: the only smooth solution of detailed momentum equation from the stellar surface to infinity

⇒ possible to derive the wind mass-loss rate and velocity profile

$$w'_{\mathrm{c}} = \frac{\alpha}{1-\alpha}$$

$$C_{\mathrm{c}} = \frac{(1-\alpha)^{\alpha-1}}{\alpha^{\alpha}}$$

CAK theory

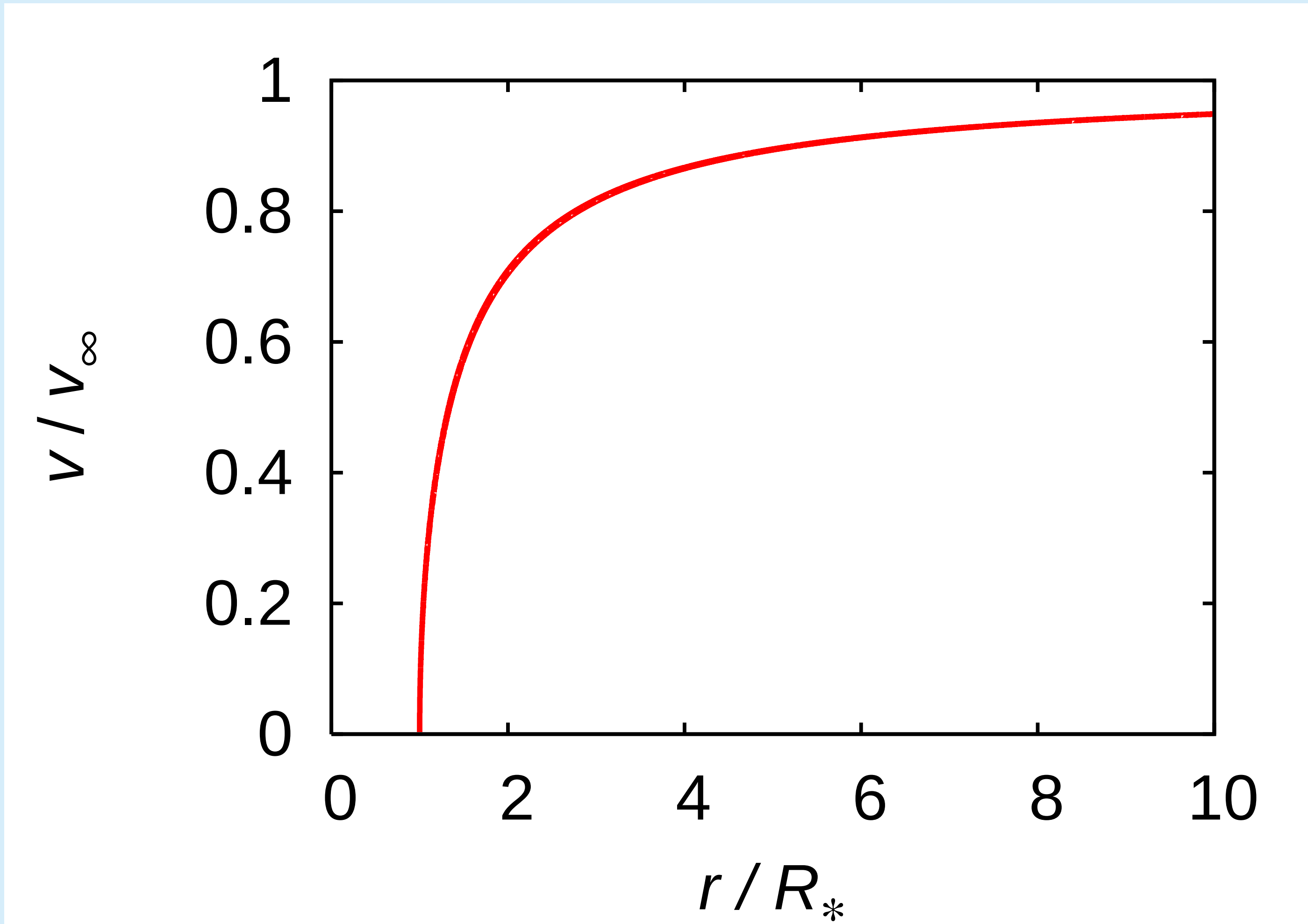
$$w_{\mathrm{c}}' = \frac{\alpha}{1-\alpha}$$

$$\Rightarrow w = \frac{\alpha}{1-\alpha} x \Rightarrow v = v_{\infty} \left(1 - \frac{R_*}{r}\right)^{1/2}$$

- where the terminal velocity

$$v_{\infty} = v_{\mathrm{esc}} \sqrt{\frac{\alpha}{1-\alpha}}$$

CAK theory



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- v_∞ scales with v_{esc}!
- as v_∞ of order of $100\,\mathrm{km\,s^{-1}}$, hot star winds are strongly supersonic!

CAK theory

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$$v_\infty = v_{\mathrm{esc}} \sqrt{\frac{\alpha}{1-\alpha}}$$

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- example: α Cam, $v_{\mathrm{esc}} = 620\,\mathrm{km\,s^{-1}}$, $\alpha = 0.61$

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- v_∞ scales with $v_{\rm esc}$!
- example: α Cam, $v_{\rm esc} = 620\,\mathrm{km\,s^{-1}}$, $\alpha = 0.61$
  $\Rightarrow$ prediction: $v_\infty = 780\,\mathrm{km\,s^{-1}}$

CAK theory

$$C_{\mathsf{c}} = \frac{(1-\alpha)^{\alpha-1}}{\alpha^{\alpha}}$$

$$\Rightarrow \quad \dot{M} = \left[\frac{4\pi m_{\mathsf{H}} G M (1-\Gamma)}{\sigma_{\mathsf{Th}}}\right]^{\frac{\alpha-1}{\alpha}} \frac{\alpha}{v_{\mathsf{th}}\,(1-\alpha)^{\frac{\alpha-1}{\alpha}}} \left(\frac{kL}{c}\right)^{\frac{1}{\alpha}}$$

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Beyond the classical CAK theory

- inclusion of the dependence of k on the ionisation equilibrium – δ parameter (Abbott 1982)

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Comparison with observations

- nice wind theory $\Rightarrow$ compare it with observations!

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no coffee time yet...

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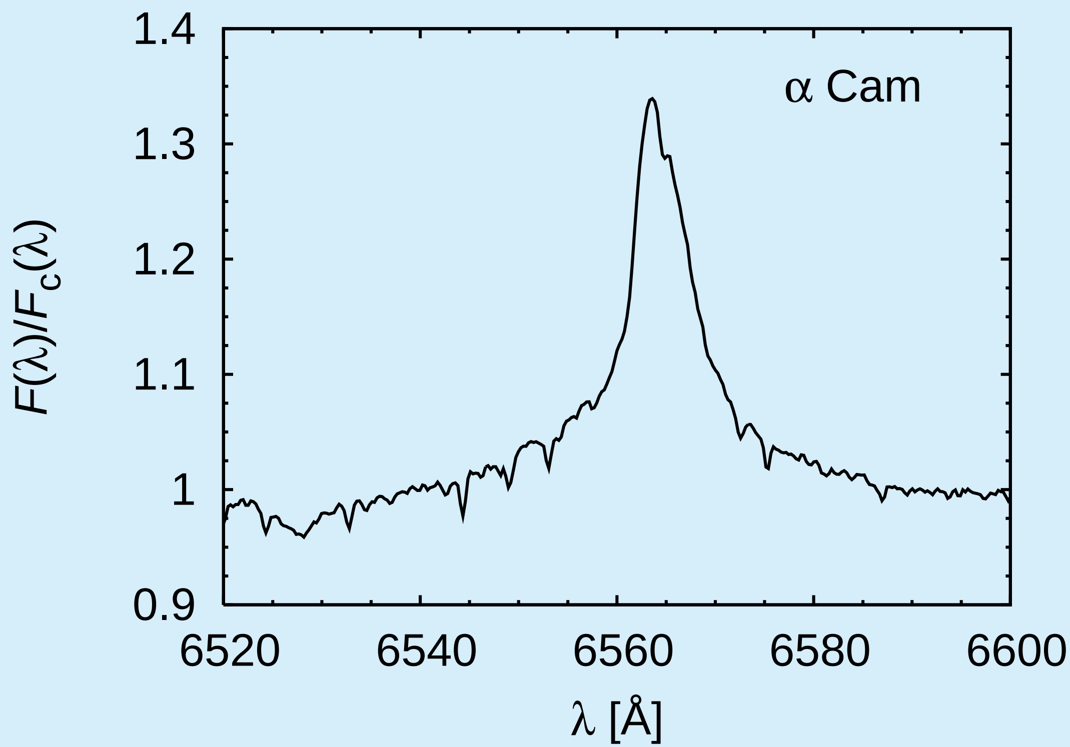
- more theory, please!

Observations: Hα line profiles

- Hα emission line of α Cam



Observations: Hα line profiles

recombination line

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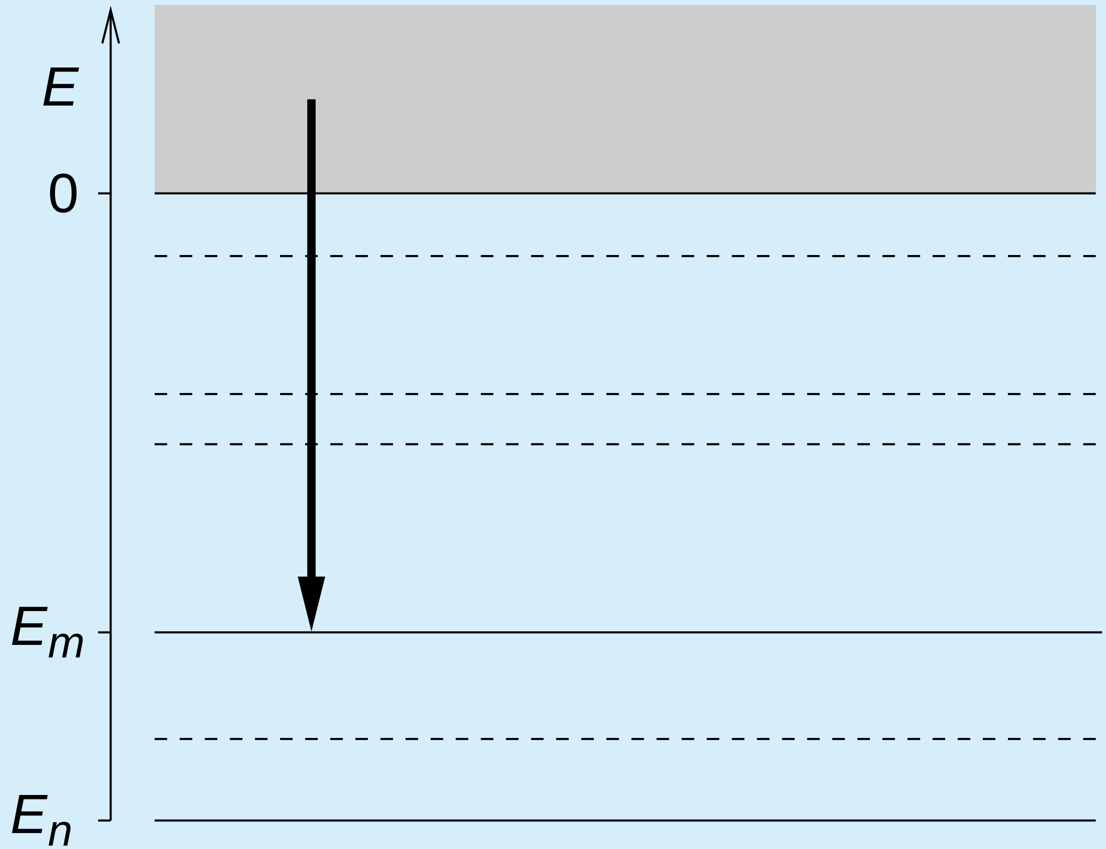

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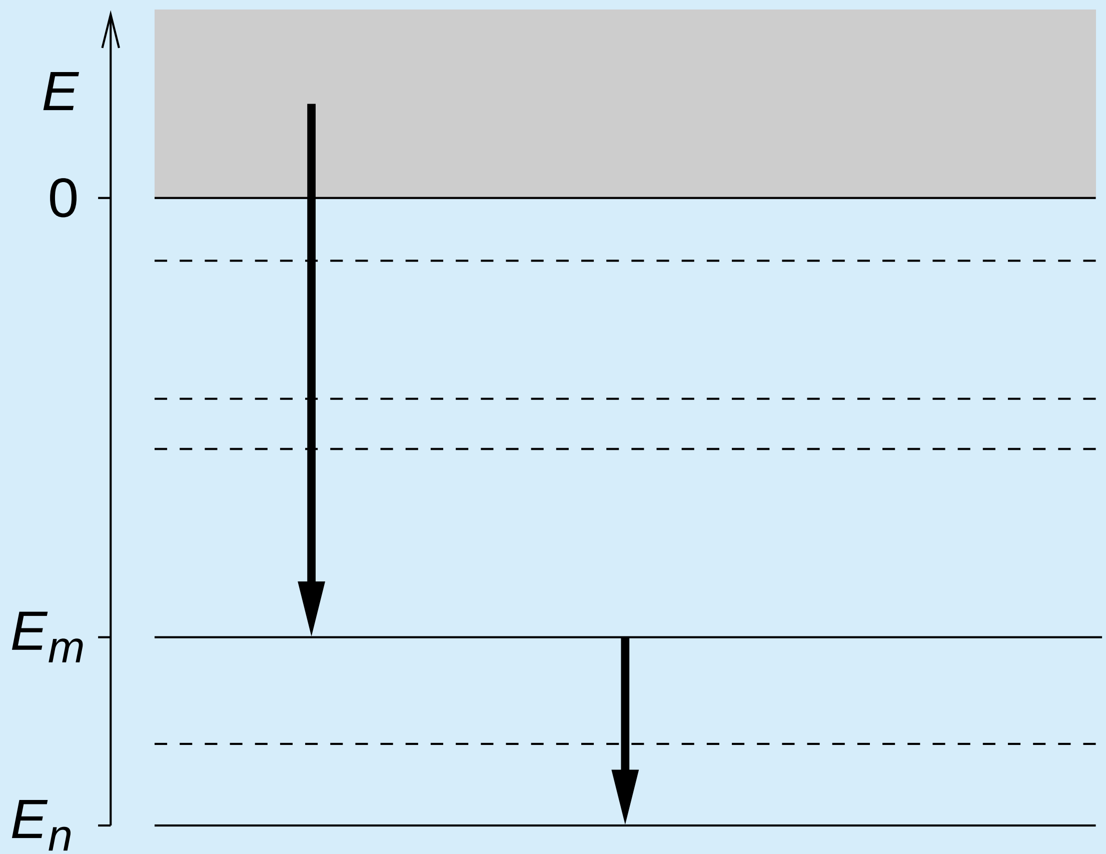

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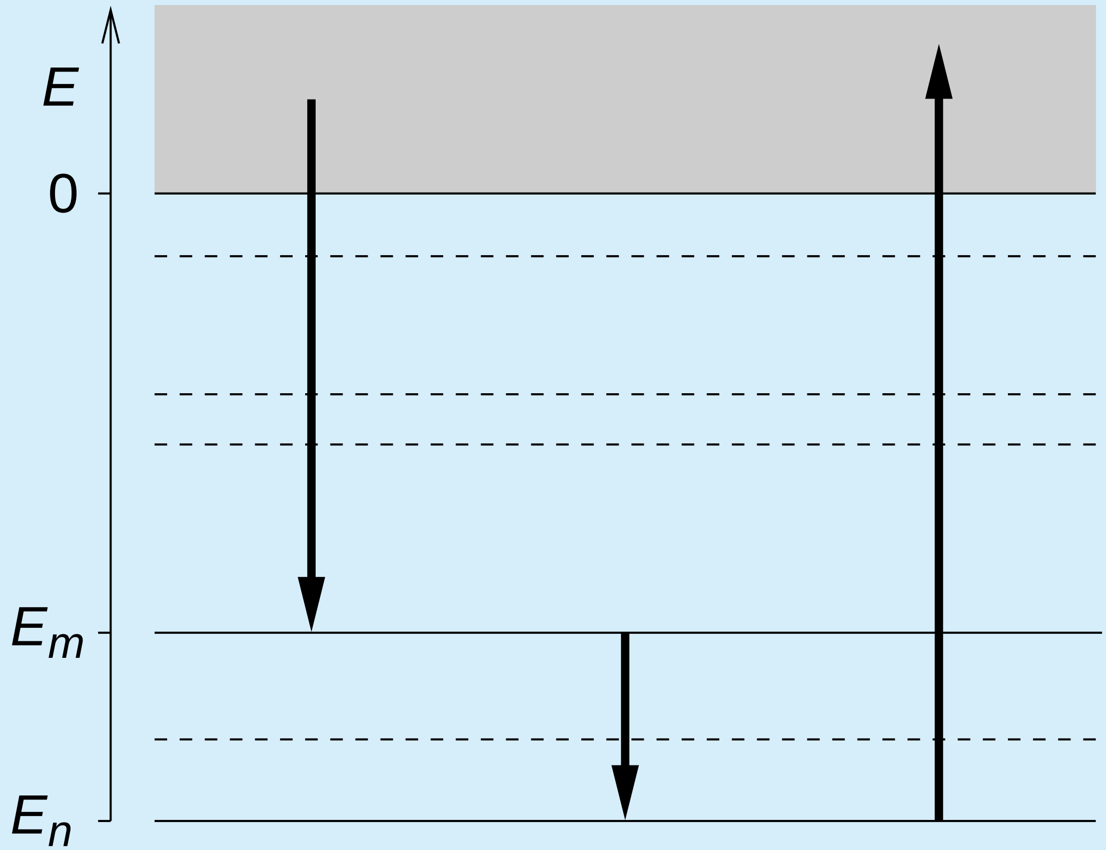
Observations: Hα line profiles



recombination line

Observations: Hα line profiles

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- number of Hα photons emitted per unit of time

$$N_{\mathrm{H}\alpha} \sim n_{\mathrm{p}} n_{\mathrm{e}}$$

- where
  - n_{p} is the number density of H^{+}
  - n_{e} is the number density of free electrons

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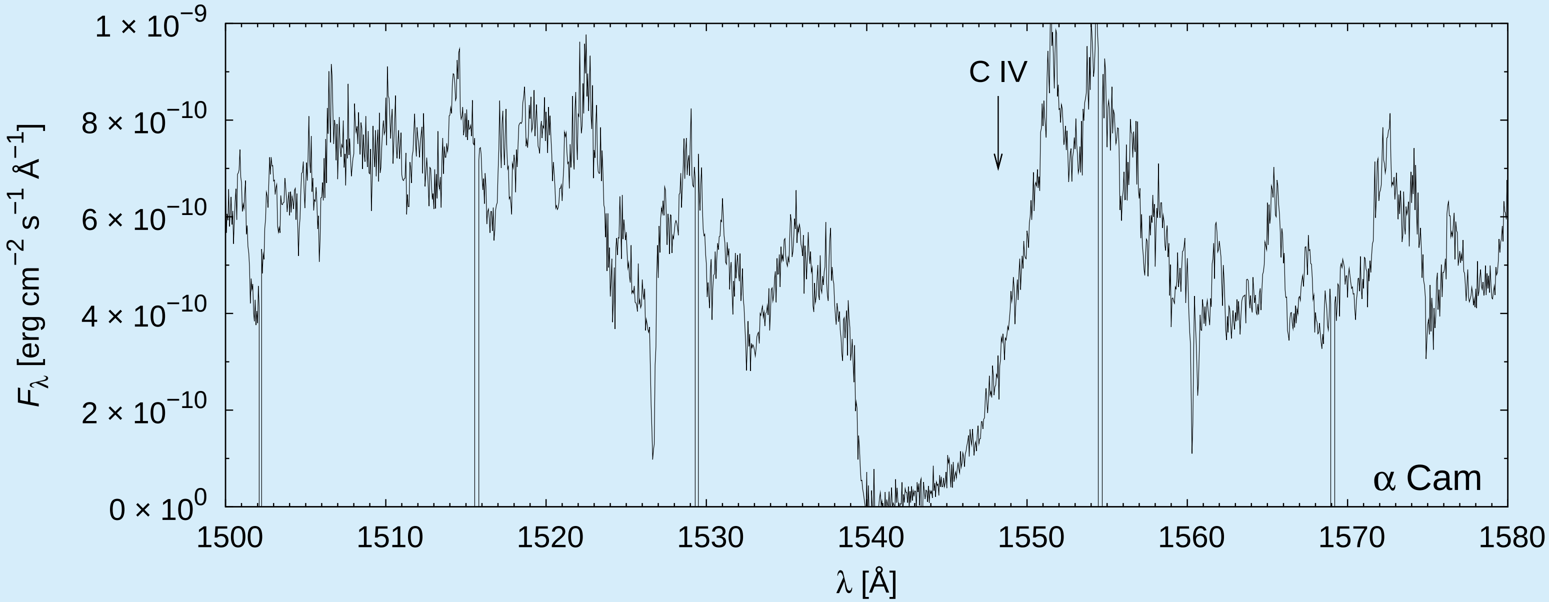
$\Rightarrow$ possibility to derive $\dot{M}$ using NLTE models

- example: α Cam
  - our estimate: $9 \times 10^{-6}\,\mathrm{M}_{\odot}\,\mathrm{yr}^{-1}$
  - theoretical prediction: $1.4 \times 10^{-6}\,\mathrm{M}_{\odot}\,\mathrm{yr}^{-1}$ (Krtička & Kubát 2007)
  - Hα line observation: $1.5 \times 10^{-6}\,\mathrm{M}_{\odot}\,\mathrm{yr}^{-1}$ (Puls et al. 2006)

Observations: P Cyg lines I.

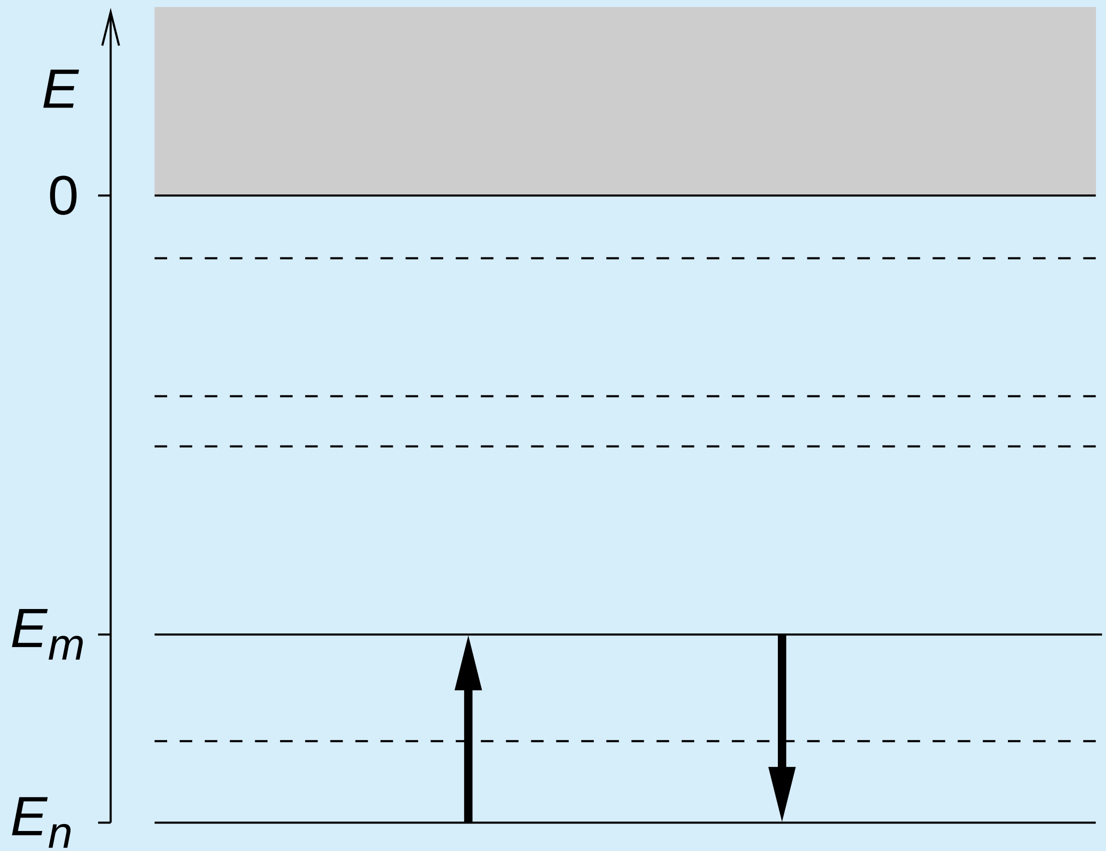
- IUE spectrum of α Cam



- saturated line profile of P Cyg type

Observations: P Cyg lines I.

- lines of the most abundant ion of a given element



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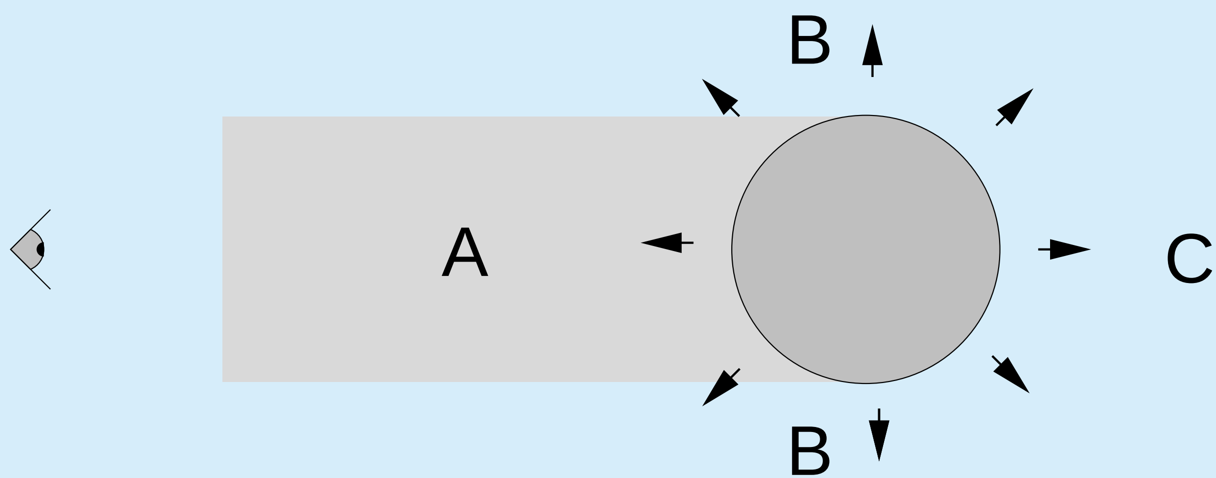

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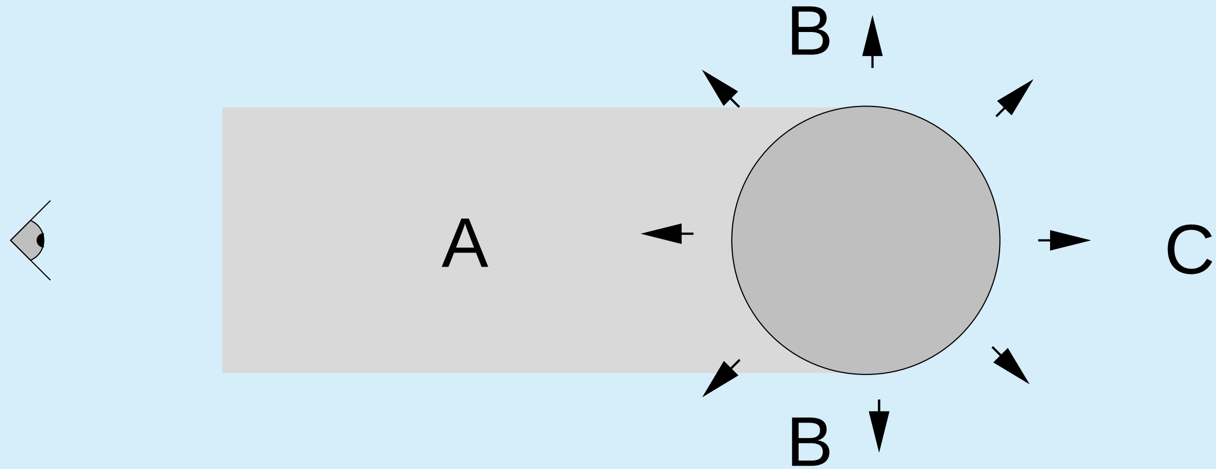

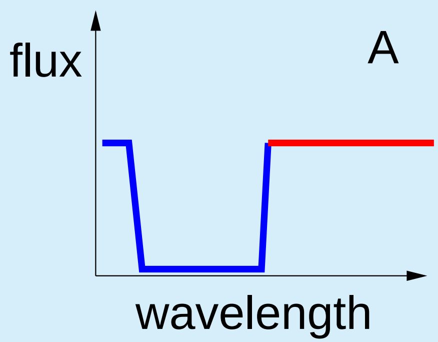

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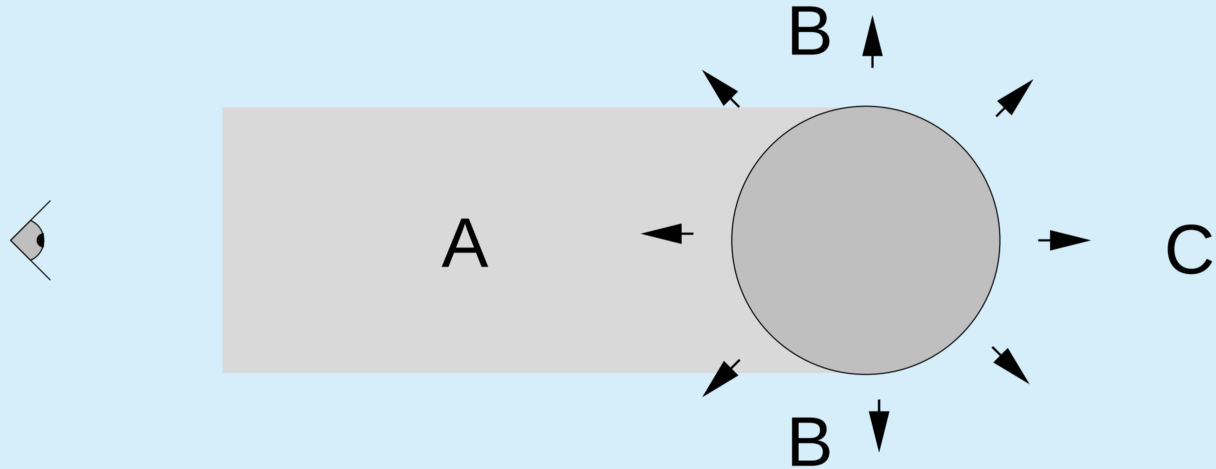

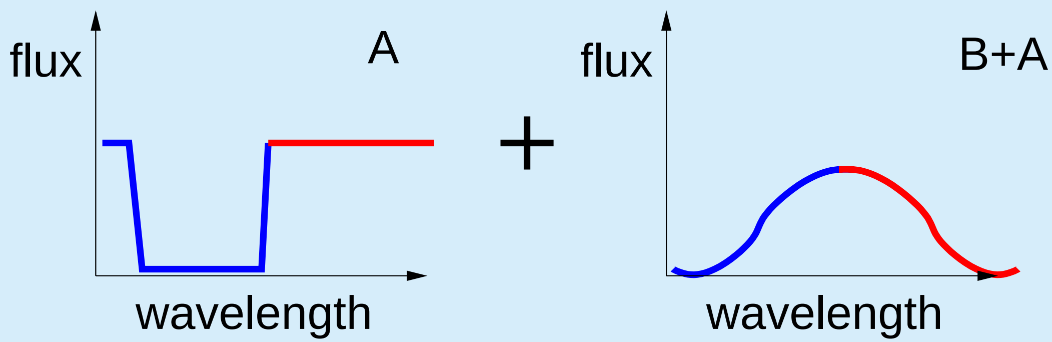

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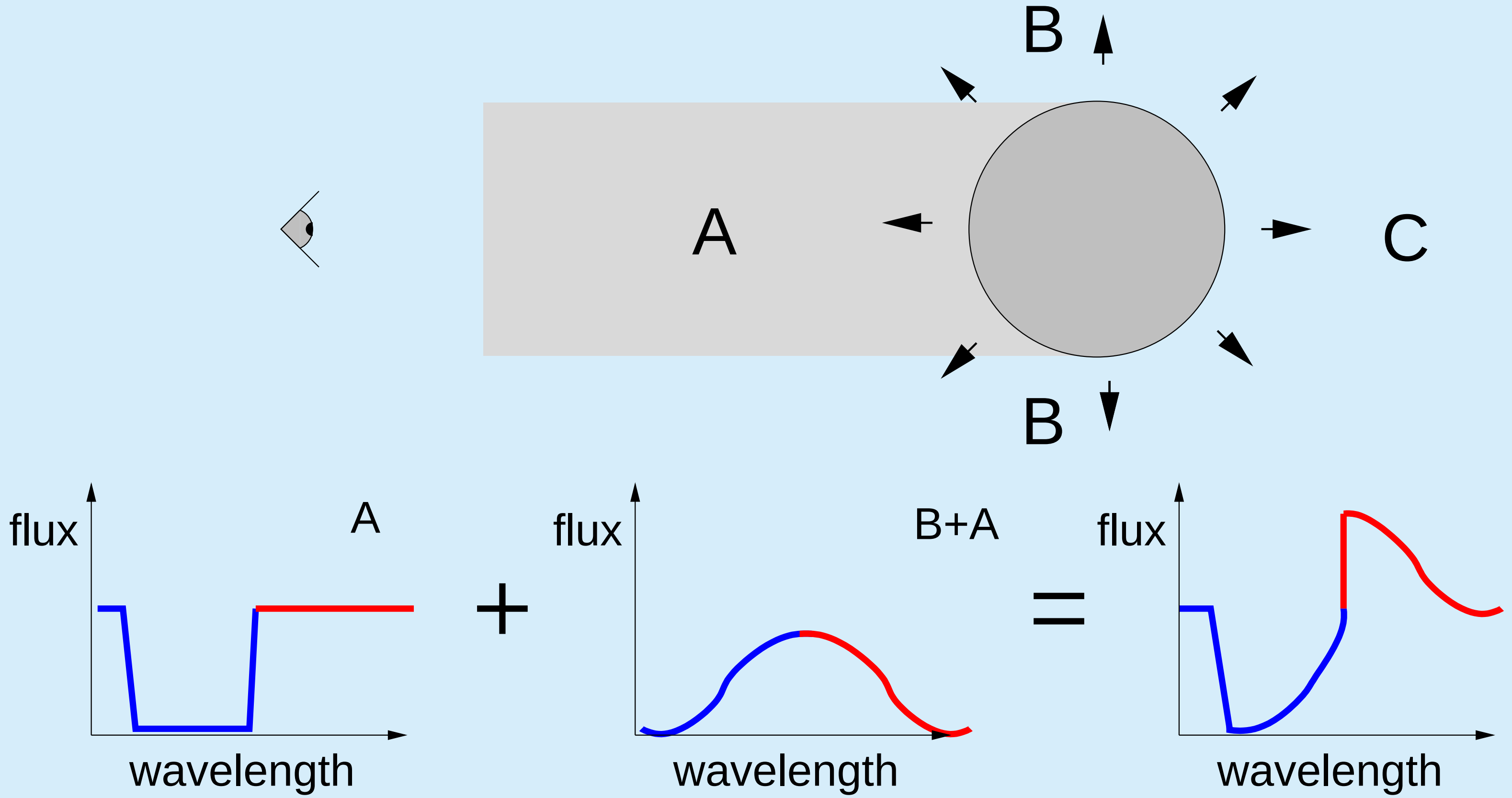
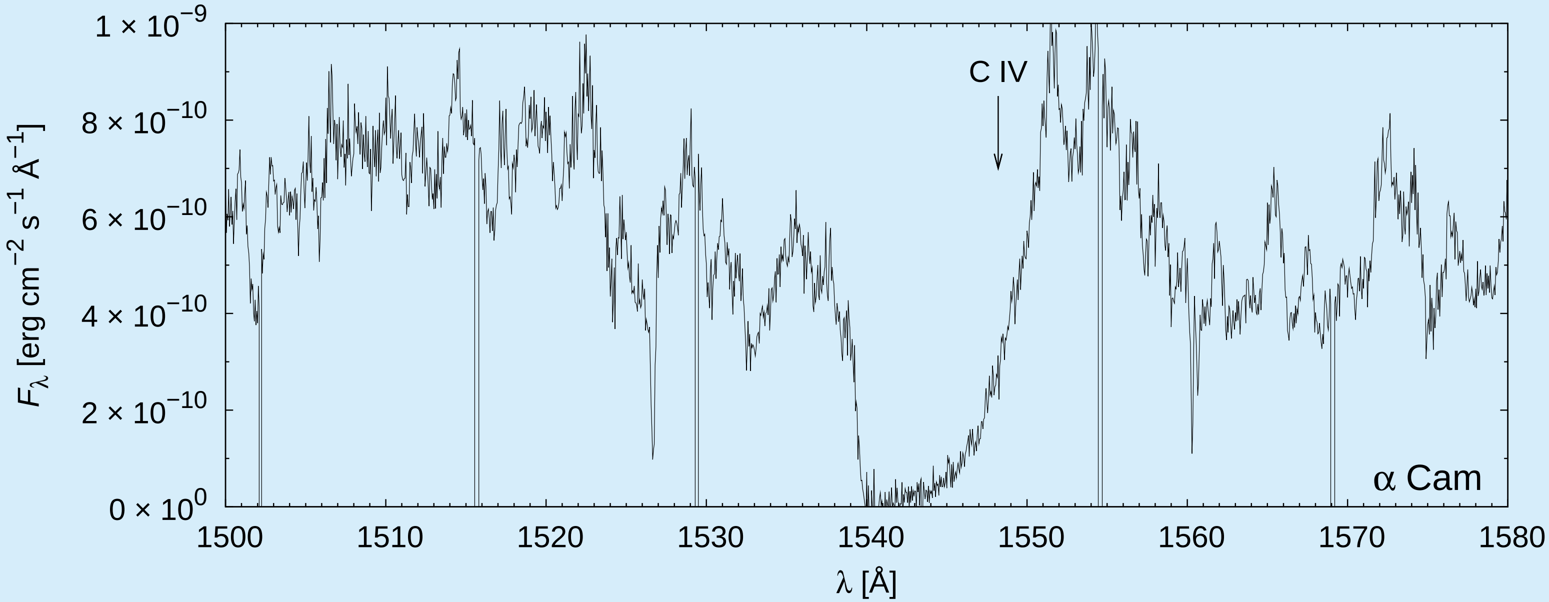


Observations: P Cyg lines I.

- IUE spectrum of α Cam



- absorption in the wind between star and observer
- emission due to the wind around the star

Observations: P Cyg lines I.

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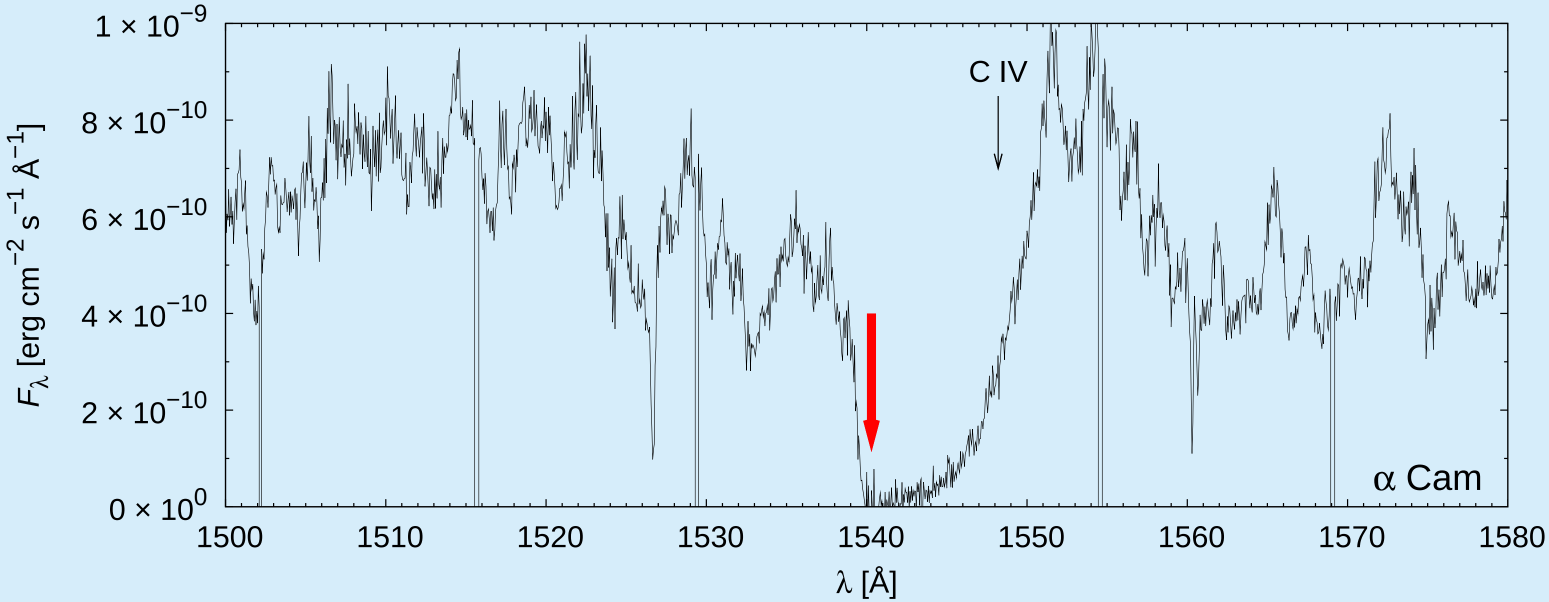

- the absorption edge originates in the wind with the highest velocity in the direction of observer

Observations: P Cyg lines I.

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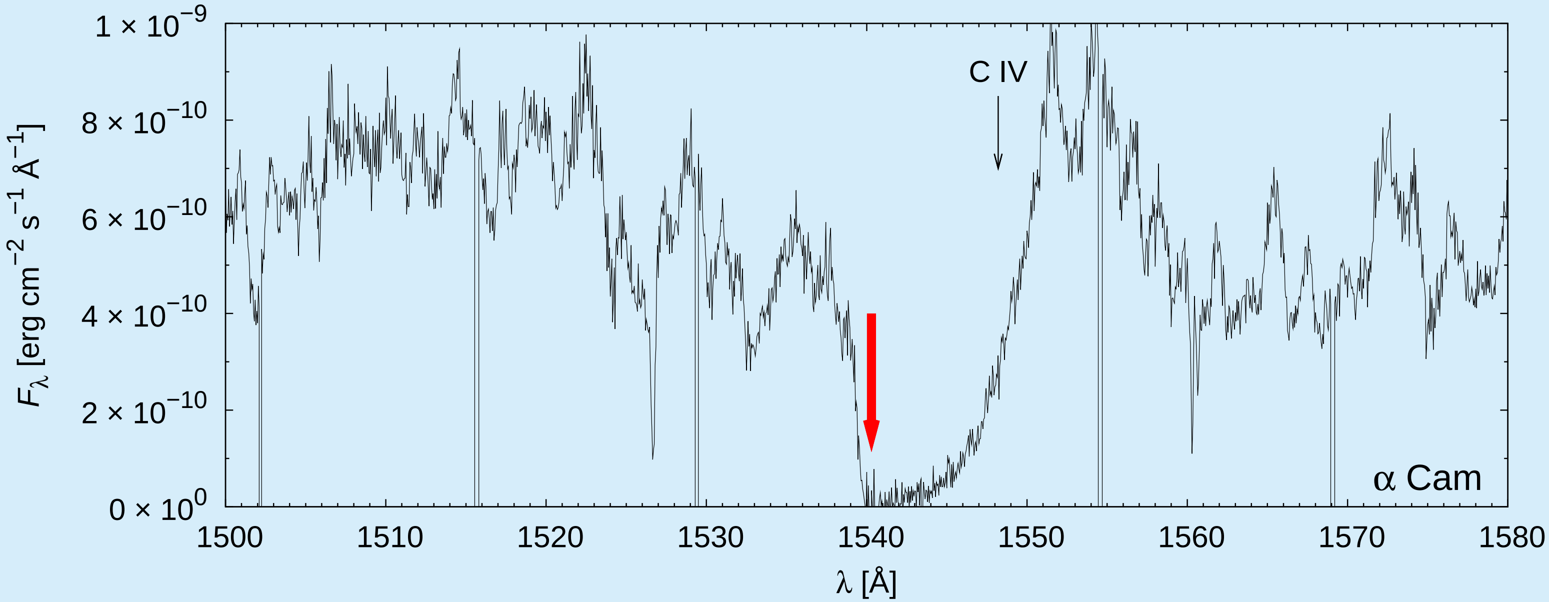

- the absorption edge originates in the wind with the highest velocity in the direction of observer
- possibility to derive the terminal velocity v_∞

Observations: P Cyg lines I.

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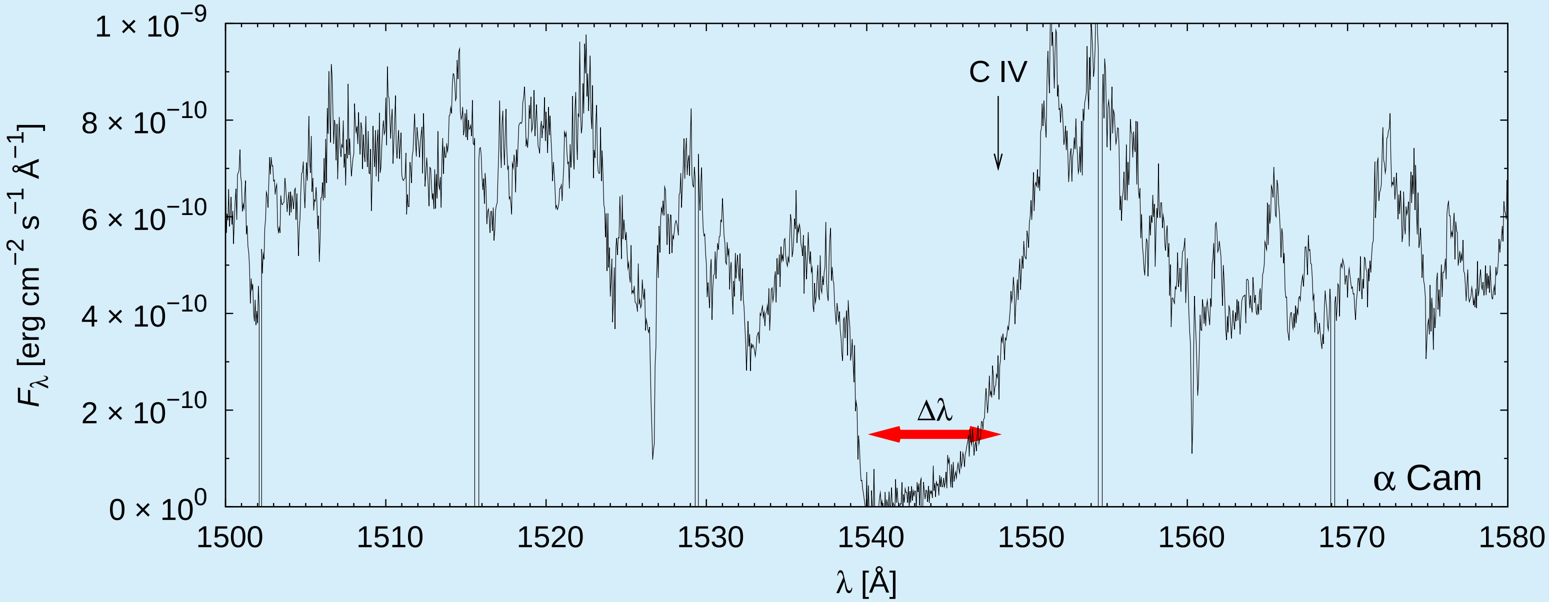

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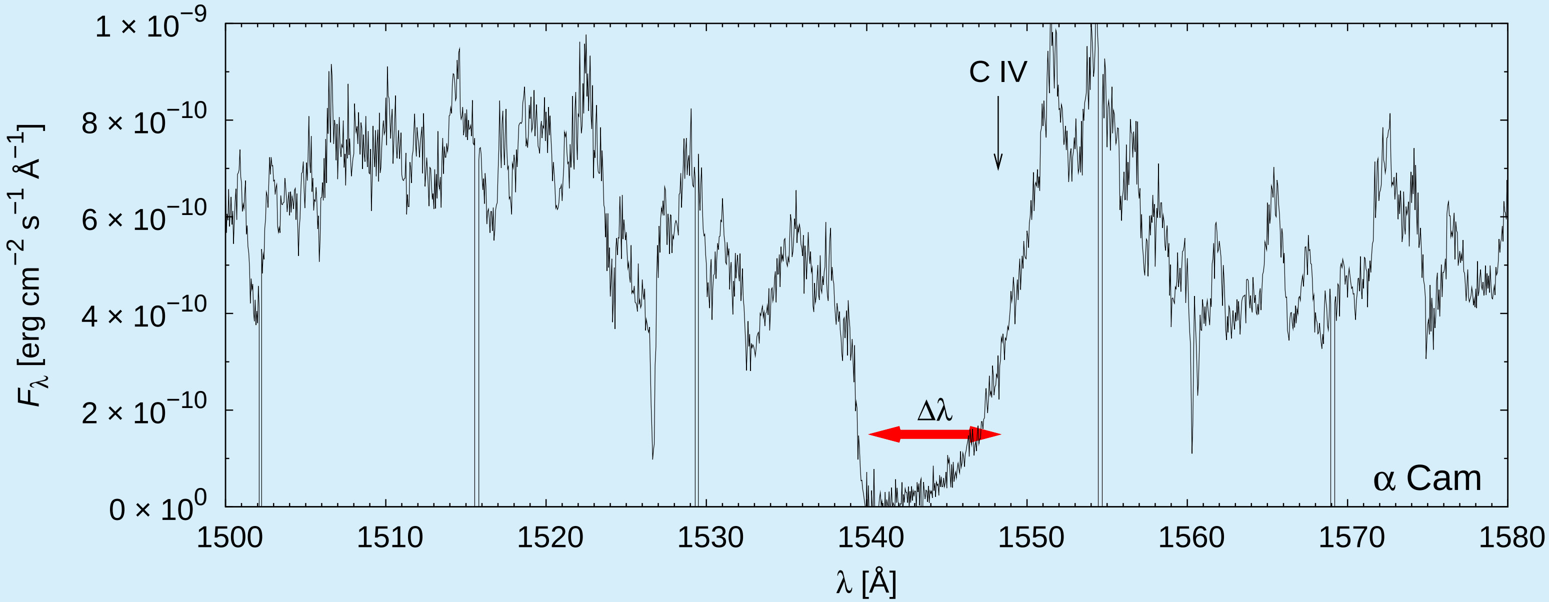

$$v_\infty = \frac{\Delta\lambda}{\lambda_0} c$$

- where λ_0 is the laboratory wavelength of a given line

Observations: P Cyg lines I.

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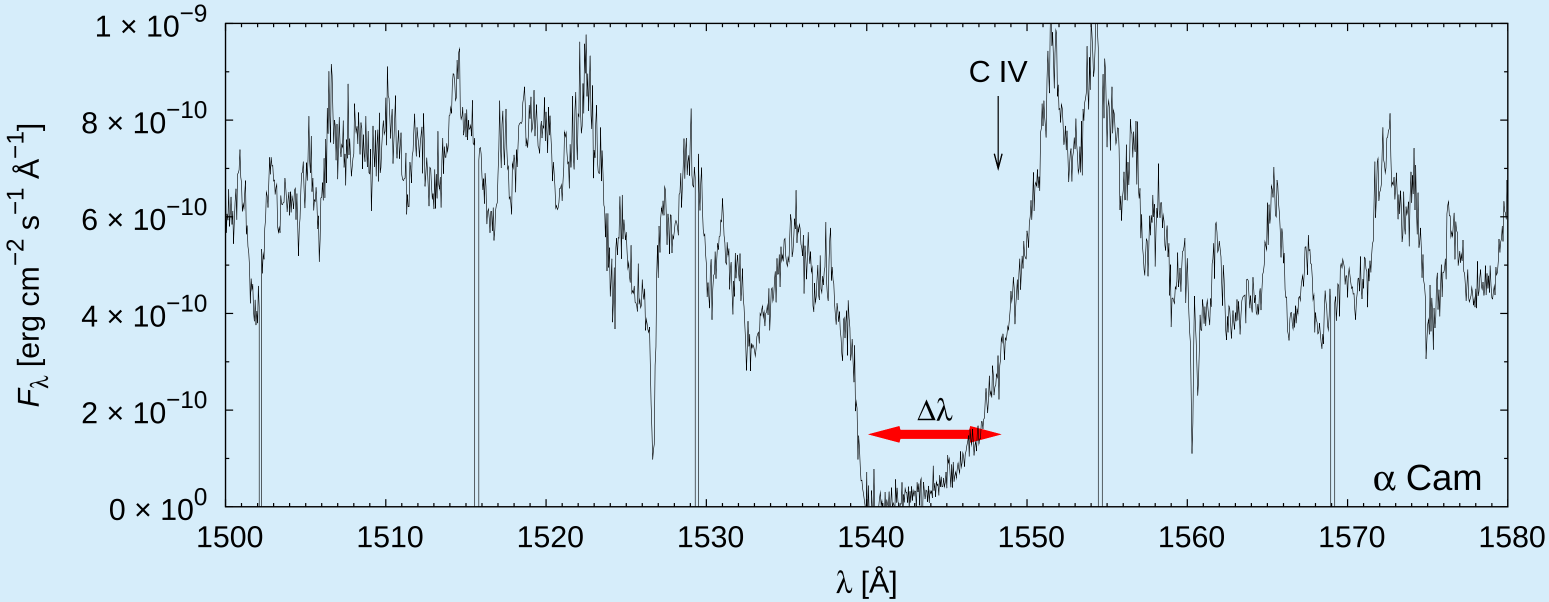
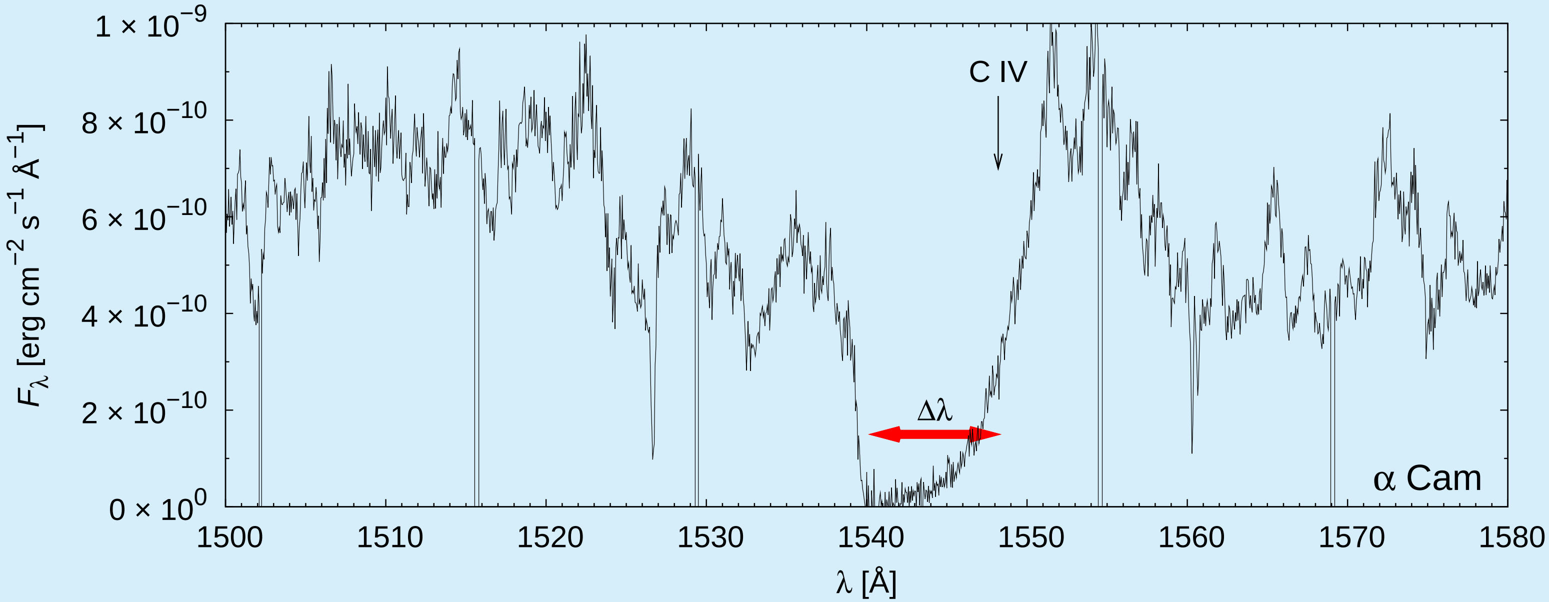


$$v_\infty = \frac{\Delta\lambda}{\lambda_0} c$$

- α Cam: $\Delta\lambda = 7.9\,\text{Å} \Rightarrow v_\infty = 1500\,\mathrm{km\,s^{-1}}$
- our estimate: $780\,\mathrm{km\,s^{-1}}$

Observations: P Cyg lines I.

- IUE spectrum of α Cam



- why is the absorption part saturated?

Observations: P Cyg lines I.

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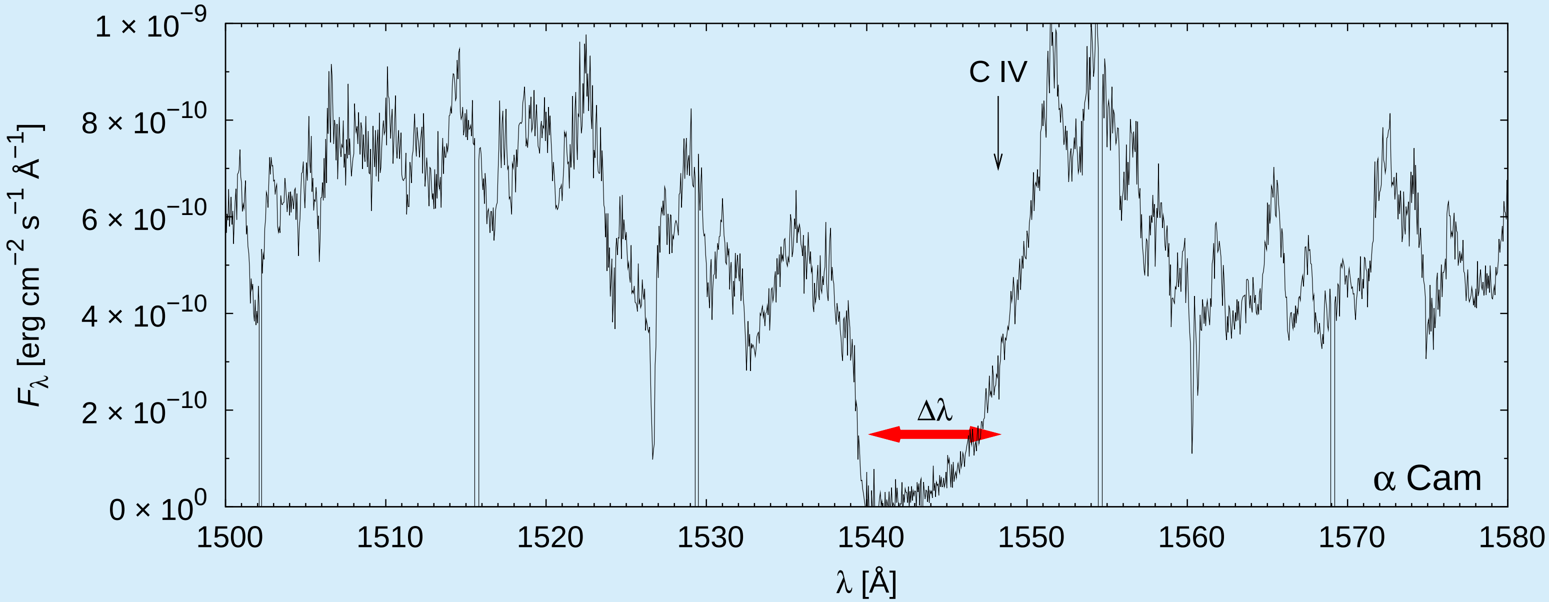


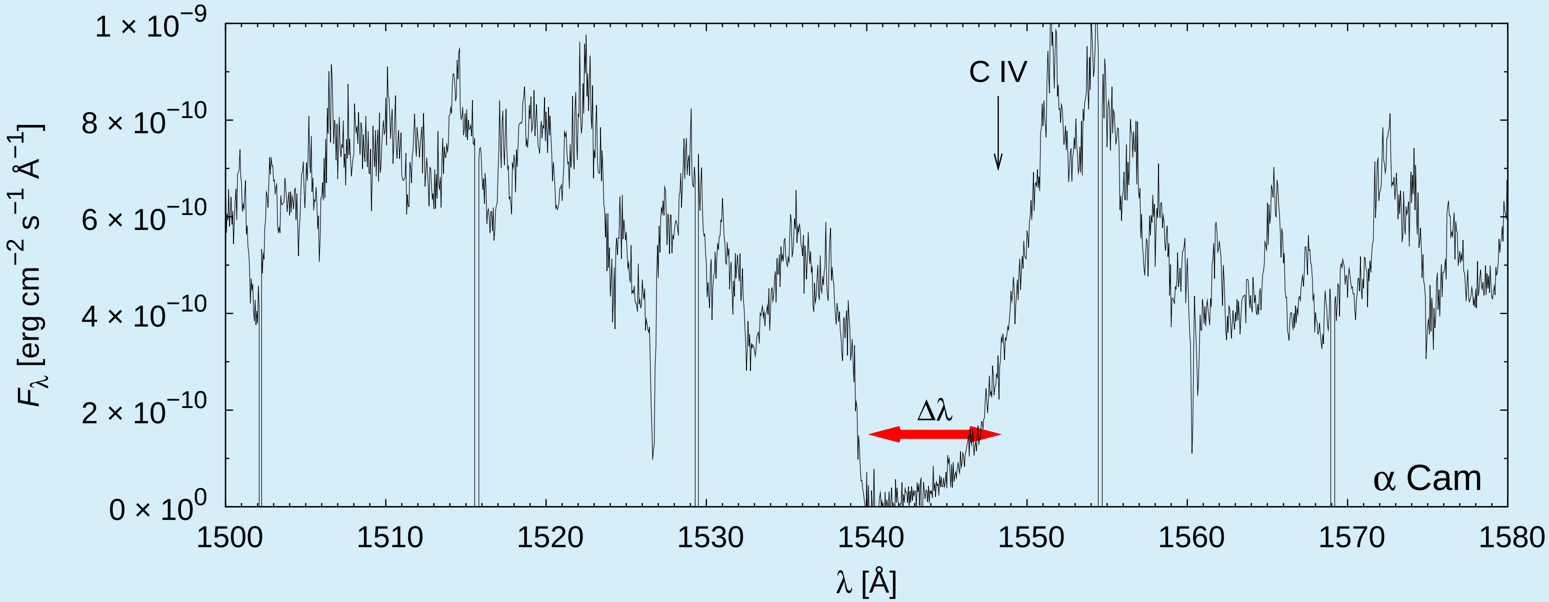
- why is the absorption part saturated?

$$I(y) = I_c(\mu) \exp\left[-\tau(\mu) y\right] + S_L \left\{1 - \exp\left[-\tau(\mu) y\right]\right\}$$

- the emergent intensity: $y \to 1$

Observations: P Cyg lines I.

- IUE spectrum of α Cam



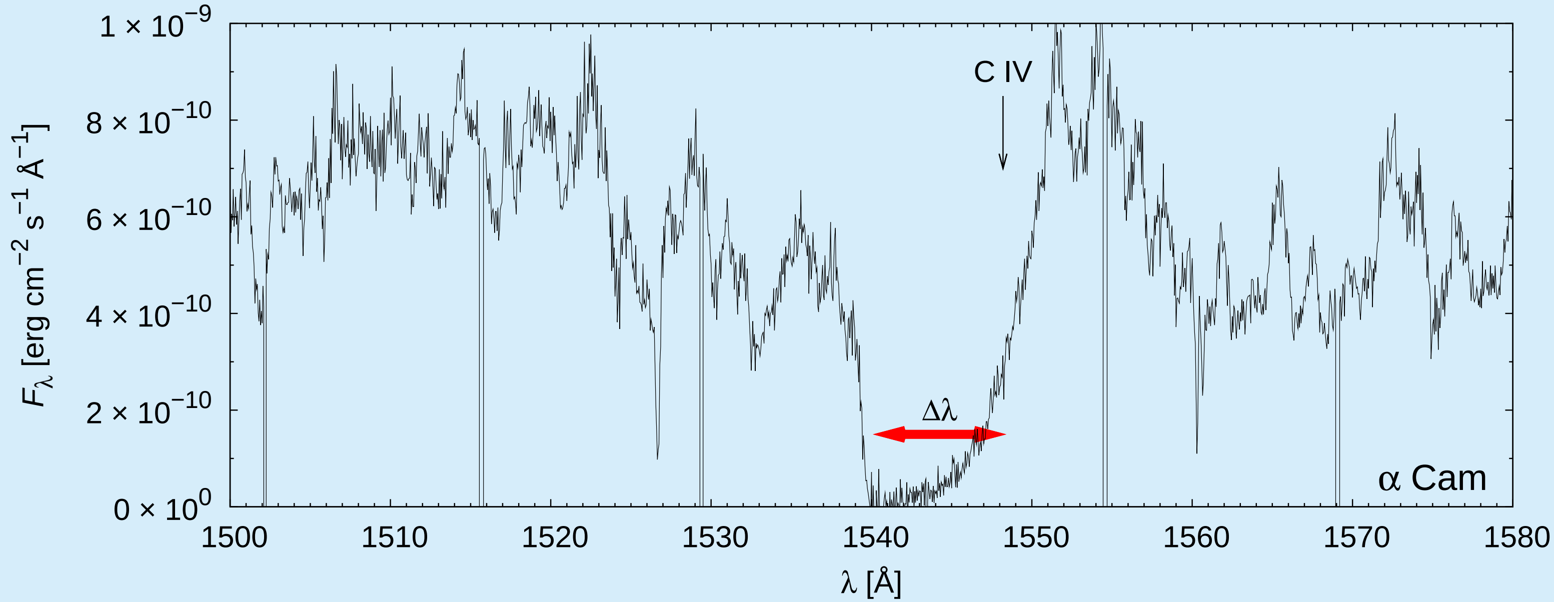
- why is the absorption part saturated?

$$I = I_{\mathrm{c}}(\mu)\exp\left[-\tau(\mu)\right] + S_{\mathrm{L}}\left\{1 - \exp\left[-\tau(\mu)\right]\right\}$$

- optically thick lines $\tau \gg 1$ with $S_{\mathrm{L}} \ll I_{\mathrm{c}} \Rightarrow I \ll I_{\mathrm{c}}$

Observations: P Cyg lines I.

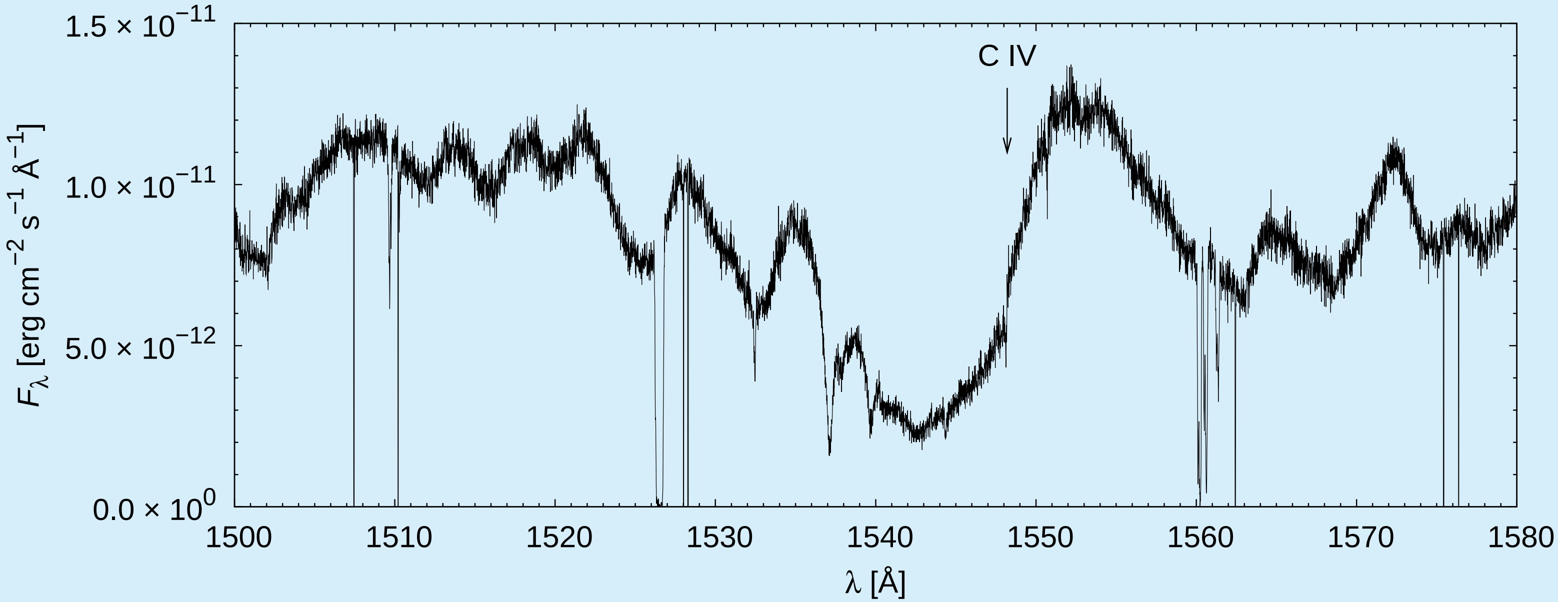
- IUE spectrum of α Cam



- for saturated lines ($\tau \gg 1$) the absorption part of the P Cyg line profile does not depend on τ
  $\Rightarrow$ determination of v_∞ possible
  $\Rightarrow$ determination of $\dot{M}$ impossible

Observations: P Cyg lines II.

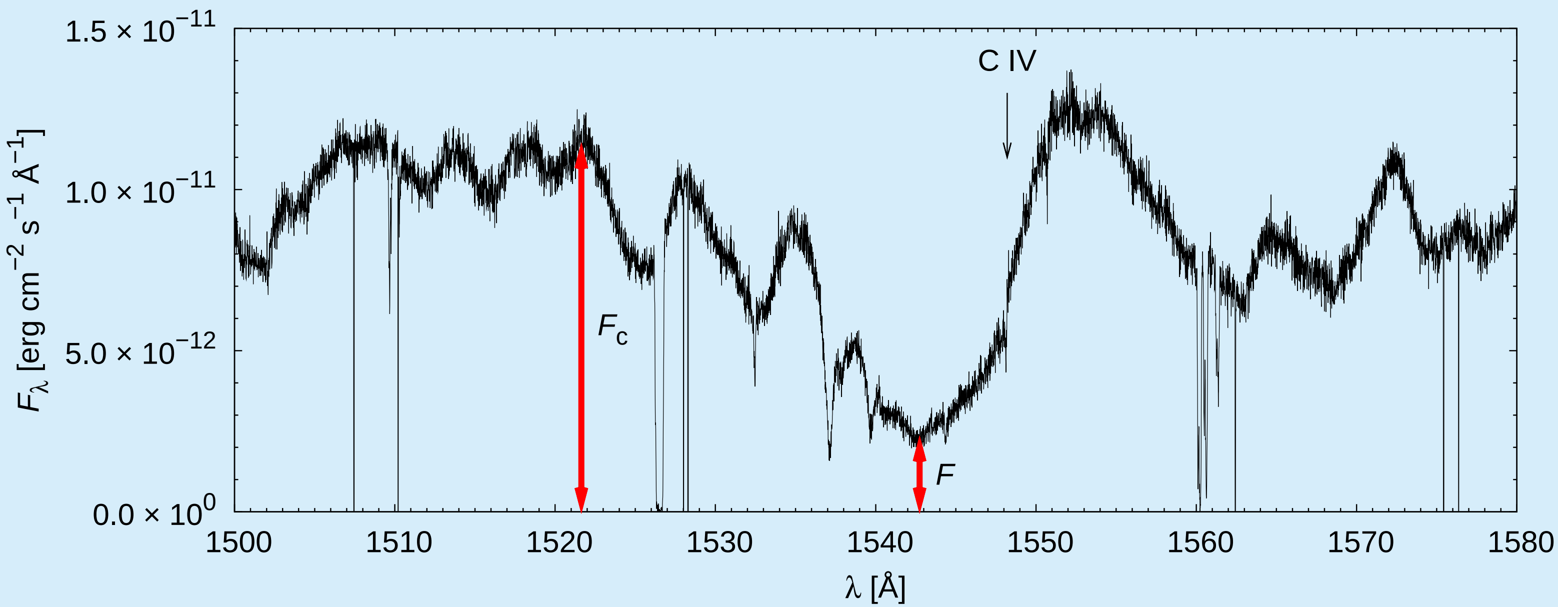
- HST spectrum of HD 13268



- unsaturated line profile of P Cyg type

Observations: P Cyg lines II.

- HST spectrum of HD 13268

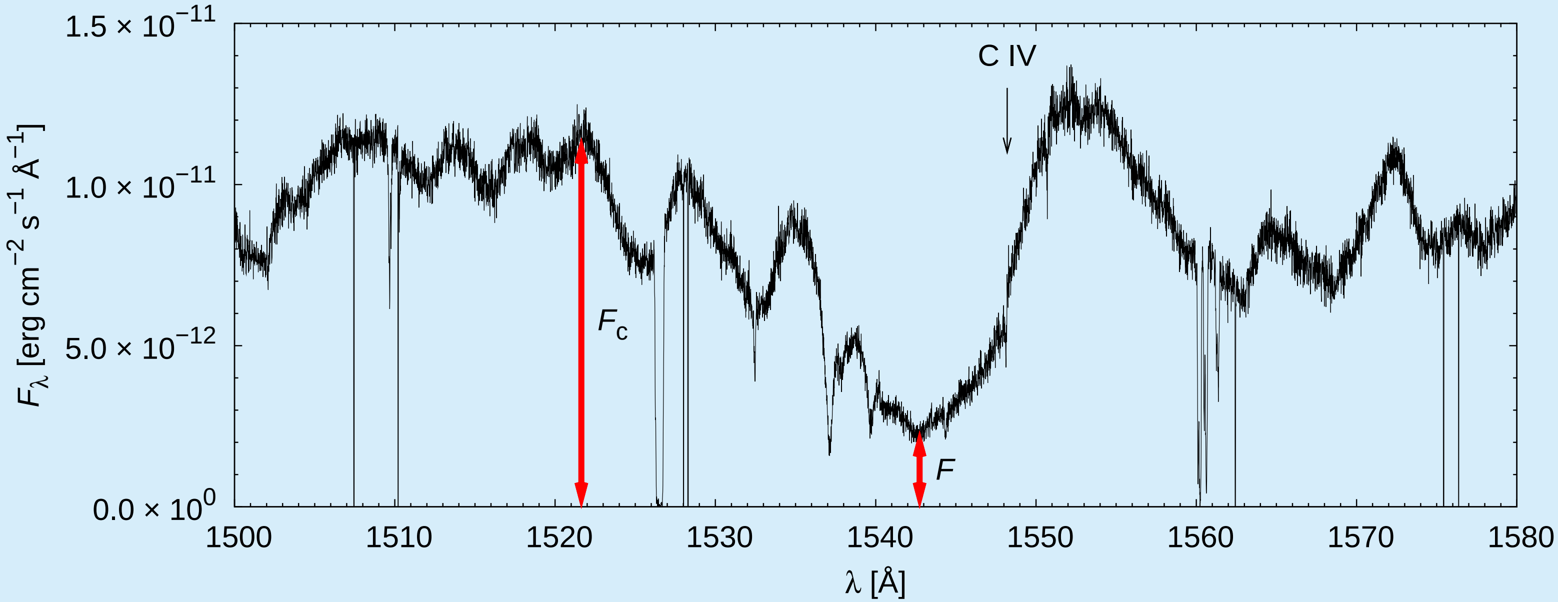


$$\frac{F}{F_\mathrm{c}} \approx \exp\left[-\tau(\mu=1)\right]$$

$$\tau(\mu=1) = \frac{\chi_\mathrm{L} c}{\nu_0}\left(\frac{\mathrm{d}v}{\mathrm{d}r}\right)^{-1}$$

Observations: P Cyg lines II.

- HST spectrum of HD 13268



$$\frac{F}{F_{\mathrm{c}}} \approx \exp\left[-\tau(\mu = 1)\right]$$

$$\tau(\mu = 1) = \frac{\pi e^2}{m_{\mathrm{e}} c} g_i f_{ij} \left(\frac{n_i(r)}{g_i} - \frac{n_j(r)}{g_j}\right) \frac{c}{\nu_0} \left(\frac{\mathrm{d}v}{\mathrm{d}r}\right)^{-1}$$

Observations: P Cyg lines II.

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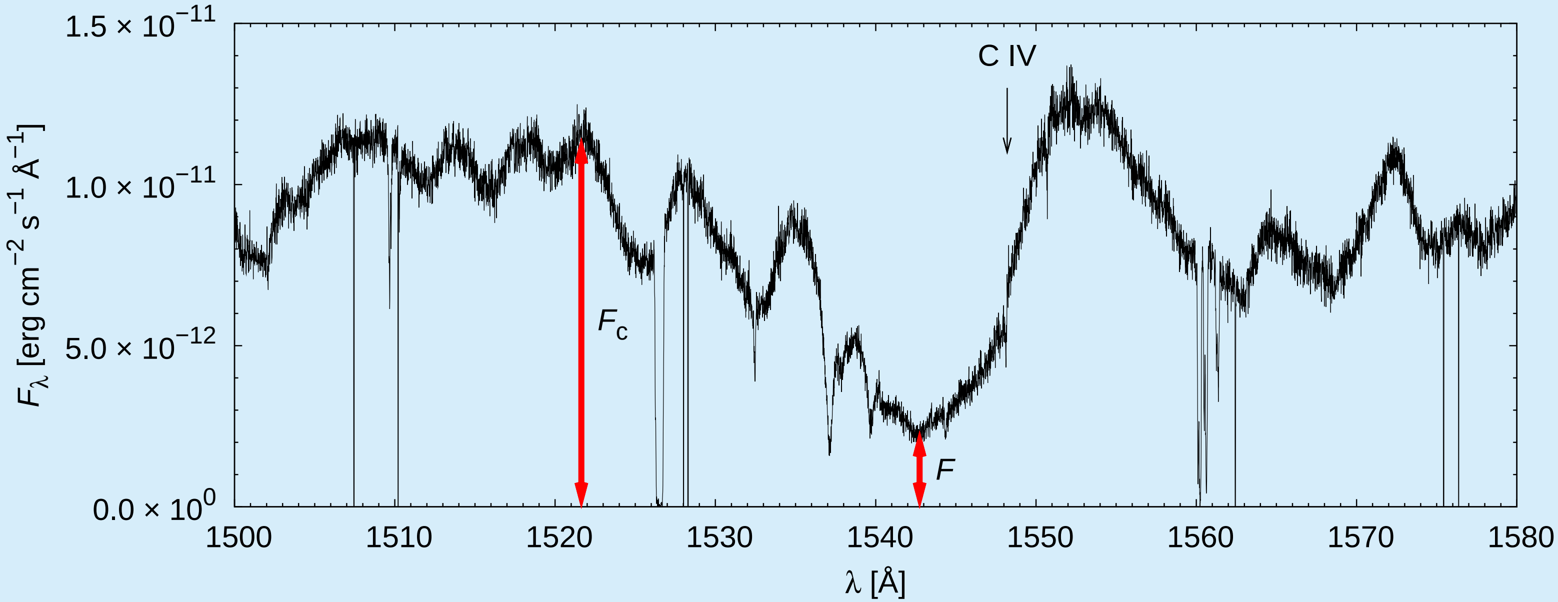

$$\frac{F}{F_c} \approx \exp\left[-\tau(\mu = 1)\right]$$

$$\tau(\mu = 1) = \frac{\pi e^2}{m_e c} \lambda_{ij} f_{ij} n_i(r) \left(\frac{dv}{dr}\right)^{-1}$$

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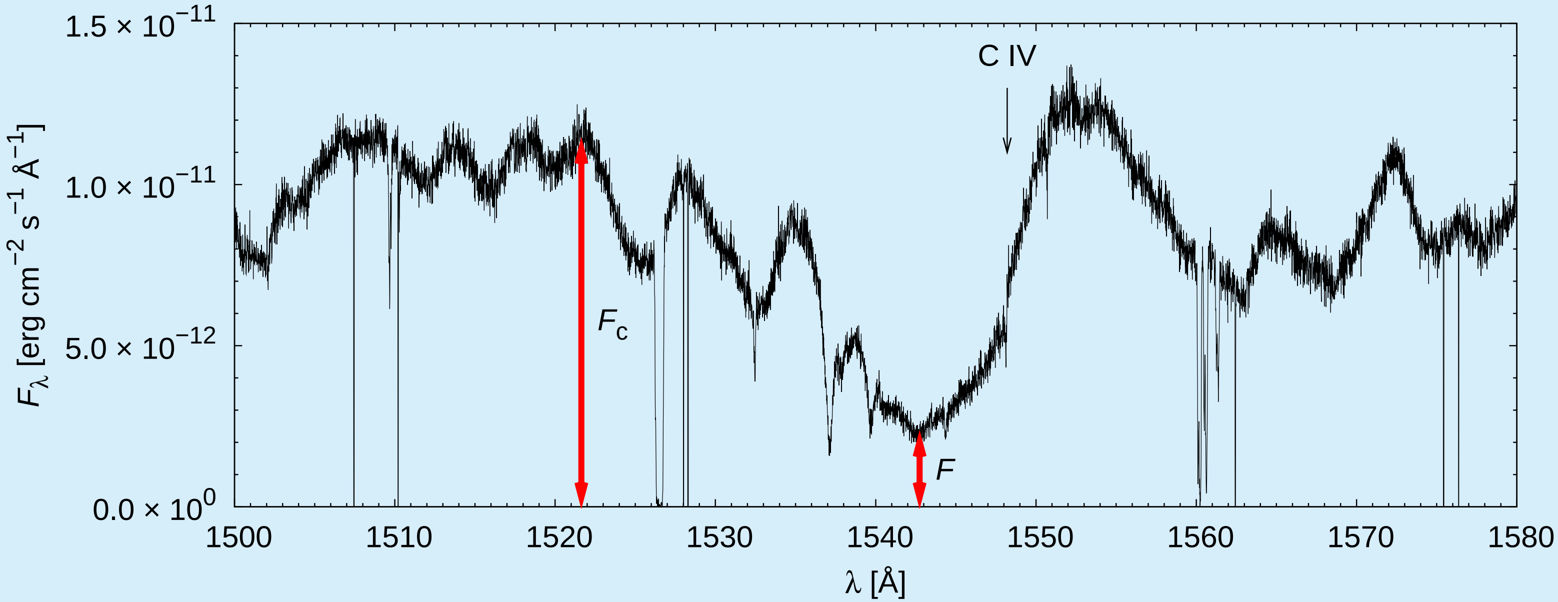
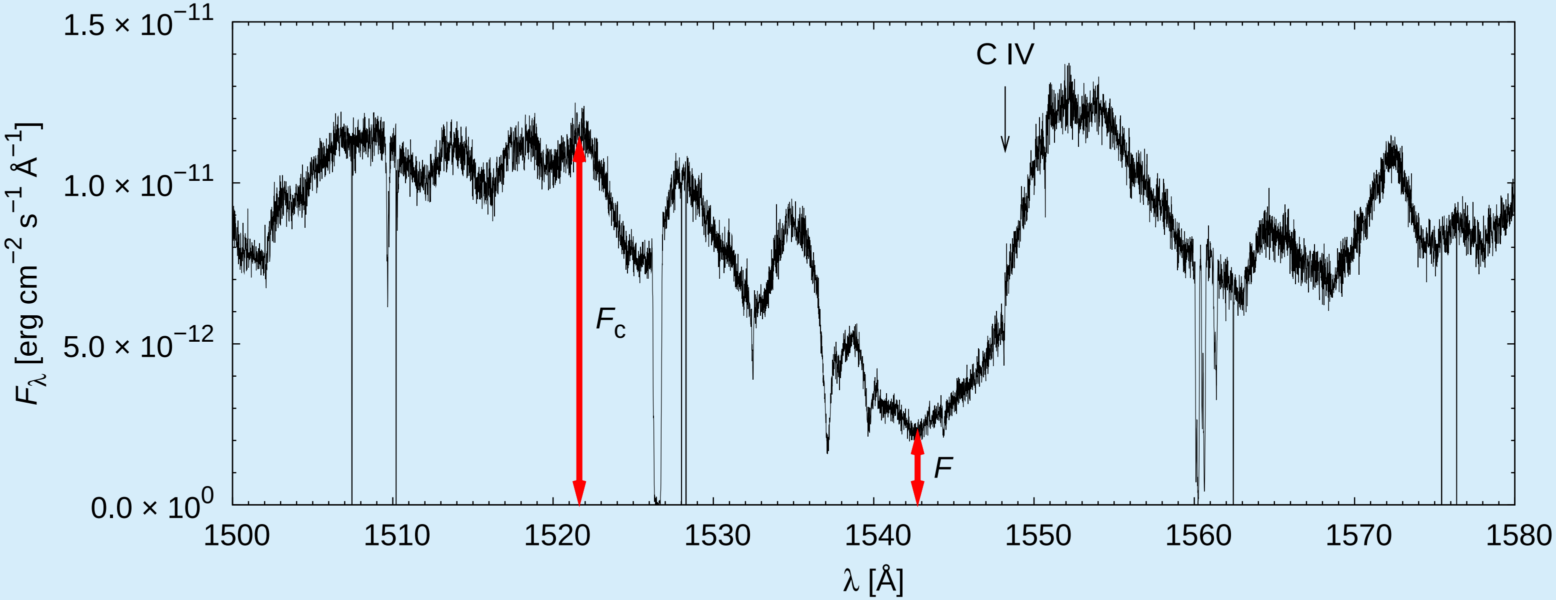


$$\tau(\mu=1)=\frac{\pi e^2}{m_e c}\lambda_{ij}f_{ij}\frac{q_{\rm CIV}Z_{\rm C}}{4\pi m_{\rm H}}\frac{\dot M}{v r^2}\left(\frac{{\rm d}v}{{\rm d}r}\right)^{-1}$$

- $Z_{\rm C}$ is the carbon number density relatively to H
- $q_{\rm CIV}$ is the ionisation fraction of CIV

Observations: P Cyg lines II.

- HST spectrum of HD 13268

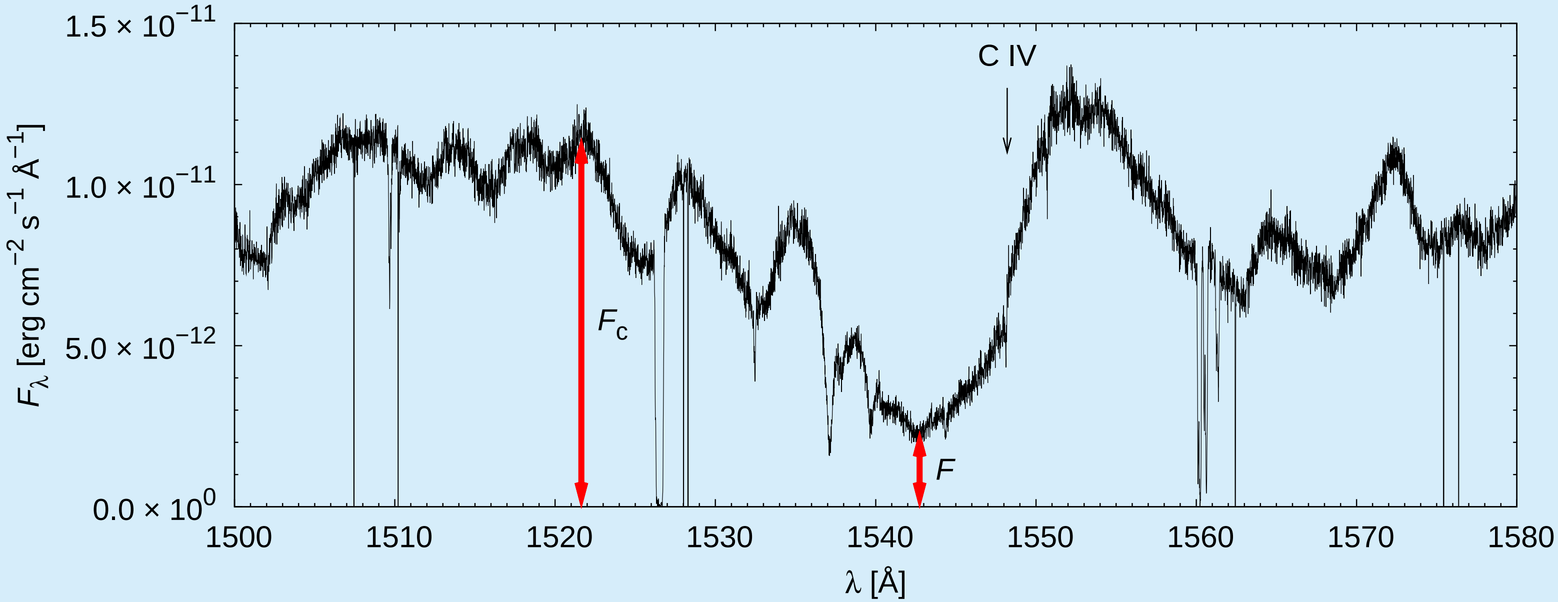


$$\tau(\mu=1)=\frac{\pi e^2}{m_\mathrm{e}c}\lambda_{ij}f_{ij}\frac{Z_\mathrm{C}}{4\pi m_\mathrm{H}}\frac{1}{v_\infty^2 R_*}q_\mathrm{CIV}\dot{M}$$

- our order-of-magnitude approximations:
  $v\to v_\infty$, $r\to R_*$, $\mathrm{d}v/\mathrm{d}r\to v_\infty/R_*$

Observations: P Cyg lines II.

- HST spectrum of HD 13268

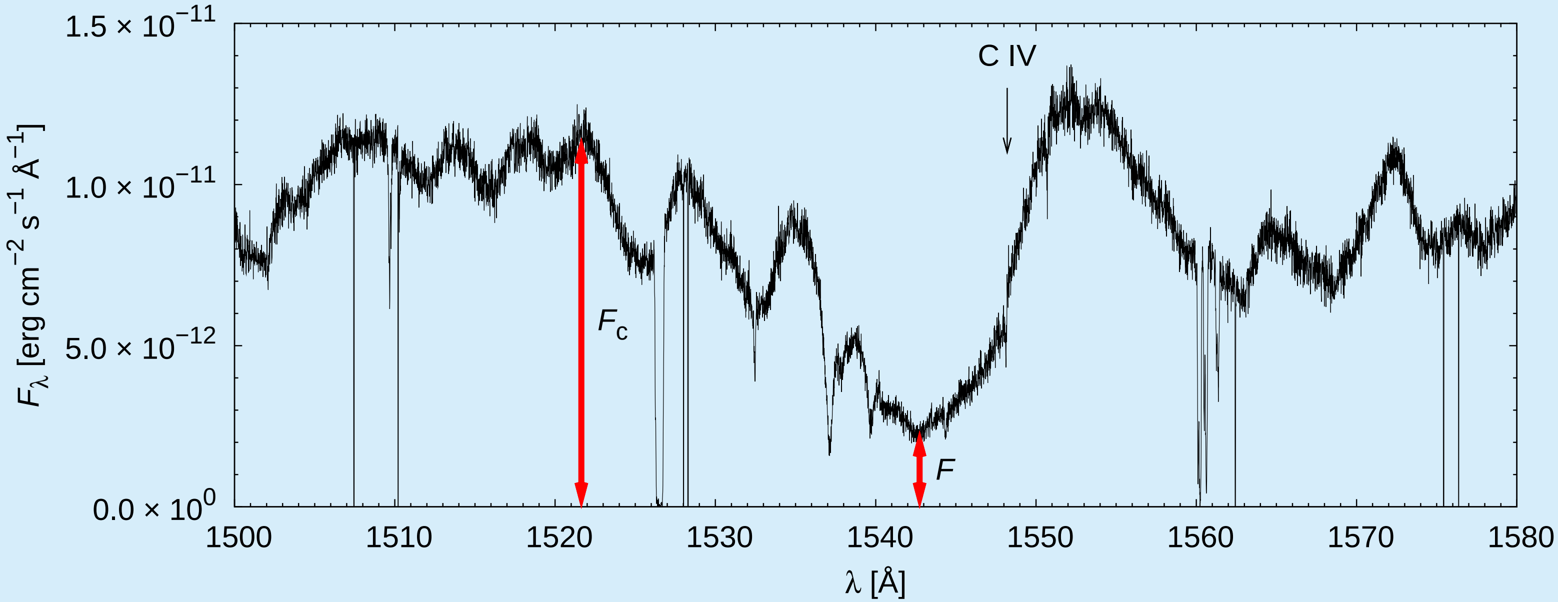


$$\tau(\mu=1)=\frac{\pi e^2}{m_{\rm e}c}\lambda_{ij}f_{ij}\frac{Z_{\rm C}}{4\pi m_{\rm H}}\frac{1}{v_\infty^2 R_*}q_{\rm CIV}\dot{M}$$

$\Rightarrow$ from unsaturated wind line profiles possible to derive $q_{\rm CIV}\dot{M}$

Observations: P Cyg lines II.

- HST spectrum of HD 13268

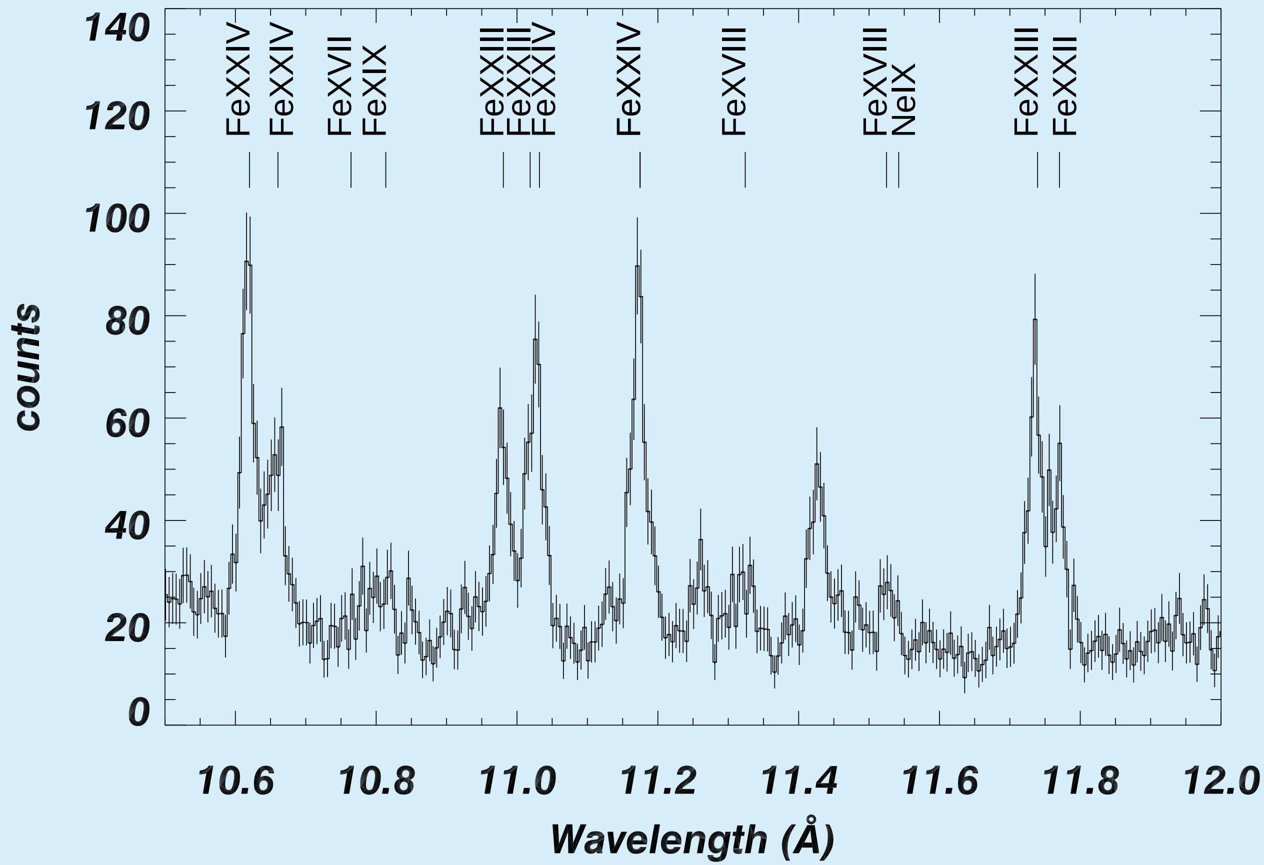


$$\tau(\mu=1)=\frac{\pi e^2}{m_\mathrm{e}c}\lambda_{ij}f_{ij}\frac{Z_\mathrm{C}}{4\pi m_\mathrm{H}}\frac{1}{v_\infty^2 R_*}q_\mathrm{CIV}\dot{M}$$

- in our case $q_\mathrm{CIV}\dot{M}=4\times10^{-10}\,\mathrm{M}_\odot\,\mathrm{yr}^{-1}$
- $\dot{M}$ can be derived with a knowledge of q_CIV

Observation: X-ray emission

- X-ray spectrum θ^1 Ori C



(CHANDRA, Schulz et al. 2003)

Observation: X-ray emission

- X-ray emission of hot stars consists of numerous lines of highly excited elements (N VI, O VII, Fe XXIV, . . .)
- signature of a presence of gas with temperatures of the order 10^6 K
- X-ray emission originates in the wind
  - how?

Observation: X-ray emission

- problem:
  - the wind temperature is of the order of the stellar effective temperature – 10^4 K (as expected from the observed ionisation structure and as derived from NLTE models, e.g., Drew 1989)
  - how can such gas emit X-ray radiation with typical temperatures $\sim 10^6$ K?

Observation: X-ray emission

- problem:
  - the wind temperature is of the order of the stellar effective temperature – 10^4 K
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- solution:
  - most of the wind material is „cool“ with temperatures of order of 10^4 K
  - only a very small fraction of the wind is very hot $\sim 10^6$ K
  - the „hot“ material quickly cools down (radiatively)

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- further problem: how is this possible?

How to create X-rays?

- hot stars have stellar wind with typical velocities $\approx 1000\,\mathrm{km\,s^{-1}}$

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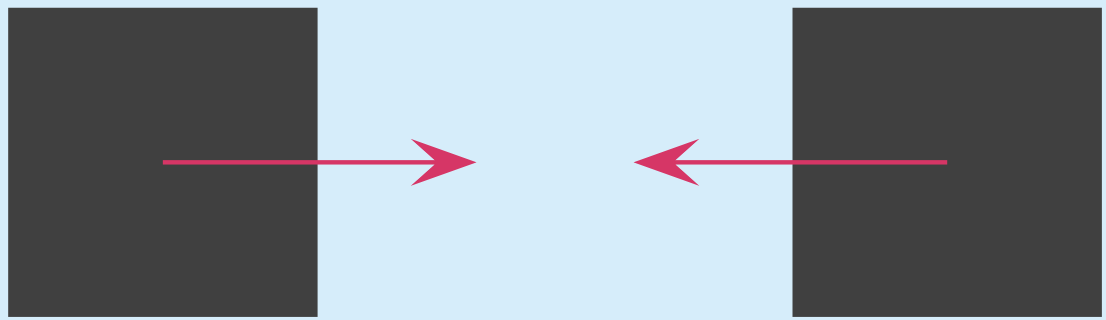
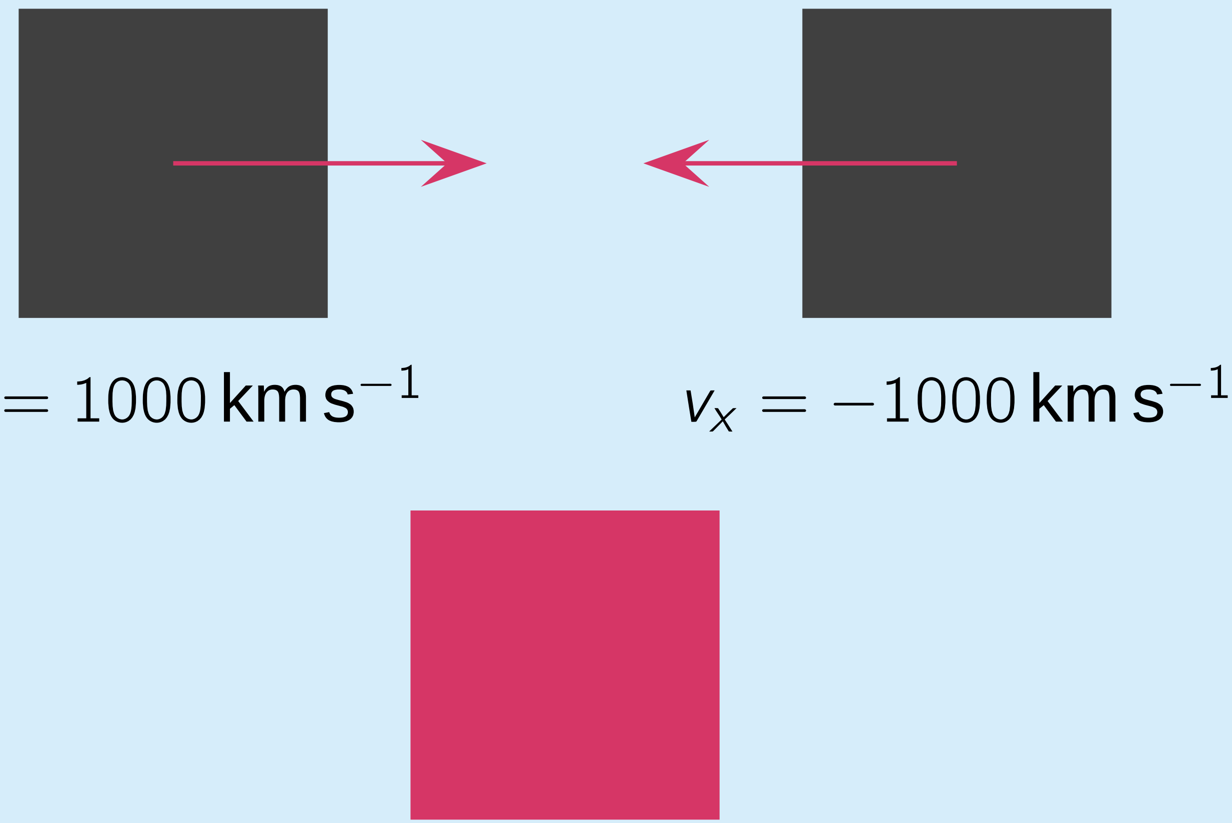


$v_x = 1000\,\mathrm{km\,s^{-1}}$ $v_x = -1000\,\mathrm{km\,s^{-1}}$

How to create X-rays?

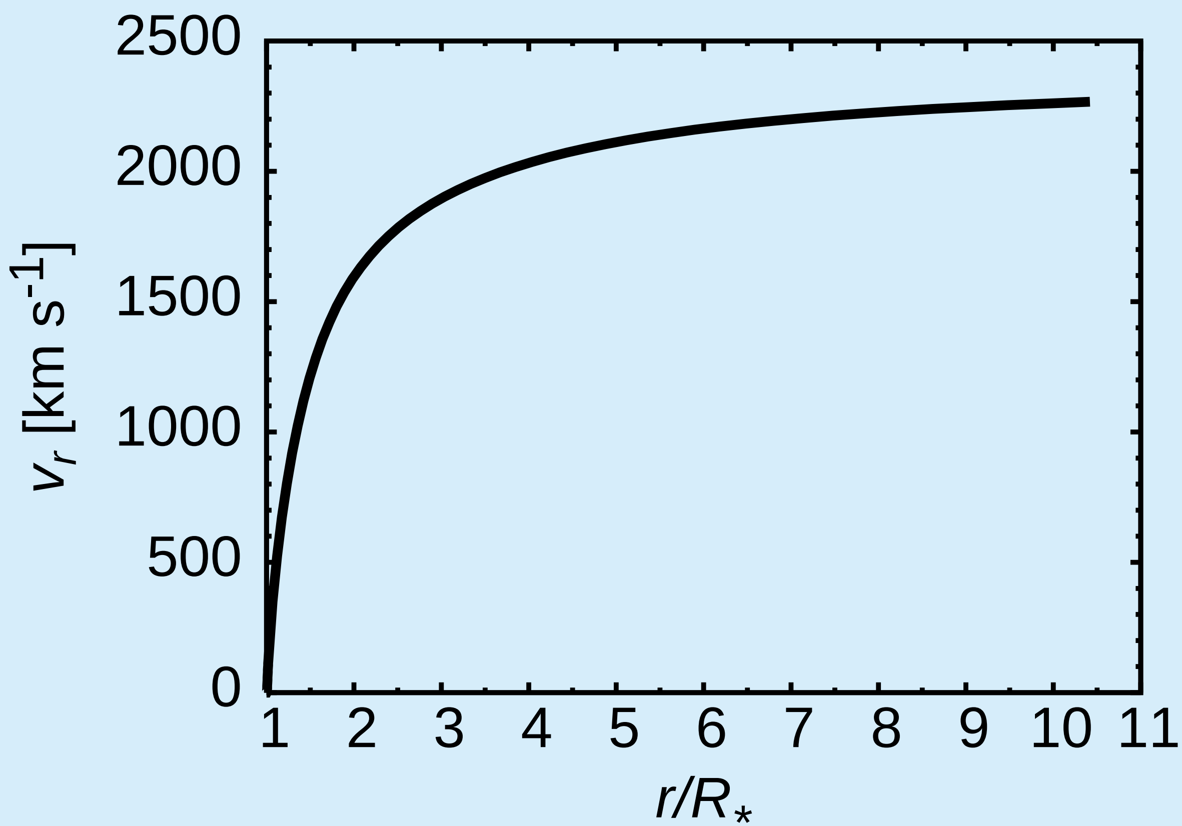
- hot stars have stellar wind with typical velocities $\approx 1000\,\mathrm{km\,s^{-1}}$



$T = 2 \cdot 10^7\,\mathrm{K}$

Can wind material collide?

- possible influence of the wind instabilities



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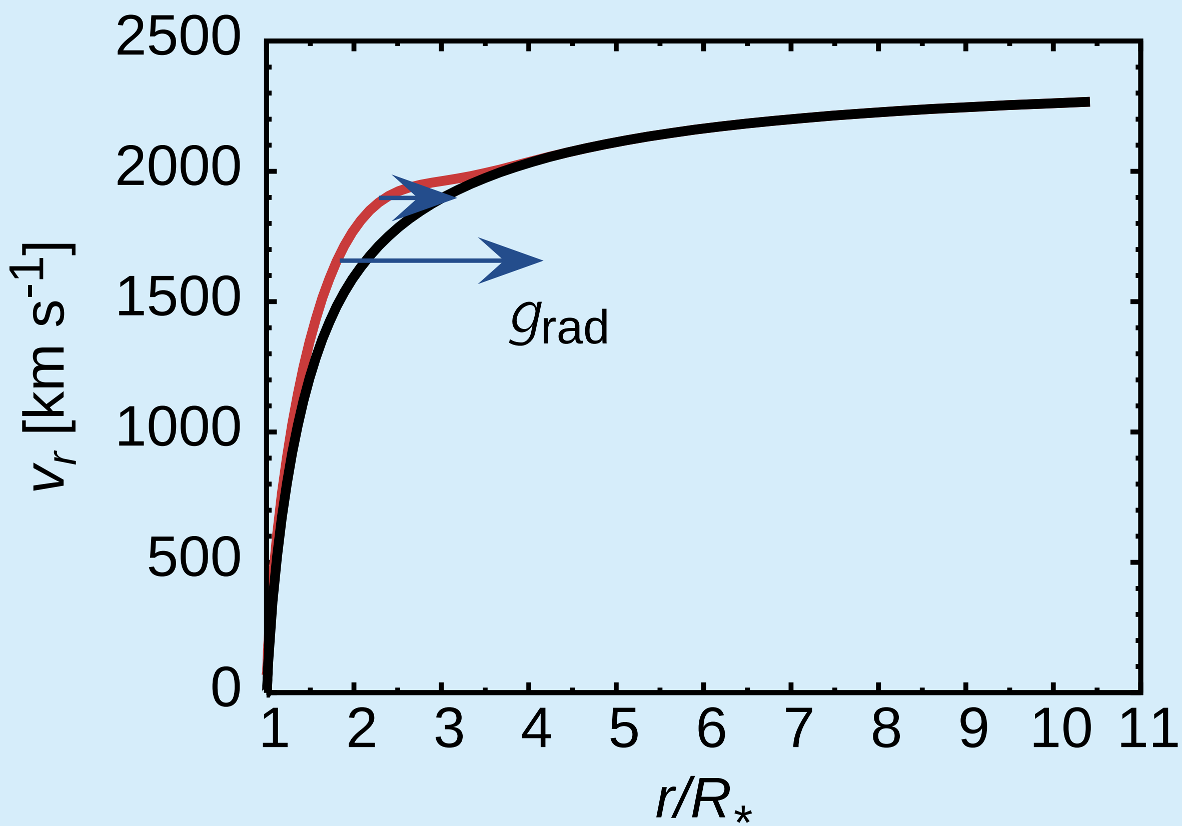

Wind instabilities I.

- main idea
  - the Sobolev approximation gives reliable prediction of wind structure

$\Rightarrow$ a sound basis for the study of instabilities

Wind instabilities I.

- time-dependent hydrodynamical equations

$$\frac{\partial \rho}{\partial t} + \frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2 \rho v\right) = 0$$

$$\rho \frac{\partial v}{\partial t} + \rho v \frac{\partial v}{\partial r} = -a^2 \frac{\partial \rho}{\partial r} + f_{\text{rad}} - \frac{\rho G M (1-\Gamma)}{r^2}$$

- ρ, v are the wind density and velocity
- a is the sound speed

Wind instabilities I.

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- comoving fluid-frame + small perturbations of stationary solution
  - $\rho = \rho_0 + \delta\rho$,
  - $v = v_0 + \delta v$, $v_0 = 0$

Wind instabilities I.

- equations for perturbations $\delta\rho$, δv

$$\frac{\partial\delta\rho}{\partial t} + \rho_0 \frac{\partial\delta v}{\partial r} = 0$$

$$\rho_0 \frac{\partial\delta v}{\partial t} = -a^2 \frac{\partial\delta\rho}{\partial r} + \delta f_{\mathrm{rad}}$$

- perturbation of the radiative force
  $\delta f_{\mathrm{rad}} = \rho_0 g'_{\mathrm{rad}}\, \delta v/\delta r$
- where $g'_{\mathrm{rad}} \equiv \partial g_{\mathrm{rad}}/\partial\left(\mathrm{d}v/\mathrm{d}r\right)$

Wind instabilities I.

- the wave equation

$$\frac{\partial^2 \delta v}{\partial t^2} = a^2 \frac{\partial^2 \delta v}{\partial r^2} + g'_{\mathrm{rad}} \frac{\partial^2 \delta v}{\partial t \partial r}$$

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- the dispersion relation

$$\omega^2 + g'_{\mathrm{rad}} \omega k - a^2 k^2 = 0$$

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- zero radiative force

$$\frac{\omega}{k} = \pm a$$

- ordinary sound waves

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- general case
  - new type of waves – radiative-acoustic (Abbott) waves (Abbott 1980, Feldmeier et al. 2008)
  - downstream (+) and upstream (-) mode

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- critical point: radial wind velocity equals to the speed of (upstream) Abbott waves

$$v_{\mathrm{c}} - \frac{1}{2} g'_{\mathrm{rad}} - \left(\frac{1}{4} g'^2_{\mathrm{rad}} + a^2 \right)^{1/2} = 0$$

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- critical point: radial wind velocity equals to the speed of (upstream) Abbott waves

$\Rightarrow$ no information can travel from the regions with $v > v_{\mathrm{c}}$ towards the stellar surface (critical surface resembles the even horizon of a black hole, Feldmeier & Shloshman 2000)

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$\Rightarrow$ mass-loss rate is determined there

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$\Rightarrow$ no instability of hot-star winds!

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- hydrodynamical simulations (Votruba et al. 2007)

Wind instabilities II.

- our stability analysis showed that the wind should be stable

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- what causes the occurrence of X-rays?

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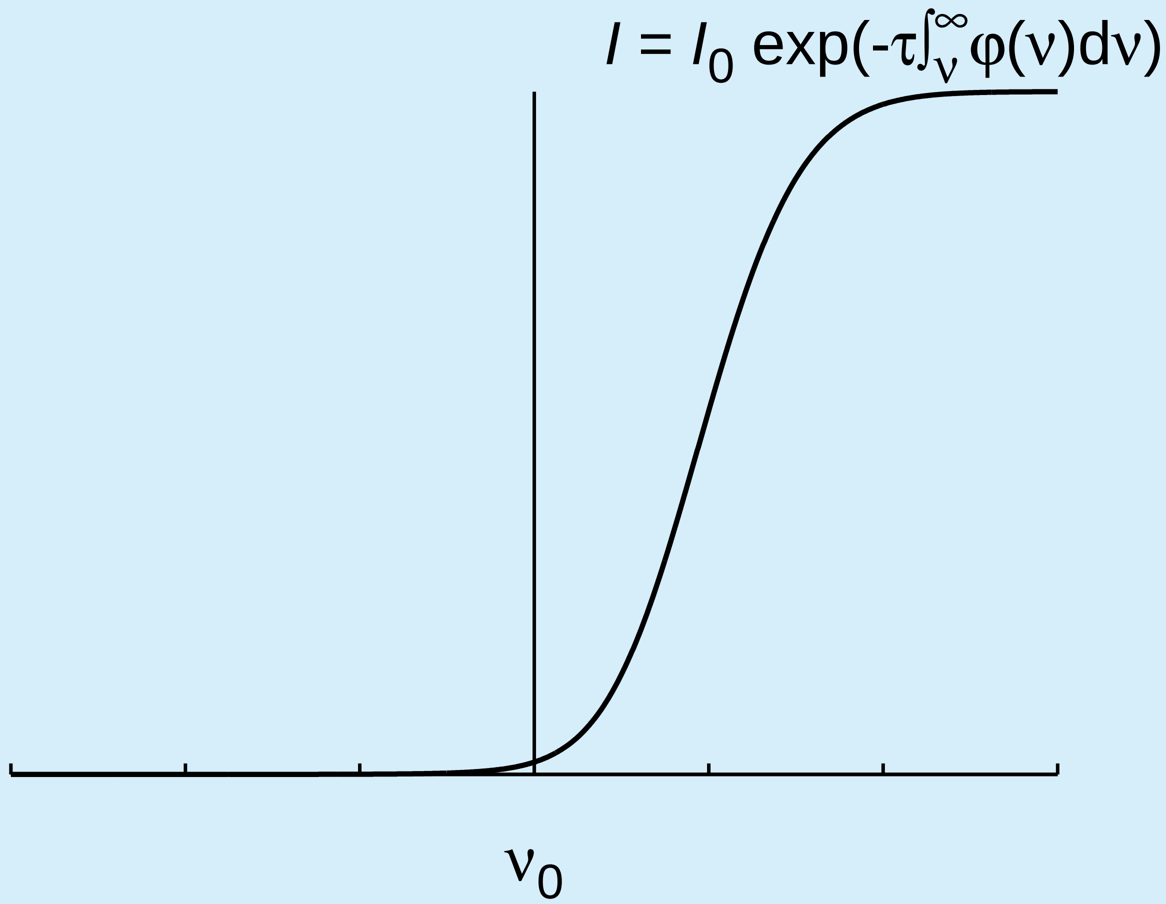
- what is wrong with our stability analysis?

Wind instabilities II.

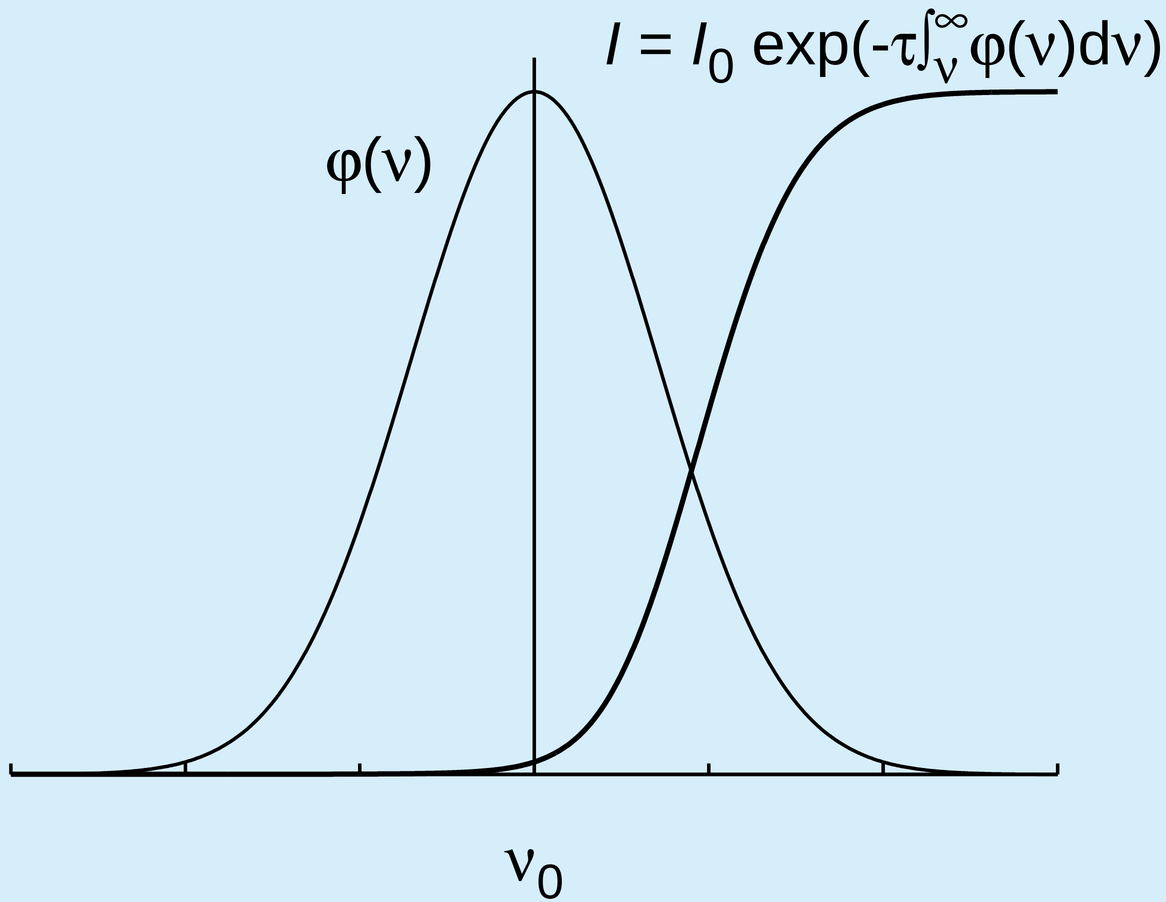
- our stability analysis showed that the wind should be stable
- what causes the occurrence of X-rays?
- what is wrong with our stability analysis?
- the Sobolev approximation is not valid for small (optically thin) perturbations!

Wind instabilities II.



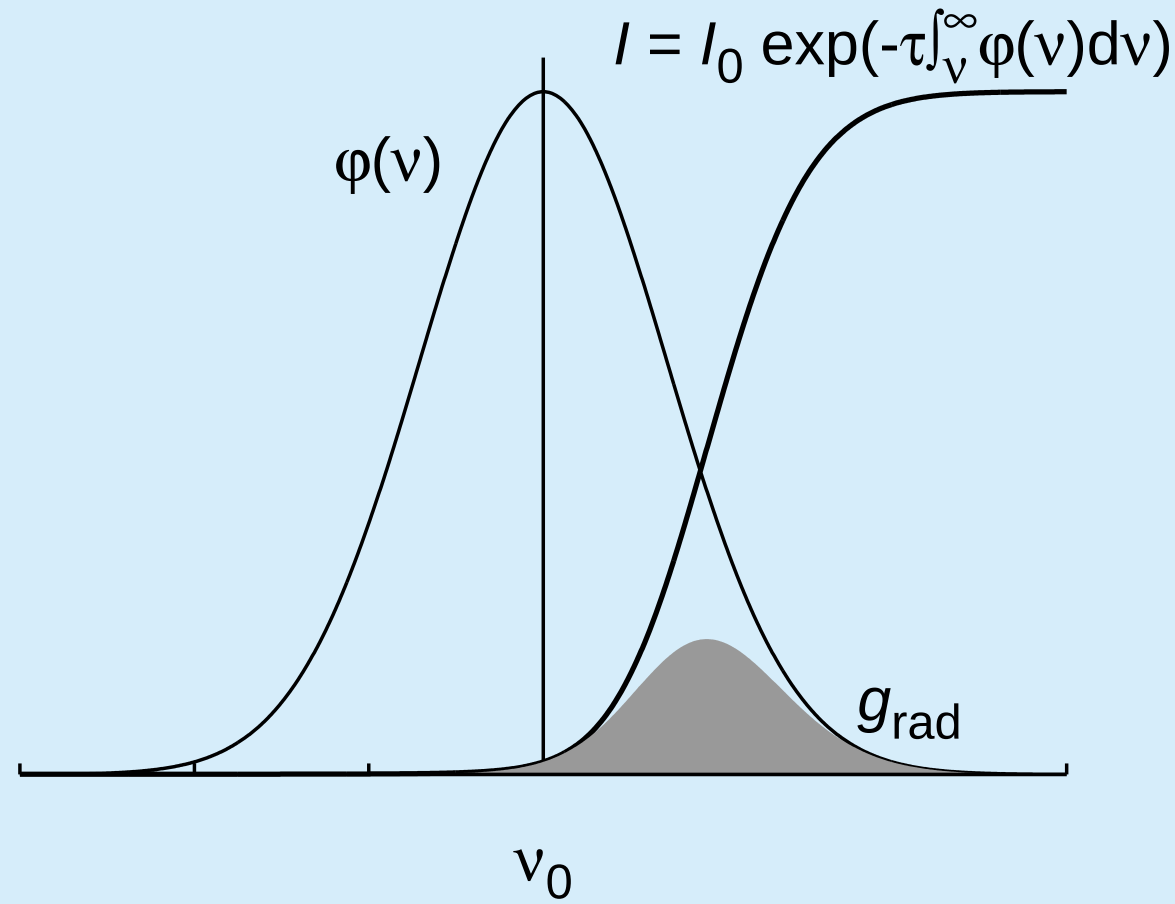
- the radiative transfer in the comoving frame

Wind instabilities II.



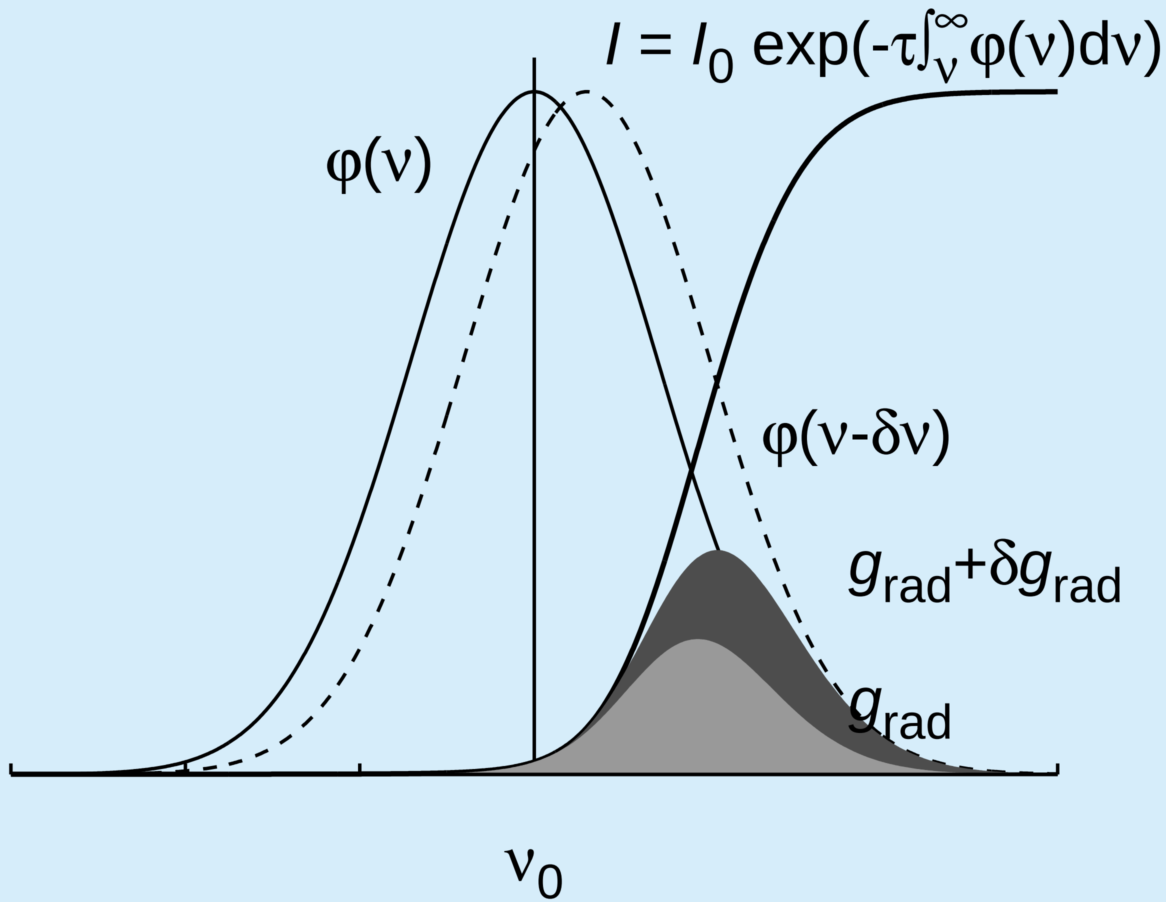
- the absorption profile in the comoving frame

Wind instabilities II.



- the line force

Wind instabilities II.



- the line force after a small change of the velocity

Wind instabilities II.

- the radiative acceleration

$$g_{\mathrm{rad}} = \frac{2\pi}{c\rho} \int_0^\infty \mathrm{d}\nu \, \chi_{\mathrm{L}}(r) \varphi_{ij}(\nu) \int_{-1}^{1} \mathrm{d}\mu \, \mu I(r,\mu,\nu)$$

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- optically thin perturbation

$$\delta g_{\mathrm{rad}} = \frac{2\pi}{c\rho} \int_0^\infty \mathrm{d}\nu \, \chi_{\mathrm{L}}(r) \delta\varphi_{ij}(\nu) \int_{-1}^{1} \mathrm{d}\mu \, \mu I(r,\mu,\nu)$$

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$$\delta\varphi_{ij}(\nu) = \frac{\mathrm{d}\varphi_{ij}(\nu)}{\mathrm{d}\nu} \delta\nu = \frac{\mathrm{d}\varphi_{ij}(\nu)}{\mathrm{d}\nu} \nu_0 \frac{\delta v}{c}$$

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$$\Rightarrow \quad \delta g_{\mathrm{rad}} = \Omega \delta v \quad (\Omega > 0)$$

Wind instabilities II.

- equations for perturbations $\delta\rho$, δv

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- the dispersion relation

$$\omega^2 + i\Omega\omega - a^2 k^2 = 0$$

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- negligible gas pressure: $\Omega^2 \gg a^2 k^2$

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- the dispersion relation (non-zero ω)

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$\Rightarrow$ strong instability of the radiative driving (Lucy & Solomon 1970, MacGregor et al. 1979, Carlberg 1980, Owocki et al. 1984)

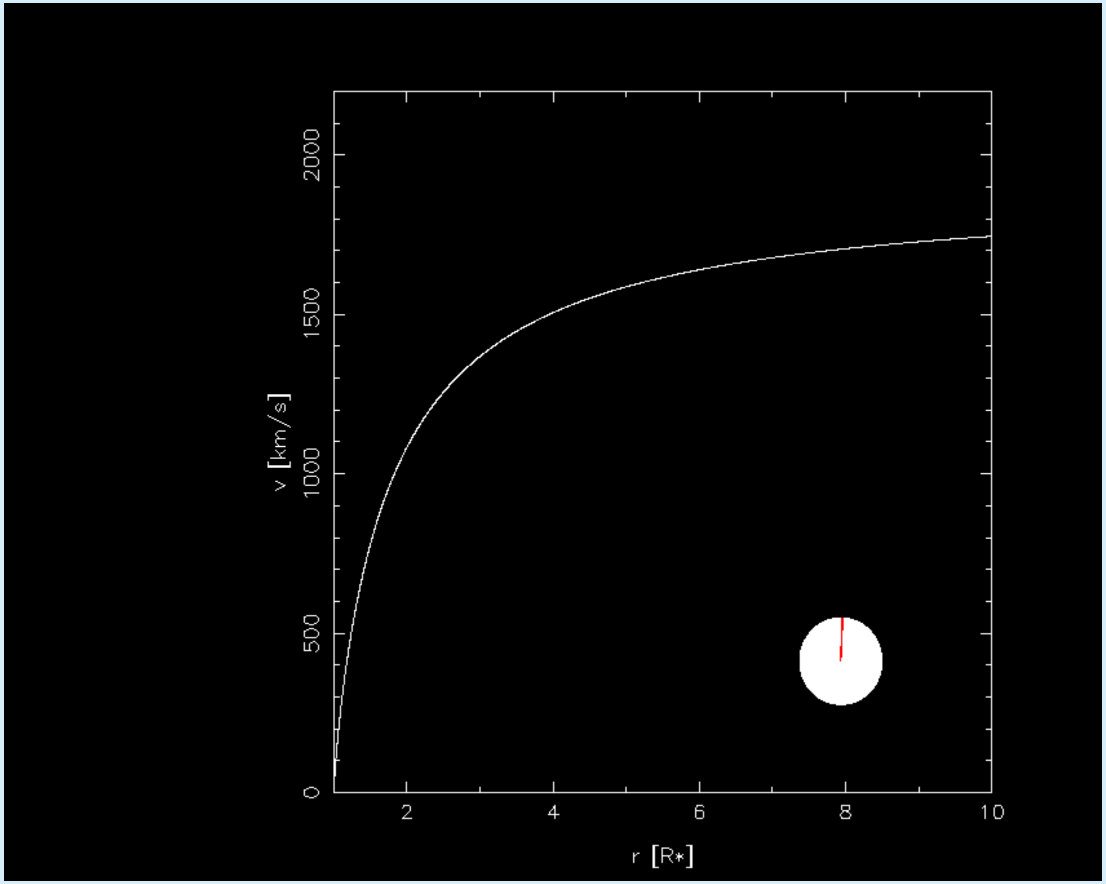
Wind instabilities III.

- our instability analysis is linear only

$\Rightarrow$ hydrodynamical simulations are necessary to describe the instability in detail (Owocki et al. 1988, Feldmeier et al. 1997, Runacres & Owocki 2002)

Wind instabilities III.

- hydrodynamical simulations (Feldmeier et al. 1997)



Wind instabilities III.

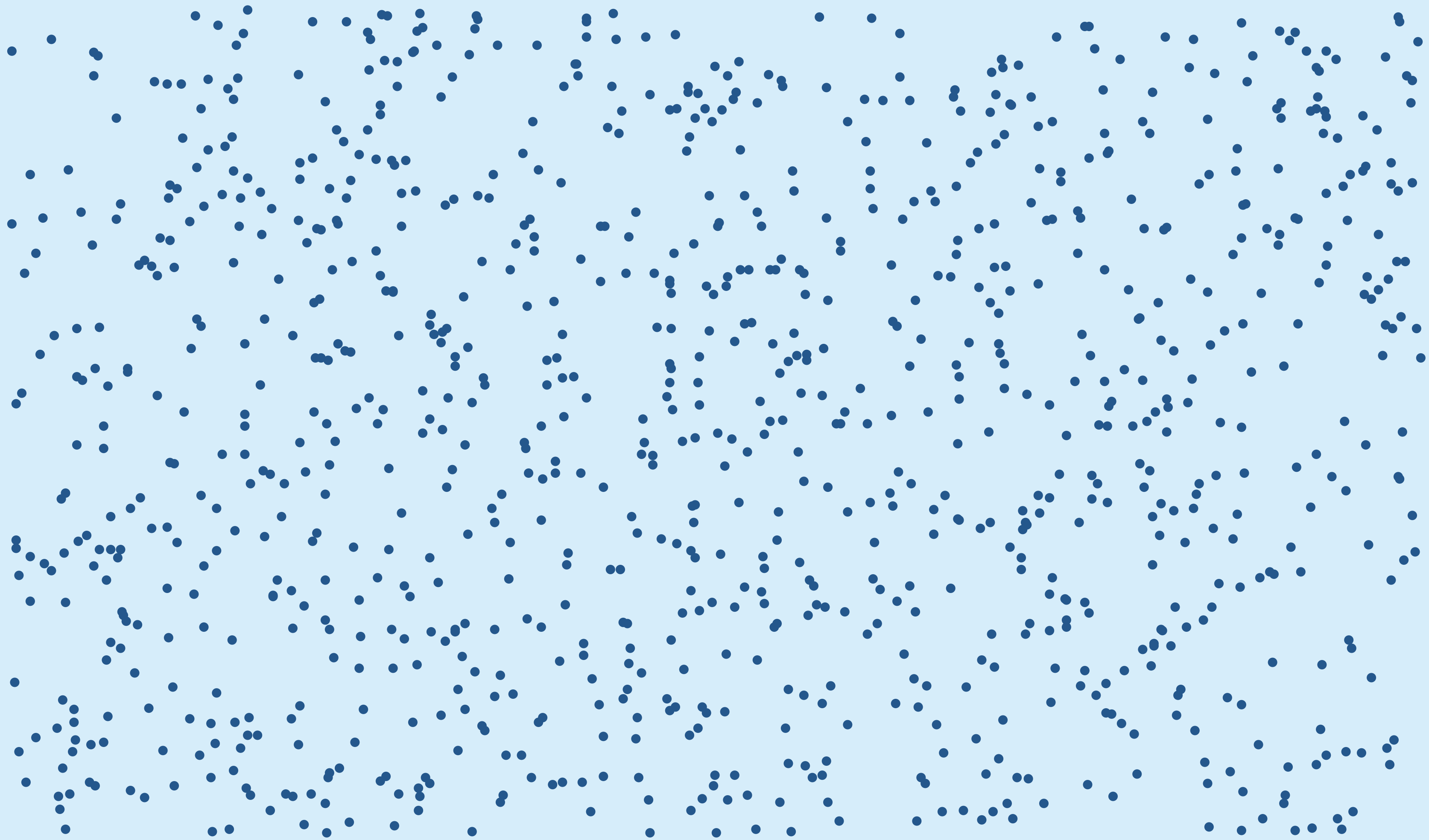
- hydrodynamical simulations are able to explain the main properties of X-ray emission of hot stars

Hot star winds: micro-view

- stellar wind of hot stars is accelerated due to the scattering of radiation in lines and on free electrons.
- how does it work on a micro-level?

Hot star winds: micro-view

Typical volume with:
1000 H ions



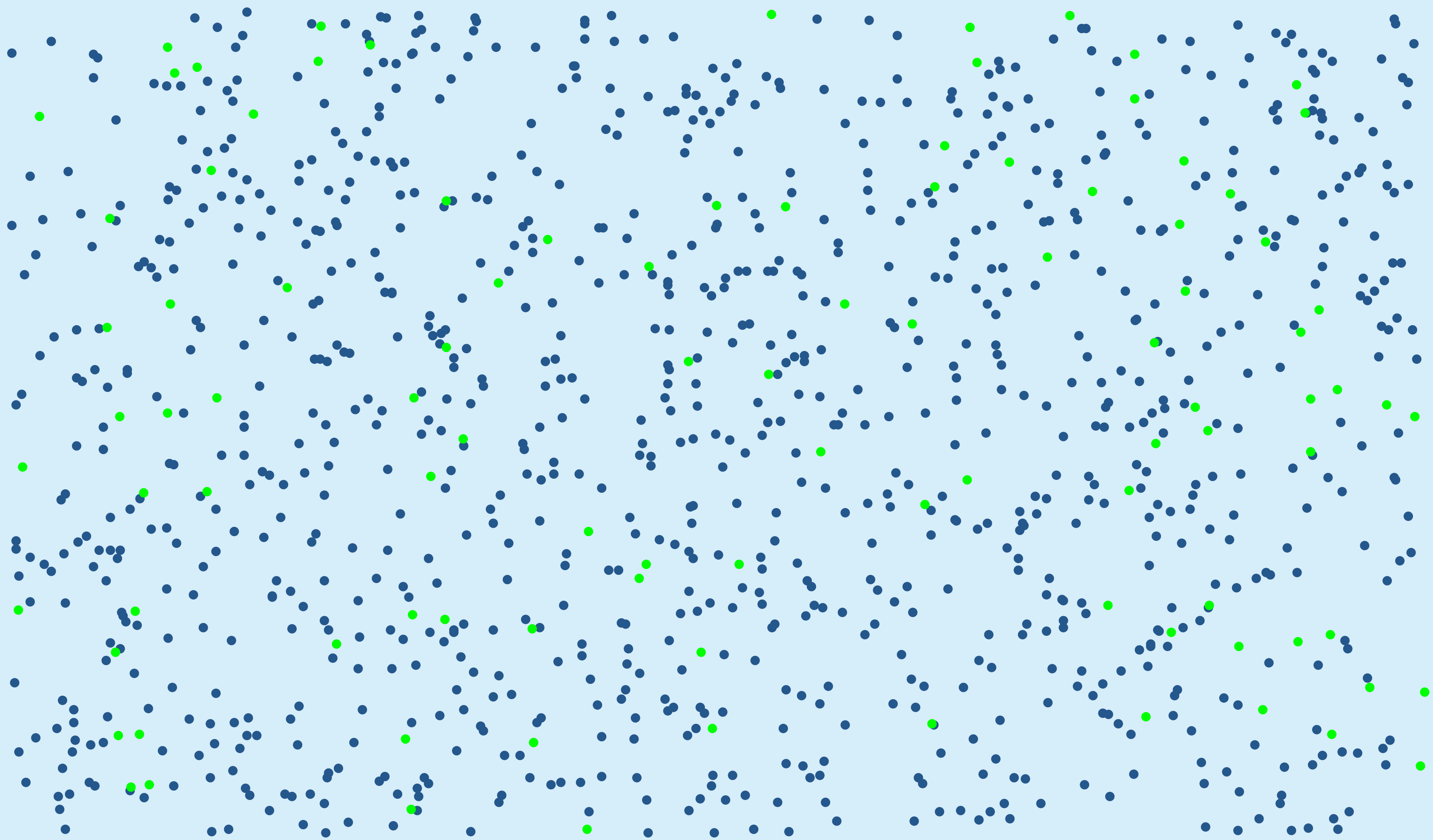
Hot star winds: micro-view

Typical volume with:
1000 H ions

- radiative acceleration due to the line absorption can be in most cases neglected
- radiative acceleration due to the free-free processes also negligible $\sigma_\mathrm{p} \ll \sigma_\mathrm{e}$

Hot star winds: micro-view

Typical volume with:
1000 H ions + 100 He ions



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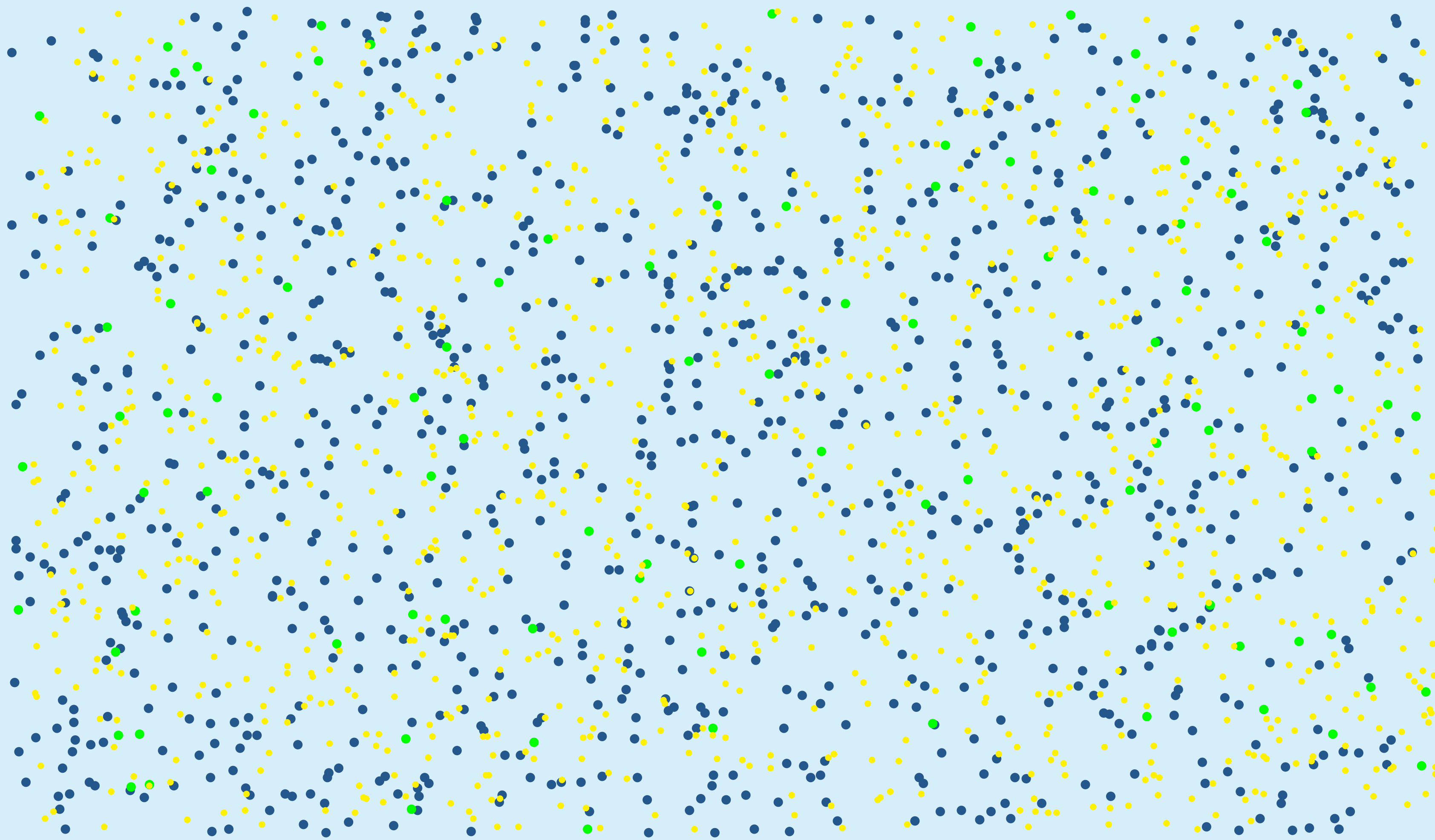
Hot star winds: micro-view

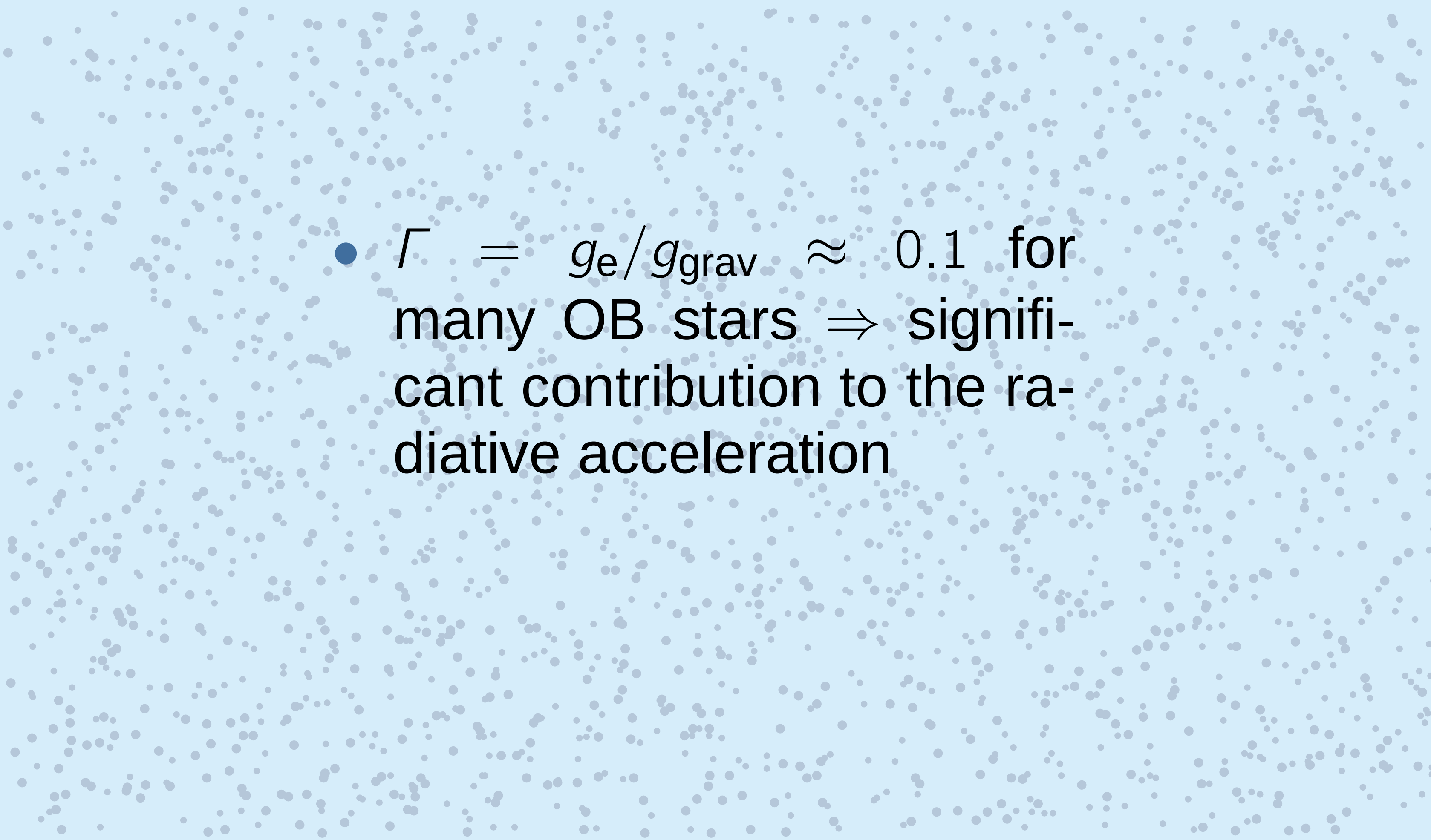
Typical volume with:

1000 H ions + 100 He ions + 1200 e^{-}



Hot star winds: micro-view

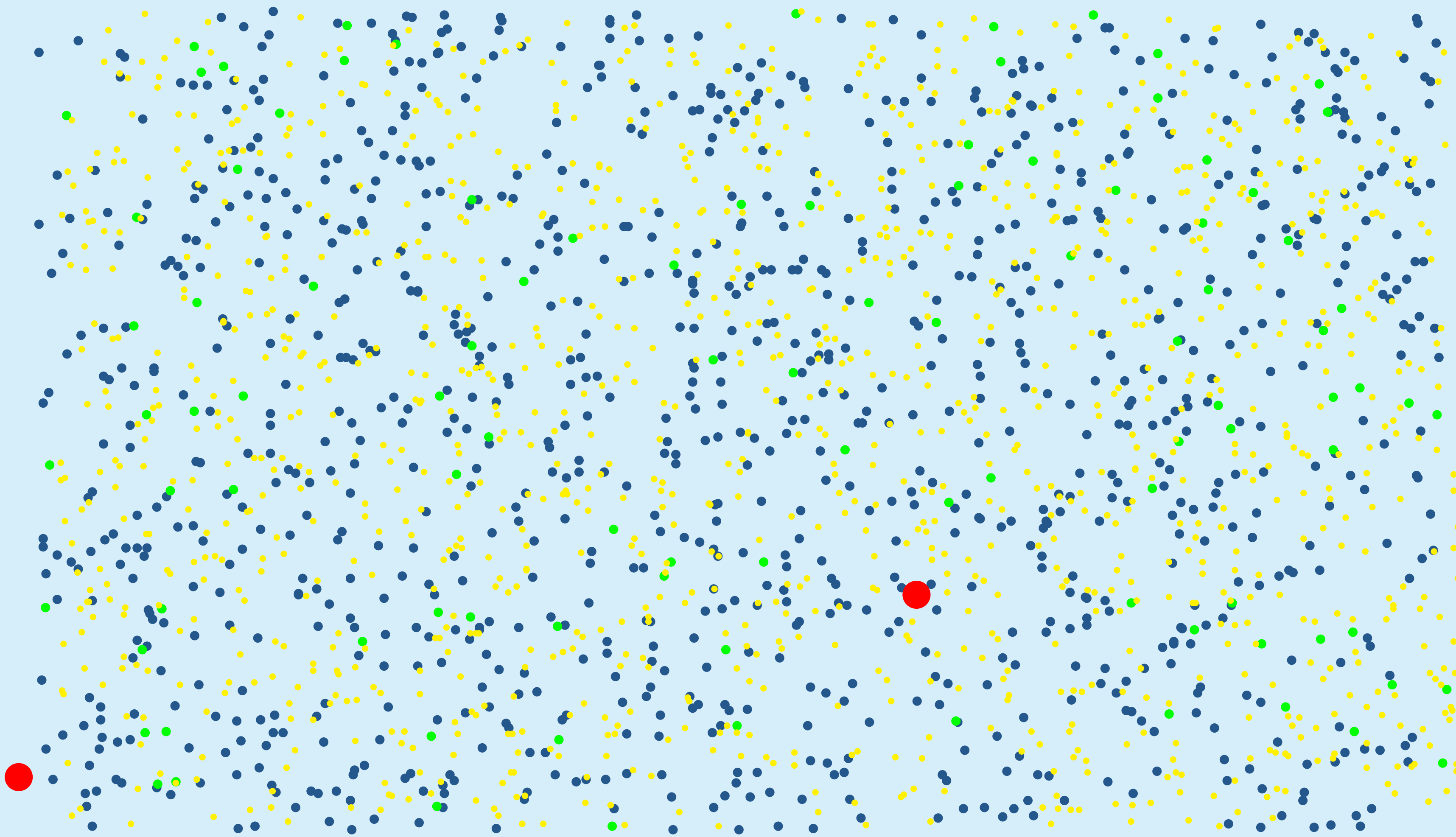
Typical volume with:
1000 H ions + 100 He ions + 1200 e^-



- $\Gamma = g_{\mathrm{e}}/g_{\mathrm{grav}} \approx 0.1$ for many OB stars $\Rightarrow$ significant contribution to the radiative acceleration

Hot star winds: micro-view

Typical volume with:
1000 H ions + 100 He ions + 1200 e$^-$ + 2 metals



Hot star winds: micro-view

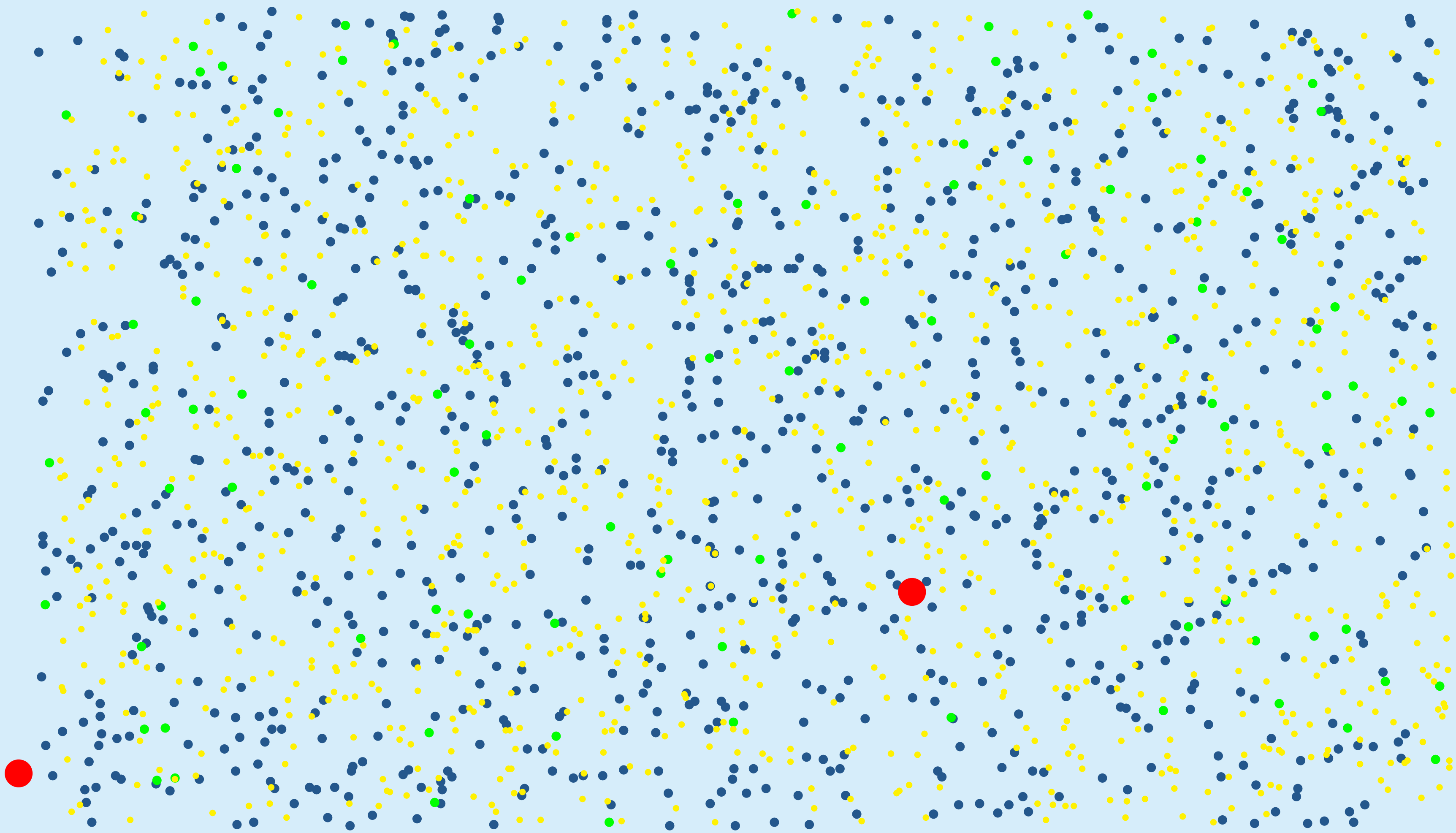
Typical volume with:
1000 H ions + 100 He ions + 1200 e^- + 2 metals

- maximum radiative acceleration due to the lines $g_{\text{line}}^{\text{max}} \approx 1000\, g_{\text{grav}}$ (Gayley 1995) $\Rightarrow$ crucial contribution to the radiative acceleration

Hot star winds: micro-view

Typical volume with:

1000 H ions + 100 He ions + 1200 e^{-} + 2 metals



How can this work?

two efficient processes necessary:

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- process which transfers momentum from radiative field to heavier ions

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two efficient processes necessary:

- process which transfers momentum from radiative field to heavier ions
- process which transfers momentum from heavier ions to the bulk flow (H, He – mostly passive component)

How to transfer momentum?

- wind is ionised $\Rightarrow$ Coulomb collisions are efficient to transfer momentum from heavier elements to the passive component.

How to transfer momentum?

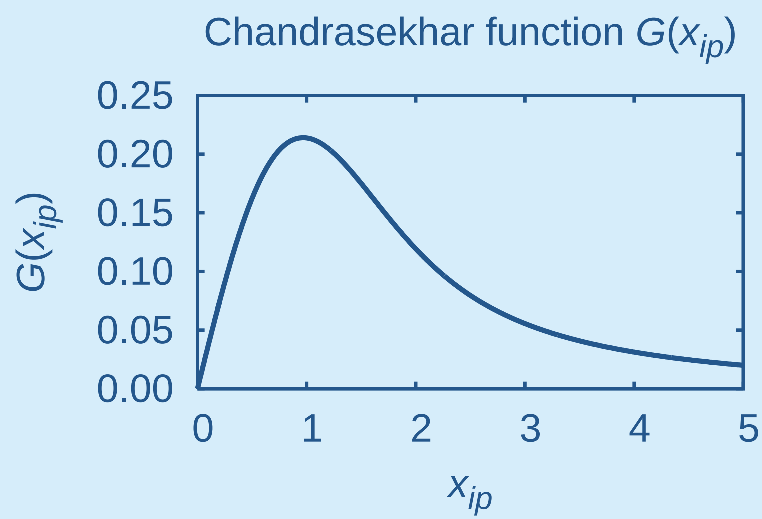
frictional force on passive component (p) due to ions (i)

$$f_{\mathrm{pi}} = \rho_{\mathrm{p}} g_{\mathrm{pi}} = n_{\mathrm{p}} n_{\mathrm{i}} \frac{4\pi q_{\mathrm{p}}^2 q_{\mathrm{i}}^2}{kT_{\mathrm{ip}}} \ln\Lambda\, G(x_{\mathrm{ip}}) \frac{v_{\mathrm{i}} - v_{\mathrm{p}}}{|v_{\mathrm{i}} - v_{\mathrm{p}}|},$$

where n_{p}, n_{i} are number densities of components, v_{i}, v_{p} are their radial velocities, and q_{p}, q_{i} their charges.

$$x_{\mathrm{ip}} = \frac{|v_{\mathrm{i}} - v_{\mathrm{p}}|}{\alpha_{\mathrm{ip}}}$$

$$\alpha_{\mathrm{ip}}^2 = \frac{2k\left(m_{\mathrm{i}} T_{\mathrm{p}} + m_{\mathrm{p}} T_{\mathrm{i}}\right)}{m_{\mathrm{i}} m_{\mathrm{p}}}$$



Momentum transfer efficiency

$$x_{\mathrm{ip}} = \frac{\left| v_{r\,\mathrm{i}} - v_{r\,\mathrm{p}} \right|}{\alpha_{\mathrm{ip}}}$$

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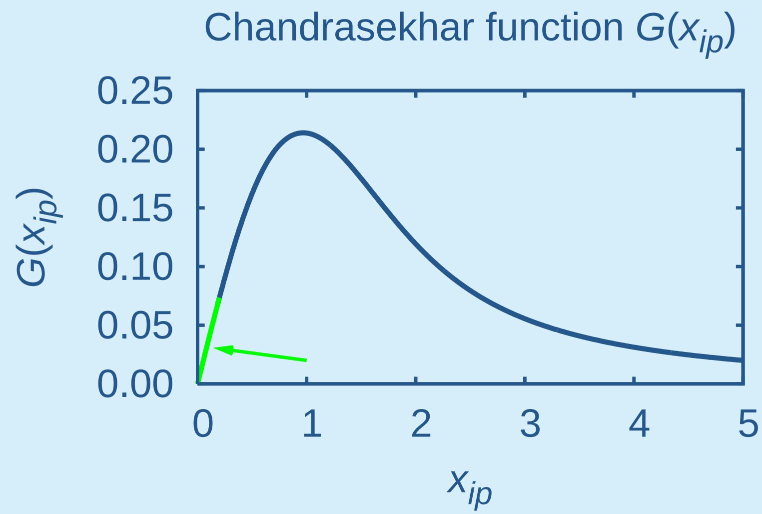

- efficient transfer of momentum from heavier ions: one-component models sufficient

Momentum transfer efficiency

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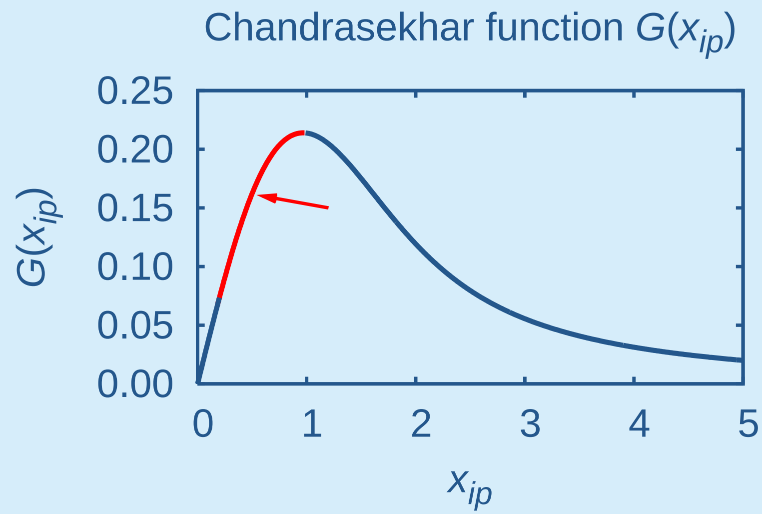

- inefficient transfer of momentum from heavier ions: $x_{\mathrm{ip}} \gtrsim 0.1$, part of energy goes to heating – frictional heating

Momentum transfer efficiency

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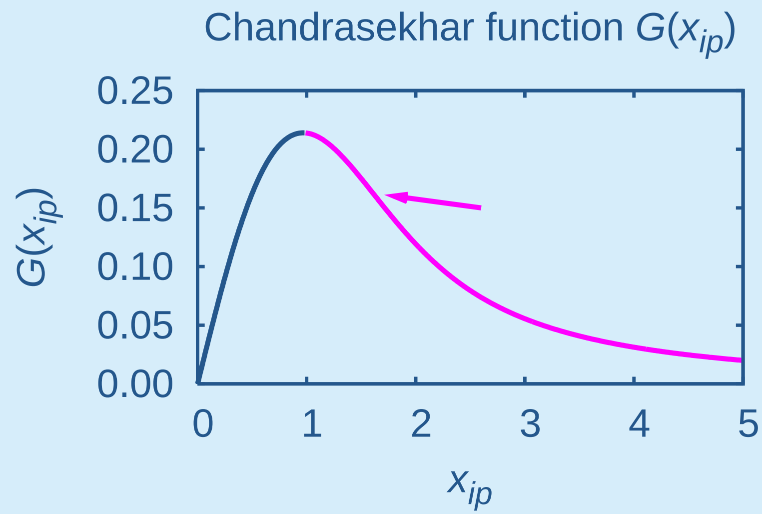

- inefficient collisions between components: $x_{\rm ip} \gtrsim 1$. Chandrasekhar function is a decreasing function of velocity difference $\Rightarrow$ dynamical decoupling of wind components
- important for low-density winds (Springmann & Pauldrach 1992, Krtička & Kubát 2001, Votruba et al. 2007).

Hot chemically peculiar stars

- hotter main sequence O stars have winds accelerated by the line transitions of heavier elements (C, N, O, Si, Fe, . . .)

Hot chemically peculiar stars

- for late B stars and A stars (of the main sequence) the radiative force is not strong enough to drive a wind

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- overabundance (or underabundance) of certain elements (He, Si, Mg, Fe, . . .) in the atmosphere (e.g., Vauclair 2003, Michaud 2005)

Hot chemically peculiar stars

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$$\text{radiative diffusion} \times \text{gravitation settling}$$

$\Rightarrow$ chemically peculiar (CP) stars

- the chemical peculiarity affects surface layers only (the initial chemical composition of the stellar core is roughly solar one)

Hot chemically peculiar stars

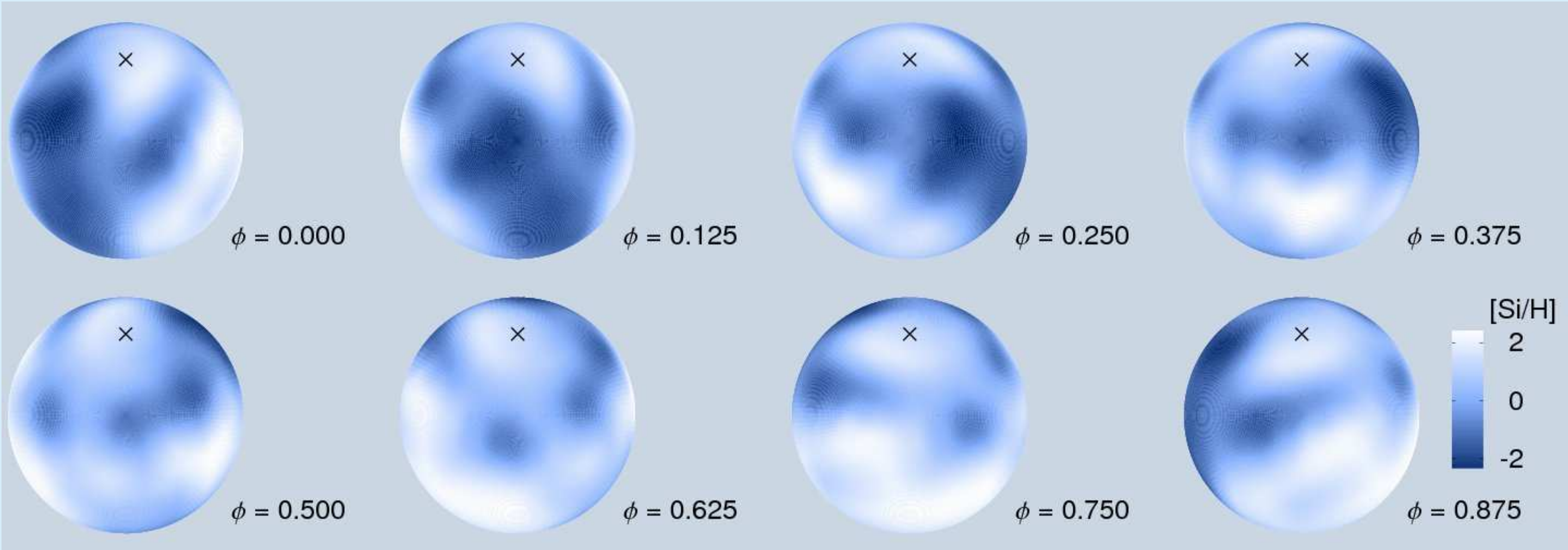
- example: HD 37776

Hot chemically peculiar stars

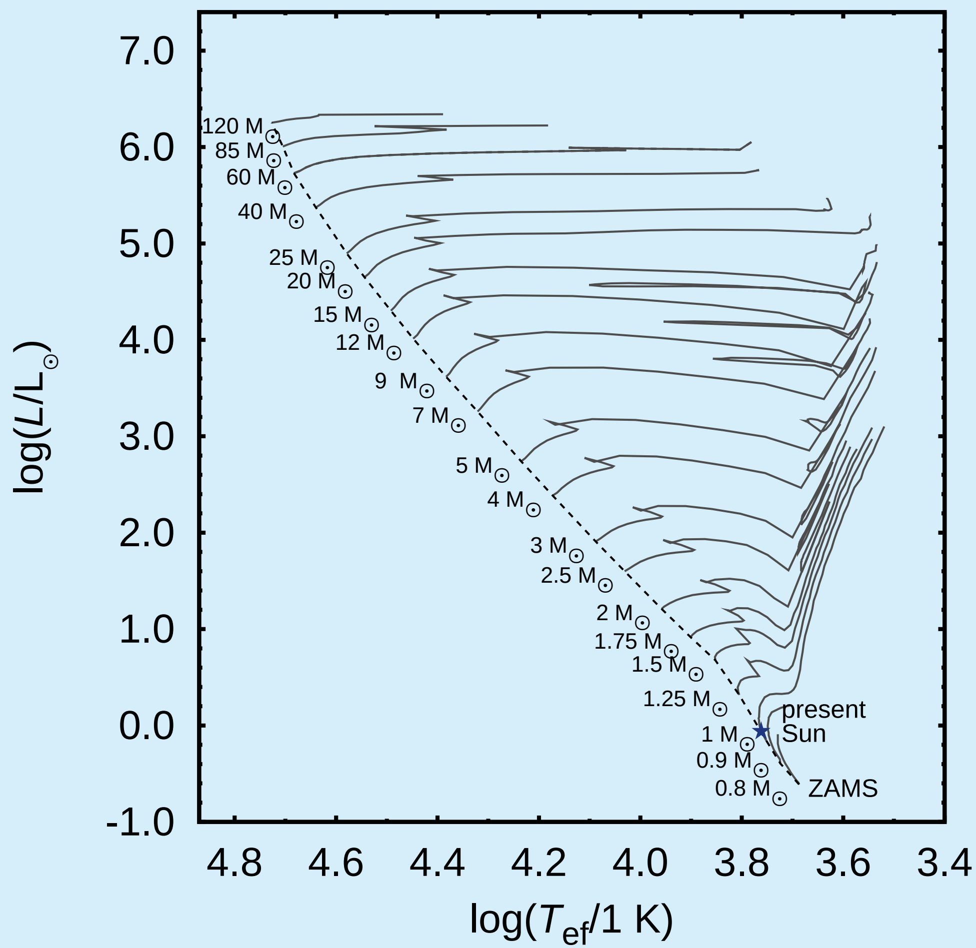
- example: HD 37776

Hot chemically peculiar stars

- example: HD 37776
- Si surface distribution (Chochlova et al. 2000)

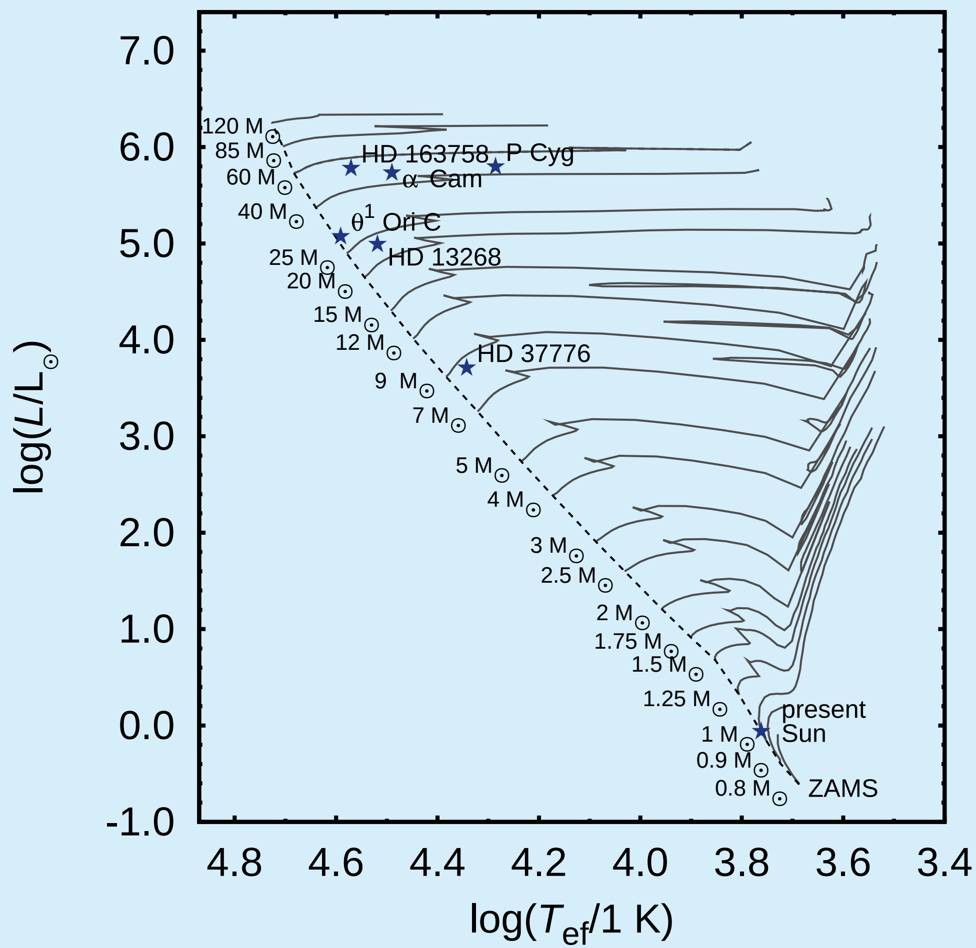


Stars in HR diagram



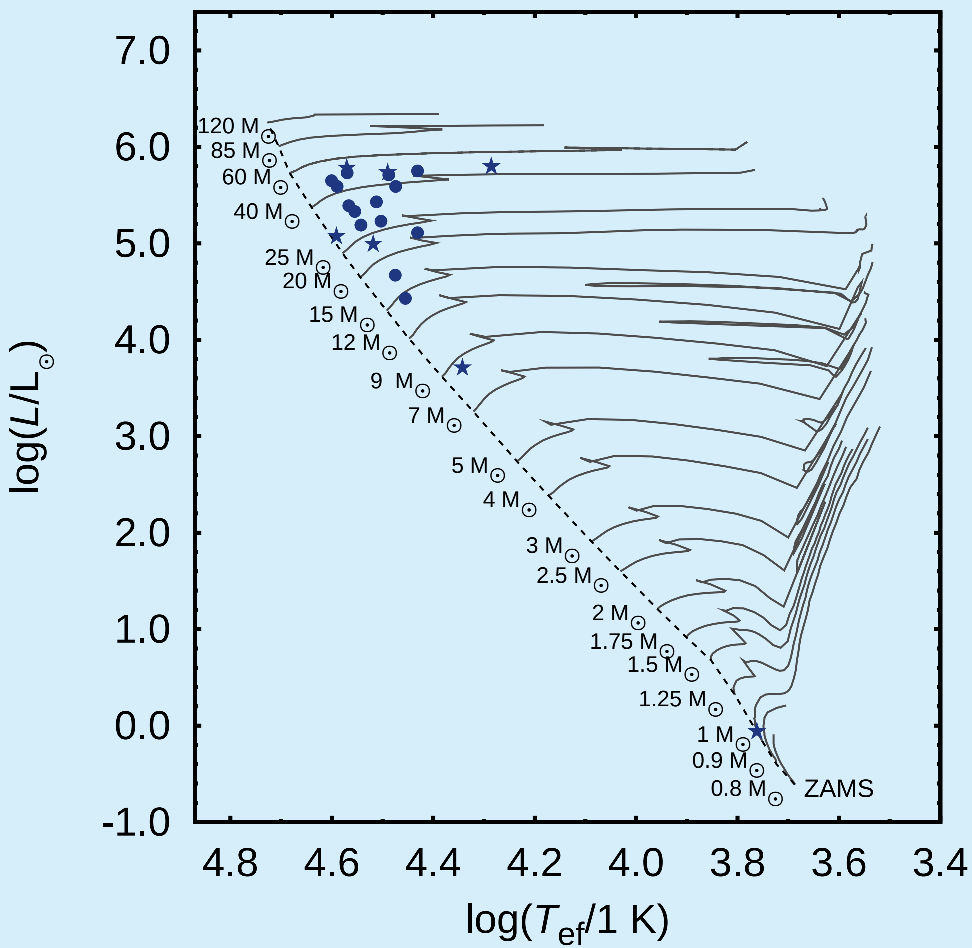
evolutionary tracks (Schearer et al. 1992)

Stars in HR diagram



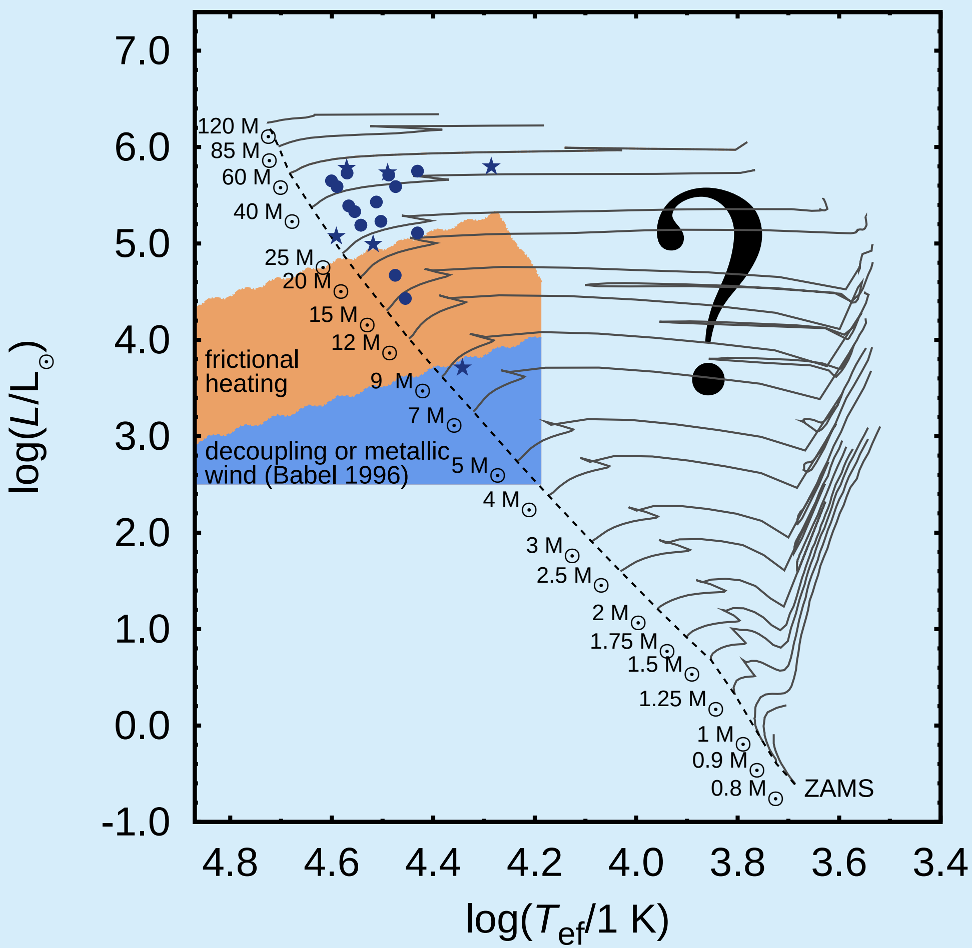
position of stars discussed here

Stars in HR diagram



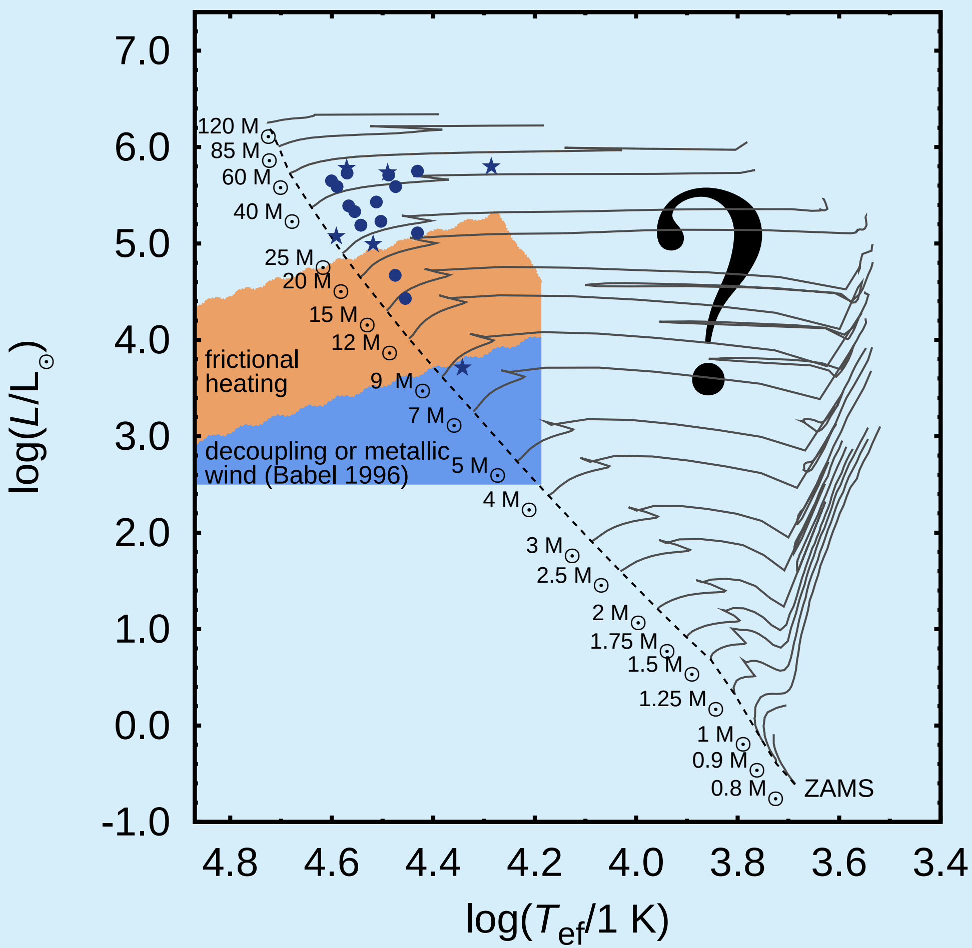
stars with P Cyg profiles (Püskülllü et al. 2008)

Stars in HR diagram



stars with different type of wind (Krtička et al. 2008)

Stars in HR diagram



stars more massive than $M \gtrsim 20\,\mathrm{M}_\odot$ have strong winds basically during all evolutionary phases

The importance of hot star wind I.

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The importance of hot star wind I.

- stars more massive than $M \gtrsim 20\,\mathrm{M}_\odot$ have strong winds basically during all evolutionary phases
- the duration of the main-sequence phase of massive stars is about $10^6\,\mathrm{yr}$
- during this time massive stars lose mass at the rate of the order of $10^{-6}\,\mathrm{M}_\odot\,\mathrm{yr}^{-1}$
- a significant part of stellar mass can be lost due to the winds

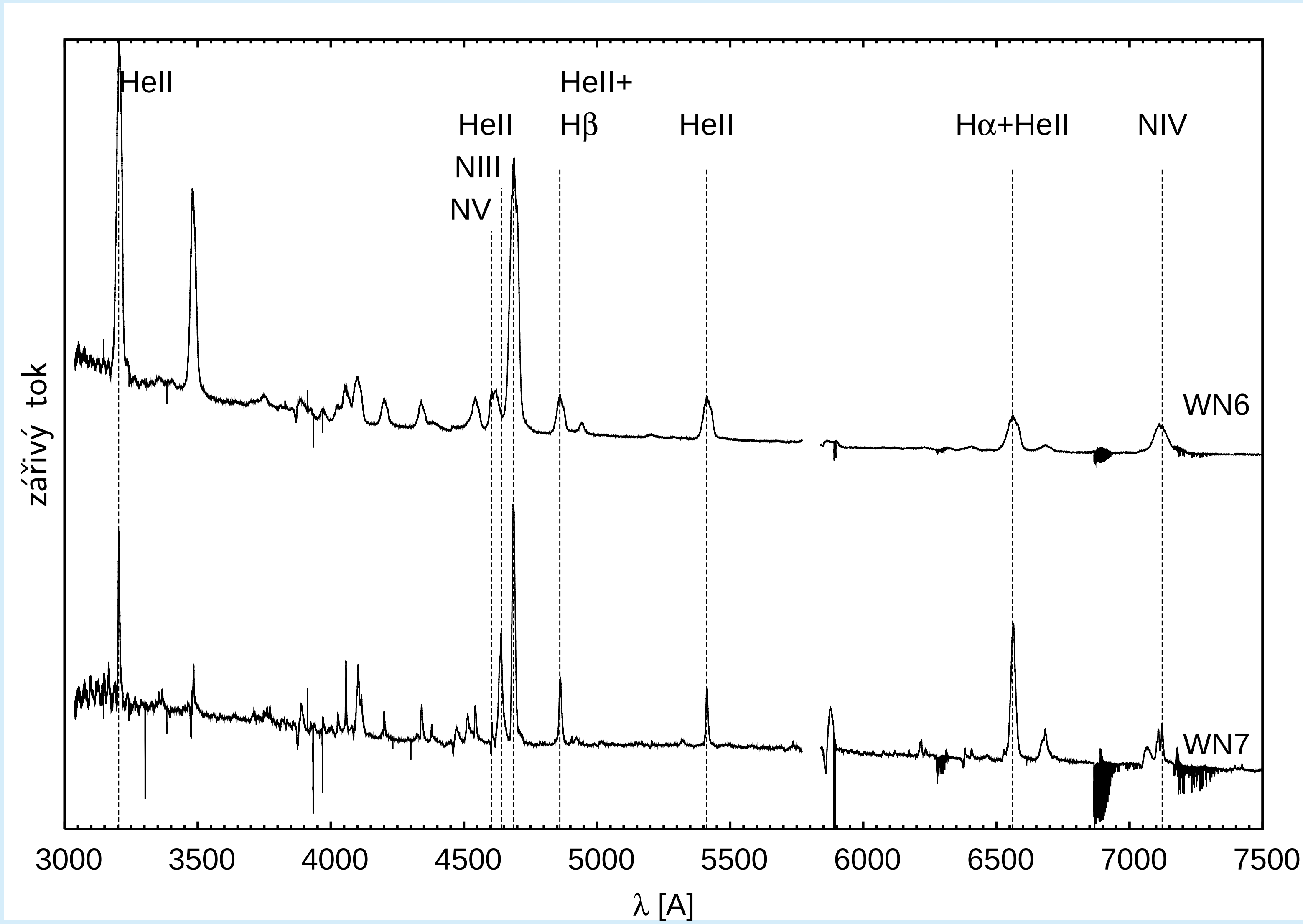
The importance of hot star wind II

- the evolutionary phases connected with the wind

The importance of hot star wind II

- the evolutionary phases connected with the wind
- Wolf-Rayett stars
  - hot stars with very strong wind (mass-loss rate could be of the order of $10^{-5}\,\mathrm{M}_\odot\,\mathrm{yr}^{-1}$)
  - wind starts already in the stellar atmosphere
  - spectrum dominated by emission lines
  - enhanced abundance of nitrogen and/or carbon and oxygen

The importance of hot star wind II



The importance of hot star wind II

- the evolutionary phases connected with the wind
- Wolf-Rayett stars
  - how can these stars originate?

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  - during the stellar evolution the core abundance of nitrogen (hydrogen burning) and carbon+oxygen (helium burning) increases

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  $\Rightarrow$ Wolf-Rayett stars

The importance of hot star wind II

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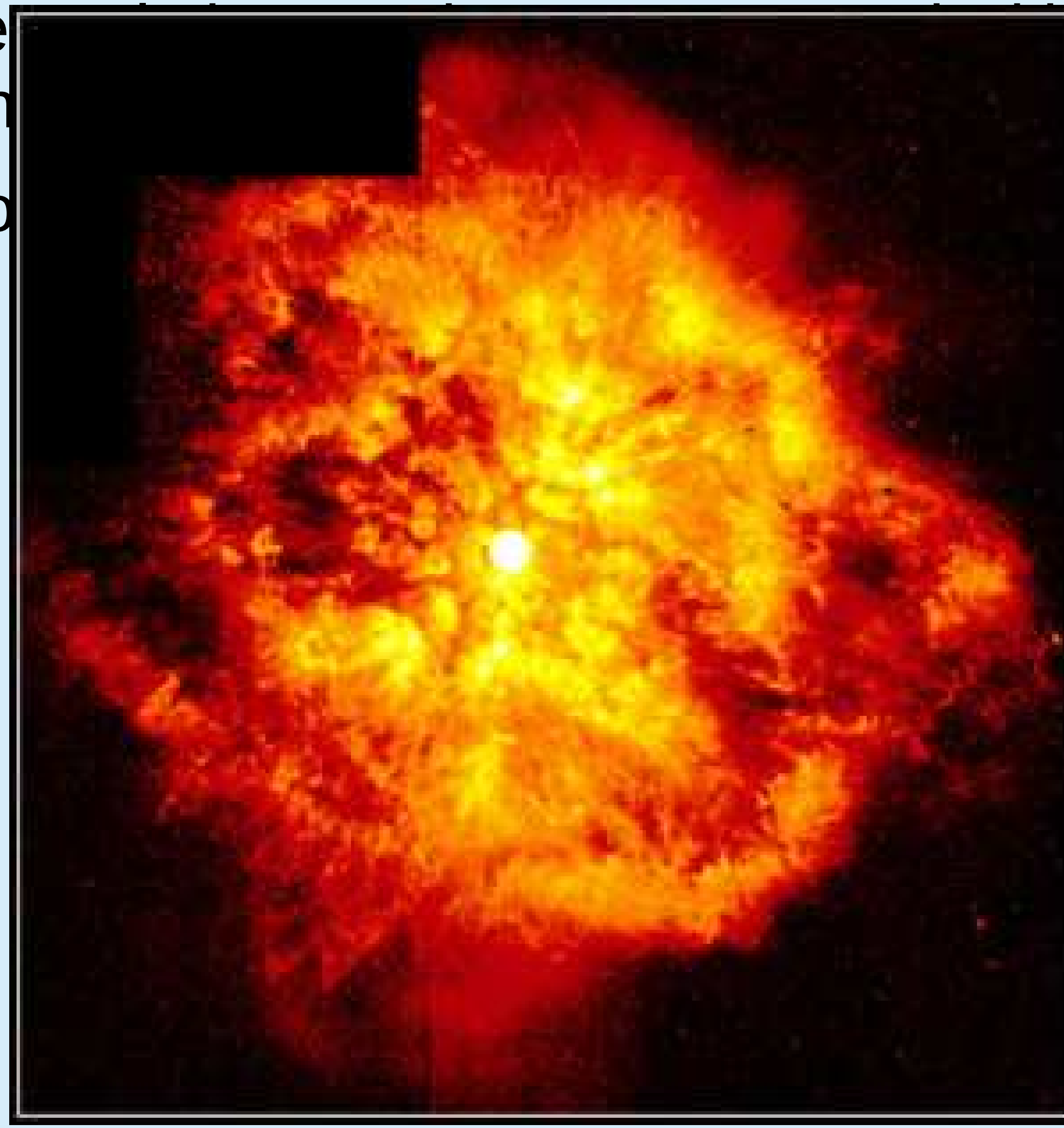

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$\Rightarrow$ planetary nebula: interaction of slow high-density and fast low-density winds

The importance of hot star wind II

- planetary nebulae
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The importance of hot star wind IV

- hot star wind influence also the interstellar environment

(e.g., Dale & Bonnell 2008)

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- hot star wind influence also the interstellar environment
  - enrichment of the interstellar medium

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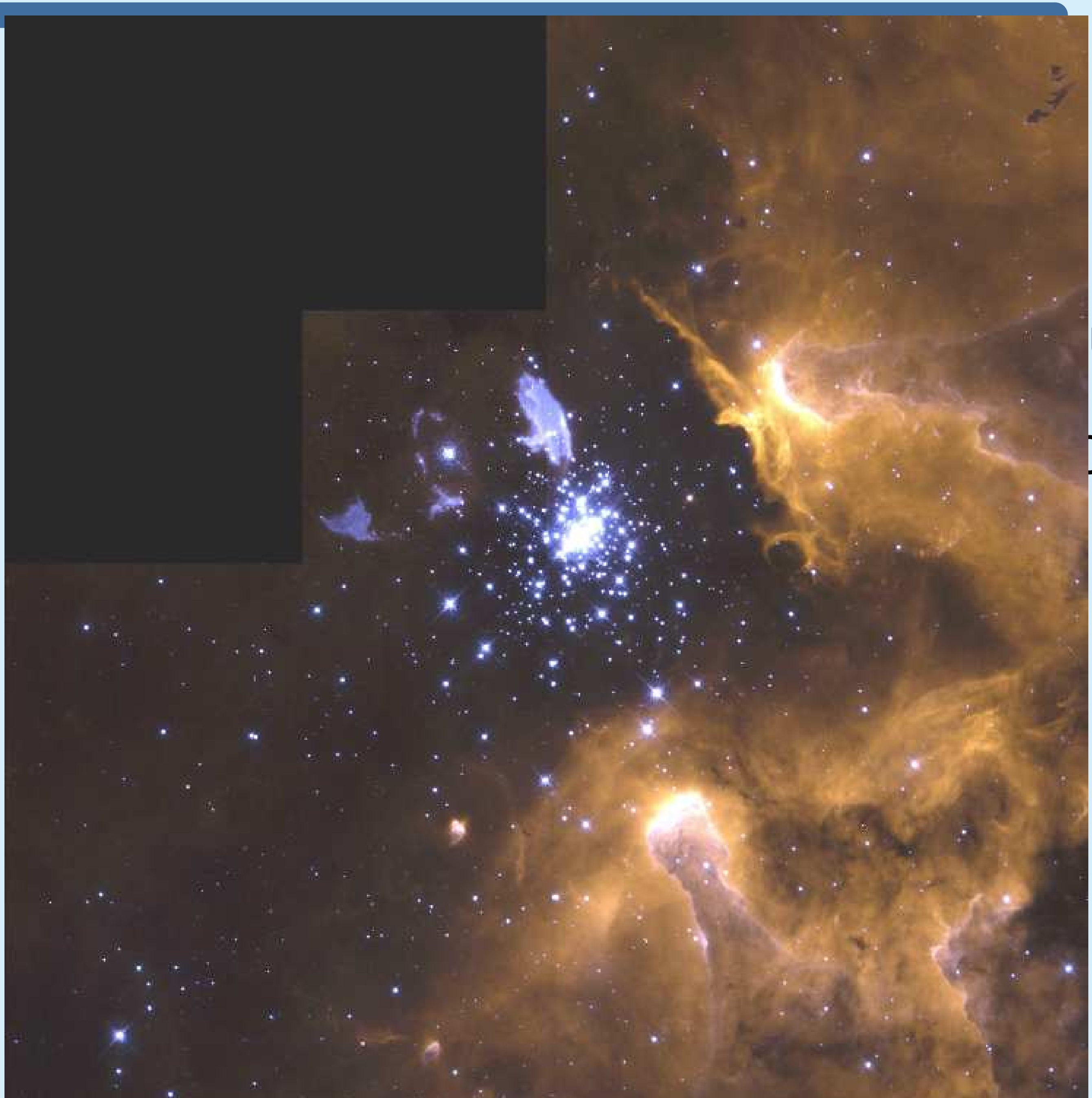
- hot star wind influence also the interstellar environment
  - enrichment of the interstellar medium
  - momentum input to the interstellar medium

(e.g., Dale & Bonnell 2008)

The importance of hot star wind IV

-



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What is unclear. . .

- chance for you!

What is unclear. . .

- the most uncertain quantity is . . .

What is unclear. . .

- the most uncertain quantity is the wind mass-loss rate!

What is unclear. . .

- the most uncertain quantity is the wind mass-loss rate!
- why?

What is unclear. . .

- mass-loss rate and observation

What is unclear. . .

- mass-loss rate and observation
- mass-loss rate can not be derived directly from observation

$$\dot{M} = 4\pi r^2 v \rho$$

  - v is fine
  - ρ is problematic

What is unclear. . .

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$\Rightarrow$ if $C > 1$ we significantly overestimate wind mass-loss rate (by a factor of $\sqrt{C}$)

What is unclear. . .

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- instability of the radiative driving $\Rightarrow$ clumpy wind

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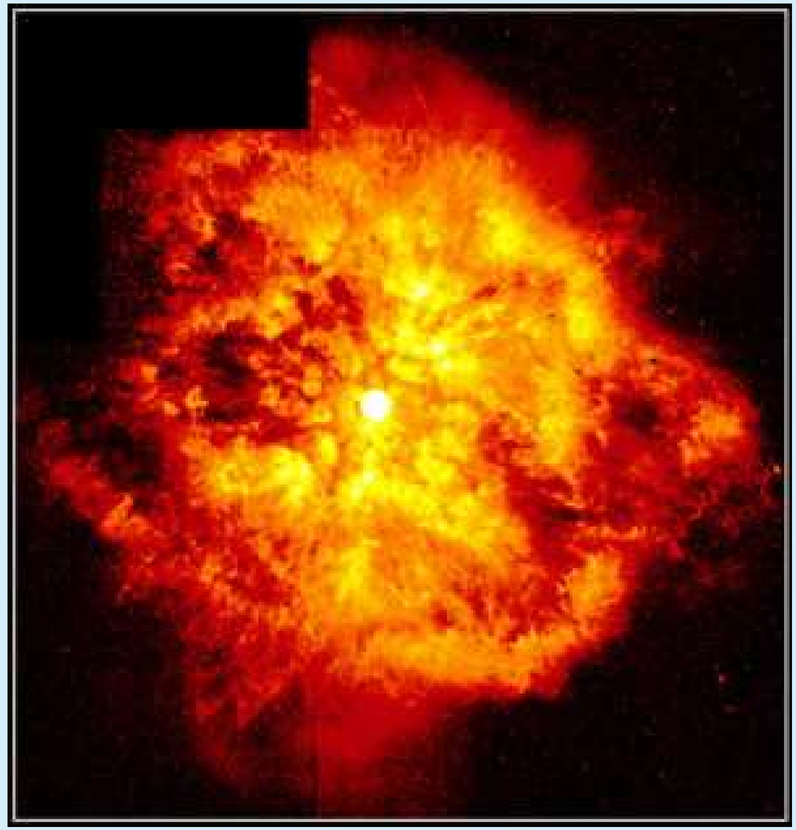
- what is the influence of inhomogeneities on the predicted mass-loss rates?

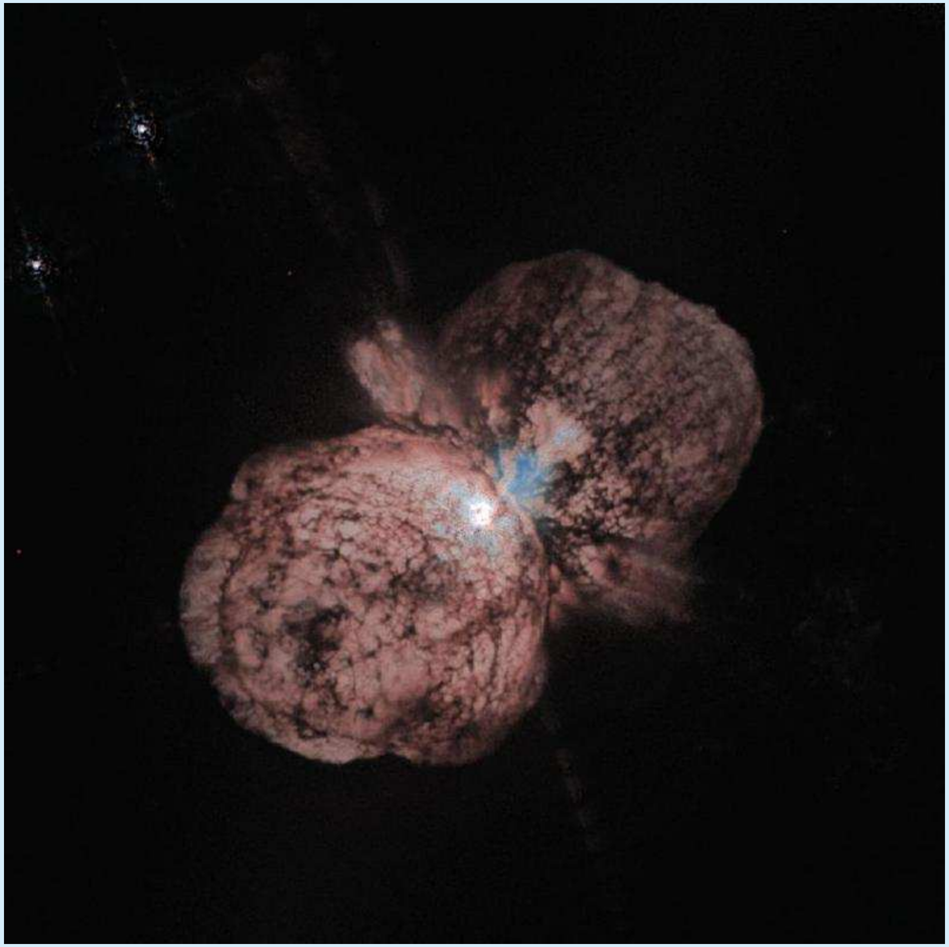
$\Rightarrow$ precise values of wind mass-loss rates can not be obtained until we underhand the influence of inhomogeneities

What is unclear II.



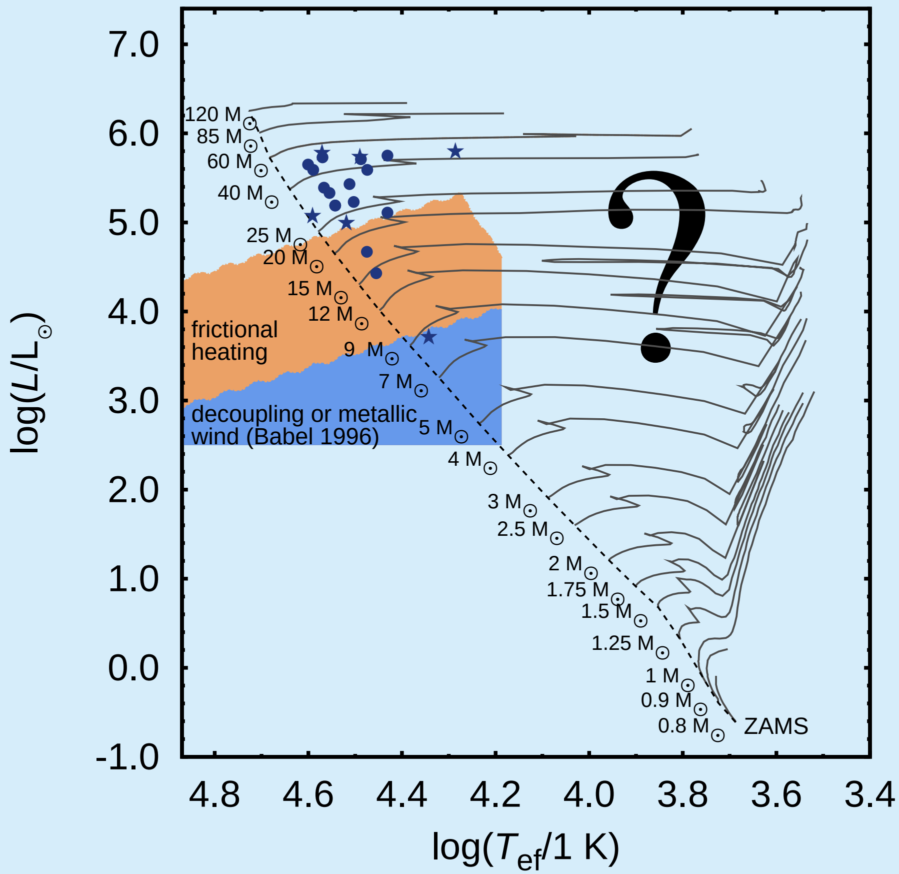
- what drives winds of WR stars? (Gräfener & Hamann 2005)

What is unclear III.



- what causes explosions like this?

What is unclear IV.



- what happens outside the well-studied regions?

More informations (papers)

- Milne, E. A. 1926, *MNRAS*, **86**, 459

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- Pauldrach A., Puls J., Kudritzki R. P. 1986, **A&A**, *164*, 86
- Gayley, K. G., Owocki, S. P. 1994, *ApJ*, **434**, 684
- Feldmeier, A., Puls, J., Pauldrach, A. W. A. 1997, *A&A*, **322**, 878

More informations (book, reviews

- Lamers, H. J. G. L. M. & Cassinelli, J. P., 1999, Introduction to Stellar Winds (Cambridge: Cambridge Univ. Press)

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- Lamers, H. J. G. L. M. & Cassinelli, J. P., 1999, Introduction to Stellar Winds (Cambridge: Cambridge Univ. Press)

- Kudritzki, R. P., Puls, J. 2000, *ARA&A*, **38**, 613 (http://www.usm.uni-muenchen.de/people/puls/Puls.html)
- Owocki, S. P. 2004, EAS Publications Series, Vol. 13, Evolution of Massive Stars, 163 (http://www.bartol.udel.edu/˜owocki/preprints)
- Krtička, J., Kubát, J. 2007, Active OB-Stars (San Francisco: ASP Conf. Ser), 153 (http://arxiv.org/abs/astro-ph/0511443)
- this lecture http://www.physics.muni.cz/˜krticka/belehrad.pdf

Conclusions

- hot star winds are accelerated by the radiative force due to the line transitions of heavier elements (carbon, nitrogen, silicon, iron, . . .)

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- the most important quantity is the mass loss rate (the amount of mass lost by the star per unit of time)

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Conclusions

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- the most important quantity is the mass loss rate
- the mass-loss rate depends mainly on the stellar luminosity
- mass-loss influences the stellar evolution and the circumstellar environment