$$d\varrho = \frac{p+\varrho}{n}\,dn + \underbrace{Tn\,ds}_{=0 \text{ (adiabatic)}}$$

$$\frac{p+\varrho}{n} \equiv w \quad \text{(enthalpy)}$$

$$\boxed{d\varrho = w\,dn} \qquad d\left(\frac{p+\varrho}{n}\,n\right) = dp + d\varrho = \underbrace{w\,dn}_{=d\varrho} + n\,dw$$

$$dp + \cancel{d\varrho} = \cancel{d\varrho} + n\,dw \qquad \boxed{dp = n\,dw}$$

$$\frac{dp}{d\varrho} \equiv c_s^2 = \frac{n\,dw}{w\,dn} = \frac{d\ln w}{d\ln n}$$

$$\boxed{\frac{n'}{n}c_s^2 = \frac{w'}{w}}$$

$$0 = u^i \nabla_i E = u^i \nabla_i (w u_t) \qquad \boxed{\frac{w'}{w} = -\frac{u_t'}{u_t}}$$

$$c_s^2 \frac{n'}{n} u_t^2 g^{tt} = -u_t u_t' g^{tt}$$

$$c_s^2 \frac{n'}{n} u_\varphi^2 g^{\varphi\varphi} = -u_\varphi u_\varphi' g^{\varphi\varphi}$$

$$c_s^2 \frac{n'}{n}\left(u_t^2 g^{tt} + u_\varphi^2 g^{\varphi\varphi}\right) = -\frac{1}{2}\left(u_t^2 g^{tt} + u_\varphi^2 g^{\varphi\varphi}\right)' + \frac{1}{2}\left[u_t^2 (g^{tt})' + u_\varphi^2 (g^{\varphi\varphi})'\right]$$

$$c_s^2 \frac{n'}{n}\left(1 - u_r^2 g^{rr}\right) = -\frac{1}{2}\left(1 - u_r^2 g^{rr}\right)' + \frac{1}{2} u_t^2 \left(\frac{\partial \mathcal{U}}{\partial r}\right)_\ell$$

$$\boxed{-u_r^2 g^{rr} \equiv V^2}$$

$$c_s^2 \frac{n'}{n}(1+V^2) = -\frac{1}{2}\cdot 2VV' + \frac{1}{2}u_t^2\left(\frac{\partial u}{\partial r}\right)_l$$

$$\boxed{c_s^2 \frac{n'}{n}(1+V^2) = -VV' + \frac{1}{2}u_t^2\left(\frac{\partial u}{\partial r}\right)_l}$$

$$0 = \nabla_i(nu^i) \Rightarrow \partial_r(r^2 u_r g^{rr} n) = 0$$

$$\frac{2}{r} + \frac{u_r'}{u_r} + \frac{(g^{rr})'}{(g^{rr})} + \frac{n'}{n} = 0$$

$$\frac{(V^2)'}{V^2} = 2\frac{V'}{V} = \frac{(g^{rr}u_r^2)'}{g^{rr}u_r^2} = 2\frac{u_r'}{u_r} + \frac{(g^{rr})'}{g^{rr}}$$

$$\frac{u_r'}{u_r} = \frac{V'}{V} - \frac{1}{2}\frac{(g^{rr})'}{(g^{rr})}$$

$$\boxed{\frac{2}{r} + \frac{V'}{V} + \frac{1}{2}\frac{(g^{rr})'}{(g^{rr})} + \frac{n'}{n} = 0}$$

$$V^2\frac{n'}{n} = -V'V - V^2\frac{2}{r} - \frac{1}{2}\frac{(g^{rr})'}{g^{rr}}V^2$$

$$-V^2\frac{n'}{n} = +V'V + V^2\frac{2}{r} + \frac{1}{2}\frac{(g^{rr})'}{g^{rr}}V^2$$

$$c_s^2 \frac{n'}{n}(1+V^2) = -VV' + \frac{1}{2}u_t^2\left(\frac{\partial u}{\partial r}\right)_l$$

$$\frac{n'}{n}\left[c_s^2(1+V^2) - V^2\right] = V^2\left(\frac{2}{r} + \frac{1}{2}\frac{(g^{rr})'}{g^{rr}}\right) + \frac{1}{2}u_t^2\left(\frac{\partial u}{\partial r}\right)_l$$

$$\left(\frac{\partial u}{\partial r}\right)_\ell = (g^{tt})' + \ell^2 (g^{\varphi\varphi})'$$

$$\ell_K^2 = -\frac{(g^{tt})'}{(g^{\varphi\varphi})'}$$

$$\left(\frac{\partial u}{\partial r}\right)_\ell = (g^{\varphi\varphi})' \left[\ell^2 - \ell_K^2\right]$$

$$(g^{\varphi\varphi})' = \left(-\frac{1}{r^2}\right)' = \frac{2}{r^3}$$

$$\left(\frac{\partial u}{\partial r}\right)_\ell = \frac{2}{r^3}\left(\ell^2 - \ell_K^2\right)$$

$$\frac{n'}{n}(1+V^2)\left[c_s^2 - \frac{V^2}{1+V^2}\right] = V^2\left(\frac{2}{r} + \frac{1}{2}\frac{(g^{rr})'}{g^{rr}}\right)$$
$$+ u_t^2 \frac{1}{r^3}\left(\ell^2 - \ell_K^2\right)$$

$$\frac{(g^{rr})'}{g^{rr}} = \frac{\left(1-\frac{r_G}{r}\right)'}{\left(1-\frac{r_G}{r}\right)} = \frac{r_G}{r^2}\left(1-\frac{r_G}{r}\right)^{-1}$$

$$\frac{n'}{n}(1+V^2)\left[c_s^2 - \frac{V^2}{1+V^2}\right] = \frac{2V^2}{r}\left[1 + \frac{1}{4}\left(\frac{r_G}{r}\right)\left(1-\frac{r_G}{r}\right)^{-1}\right]$$
$$+ u_t^2 \frac{\ell^2 - \ell_K^2}{r^3}$$

$$\boxed{\frac{2V^2}{r}\frac{1-\frac{r_G}{r}+\frac{1}{4}\frac{r_G}{r}}{1-\frac{r_G}{r}} = \frac{2V^2}{r}\frac{1-\frac{3}{4}\frac{r_G}{r}}{1-\frac{r_G}{r}}}$$

not a good idea!

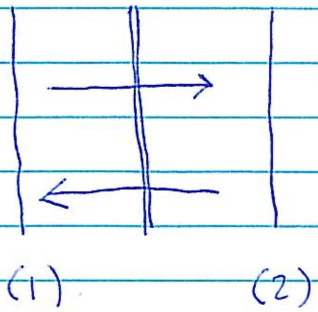
$$\frac{n'}{n}(1+V^2)\left(c_s^2 - \frac{V^2}{1+V^2}\right)$$

$$= \frac{2V^2}{r}\left[1 + \frac{1}{4}\left(\frac{r_G}{r}\right)\left(1 - \frac{r_G}{r}\right)^{-1}\right] + u_t^2 \frac{l^2 - l_K^2}{r^3}$$

positive

this must be negative at the sonic point

$$\boxed{l < l_K}$$ AT THE SONIC POINT



$\dot{M}_+ \; ; \; \dot{M}_-$

$\dot{J}_+ \; ; \; \dot{J}_-$

$$\dot{M} = \dot{M}_0 = \dot{M}_+ - \dot{M}_-$$

$$\dot{J}_+ = \dot{M}_+ \dot{j}_+ \; ; \; \dot{J}_- = \dot{M}_- \dot{j}_-$$

$$\dot{J} = \dot{M}_+ \dot{j}_+ - \dot{M}_- \dot{j}_-$$

$$\dot{J}_0 = \dot{M}_0 \dot{j}_0$$

$$\dot{j}_0 = \frac{\dot{M}_+ \dot{j}_+ + \dot{M}_- \dot{j}_-}{\dot{M}_+ + \dot{M}_-}$$

$$\dot{J} - \dot{J}_0 = \text{torque}$$

$$\text{torque} = \dot{M}_+ \dot{j}_+ - \dot{M}_- \dot{j}_- - (\dot{M}_+ - \dot{M}_-)\left(\frac{\dot{M}_+ \dot{j}_+ + \dot{M}_- \dot{j}_-}{\dot{M}_+ + \dot{M}_-}\right)$$

$$= \frac{(\dot{M}_+ \dot{j}_+ - \dot{M}_- \dot{j}_-)(\dot{M}_+ + \dot{M}_-) - (\dot{M}_+ - \dot{M}_-)(\dot{M}_+ \dot{j}_+ + \dot{M}_- \dot{j}_-)}{\dot{M}_+ + \dot{M}_-}$$

$$= \frac{\dot{M}_+^2 \dot{j}_+ - \dot{M}_+^2 \dot{j}_+ + \dot{M}_+ \dot{M}_- \dot{j}_+ + \dot{M}_+ \dot{M}_- \dot{j}_+}{(\dot{M}_+ + \dot{M}_-)}$$

$$- \frac{\dot{M}_-^2 \dot{j}_- + \dot{M}_-^2 \dot{j}_- - \dot{M}_+ \dot{M}_- \dot{j}_- - \dot{M}_+ \dot{M}_- \dot{j}_-}{(\dot{M}_+ + \dot{M}_-)}$$

$$= \frac{2 \dot{M}_+ \dot{M}_- (\dot{j}_+ - \dot{j}_-)}{\dot{M}_+ + \dot{M}_-}$$

$$u_t^2 = \text{const}$$

$$u_t^2 = (u^t g_{tt})^2 = \frac{g_{tt}^2}{g_{tt} + \Omega^2 g_{\varphi\varphi}}$$

$$\Omega^2 = \left(l \frac{g_{tt}}{g_{\varphi\varphi}}\right)^2 = l^2 \frac{g_{tt}^2}{g_{\varphi\varphi}^2}$$

$$u_t^2 = \frac{g_{tt}^2}{g_{tt} + l^2 \frac{g_{tt}^2}{g_{\varphi\varphi}^2} g_{\varphi\varphi}}$$

$$= \frac{g_{tt}}{1 + l^2 \frac{g_{tt}}{g_{\varphi\varphi}}} = \boxed{\frac{g_{tt}\, g_{\varphi\varphi}}{g_{\varphi\varphi} + l^2 g_{tt}} = a_0}$$

$$-\left(1 - \frac{r_g}{r}\right) r^2 \sin^2\theta = a_0 \left(-r^2 \sin^2\theta + l^2 \left(1 - \frac{r_g}{r}\right)\right)$$

$$\left[-\left(1 - \frac{r_g}{r}\right) r^2 + a_0 r^2\right] \sin^2\theta = a_0 l^2 \left(1 - \frac{r_g}{r}\right)$$

$$\sin^2\theta = \frac{a_0 l^2 \left(1 - \frac{r_g}{r}\right)}{r^2 \left(a_0 - 1 + \frac{r_g}{r}\right)}$$

$$a_0 l^2 \left(1 - \frac{r_g}{r}\right) = r^2 \left(a_0 - 1 + \frac{r_g}{r}\right)$$

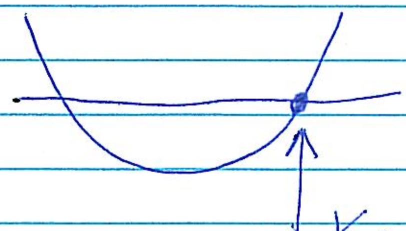
$$r a_0 l^2 - a_0 l^2 r_g = r^3 a_0 - r^3 + r_g r^2$$

$$\boxed{r^3 (a_0 - 1) + r_g r^2 - r a_0 l^2 + a_0 l^2 r_g = 0}$$

$$\frac{1}{u_t} \qquad \frac{dp}{p+\rho} = dW$$

$$W =$$

$$\boxed{U_{eff}(r,\theta) = U^0_{eff}}$$



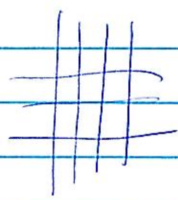
Keplerian at the center

$$U_{eff}(r,\theta) = U^0_{eff} + \frac{1}{2}\left(\frac{\partial^2 U}{\partial r^2}\right)\delta r^2 + \frac{1}{2}\left(\frac{\partial^2 U}{\partial r^2}\right)\delta\theta^2 + \ldots$$

$$U_{eff} - U^0_{eff} = \frac{1}{2}\omega_r^2(r - r_0)^2 + \omega_\theta^2\theta^2$$

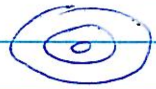
r_0^2

$$a^2\xi^2 + b^2\eta^2 = c^2$$

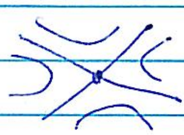


$$-a^2\xi^2 + b^2\eta^2 = c^2$$

$$a^2x^2 + b^2y^2 = c^2$$



$$-a^2x^2 + b^2y^2 = c^2$$

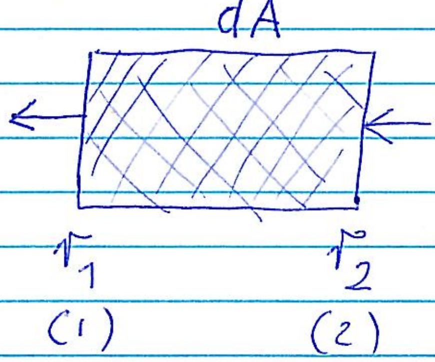


$$L_i \equiv T^{K}{}_{i}\,\xi_K \qquad \nabla_i L^i = 0$$

$$E_i \equiv T^{K}{}_{i}\,\eta_K \;;\quad \nabla_i E^i = 0$$

$$T^{i}{}_{K} = (p+\varrho)u^i u_K - \delta^{i}_{K} p$$
$$+\, a\sigma^{i}{}_{K} + f^i u_K + f_K u^i$$

$$0 = \int (\nabla_i L^i)\, dV$$



$$0 = \oint_{2} L^i dN_i - \oint_{1} L^i dN_i + \oint_{A} L^i dN_i$$

$$\oint_{1,2} L^i dN_i = \oint (p+\varrho)\, u^i (u\xi)\, dN_i$$
$$+ \oint p\, \xi^i dN_i$$
$$+ \oint a\sigma^{i}{}_{K}\, \xi^K dN_i$$
$$+ \oint (f^i u_K + f_K u^i)\, \xi^K dN_i$$

$$(j_2 - j_1)\dot M + T_2 - T_1 + u_\varphi f = 0$$

$$(e_2 - e_1)\dot M + \Omega_2 T_2 - \Omega_1 T_1 + u_t f = 0$$

$$\cancel{(e_2 - e_1)\dot M + \Omega_2 (j_2 - j_1)\dot M + u_t f = 0}$$

$$T_2 = T_1 - (j_2 - j_1)\dot M$$

$$(e_2 - e_1)\dot M + \Omega_2 T_1 - \Omega_1 T_1$$

$$- \Omega_2 (j_2 - j_1) + u_t f = 0$$

$$\frac{de}{dr}\dot M + \frac{d\Omega}{dr} T_1 - \Omega \frac{dj}{dr}\dot M + u_t f = 0$$

$$\Omega \frac{dj}{dr}$$

$$0 = \nabla_i \left(T^i_{\ k} \xi^k \right)$$

$$\nabla_i \Big[(p+\rho)\, u^i (u\xi) + \xi^i p + a \sigma^i_{\ k} \xi^k + f^i (u\xi) + u^i (\xi f) \Big]$$

$$u^i \nabla_i \left[\frac{p+\rho}{n} (u\xi) \right] + \nabla^i \left(a \sigma^i_{\ k} \xi^k \right) + \nabla_i \left(f^i (u\xi) \right) = 0$$

$$\nabla_i \left[\frac{p+\rho}{n} (u\xi)\, u^i \right] + \nabla_i \left(a \sigma^i_{\ k} \xi^k \right) + \underbrace{\nabla_i \left(f^i (u\xi) \right)}_{=0} = 0$$

$$\dot{M} \left(j_1 - j_2 \right) = T_1 - T_2$$

vertically integrated.

REPETITION

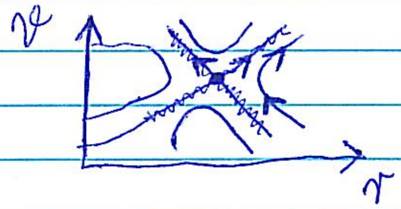
(1) The equipotential structure, ROCHE LOBE, THE CUSP

* new point — thin disks, vertical and radial gradients

(2) The sonic radius

$$\frac{r}{n}\frac{dn}{dr} = \frac{\mathcal{D}(n, v; r)}{c_s^2 - \tilde{v}^2}$$

EIGENVALUE
EIGENSOLUTION



Location of the sonic point
Location of the cusp

* new point: NO TORQUE AT THE SONIC POINT

$\rightarrow \dot{M}_+, \dot{j}_+$
$\leftarrow \dot{M}_-, \dot{j}_-$

$$\dot{M}_0 = \dot{M}_+ - \dot{M}_-$$

$$\dot{J} = \dot{M}_+ \dot{j}_+ - \dot{M}_- \dot{j}_-$$

$$\dot{j}_0 = \frac{\dot{M}_+ \dot{j}_+ + \dot{M}_- \dot{j}_-}{\dot{M}_+ + \dot{M}_-}$$

$$\dot{J}_0 = \dot{M}_0 \dot{j}_0$$

$$\boxed{\mathbb{T} = \dot{J} - \dot{J}_0} \leftarrow \text{torque}$$

$$\mathbb{T} = \dot{J} - \dot{J}_0 = \dot{M}_+ \dot{j}_+ - \dot{M}_- \dot{j}_- - (\dot{M}_+ - \dot{M}_-)\dot{j}_0$$

$$= \dot{M}_+ \dot{j}_+ - \dot{M}_- \dot{j}_- - \frac{(\dot{M}_+ - \dot{M}_-)(\dot{M}_+ \dot{j}_+ + \dot{M}_- \dot{j}_-)}{\dot{M}_+ + \dot{M}_-} =$$

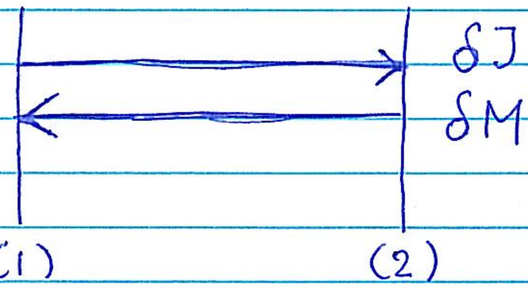
$$= \frac{1}{\dot{M}_+ + \dot{M}_-}\Big[(\dot{M}_+ j_+ - \dot{M}_- j_-)(\dot{M}_+ + \dot{M}_-) - (\dot{M}_+ - \dot{M}_-)(\dot{M}_+ j_+ + \dot{M}_- j_-)\Big]$$

$$= \frac{1}{\dot{M}_+ + \dot{M}_-}\Big[\cancel{\dot{M}_+^2 j_+} + \dot{M}_+ \dot{M}_- j_+ - \dot{M}_- \dot{M}_+ j_- - \cancel{\dot{M}_-^2 j_-} - \cancel{\dot{M}_+^2 j_+} - \dot{M}_+ \dot{M}_- j_- + \dot{M}_- \dot{M}_+ j_+ + \cancel{\dot{M}_-^2 j_-}\Big]$$

$$\boxed{= 2\,\frac{\dot{M}_+ \dot{M}_-}{\dot{M}_+ + \dot{M}_-}\,(j_+ - j_-) = \mathbb{T}}$$

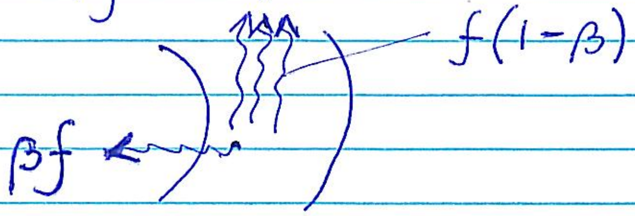
Torque is zero at the sonic point.

③ The flux formula



$$(\text{flux}) = \frac{\dot{M}}{2\pi r}\,\frac{d\Omega}{dr}\,(j - j_0)$$

Entropy complication



November 29 (3)

$$(\text{flux}) = \frac{(\text{energy})}{(\text{area})(\text{time})} = \frac{(\text{momentum})\cdot c}{(\text{area})(\text{time})}$$

$$= \underbrace{\left[\frac{(\text{momentum})}{(\text{time})}\right]}_{\text{force}} \frac{c}{\sigma}$$

$$\boxed{(\text{flux}) = \frac{c}{\sigma}(\text{force})}$$

definition of the cross-section

$$(\text{force}) = \frac{\sigma}{c}(\text{flux})$$

$$\frac{GMm_p}{R^2} = \frac{\sigma}{c}(\text{flux})$$

$$\frac{GMm_pc}{\sigma R^2} = (\text{flux})$$

$$\frac{4\pi GMm_pc}{\sigma} = \text{luminosity}$$

$$\boxed{\frac{4\pi GM_\odot m_pc}{\sigma}\left(\frac{M}{M_\odot}\right) = L_{Edd}}$$

$= 10^{38}$ erg/sec (10^{33}, why)

November 29 (4)

$$\dot{M}_{Edd} c^2 = L_{Edd}$$

$$\boxed{\dot{M}_{Edd} = L_{Edd}/c^2}$$

A propos: c^5/G $\quad$ (10^{54} Watt)

$\boxed{\text{EQUATIONS (NEWTONIAN) VERTICALLY INTEGRATED}}$

$$\rho v \frac{dv}{dR} = \rho(\Omega^2 - \Omega_K^2) R - \frac{dP}{dR}$$

$$\dot{M}\left[\frac{dU}{dR} + p\, dV\right] = 4\pi R^2 H (\tau_{R\phi}) \frac{d\Omega}{dR} + 4\pi R F$$

$$\dot{M} = 4\pi R H \rho v$$

$$\dot{M}(j - j_0) = 4\pi R^2 H (\tau_{R\phi})$$

$$F = \frac{acT^4}{\kappa \rho H} \; ; \; \tau_{R\phi} = \rho [\alpha c_s H] R \frac{d\Omega}{dR}$$

$$\frac{c_s}{\Omega_K R} = \frac{H}{R} \quad \text{(derive this)}$$

$p = \ldots \qquad V = \ldots \qquad c_s = \ldots$

$\kappa = \ldots \qquad j = \Omega R^2$

November 29 5

Shakura - Sunyaev { gas
radiation

Slim

Adaf

... and the FINAL GRAPH