White Dwarfs

Their evolution and structure

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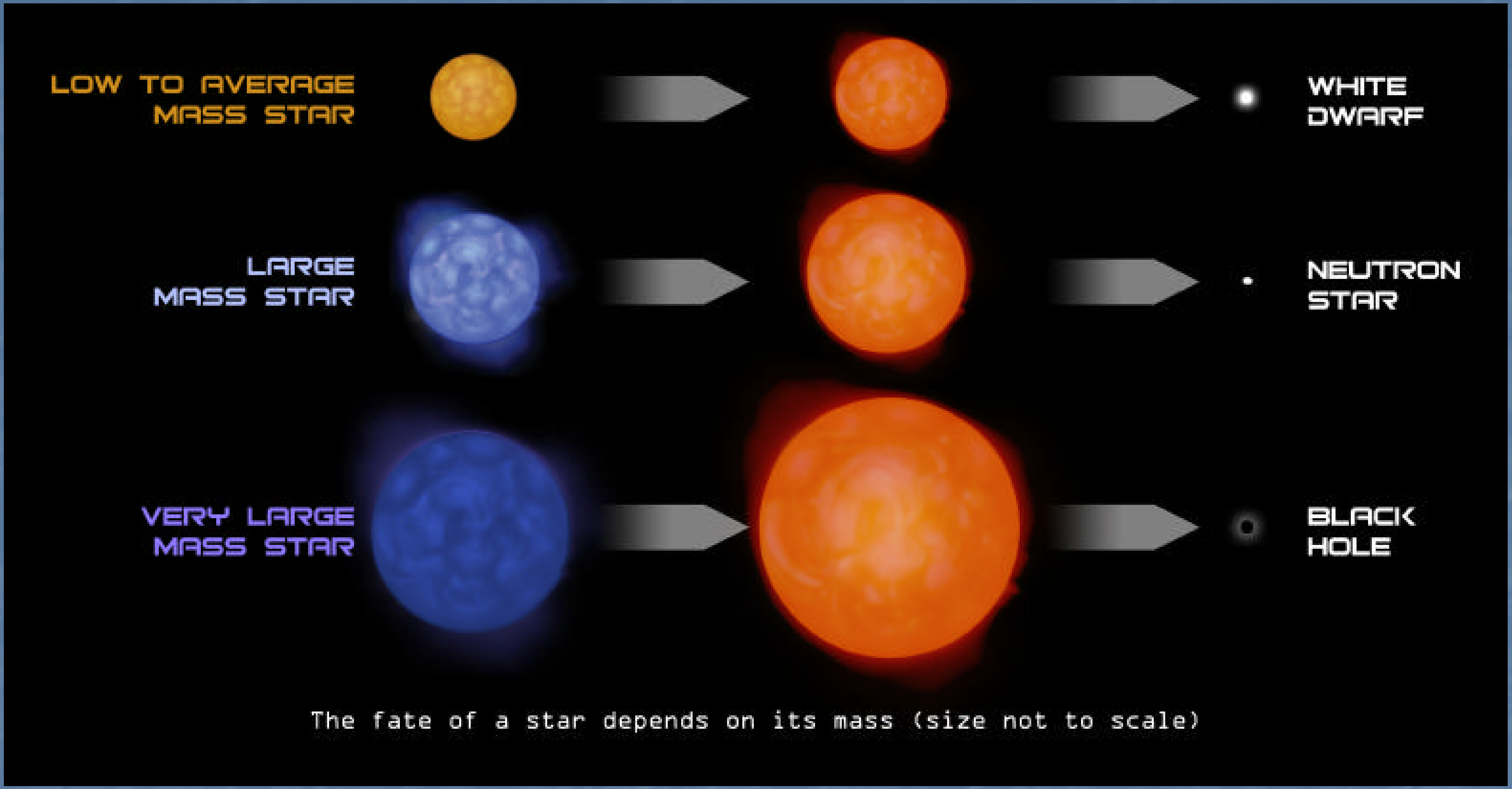
Astronomický ústav AV CR

Outline

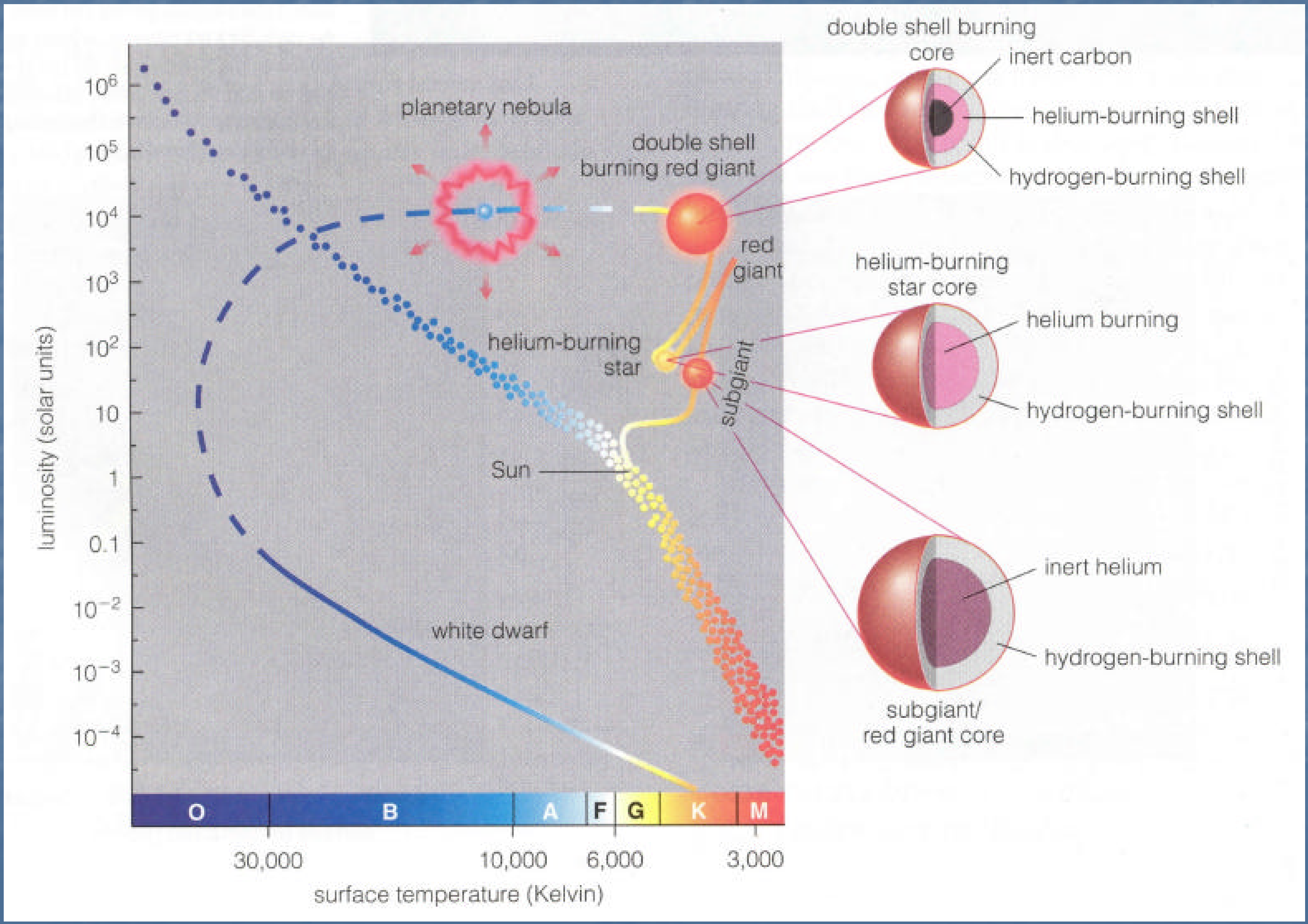
- Evolution toward a white dwarf.
- White dwarf properties.
- Brief history of the discovery and observations of white dwarf stars.
- White dwarf structure.
- Atmospheres.
- Evolution – cooling.
- White dwarfs in binary systems.
- Their distribution and importance in the Galaxy.
- White dwarfs in globular clusters.

Evolution



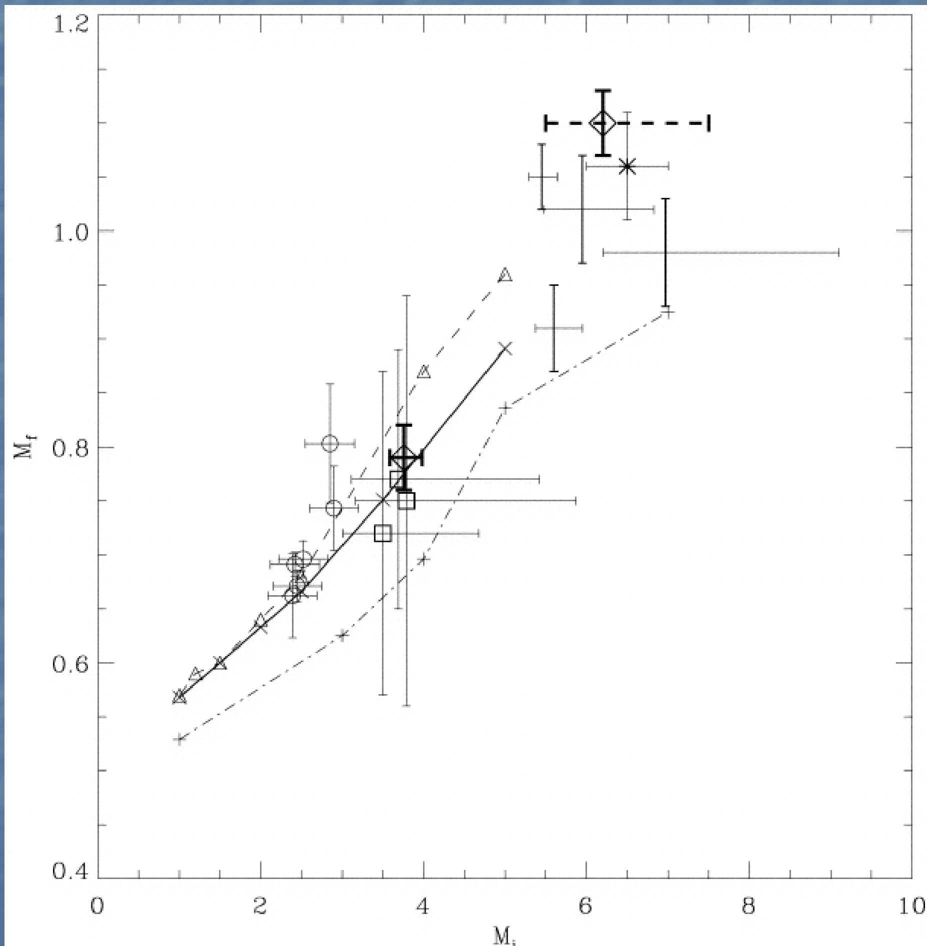
- About 90% of stars will evolve into white dwarfs.
- Stars with a mass below ~8 $M_{\odot}$ will evolve into white dwarfs.
- More than 10% of stars are white dwarfs.

Evolution



planetary nebula
double shell burning red giant
red giant
helium-burning star
subgiant
Sun
white dwarf
luminosity (solar units)
10^6
10^5
10^4
10^3
10^2
10
1
0.1
10^{-2}
10^{-3}
10^{-4}
O
B
A
F
G
K
M
30,000
10,000
6,000
3,000
surface temperature (Kelvin)
double shell burning core
inert carbon
helium-burning shell
hydrogen-burning shell
helium-burning star core
helium burning
hydrogen-burning shell
inert helium
hydrogen-burning shell
subgiant/ red giant core

Initial-to-final mass relations



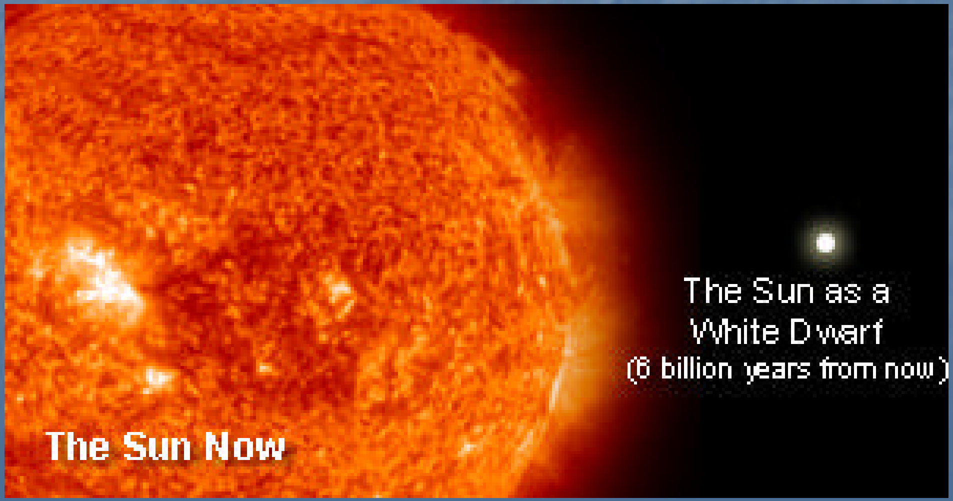
Finley & Koester 1997

- From cluster evolution and binary stars:
  - PG 0922+162,
  - Pleiades,
  - NGC 2516,
  - NGC 3532.
- And theoretical relations.

White Dwarf Properties

HST photo of the Helix nebula (NGC 7293)

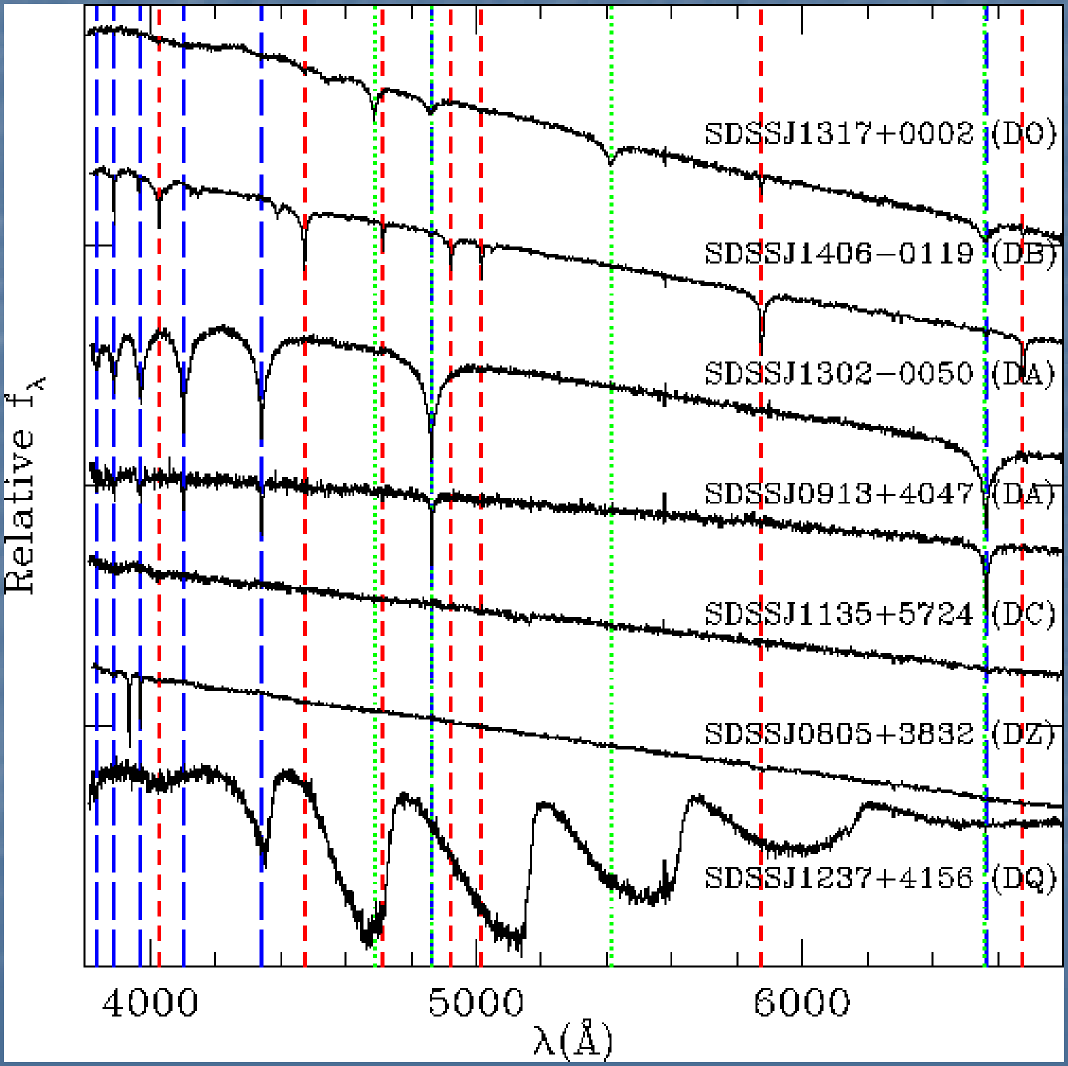
- They are very compact objects with $\rho = 10^6 - 10^9$ gcm^{-3}.
- They are supported by electron degenerate pressure.
- They have low luminosities due to their small radii ($\sim 10^9$ cm).



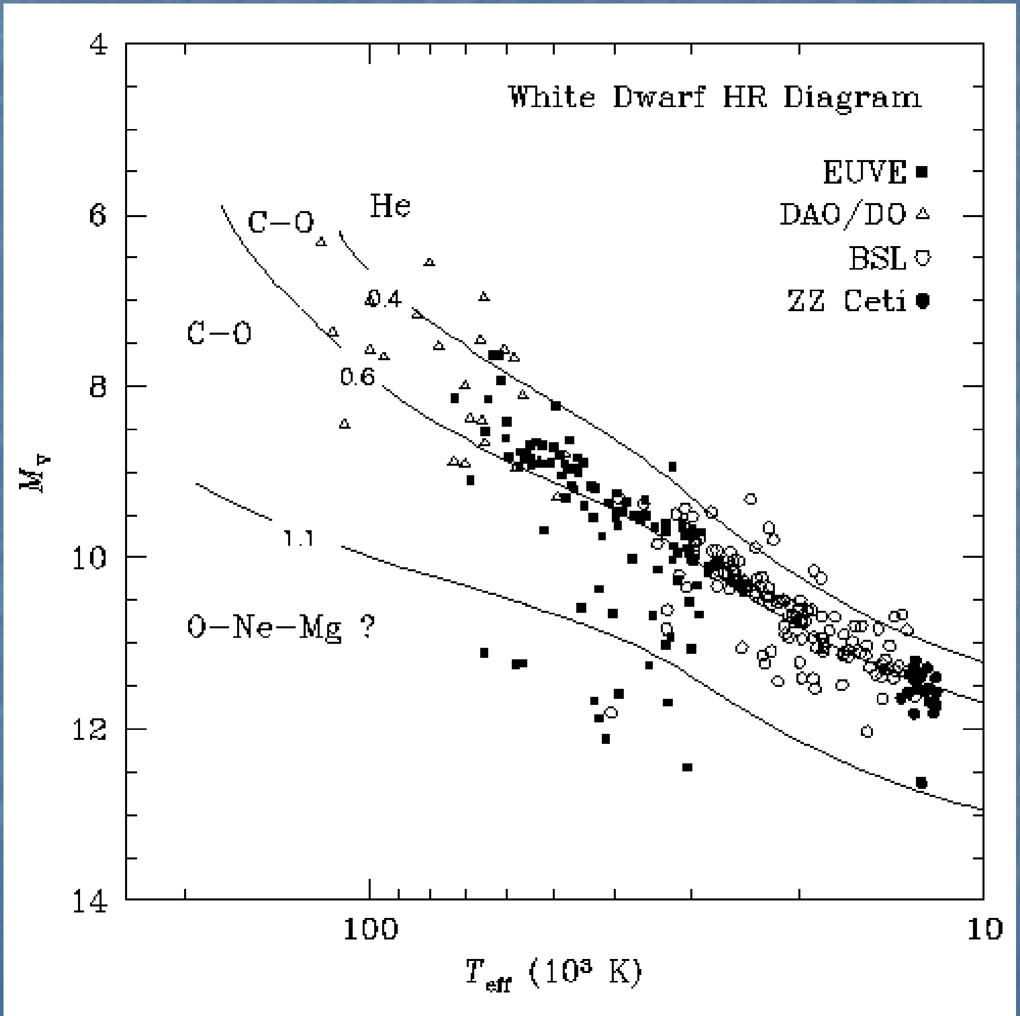
Atmospheric Composition

- DA (hydrogen-rich)
- DO (helium-rich: He II)
- DB (helium-rich: He I)
- DC (continuous spectrum)
- DQ (carbon features)
- DZ (metal lines)
- Other classifications:
  - H – magnetic
  - P – polarized magnetic
  - V – variable.
- $\theta = 50400/T$

(Sion et al. 1983, ApJ, 269, 253)

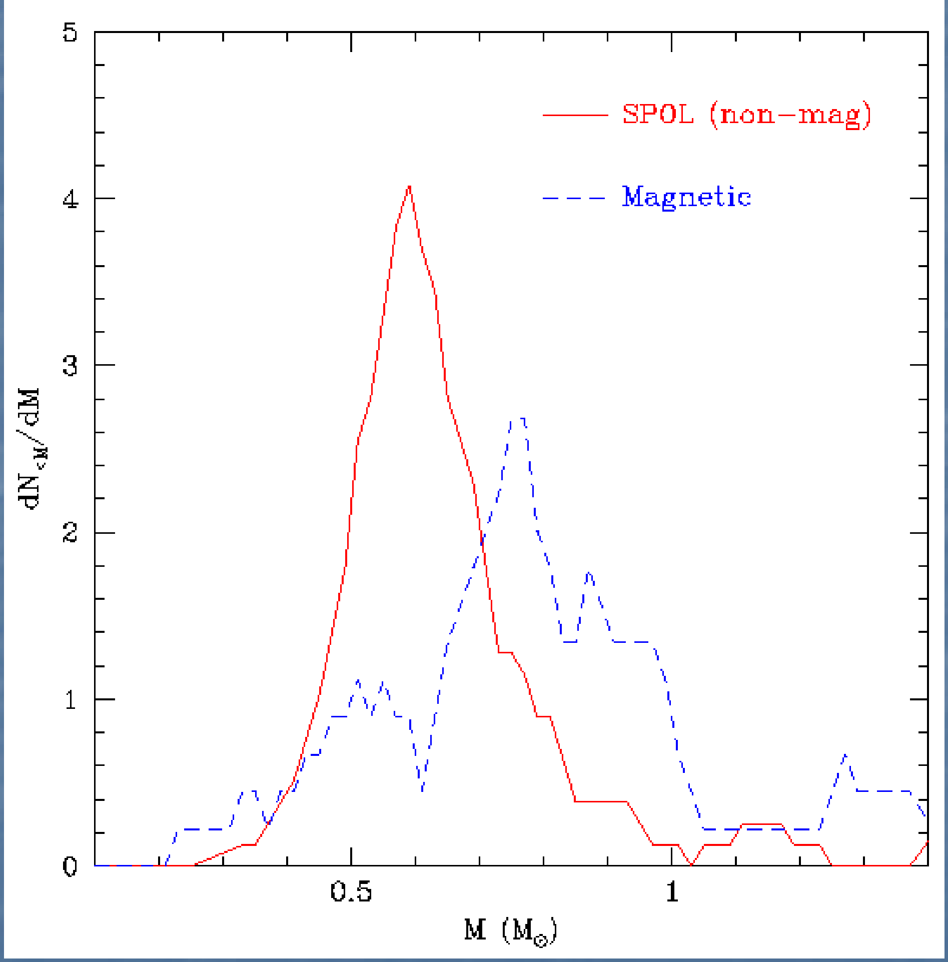


Temperature and Cooling Age

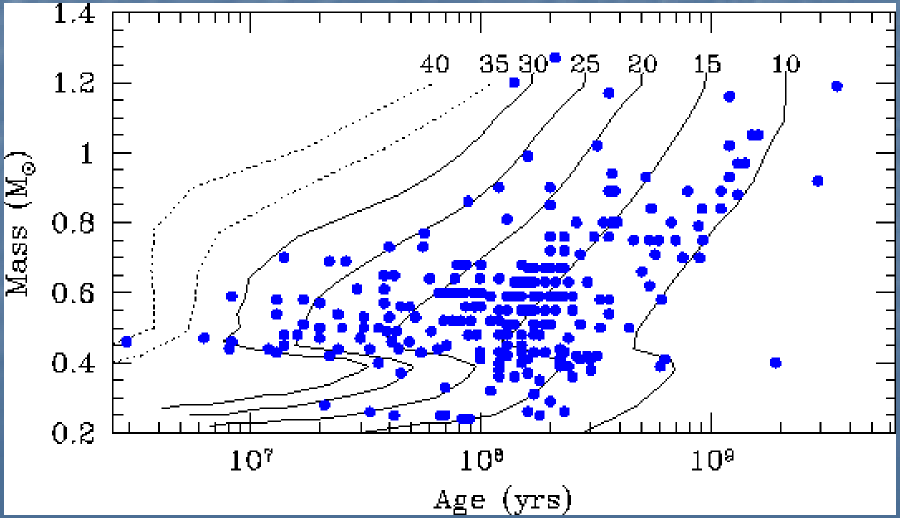


- Cooling age and mass: can be derived from the effective temperature and surface gravity using theoretical mass-radius relations.
- Temperature ranges from ~100 000 K down to about 3000 K.
- Corresponding to 10^{10}yrs of cooling.

Mass Distribution

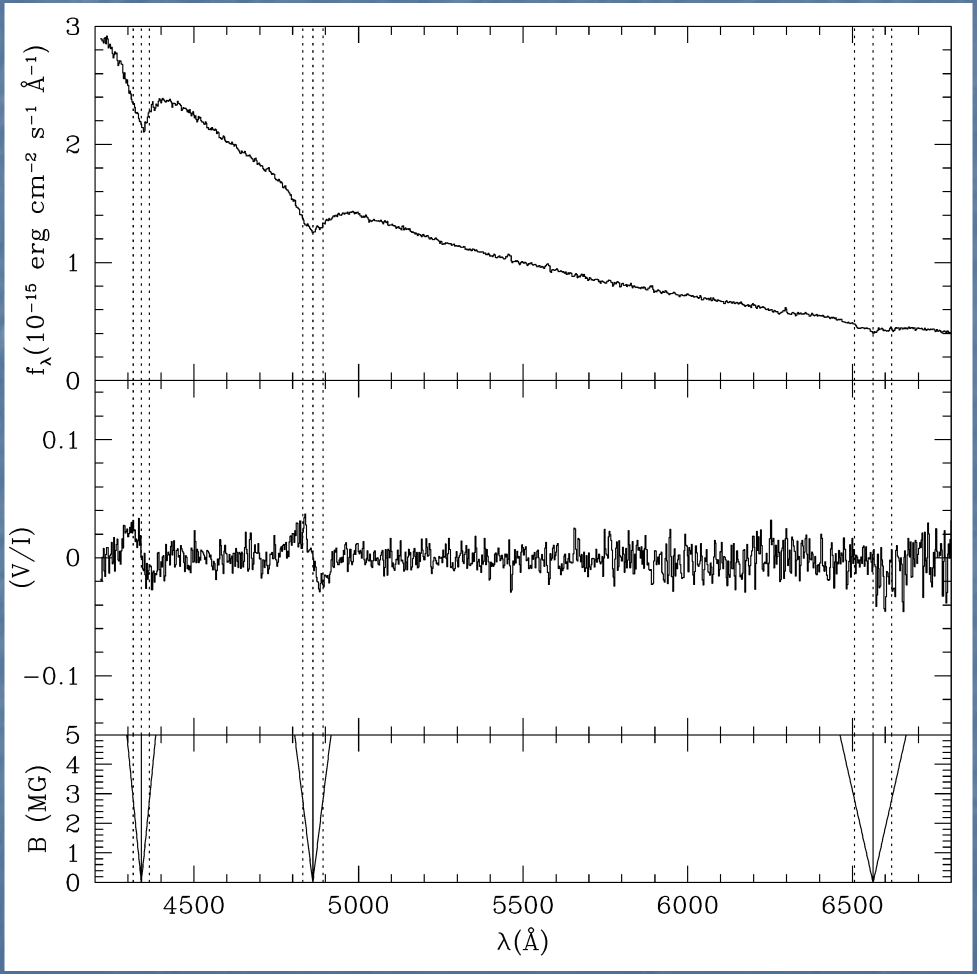


- Average mass of a white dwarf is ~$0.6M_\odot$.
- For most white dwarfs $0.4M_\odot < M_{WD} < 1.2M_\odot$.
- Very low mass white dwarfs must have evolved through close binary stars.



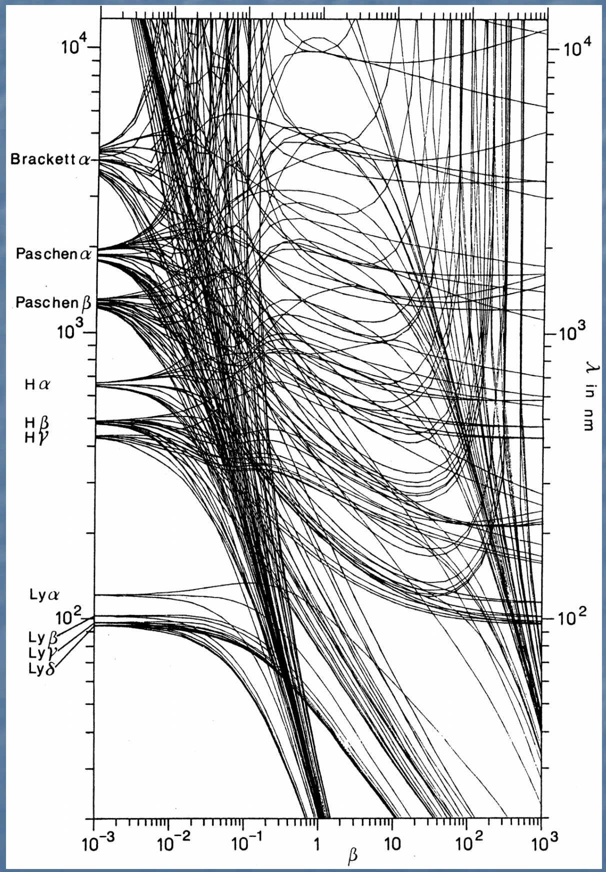
Magnetic Fields

- About 20% of white dwarfs have magnetic fields (~1 kG – 1000MG).
- Magnetic field appear to be fossil remains.
- The progenitors of magnetic white dwarfs are believed to be Ap/Bp stars (~1kG – 10kG).
- Our sun has a global, roughly dipolar field of ~1G and near sunspots magnetic loops are seen to have B ~ 1 - 10 kG.

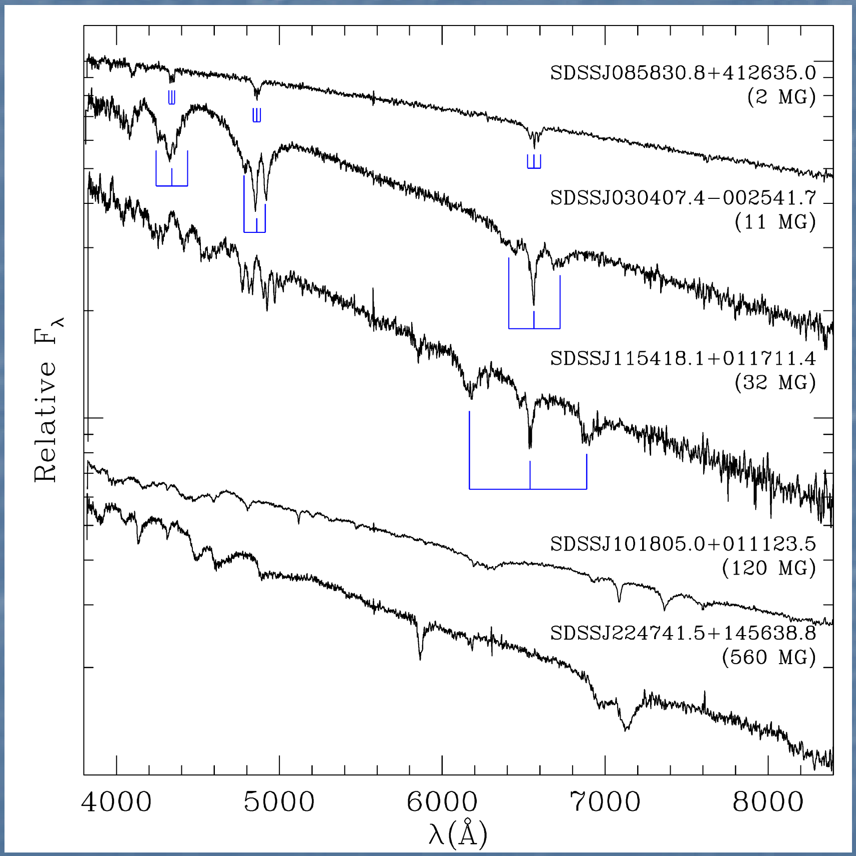


EUVE J0823-254: B_p = 3.5 MG, B_l = 0.6 MG

Magnetic Fields

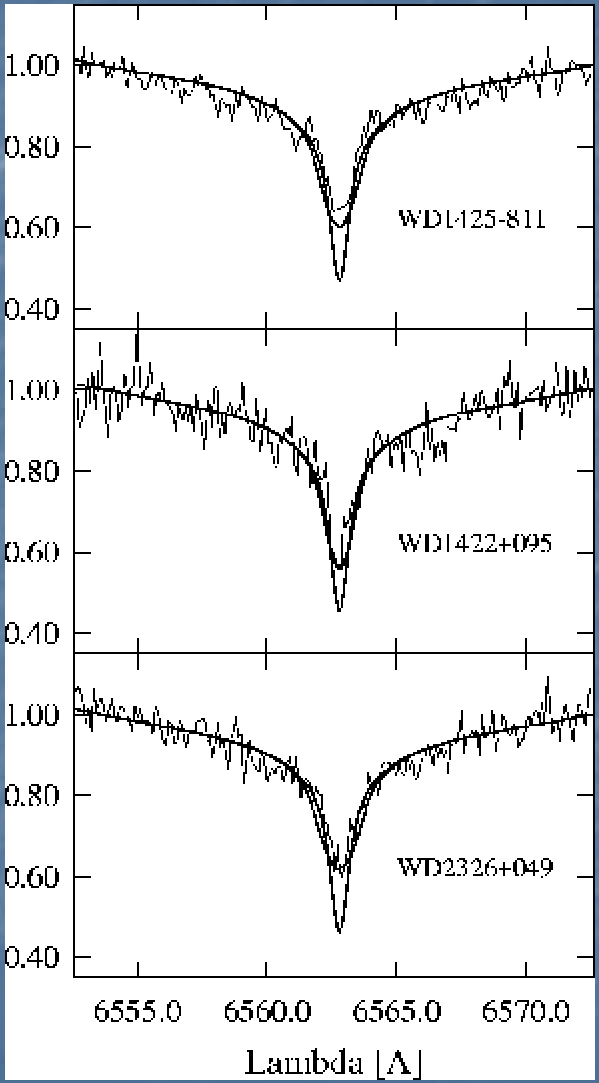


(Wunner 1990)



Rotation

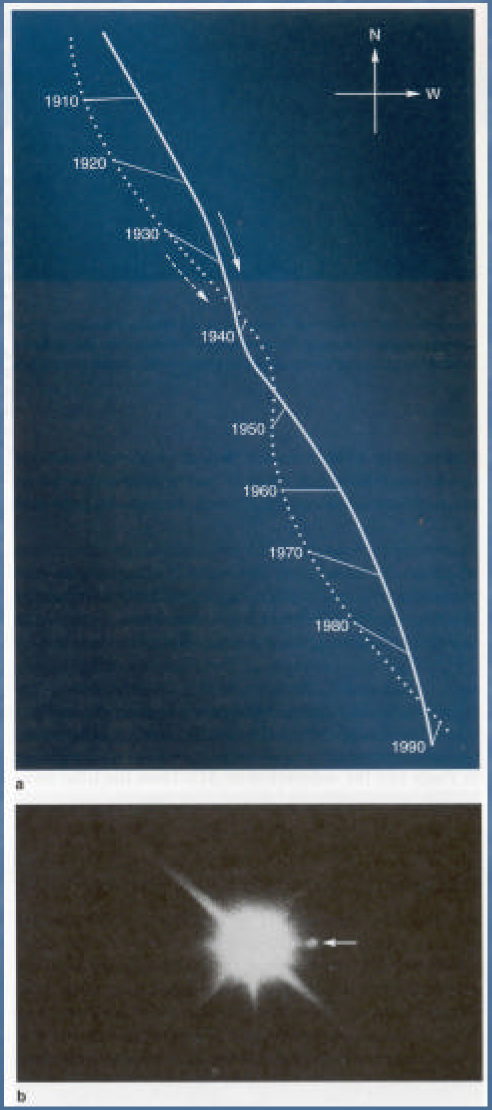
- White dwarfs are generally slow rotators (8-40 km s^{-1}).
- We can measure the rotation rates (v sin$i$):
  - From the shape of hydrogen line cores.
  - Variation of polarization in magnetic white dwarfs.
  - Asteroseismology.
- WD1425-811: 38±3 km s^{-1}
- WD1422+095: 29±7 km s^{-1}
- WD2326+049: 45±5 km s^{-1}



Koester et al. 1998

Discovery of white dwarfs

- Friedrich W. Bessel observed Sirius between 1834 and 1844 and combined with observations dating back to 1755 noticed that Sirius was oscillating about its apparent path on the sky and concluded that it must have a companion (1844, MNRAS, 6, 136).
- Sirius B was the *dark* companion to Sirius A until it was observed by Alvan G. Clark in 1862 while testing his father's new 18-inch refractor.
- Bessel also found Procyon to have a *dark* companion.
- Procyon B was first observed in 1896 by John M. Schaeberle using the 36-inch refractor at Lick Observatory.



Sirius B and Procyon B

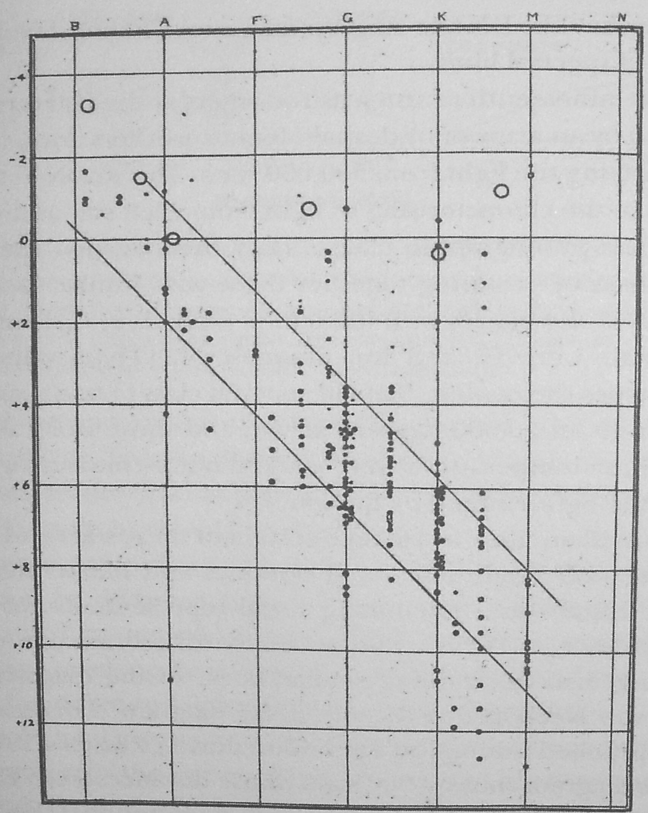
Sirius

- Sirius A is an A1 star.
- The brightest white dwarf next to the brightest star in the sky (V ~ 8 mag compared to V ~ -1.5 mag).
- The orbital period is 50 years (modern value is 49.9 years).
- And its distance is 2.63 pc (8.7 light years).

Procyon

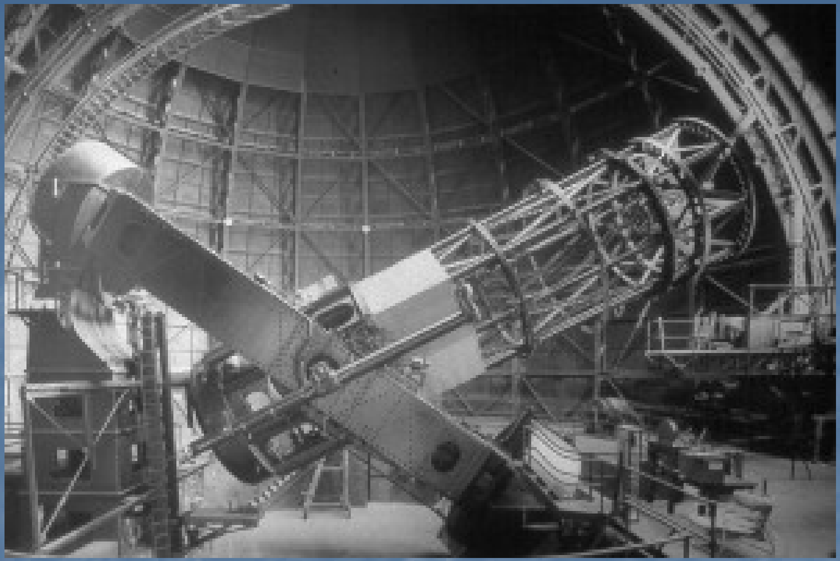
- Procyon A is an F5 star.
- The primary is at V ~ 0.3 mag compared to V ~ 10.7 mag for the white dwarf.
- The orbital period is 40 years (modern value is 40.8 years).
- And its distance is 3.5 pc (11.4 light years).

40 Eridani B



The original Hertzsprung-Russell (H-R) diagram, with spectral types listed along the top and absolute magnitudes on the left. (From Henry Norris Russell, *Nature* 93, 1914)

- 40 Eridani B was the first white dwarf for which a spectrum was obtained as part of the Henry Draper Catalogue (Cannon & Pickering 1918, Harvard Annals, 92).
- The star was classified as an A star, but Henry N. Russell disregarded because its "... spectrum is very doubtful"
- In the same year W.S. Adams noted its peculiarity.

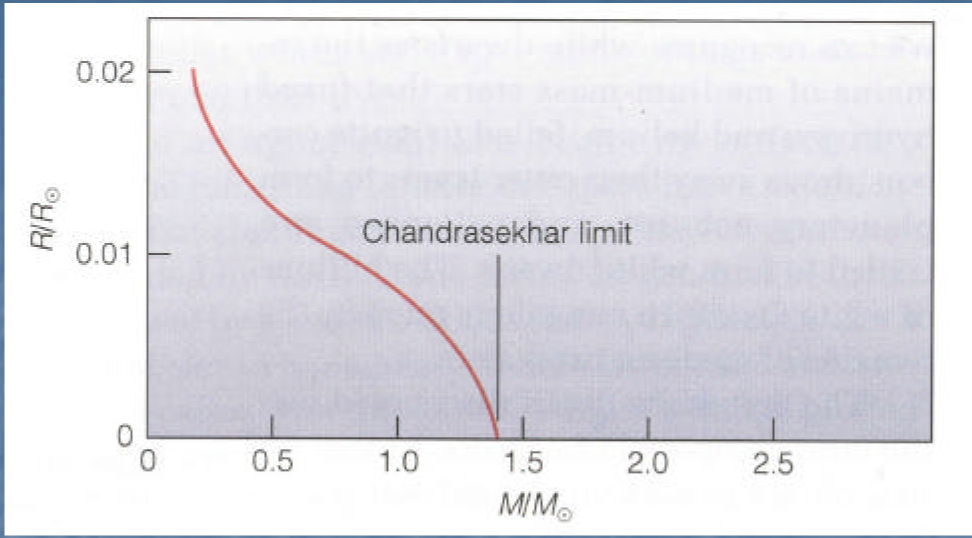


Sirius B

- The first spectrum of Sirius B was obtained by Walter S. Adams using the 60-inch reflector at Mt. Wilson Observatory in 1914.

- Adams obtained another spectrum using the 100-inch reflector and used it to determine a gravitational redshift of 21 km s^{-1} (1925, Proc. Natn. Acad. Scie., 11, 382).
- This measurement agreed with the predicted value +20 km s^{-1} by Arthur S. Eddington (1924, MNRAS, 84, 308).
- However, Eddington's calculation was based on the contemporary knowledge of white dwarfs, i.e., he used general relativity, but his assumed structure of the white dwarf was wrong.

Sirius B



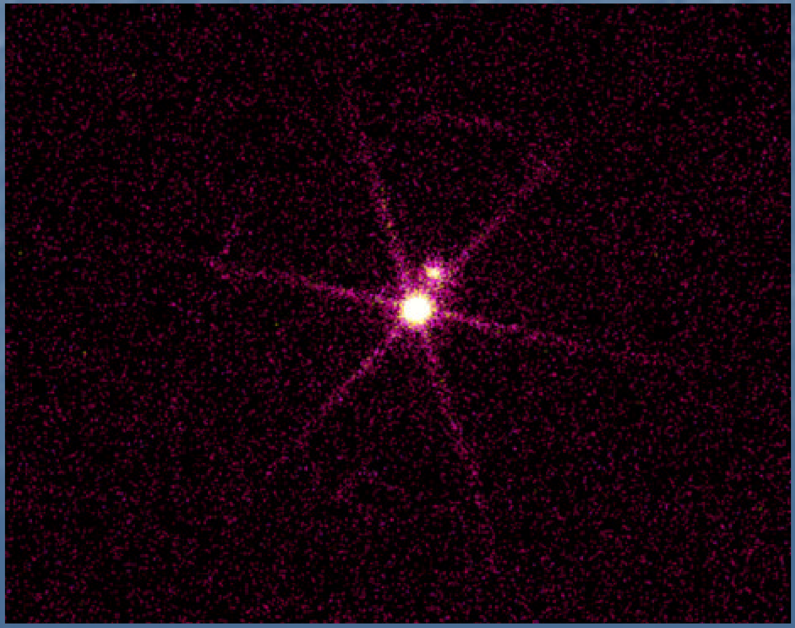
- In 1935, Subrahmanyan Chandrasekhar presented his theory that there exists an upper limit on the mass of a white dwarf.
- The pressure that prevents a white dwarf from collapsing on itself is electron degeneracy.
- Assuming M = 1.0 $M_\odot$ and R = 0.008 $R_\odot$, we should observe a gravitational redshift of 78 km s^{-1}.

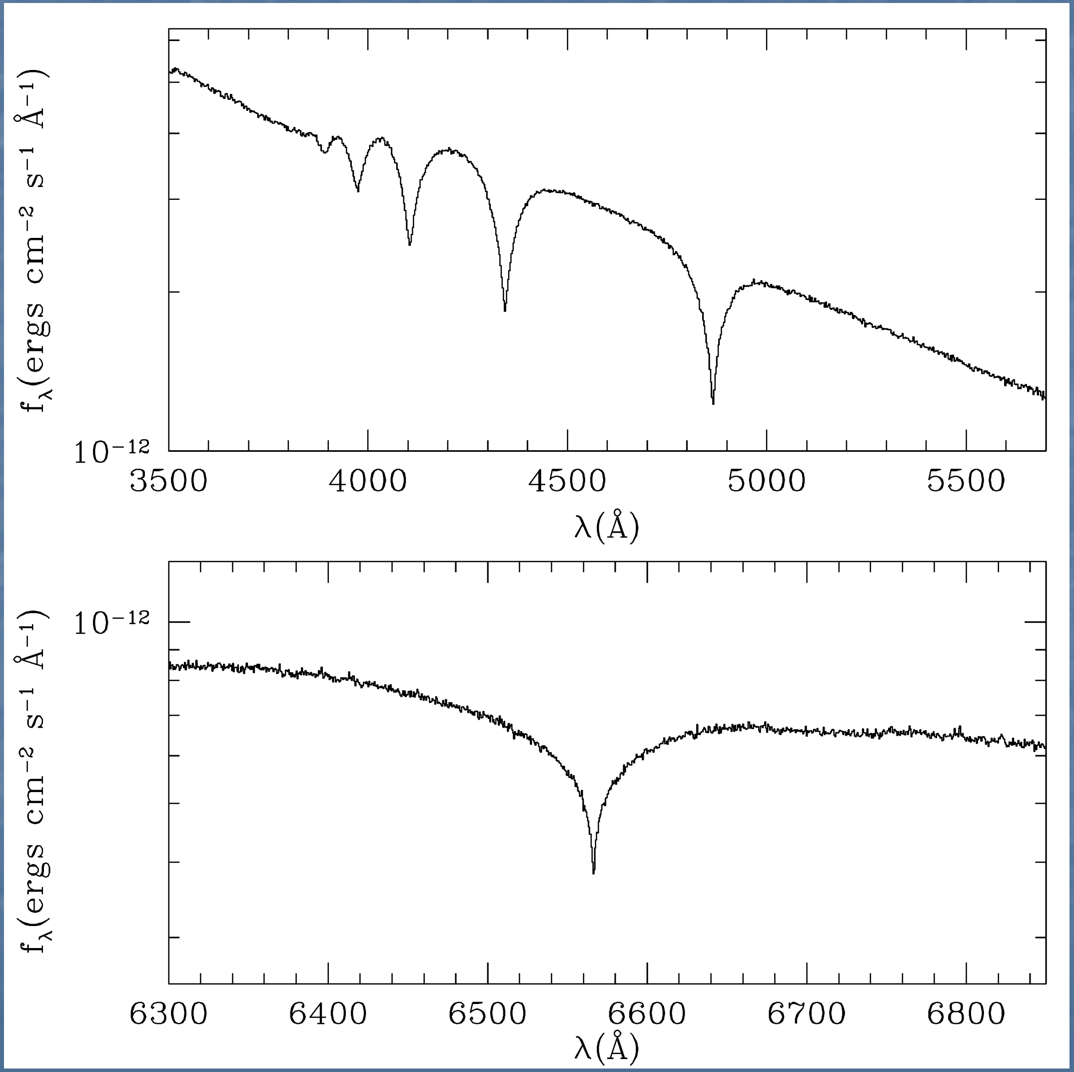
$$\frac{v}{c} = \frac{\Delta l}{l} = \frac{GM}{c^2 R}$$

- In 1971, J.L. Greenstein, J.B. Oke & H.L. Shipman obtained a new gravitational redshift of 89±16 km s^{-1}.

Sirius B

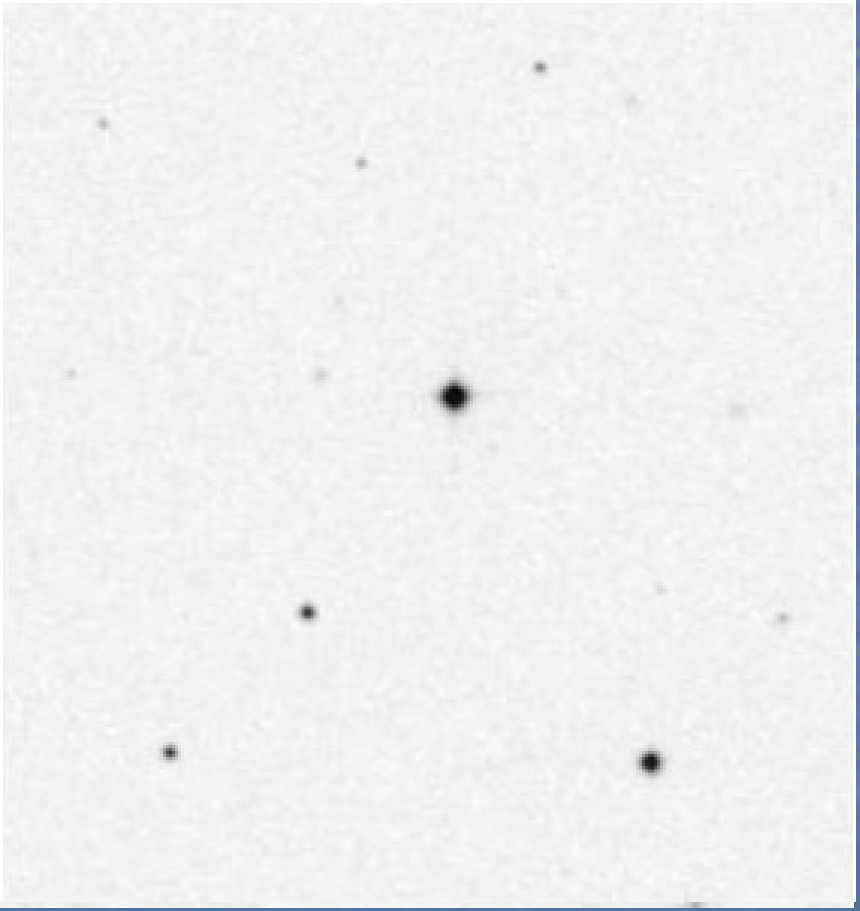
- The most recent spectrum of Sirius B was obtained with the Hubble Space Telescope.
- $v_{GR} = 80 \pm 5$ km s^{-1} (Barstow et al. 2005).
- T=25000 K, M=1.02 $M_{\odot}$.





Proper Motion

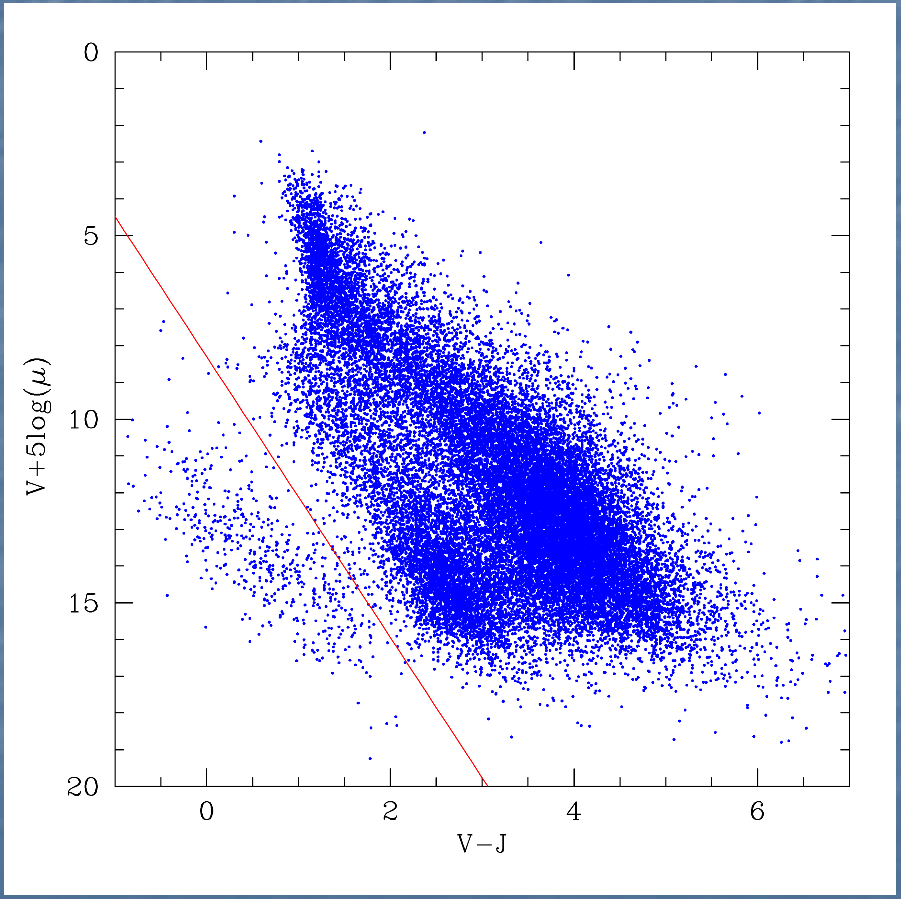
- The 3rd white dwarf (WD 0046+051) was discovered in 1917 by Adriaan van Maanen.
  - μ= 3” yr^{-1}.
  - DZ white dwarf.
- Due to their intrinsic faintness they will be close to the Sun.
- Therefore, they will have large tangential motion relative to the Sun, i.e., large proper motion.



Searching for more WDs

- By 1941, 38 white dwarfs were known (Kuiper 1941, PASP, 53, 248).
- Several such surveys were conducted, most notable of these are the surveys of Willem J. Luyten.
  - Bruce Proper Motion Survey (BPM)
  - Luyten Two Tenths Catalog (LTT)
  - Luyten Half-Second Catalog (LHS)
  - New Luyten Two Tenths Catalog (NLTT)
  - Luyten Five tenths Catalog (LFT)
- Another major proper-motion survey was the Lowell Proper motion survey (Giclas, Thomas & Burnham).
- Luyten also obtained photographic colors for many of his stars, resulting in the PHL catalog (Palomar-Haro-Luyten) and the white dwarf catalogs.

Searching for more WDs



- This is a Reduced Proper Motion diagram.
- Luyten used a diagram similar to this one to select his white dwarf candidates.
- However, he used *B* and *R* photographic bands.
- V + 5 log(μ) is an approximation for the absolute magnitude:
  - $M = m + 5 \log\pi + 5$

Searching for more WDs

- Colorimetric surveys, which aimed at identifying objects with blue emission excess:
  - Palomar-Green (PG),
  - Edinburgh-Cape (EC),
  - Hamburg/ESO survey(HE),
  - Montreal-Cambridge-Tololo (MCT).
- Many hot white dwarfs, which are bright in the ultraviolet were discovered using the
  - International Ultraviolet Explorer (IUE),
  - Extreme Ultraviolet Explorer (EUVE) and
  - Roentgen Satellite (ROSAT).
- All sky-surveys such as the Sloan Digital Sky Survey (SDSS) and GALEX (Galaxy Evolution Explorer) are finding white dwarfs that are fainter and further away.

Structure of a white dwarf

- Eddington understood that the density of white dwarfs must be very high.
- However, a subtle paradox was encountered in trying to understand the structure of white dwarfs.
- Applying the ionization equation implied that ions and electrons recombine at low temperatures.
- And since un-ionized matter has a low density, which means that that the white dwarf should expand as it cools.
- However, there is not enough thermal energy to do the necessary gravitational work.

Structure of a white dwarf

- In 1926, Fermi and Dirac showed that electrons obey what is now called the Fermi-Dirac statistics, which take into account the Pauli exclusion principle.
- Ralph H. Fowler (1926, MNRAS, 87, 114) applied this new rule to show that the pressure supporting white dwarfs against gravity is the electron degenerate pressure.
- For a non-relativistic degenerate electron gas, the pressure is:

$$P_e = \frac{h^2}{5m}\left(\frac{3}{8\pi}\right)^{\frac{2}{3}}\left(\frac{\rho}{\mu_e M_\mu}\right)^{\frac{5}{3}}$$

- The mass radius relationship of a white dwarf can be understood by assuming a uniform density throughout the star, i.e., $\rho = M/(^{4}/_{3}\pi R^{3})$.
- The pressure at the center of the star equals the weight per unit area of the material on top:

$$P_c = \frac{Mg}{A} = \frac{M}{4\pi R^2}\frac{GM}{R^2} = \frac{GM^2}{4\pi R^4}$$

- In a white dwarf it is the degenerate electron pressure that prevents the star from collapsing on itself, i.e., $P_c = P_e$.
- Assuming a non-relativistic electron gas, then:

$$\frac{M^2}{R^4} \propto \left(\frac{M}{R^3}\right)^{5/3}$$

$$R \propto M^{-1/3}$$

$$R \propto M^{-1/3}$$

- Therefore, as the mass of a white dwarf increases, its radius decreases.
- However the mass radius relationship is a little more complicated.
  - The density is not uniform through out the star.
  - As the white dwarf becomes more massive, the large gravities compress the particles into such high densities that the electrons gain very high-momenta and hence velocities that begin to approach the speed of light.
- Under such conditions it is necessary to adopt the fully relativistic and degenerate equation for the pressure of the electron gas:

$$P_e = \frac{ch}{8}\left(\frac{3}{\pi}\right)^{\frac{1}{3}}\left(\frac{\rho}{\mu_e M_\mu}\right)^{\frac{4}{3}}$$

Polytropes

- White dwarfs are polytropic stars, that is that they satisfy:

$$P = K\ \rho^{(n+1)/n}$$

$$P_e = \frac{h^2}{5m}\left(\frac{3}{8\pi}\right)^{\frac{2}{3}}\left(\frac{\rho}{\mu_e M_\mu}\right)^{\frac{5}{3}}$$

- For a non-relativistic degenerate electron gas n=3/2.

$$P_e = \frac{ch}{8}\left(\frac{3}{\pi}\right)^{\frac{1}{3}}\left(\frac{\rho}{\mu_e M_\mu}\right)^{\frac{4}{3}}$$

- For a fully-relativistic degenerate electron gas n=3.

White Dwarf Structure

- To determine the structure of a star, we require that:
  1. The star is in hydrostatic equilibrium

$$\frac{dP}{dr} = -\rho\ g = -\rho \frac{G\ M(r)}{r^2}$$

  2. Mass is conserved (mass-continuity)

$$\frac{M(r)}{dr} = 4\pi r^2 \rho$$

- To solve for the structure of the white dwarf we first need to combine these equations.

White Dwarf Structure

- We can rewrite the hydrostatic equilibrium equation:

$$\frac{r^2}{\rho}\frac{dP}{dr} = -GM(r)$$

- Then taking the derivative and substituting in the mass-continuity equation:

$$\frac{d}{dr}\left(\frac{r^2}{\rho}\frac{dP}{dr}\right) = -G\,4\pi r^2\rho$$

White Dwarf Structure

$$\frac{1}{r^2}\frac{d}{dr}\left(\frac{r^2}{\rho}\frac{dP}{dr}\right) = -4\pi G\rho$$

- Lane and Emden suggested that a family of solutions may be obtained if one assumes that the pressure as a function of the density is a polytropic function ($P = K\rho^{(n+1)/n}$).
- To solve the Lane and Emden (L.E.) equation, we first scale ρ:

$$\rho = \lambda\Phi^n$$

And the pressure becomes:

$$P = K(\lambda\Phi^n)^{(n+1)/n} = K\lambda^{(n+1)/n}\Phi^{n+1}$$

White Dwarf Structure

- Which we substitute into the L.E. equation to get:

$$\frac{1}{r^2}\frac{d}{dr}\left(\frac{r^2}{\lambda\Phi^n}\frac{d(K\lambda^{(n+1)/n}\Phi^{n+1})}{dr}\right) = -4\pi G\lambda\Phi^n$$

- And working out the derivatives and simplifying we get:

$$\frac{K\lambda^{(1-n)/n}(n+1)}{4\pi G}\frac{1}{r^2}\frac{d}{dr}\left(r^2\frac{\Phi}{dr}\right) = -\Phi^n$$

- Here we define the "length" a:

$$a^2 = \frac{K\lambda^{(1-n)/n}(n+1)}{4\pi G}$$

White Dwarf Structure

$$\frac{a^2}{r^2}\frac{d}{dr}\left(r^2\frac{d\Phi}{dr}\right) = -\Phi^n$$

- And finally we define a dimensionless scaling factor, $\xi = r/a$, and the L.E. equation becomes:

$$\frac{1}{\xi^2}\frac{d}{d\xi}\left(\xi^2\frac{d\Phi}{d\xi}\right) = -\Phi^n$$

- We can now define the boundary conditions.
  - At the center of the star: $\xi = 0$ and hence $r = 0$,
  - the scaling factor, $\lambda = \rho_c$, will be the central density and $\Phi = 1$ ($\rho = \lambda\Phi^n = \rho_c$)

White Dwarf Structure

- The second boundary condition can be determined by rearranging the L.E. equation:

$$\frac{d}{d\xi}\left(\xi^2\frac{d\Phi}{d\xi}\right) = -\Phi^n\xi^2$$

- If we differentiate and solve at $\xi = 0$, then we get $d\Phi/d\xi = 0$.
- Now with the zeroth and first derivatives at $\xi = 0$ we can integrate outward and evaluate these functions until we reach the first zero of Φ which corresponds to $\rho = 0$, the surface of the star, where:

$$\Phi(\xi_1) = 0$$

White Dwarf Structure

- Remember that we defined $\xi = r/a$ and that:

$$a^2 = \frac{K\lambda^{(1-n)/n}(n+1)}{4\pi G}$$

- Therefore the stellar radius is:

$$R = a\xi_1 = \sqrt{\frac{(n+1)K\lambda^{(1-n)/n}}{4\pi G}}\ \xi_1$$

- To obtain the mass, we need to integrate the mass-continuity equation ($dM = 4\pi r^2 \rho dr$) into which we substitute for r and ρ:

$$dM = 4\pi(a\xi)^2(\lambda\Phi^n)ad\xi = 4\pi a^3\lambda\Phi^n\xi^2 d\xi$$

White Dwarf Structure

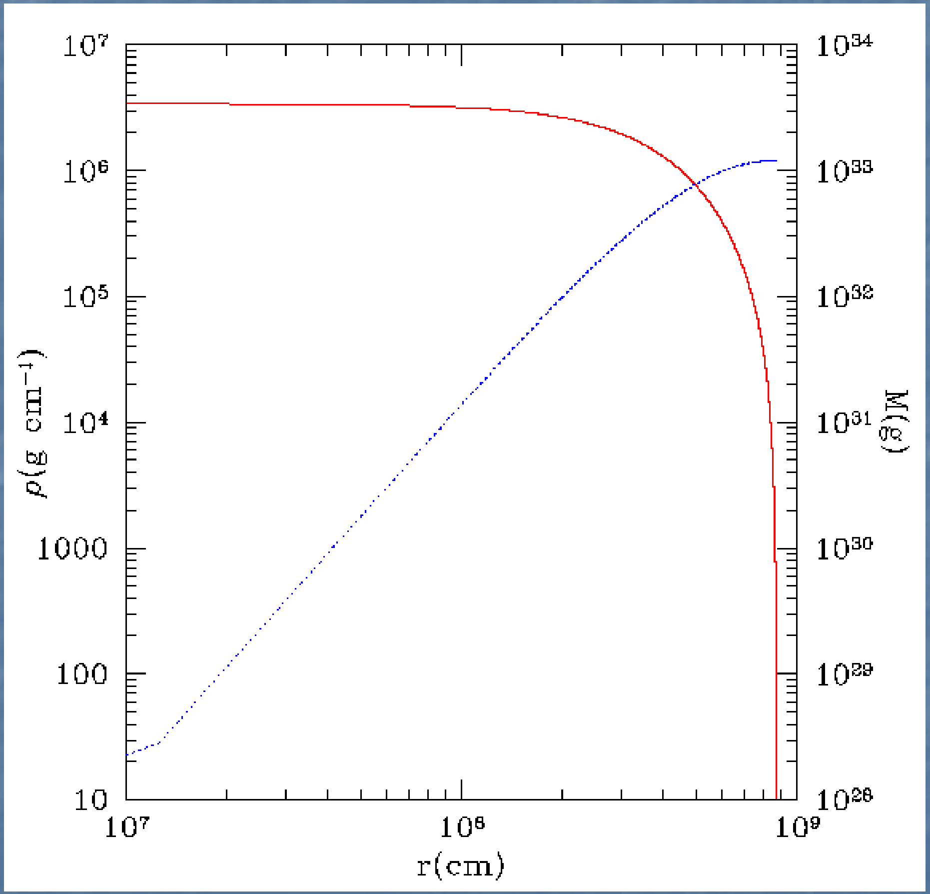
$$dM = 4\pi(a\xi)^2(\lambda\Phi^n)ad\xi = 4\pi a^3\lambda\Phi^n\xi^2 d\xi$$

- Substituting in the L.E. equation: $\Phi^n \xi^2 d\xi = -d(\xi^2 (d\Phi/d\xi))$
- And integrating from M(0) to M(x) we get:

$$M(\xi) = -4\pi a^3\lambda\xi^2\frac{d\Phi}{d\xi}$$

- The total mass of the star will be: $M(R) = M(\xi_1)$.
- Analytical solutions exist for n= 0, 1, 5 however for n= 3/2, 3 are numerical.

White Dwarf Structure



- Using the Runge-Kutta 4th order method to calculate the structure.
- $\mu_e = 2.0$,
- $\rho_c = 3.39 \times 10^6$ g cm^{-3},
- R = 7.28×10^8 cm = $0.0105\ R_\odot$,
- M = 1.194×10^{33} g = $0.6\ M_\odot$.

Chandrasekhar Limit

- We know the mass of a white dwarf:

$$M(\xi) = -4\pi a^3 \lambda \xi^2 \frac{d\Phi}{d\xi}$$

- And the pressure for a fully relativistic and degenerate electron gas:

$$P_e = \frac{ch}{8}\left(\frac{3}{\pi}\right)^{\frac{1}{3}}\left(\frac{\rho}{\mu_e M_\mu}\right)^{\frac{4}{3}}$$

- Again remember $P = K\rho^{(n+1)/n}$, we can see that $n = 3$, therefore:

$$a = \frac{1}{\lambda^{1/3}}\sqrt{\frac{K}{\pi G}}$$

$$K = \frac{ch}{8}\left(\frac{3}{\pi}\right)^{1/3}\left(\frac{1}{\mu M_\mu}\right)^{4/3}$$

Chandrasekhar Limit

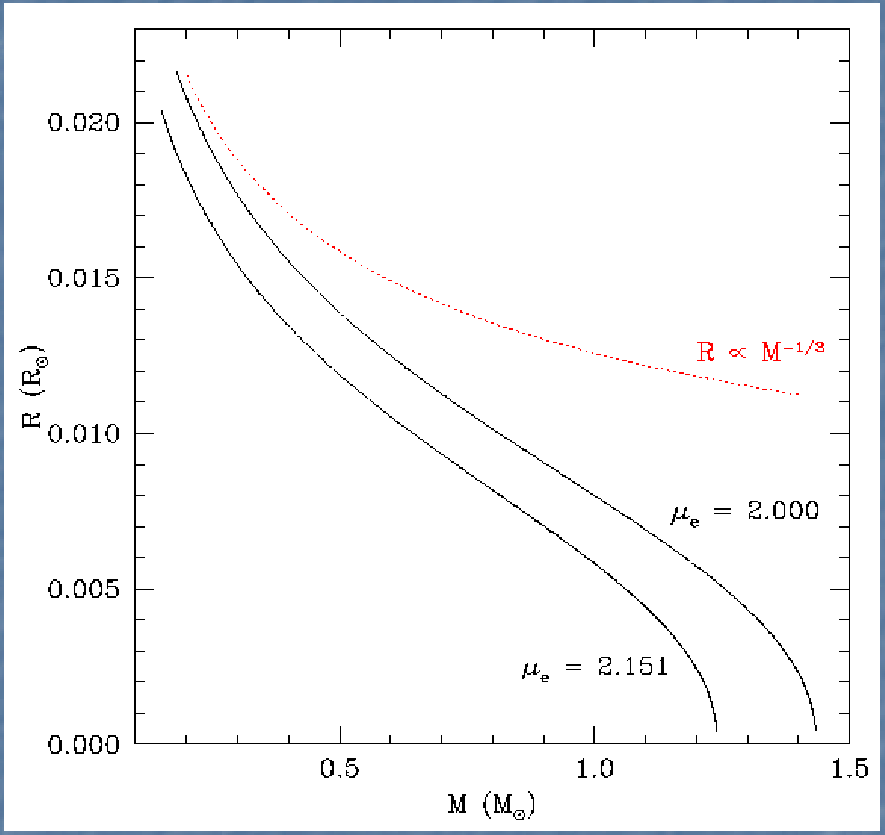
- Through substitution into the mass equation we get:

$$M(\xi_1) = -\frac{4\sqrt{3}}{\pi}\left(\frac{hc}{8G}\right)^{3/2}\left(\frac{1}{\mu_e M_\mu}\right)^2 \xi^2 \frac{d\Phi}{d\xi}$$

- The mass is independent of the central density ($\lambda = \rho_c$) and therefore if every single electron is relativistic ($v = c$) then a single maximum mass is obtained.
- And after substituting for all the constants (where $\xi_1^{\,2}(d\Phi/d\xi)_1 = -2.01824$ for n=3), we get:

$$M = \frac{5.83}{\mu_e^2} M_\odot$$

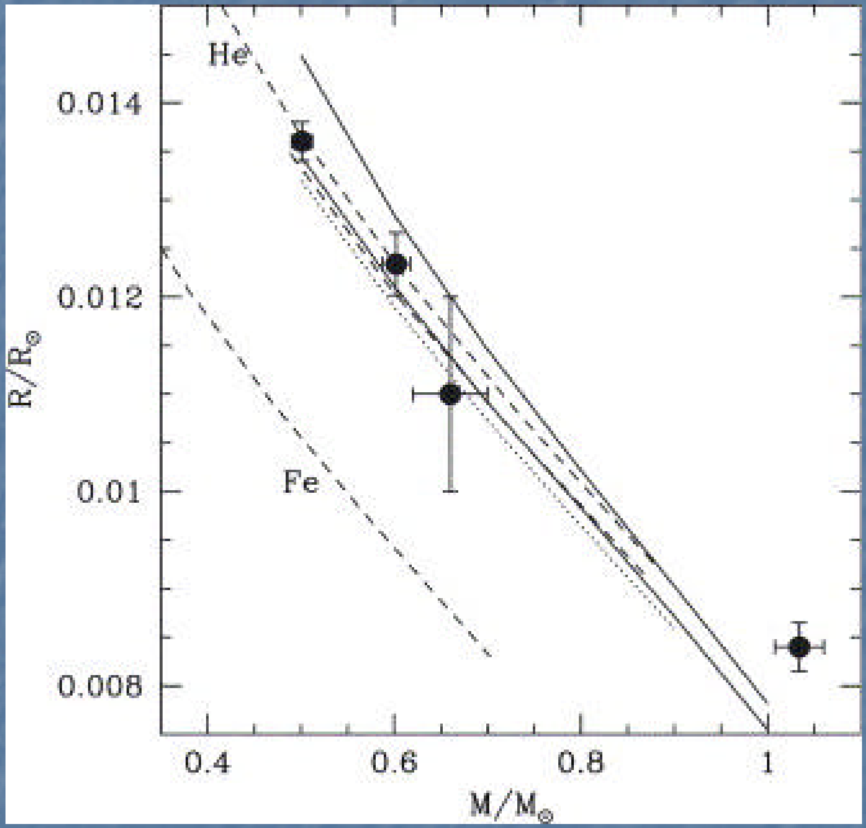
WD Mass-Radius Relation



- These are zero-temperature models.
- However, white dwarfs have surface temperatures, hence they are cooling, implying a temperature gradient.
- And higher interior temperatures.
- Need to consider the temperature.

- μ_e = 2.0: He, C, N, 0; μ_e = 2.151: Fe

Testing the Mass-Radius relationship



Hansen 2004

- Models of white dwarfs make assumptions on the interior composition (i.e., C/O cores).
- We need independent measurements of the mass and radius.
  - Visual binaries,
  - gravitational redshift,
  - parallax,
  - asteroseismology.