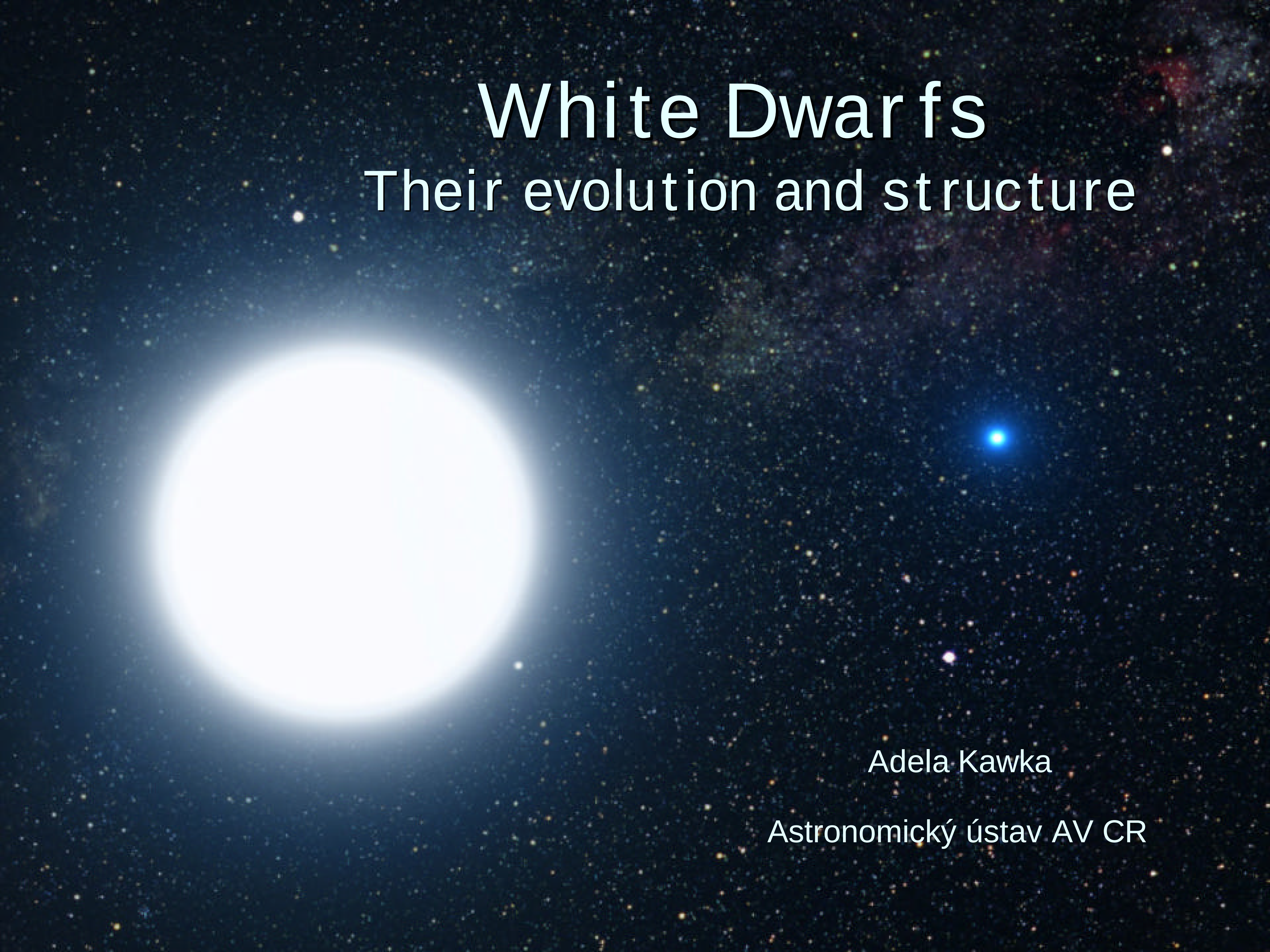

White Dwarfs
Their evolution and structure

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Astronomický ústav AV CR

Outline

- Evolution toward a white dwarf.
- White dwarf properties.
- Brief history of the discovery and observations of white dwarf stars.
- White dwarf structure.
- Evolution – cooling.
- Atmosphere.
- Variable white dwarfs.
- White dwarfs in binary systems.
- Their distribution and importance in the Galaxy.
- White dwarfs in globular clusters.

http://sunstel.asu.cas.cz/~kawka/notes.html

Revision

- Stars with a mass of ~8 $M_\odot$ will evolve into white dwarfs.
- They are very compact objects with $\rho = 10^6 - 10^9$ g cm^{-3}.
- They have low luminosities due to their small radii.
- They are supported by degenerate electron pressure.
- White dwarfs can be characterized by several properties:
  - Atmospheric composition – DA, DO, DB, DC, DZ, DQ
  - Effective temperature – cooling age
  - Surface gravity – mass, radius
  - Magnetism – 20% of white dwarfs are magnetic - ~1 kG – 1000 MG
  - Rotation – white dwarfs are generally slow rotators (< 40 km s^{-1}).
- Most white dwarfs have been discovered in proper-motion surveys.

Revision

- For the structure of a white dwarf to be determined, it is necessary that:
  - The star is in hydrostatic equilibrium.

$$\frac{dP}{dr} = -\rho\, g = -\rho \frac{G\; M(r)}{r^2}$$

  - Mass is conserved (mass-continuity).

$$\frac{M(r)}{dr} = 4\pi r^2 \rho$$

- White dwarfs are polytropic stars, where they satisfy:

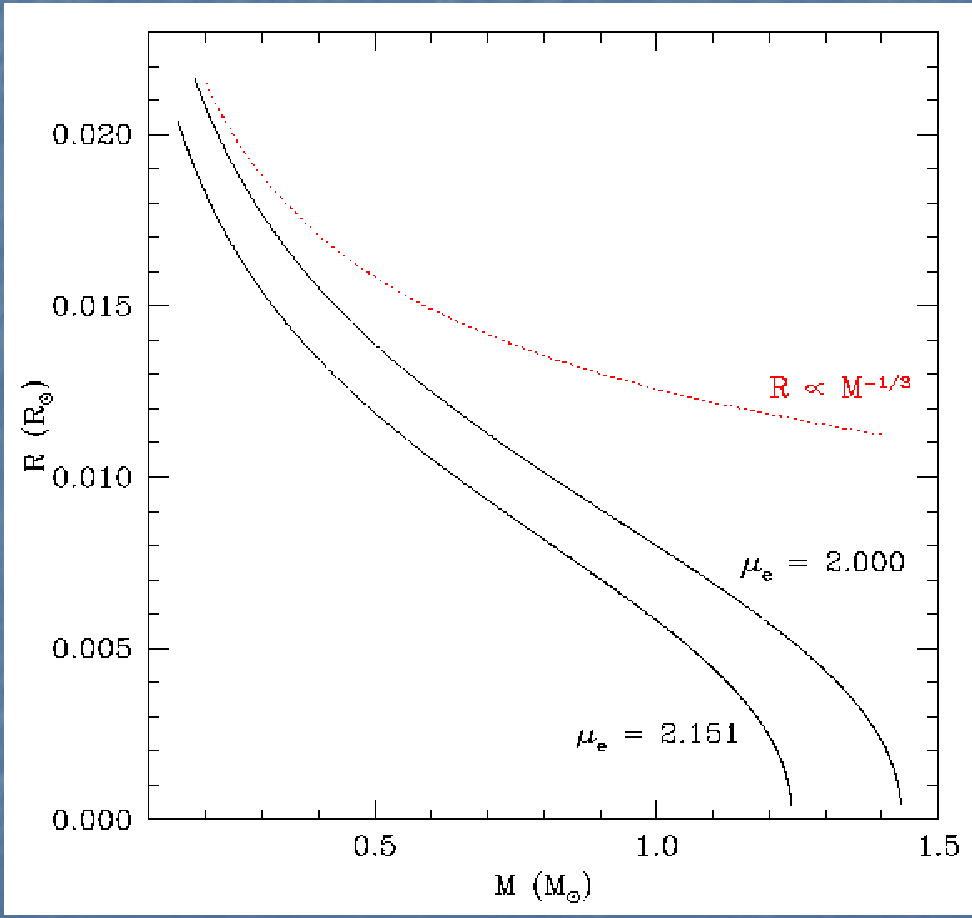
$$P = K\; \rho^{(n+1)/n}$$

- Non-relativistic degenerate electron gas: $P_e \propto \rho^{5/3}$ (n=3/2).
- Fully-relativistic degenerate electron gas: $P_e \propto \rho^{4/3}$ (n=3).

Revision

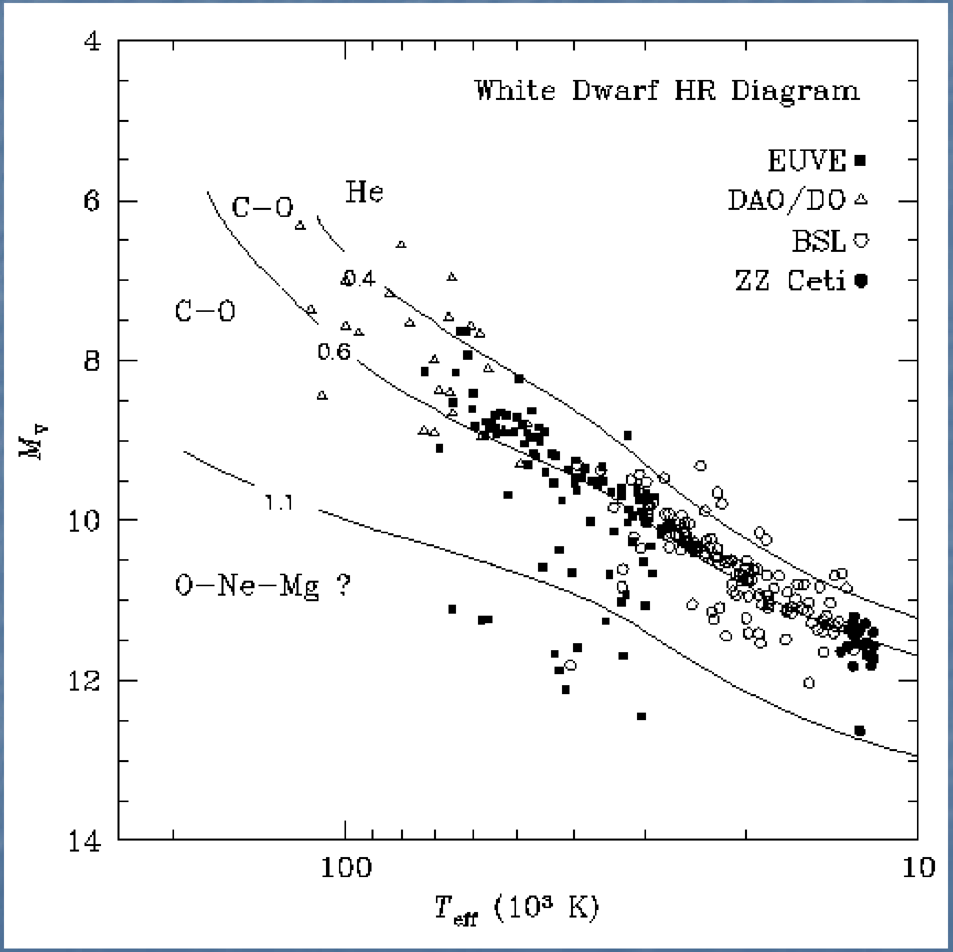
- When applying the equation for pressure of a fully-relativistic and degenerate electron gas to the solution of the Lane-Emden equation then a single (maximum) mass is obtained – Chandrasekhar limit.

$$M = \frac{5.83}{\mu_e^2} M_\odot$$



White dwarf evolution

- No nuclear reactions occur in the interior of a white dwarf.
- White dwarfs cool by slowly releasing their supply of thermal energy.
- It is important to understand the rate at which a white dwarf cools so that its age can be determined.

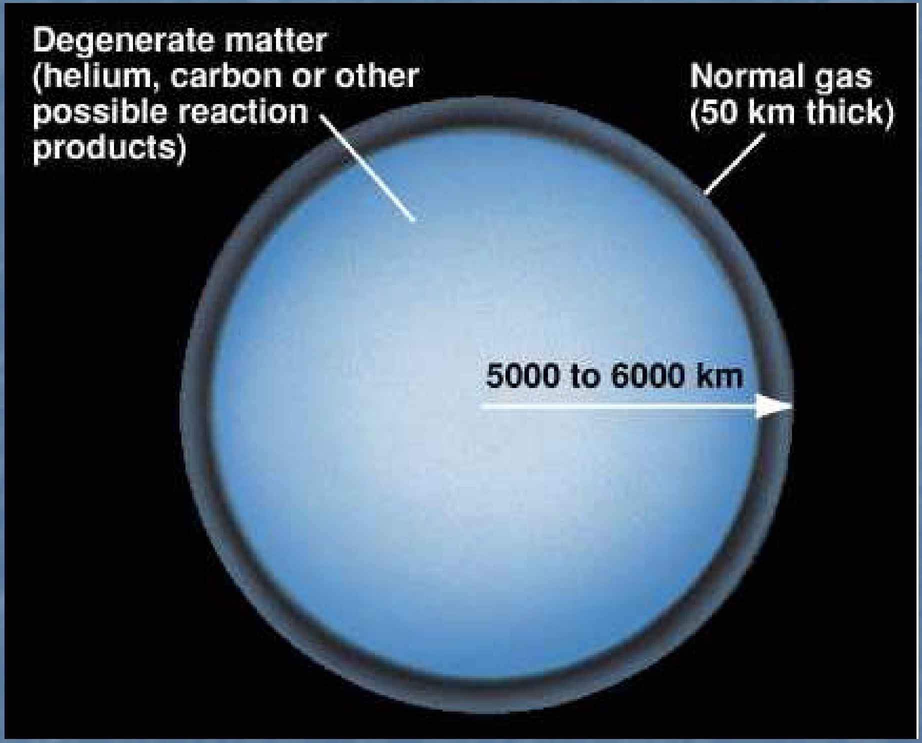


White Dwarf Cooling

- In ordinary stars, photons travel much further than atoms before they collide with an atom and lose their energy.
- White dwarfs are very dense, and therefore photons can travel only very short distances before they collide with a nucleus and lose their energy.
- Degenerate electrons can travel long distances before losing energy in a collision with a nucleus.
- That is possible because the majority of the lower energy electron states are already occupied.
- In a white dwarf, energy is transported via electron conduction.

White Dwarf Cooling

- The modern theory of white dwarf cooling was established by Mestel (1952).
- A white dwarf loses its thermal energy through a thin layer of non-degenerate atmosphere.
- The energy is stored in the thermal motions of the ions.



White Dwarf Cooling

- The evolution of a white dwarf is driven by the rate of change of the internal energy as heat is radiated out.
- From thermodynamics, the internal energy *u* is related to the entropy *s* by $du = Tds$ (at constant volume).
- Rate of energy dissipated as heat is:

$$\frac{du}{dt} = T\frac{ds}{dt} = T\left(\frac{\partial s}{\partial T}\right)_\rho \frac{\partial T}{\partial t} + T\left(\frac{\partial s}{\partial \rho}\right)_T \frac{\partial \rho}{\partial t}$$

- And the specific heat (c_V) of a gas at a constant volume is $(\partial u/\partial T)_V$.
- We will assume that there is no gravitational contraction, i.e., $(\partial \rho/\partial t)_V = 0$

White Dwarf Cooling

$$\frac{du}{dt} = c_V \frac{\partial T}{\partial t}$$

- In the interior of a white dwarf, the electrons are degenerate but the ions are not.
- The heat capacity per unit volume of a gas that consists of non-degenerate ions and non-relativistic electrons is:

$$c_V = \frac{3}{2} n_i k + \frac{\pi^2}{2} n_e k \left(\frac{kT}{E_F} \right)$$

- The electrons do not contribute significantly to the heat capacity inside the white dwarf because the electrons are strongly degenerate ($E_F >> kT$).

White Dwarf Cooling

- Since the luminosity of a white dwarf is maintained by the loss of thermal energy of the ions, then:

$$L = -\frac{dU}{dt} = -\frac{d}{dt}\int\int c_V\, dT\, dV$$

- To calculate an evolutionary scenario, we require a relationship between the core temperature and the surface temperature.
  - The interior of a white dwarf is assumed to be isothermal due to the very high electron conductivity.
  - Temperature change from the interior to the surface occurs within a thin layer of essentially non-degenerate gas.
  - Energy transport through this layer is assumed to be due to radiative diffusion.

White Dwarf Cooling

- The radiative transfer of energy in this layer requires that we know about the opacity of the gas.
- We can use Kramer's opacity law:

$$\kappa = \kappa_0 \rho T^{-3.5}$$

- The equation for radiative transfer in these layers is:

$$\frac{L}{4\pi r^2} = -\frac{4ac}{3\rho\kappa} T^3 \frac{dT}{dr}$$

- The radiation field in these layers have been assumed to be that of a blackbody.

White Dwarf Cooling

- With the assumptions that we have made, the equations of hydrostatic equilibrium and radiative transfer can be integrated to derive a temperature-pressure relationship.

$$P^2 = \frac{2}{8.5}\frac{4\pi G k}{\kappa_0 \mu m_p}\frac{M}{L}T^{8.5}$$

- This is used to connect the central temperature at the transition between the isothermal core and the non-degenerate envelope. That is at the density where the Fermi energy $E_F \sim kT_c$.
- Therefore we need to define the above equation in terms of density.

$$\rho = \left(\frac{2}{8.5}\frac{4ac}{3}\frac{4\pi G M}{\kappa_0 L}\frac{\mu m_p}{k}\right)^{1/2} T^{3.25}$$

White Dwarf Cooling

$$\rho = K_1 \left(\frac{M}{L}\right)^{1/2} T^{3.25}$$

- This equation assumed a non-degenerate gas, which becomes invalid at densities where electron density becomes important.
- At the boundary of the isothermal core with the outer non-degenerate envelope, we can assume that the pressure of the non-degenerate electron gas is equal to the pressure of a completely degenerate electron gas.
- The temperature at this boundary will be the temperature of the isothermal core.

$$\frac{\rho_c k T_c}{\mu_e M_\mu} = \frac{h^2}{5m}\left(\frac{3}{8\pi}\right)^{\frac{2}{3}}\left(\frac{\rho_c}{\mu_e M_\mu}\right)^{\frac{5}{3}}$$

White Dwarf Cooling

$$\rho_c = K_2 T_c^{3/2}$$

- Earlier we derived the density for the fully ionized ions:

$$\rho = K_1 \left(\frac{M}{L}\right)^{1/2} T^{3.25}$$

- Lets assume that this equation is valid at the core boundary, that is at $\rho = \rho_c$ and $T = T_c$, then:

$$K_1 \left(\frac{M}{L}\right)^{1/2} T_c^{3.25} = K_2 T_c^{3/2}$$

$$L = \left(\frac{K_1}{K_2}\right)^2 M T_c^{3.5}$$

White Dwarf Cooling

- The available thermal energy is provided mainly by the non-degenerate ions:

$$U = \int c_V T dV = \int \frac{c_V}{\rho} dM \simeq \frac{\bar{c}_V}{\bar{\rho}} T_c M$$

- The rate at which a white dwarf loses its thermal energy is its luminosity.

$$L = -\frac{dU}{dt}$$

White Dwarf Cooling

$$\tau = \frac{2}{5}\frac{\bar{c}_V}{\bar{\rho}}\left(\frac{K_2}{K_1}\right)^{4/7}\left(\frac{M}{L}\right)^{5/7}$$

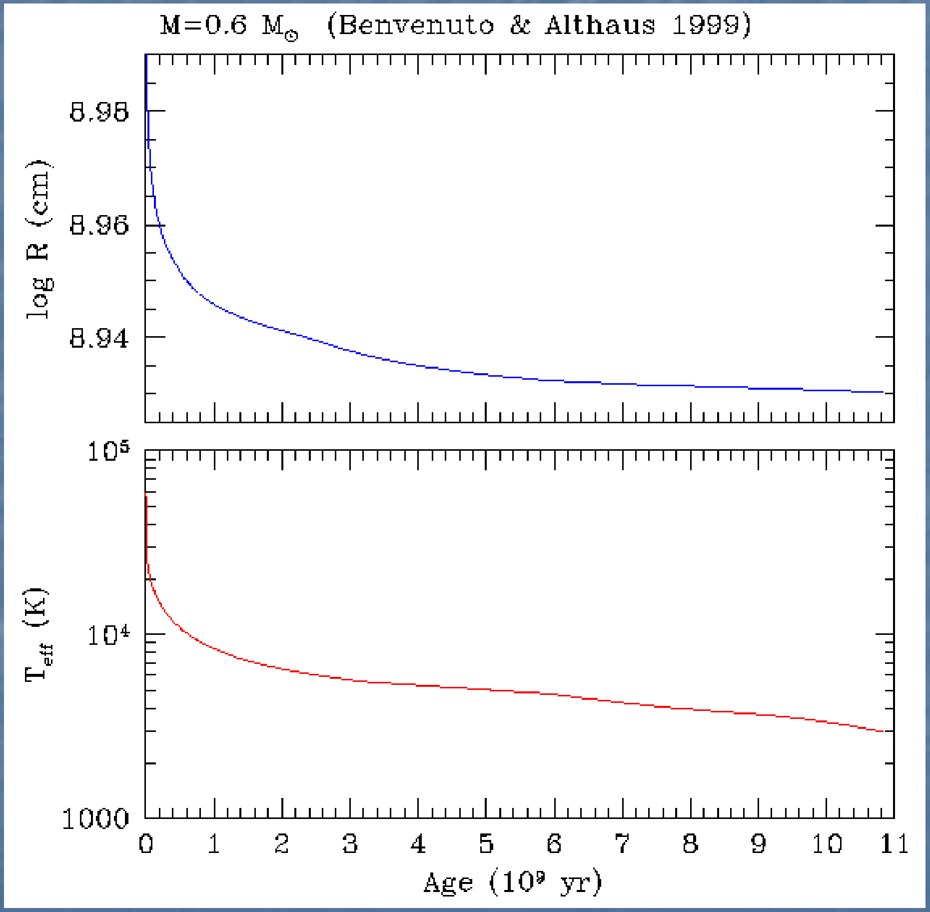
- τ is the so called cooling age, which is the time required for the luminosity of a white dwarf to go from L_0 to L.
- And the luminosity as a function of the age is:

$$L \approx 8.4 \times 10^{-4} L_{\odot}(M/M_{\odot})\ \tau_9^{-7/5}$$

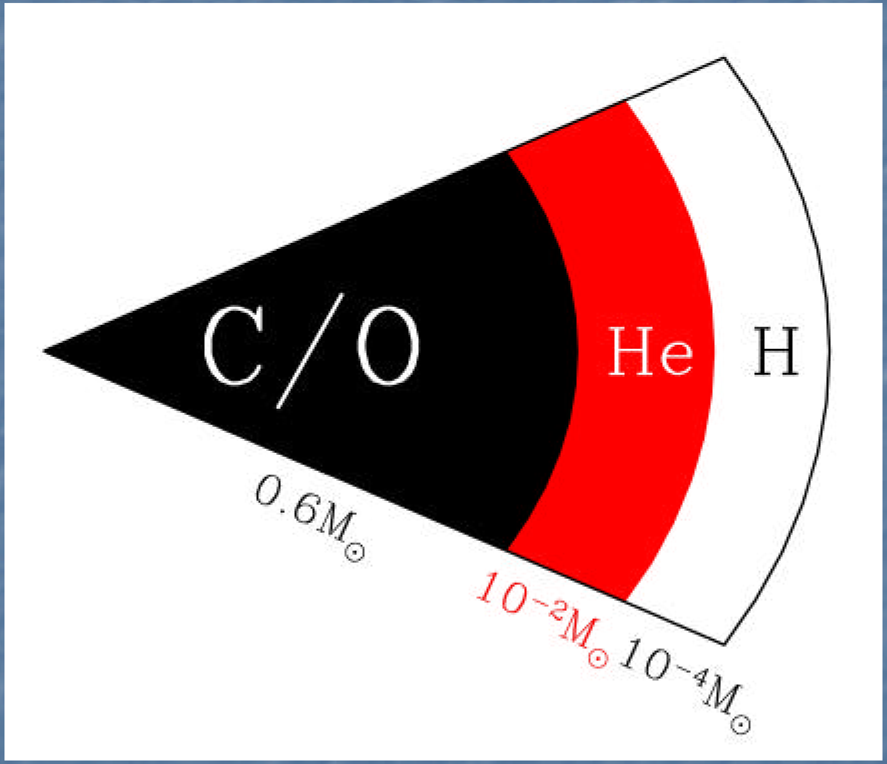
- This simple power law cooling model is a good representation of how a white dwarf cools.
- However, as with many models, reality is a little more complex.

Gravitational Energy

- In our model we have assumed that there is no gravitational contraction.
- The star is losing thermal energy, and it is actually shrinking and hence releasing gravitational energy that is contributing to the energy loss.

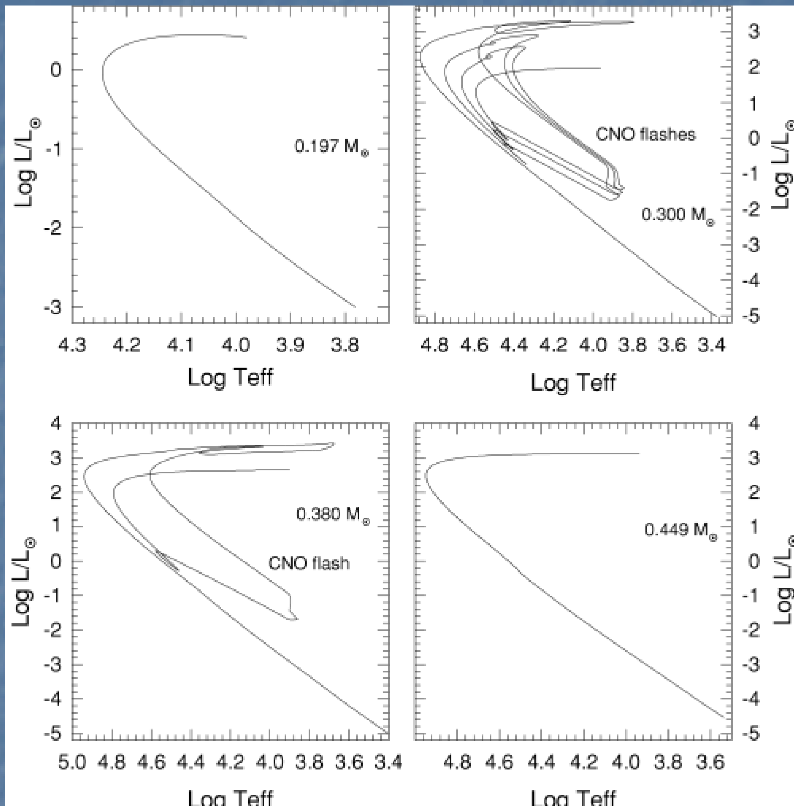


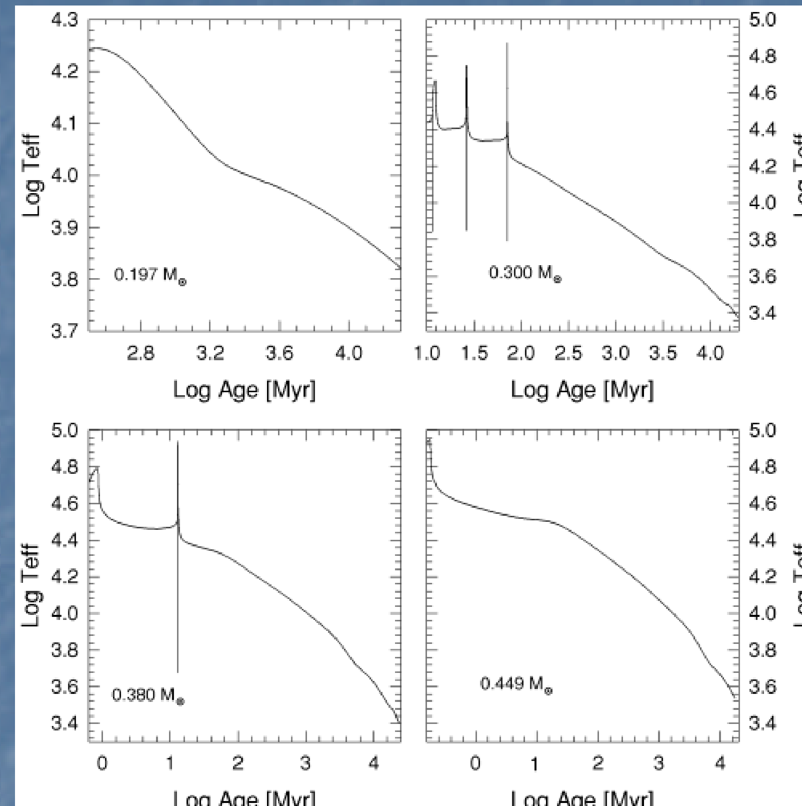
Nuclear Energy



- When the white dwarf contracts, the strong gravity causes the rapid diffusion of heavier elements toward the center and lighter elements toward the surface.
- This may lead to further CNO burning and hence reducing the hydrogen content in the atmosphere.

Nuclear Energy



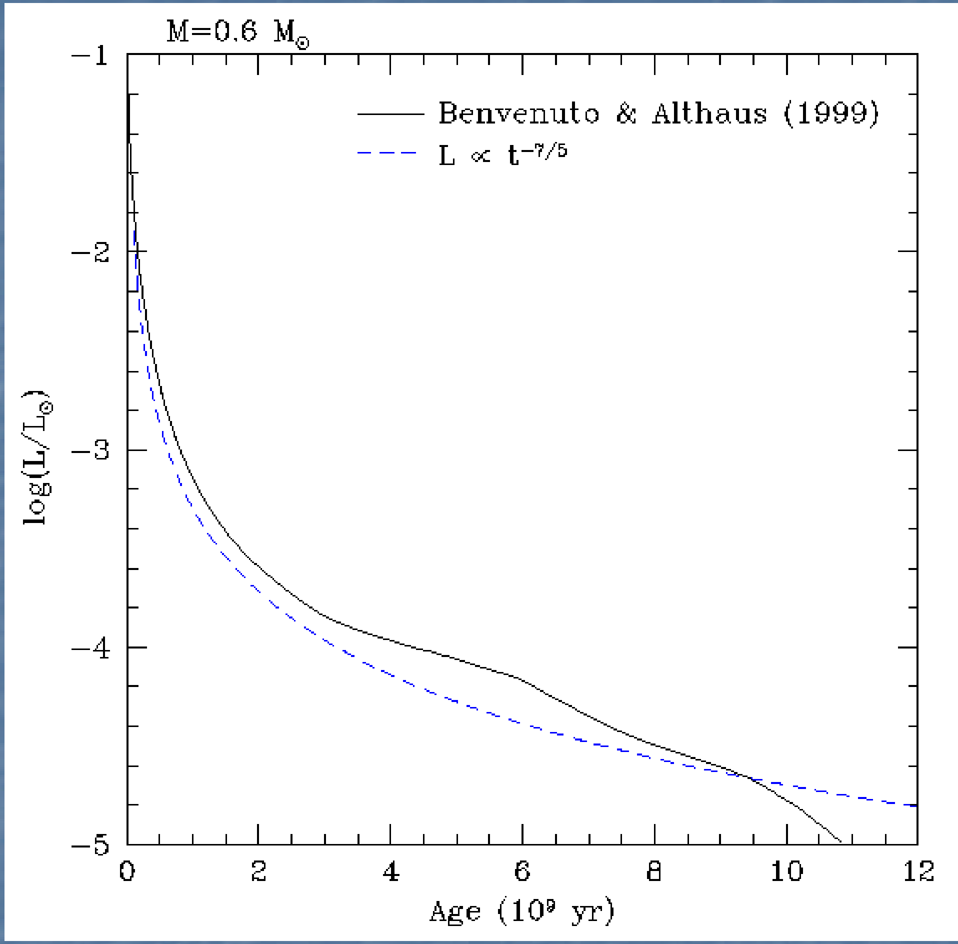


Serenelli et al. (2002)

- The cooling age can be affected by any residual hydrogen burning on the surface of the white dwarf.

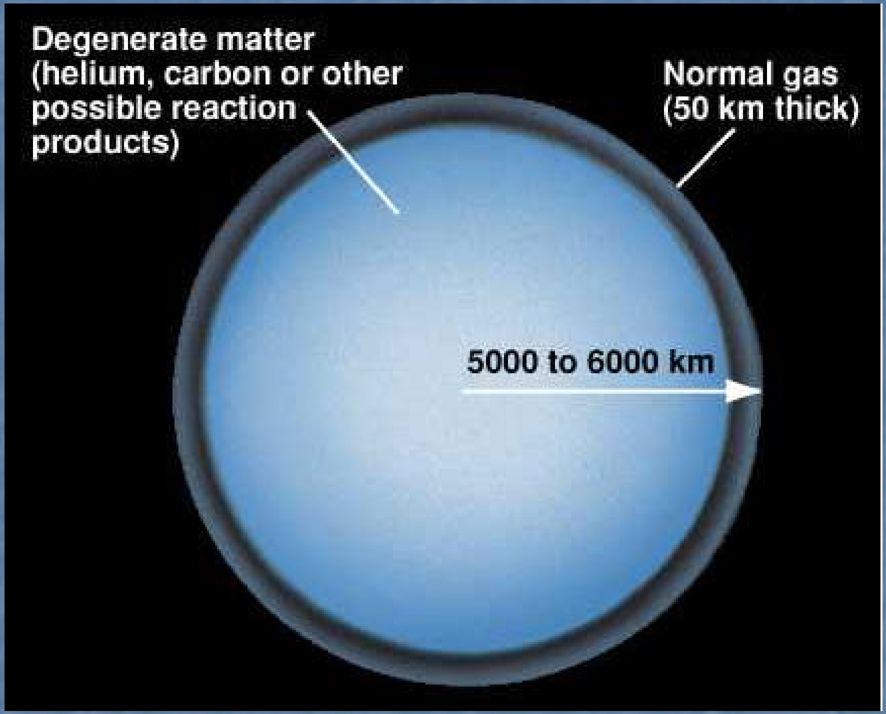
Crystallization

- In a white dwarf, the heat content is regulated by the ions.
- Very young white dwarfs are very hot and therefore the ions can be assumed to behave like an ideal gas.
- However, as the white dwarf cools, the ions begin to crystallize from the center outward.



- The ions are essentially changing phase, and therefore release latent heat.

White Dwarf Cooling



- So far we have considered the sources of energy that drive the luminosity.
- The processes by which energy is transported to the surface also needs to be considered.
- We can separate the degenerate core and the thin non-degenerate envelope.
- This envelope is very important in how a white dwarf cools because this is where the energy transport is slowest, i.e., a bottleneck for the cooling.

Energy Transport inside the core

- The transfer of energy inside the interior of a white dwarf is primarily via electron conduction.
- The electron mean free path is dependent on the scattering of the ions.
- Therefore there is a coupling between the electron conduction and the thermal reservoir of the ions.
- The coupling between electrons and neutrinos in electroweak theory suggests that neutrino radiation in a white dwarf is a possibility.
- Neutrino luminosity becomes dominant only when the white dwarf begins to contract after the final stages of AGB evolution.

Energy Transport in the Atmosphere

- Even though the envelope is very thin and its mass is a very small fraction of the white dwarf’s total mass, it is here where the energy transport is slowest and hence determines the cooling age.
- The energy transport in the thin, non-degenerate envelope (atmosphere) can be either:
  - Radiative diffusion – Hot white dwarfs
  - Convective energy transport – cooler white dwarfs (for DA white dwarfs convection begins to develop at ~12 000 K).
- And the atmosphere is the observable region of the white dwarf.

Model Atmosphere

- The total radiated flux emitted at the surface of the star is frequency dependent and is expressed as the *Eddington* flux:

$$\mathcal{F}_{\rm total} = 4\pi H_{\rm total} = 4\pi \int_0^\infty H_\nu d\nu = \sigma_R\, T_{\rm eff}^4$$

- A model atmosphere is represented by a set of non-linear equations that provide a physical description of the observable region of the star.
  - Radiative/convective energy transfer
  - Radiative equilibrium (= flux conservation)
  - Hydrostatic equilibrium – gas pressure balances the gravitational forces.
  - Equation of state – N_i (i = 1, *nlevel*) population levels
  - Charge and particle conservation – total number of particles is conserved and that the net electric charge is zero.

Model Atmosphere

- Geometry – spherical? <u>plane-parallel</u>

$$\frac{h_{\text{atmos}}}{R_{\text{star}}} < 0.001$$

- Choose the independent variable – optical depth τ? mass loading *m*? Langrangian *m* is best since it simplifies the equation for hydrostatic equilibrium.

$$dm = -\rho dz$$

$$d\tau = -\chi dz$$

$$d\tau = \left(\frac{\chi}{\rho}\right) dm$$

Model Atmosphere

- We solve the problem numerically.
- Therefore, discrete variables need to be used i.e., slice the atmosphere into slices and the spectrum in discrete frequencies.

$$m \longrightarrow m_d \qquad (d = 1, \mathrm{ND})$$

$$T = T(m_d) \longrightarrow T_d \qquad (d = 1, \mathrm{ND})$$

$$H_\nu \longrightarrow H_{d,j} \qquad (d = 1, \mathrm{ND}; j = 1, \mathrm{NJ})$$

Model Atmosphere

- Radiative transfer:

$$\frac{\partial H_\nu}{\partial z} = \chi_\nu (S_\nu - J_\nu), \qquad S_\nu = \frac{\eta_\nu}{\chi_\nu}$$

- Which integrated over frequencies leads to the radiative equilibrium (flux/energy conservation).

$$\frac{\partial H}{\partial z} = 0 = \int \chi_\nu (S_\nu - J_\nu) d\nu$$

- And we need to define the boundary condition:

$$\text{Total Flux} = \mathcal{F} = 4\pi H = 4\pi \int_0^\infty H_\nu d\nu = \sigma_R \, T_{\text{eff}}^4$$

Model Atmosphere

- Hydrostatic equilibrium

$$\frac{dP}{dz} = -\rho g \longrightarrow \frac{dP}{dm} = g$$

- And using discrete variables:

$$\frac{dP}{dm} = g \longrightarrow \frac{P_d - P_{d-1}}{m_d - m_{d-1}} = g$$

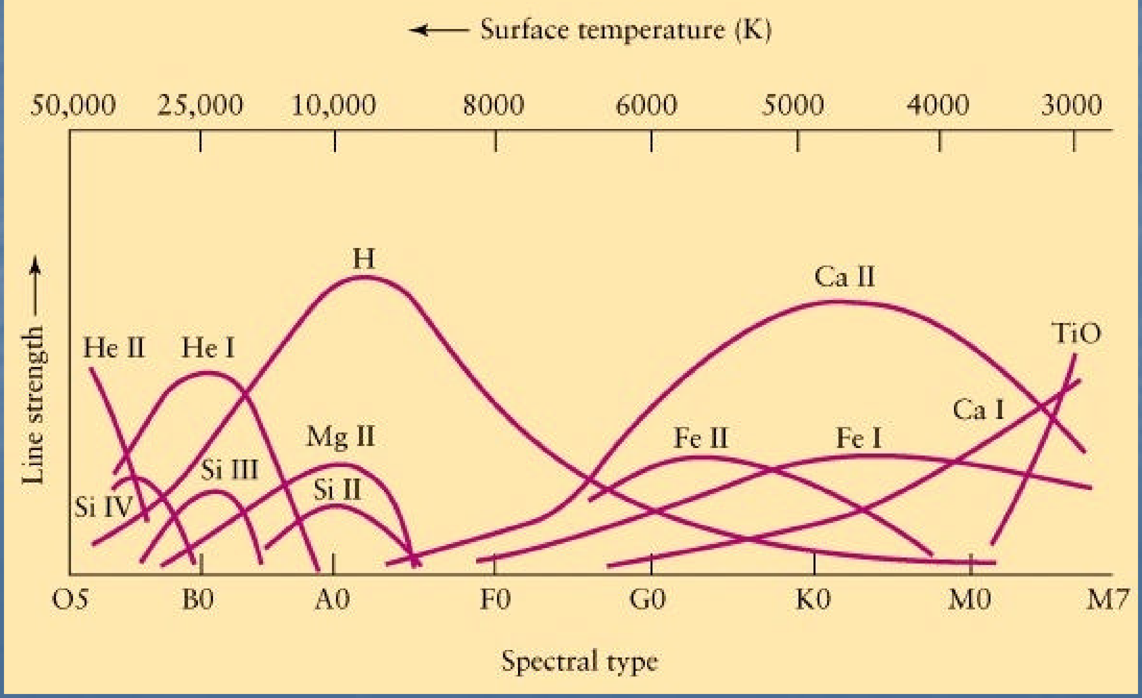
- And in the atmosphere we can assume the ideal gas law:

$$PV = NkT \longrightarrow P = nkT$$

Model Atmosphere

- Equation of state – statistical equilibrium, that is the atomic level populations for hydrogen (and/or helium and heavier elements) assuming that Saha-Boltzmann fractions hold.

$$\frac{N_i}{N_{i+1}} = \frac{u_i}{u_{i+1}} N_e \Phi(T)$$



Model Atmosphere

- Charge Conservation (assuming hydrogen only):

$$-N_e + N_p = 0$$

- Particle conservation:

$$N_{\text{total}} = N_p + N_e + \sum_{i}^{\text{nlevel}} N_i$$

Model Atmosphere

- When the radiative temperature gradient exceeds the adiabatic gradient, convective energy transfer sets in.

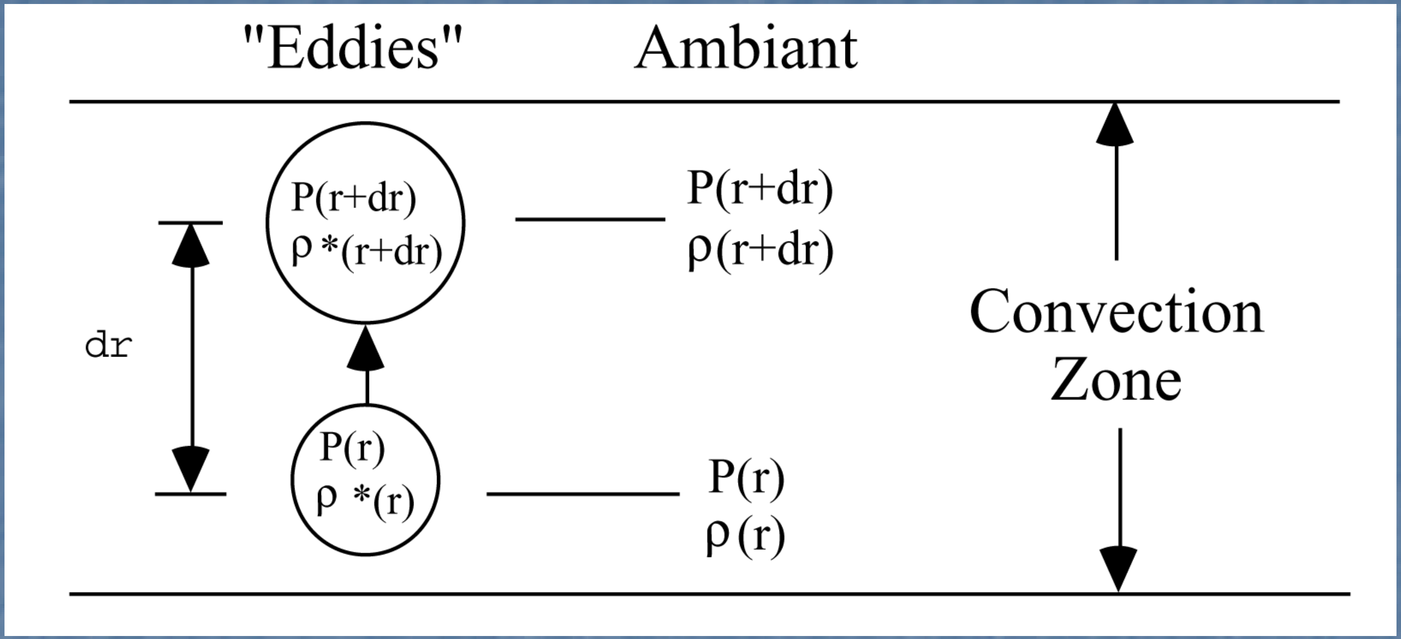
$$\nabla_{\mathrm{R}} = \left(\frac{d\ln T}{d\ln P}\right)_{\mathrm{R}} > \left(\frac{d\ln T}{d\ln P}\right)_{\mathrm{A}} = \nabla_{\mathrm{A}}$$

- For atmospheres in radiative/convective equilibrium:

$$\text{Total Flux} = \sigma T_{\mathrm{eff}}^4 = \mathcal{F}_{\mathrm{radiative}} + \mathcal{F}_{\mathrm{convective}}$$

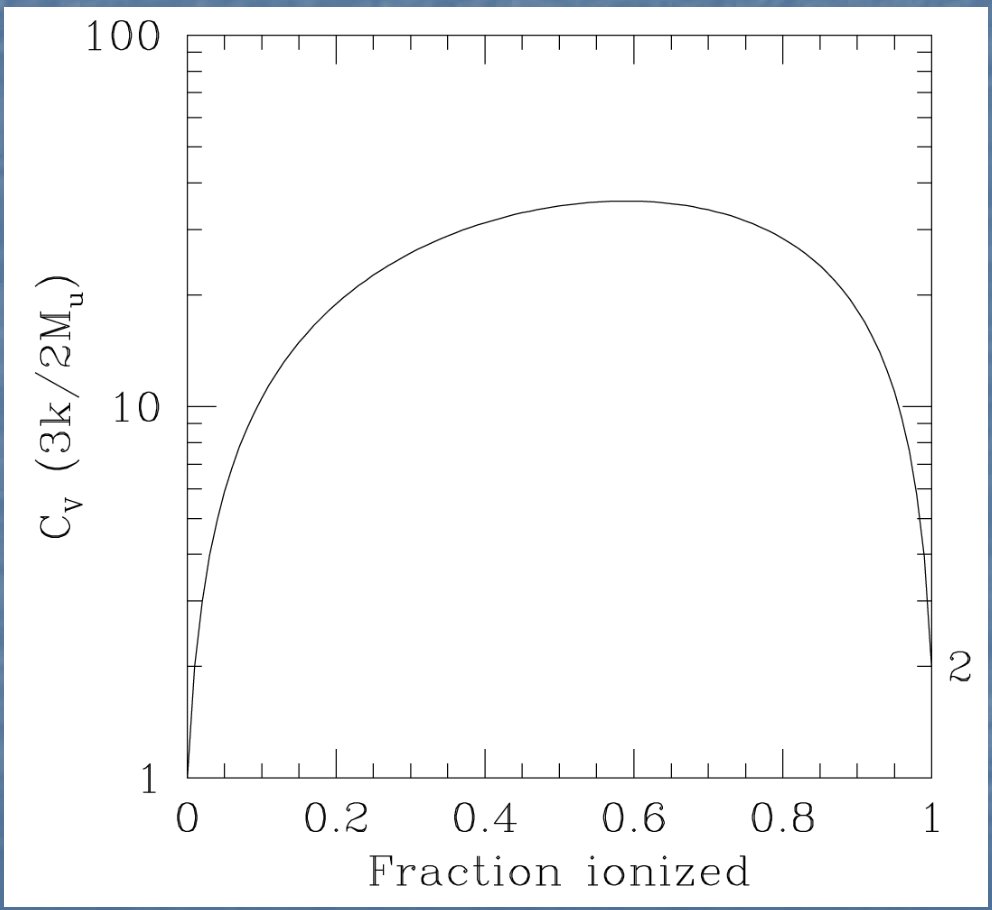
- The convective flux is a function of local variables: T and the ionization fraction $f = N_p/N$ where $N = \Sigma N_i$.
- f is a very important quantity in calculating the heat capacity C_p.

Model Atmosphere



- The energy carrier in the convection process is called an eddy or a cell (Mixing length approximation).
- In stars, eddies consist of a gas region expanding adiabatically – it does not exchange energy with the surrounding.

Model Atmosphere



- Eddies/cells are characterized by:
  - the density ρ,
  - heat capacity at constant pressure C_p,
  - transport velocity v and
  - the temperature difference with the surroundings ΔT.

$$\mathcal{F}_{\text{convective}} = \rho C_P \overline{v} \Delta T$$

Model Atmosphere

- We have defined the equations that need to be solved.
  - Radiative/convective energy transfer
  - Radiative equilibrium
  - Hydrostatic equilibrium
  - Equation of state, i.e., population levels
  - Charge and particle conservation
- We need to calculate the vector solution at each point in the atmosphere.

$$\phi(T, N_e, N_p, \ldots, N_i, \ldots, N_{\mathrm{nlev}}, H_\nu)$$

- To solve the set of non-linear equations, we need to linearize them.

Model Atmosphere

- We start with a zero order approximation for all layers in the atmosphere (d = 1, ND):

$$\phi^0(T^0, N_e^0, N_p^0, N_{i=1,\mathrm{nlev}}^0, H_\nu^0)$$

- And solve for:

$$\delta\phi(\delta T, \delta N_e, \delta N_p, \delta N_{i=1,\mathrm{nlev}}, \delta H_\nu)$$

- Once an equation *f* with ϕ_i dependent variables is solved, we always have:

$$f(\phi) = 0$$

- And starting with the approximation ϕ^0, we demand that the corrections $\delta\phi$ satisfy

$$f(\phi^0 + \delta\phi) = 0$$

Model Atmosphere

- We solve for $\delta\phi$ by linearizing *f* :

$$f(\phi^0) + \sum_i \frac{\partial f}{\partial \phi_i}\delta\phi_i = 0$$

- For example, the equation for *hydrostatic equilibrium* is already linear:

$$\frac{dP}{dm} = g \longrightarrow \frac{P_d - P_{d-1}}{m_d - m_{d-1}} = g$$

$$N_{\mathrm{tot},d}kT_d - N_{\mathrm{tot},d-1}kT_{d-1} - g(m_d - m_{d-1}) = 0$$

- However it should be linearized anyway because the radiative transfer equation is non-linear ...

$$\begin{gathered} N^0_{\mathrm{tot},d}kT^0_d - N^0_{\mathrm{tot},d-1}kT^0_{d-1} - g(m_d - m_{d-1}) \\ +N_{\mathrm{tot},d}k\delta T_d + \delta N_{\mathrm{tot},d}kT_d - N_{\mathrm{tot},d-1}k\delta T_{d-1} - \delta N_{\mathrm{tot},d-1}kT_{d-1} = 0 \end{gathered}$$

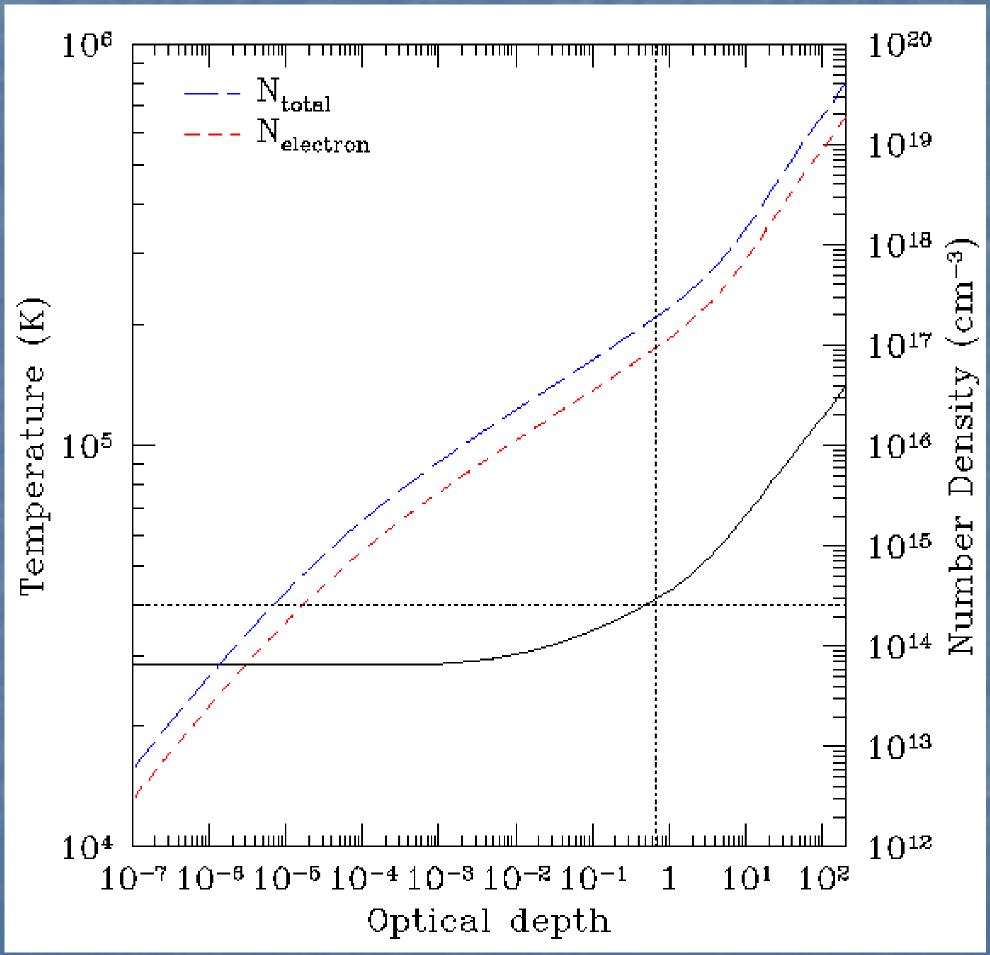
Model Atmosphere

- And finally a matrix is built using all 5 equations and solved using the Feautrier method of forward eliminating and back substitution of the tri-diagonal matrix:

$$
\begin{gathered}
T^1 = T^0 + \delta T \\
N_e^1 = N_e^0 + \delta N_e \\
N_p^1 = N_p^0 + \delta N_p \\
\dots \\
T^i = T^{i-1} + \delta T \\
N_e^i = N_e^{i-1} + \delta N_e \\
N_p^i = N_p^{i-1} + \delta N_p \\
\dots
\end{gathered}
$$

- Then we iterate until the solution has converged.

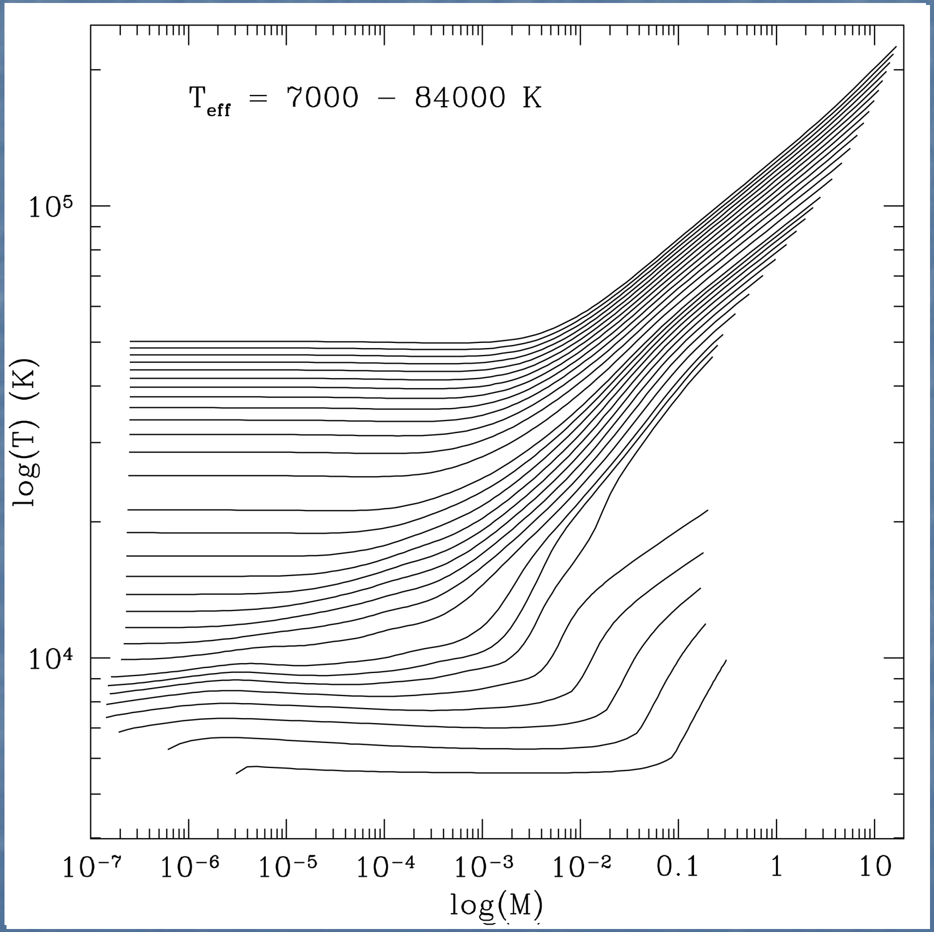
Model Atmosphere



- Model atmosphere density and temperature versus the optical depth:
  - T_{eff} = 40 000 K
  - log g = 8.0
- $e^{-\tau}$ is the photon escape probability
- Temperature at τ = 2/3 is defined as T_{eff}.

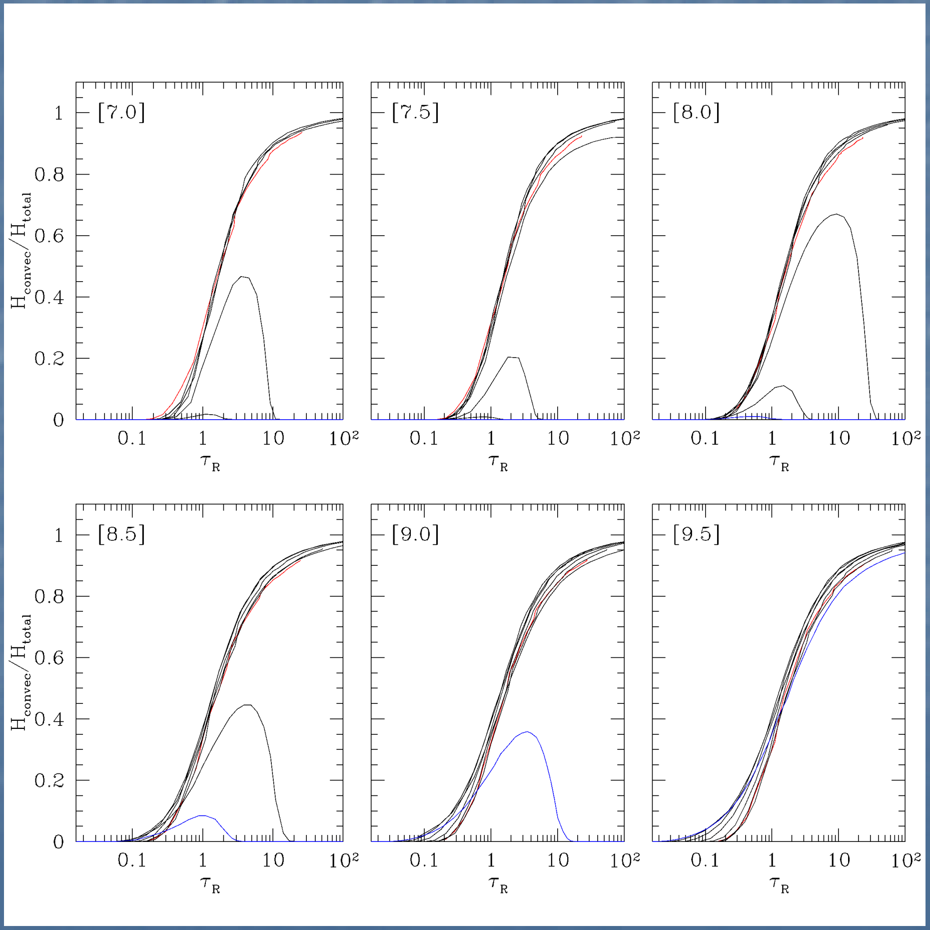
Model Atmosphere

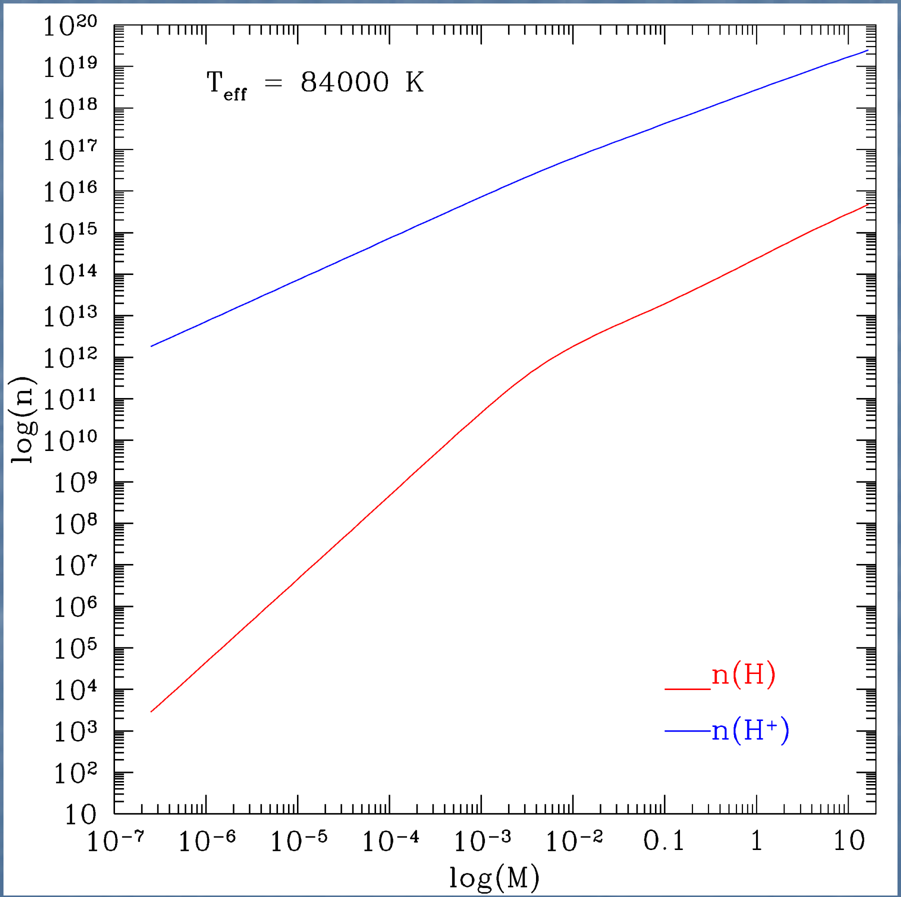
- Temperature structure as a function of depth.
- Note the sudden temperature drop between T_{eff} = 11 000 and 12 000 K.
- The effect of ionization of hydrogen.
- Optically thin outer layers almost isothermal.



Model Atmosphere

- Note the partial ionization around 12 000 K.





Line profiles

- We will assume that the atmosphere of the white dwarf is dominated by hydrogen.
- We need to consider the following absorption processes that will determine the total opacity of the hydrogen gas:
  - Absorption by neutral hydrogen between bound levels (bound-bound) – between principal quantum numbers n=l (lower level) and n=u (upper level): $E_n = 13.595\ \mathrm{eV}/n^2$.
  - Absorption by neutral hydrogen between a bound level and the continuum (bound-free), and between two free states (free-free).
  - Bound-free and free-free absorption by the negative hydrogen ion (H^-).
  - Scattering of light by neutral hydrogen (H I Rayleigh), and by e^- (Thomson).
- Due to spontaneous decay, energy levels have a finite lifetime, and therefore, a finite energy width $\Gamma_{nat} = \Delta E_l/h$ (natural width).

Line Profiles

- In white dwarfs, pressure broadening dominates over thermal broadening because of the high-density encountered in their atmospheres.
- Collisions, or interactions between particles cause the energy levels in an atom to be perturbed, and hence in the electromagnetic frequency of atomic transitions:

$$\frac{\Delta E}{h} = \Delta \nu = \frac{C_n}{R^n}$$

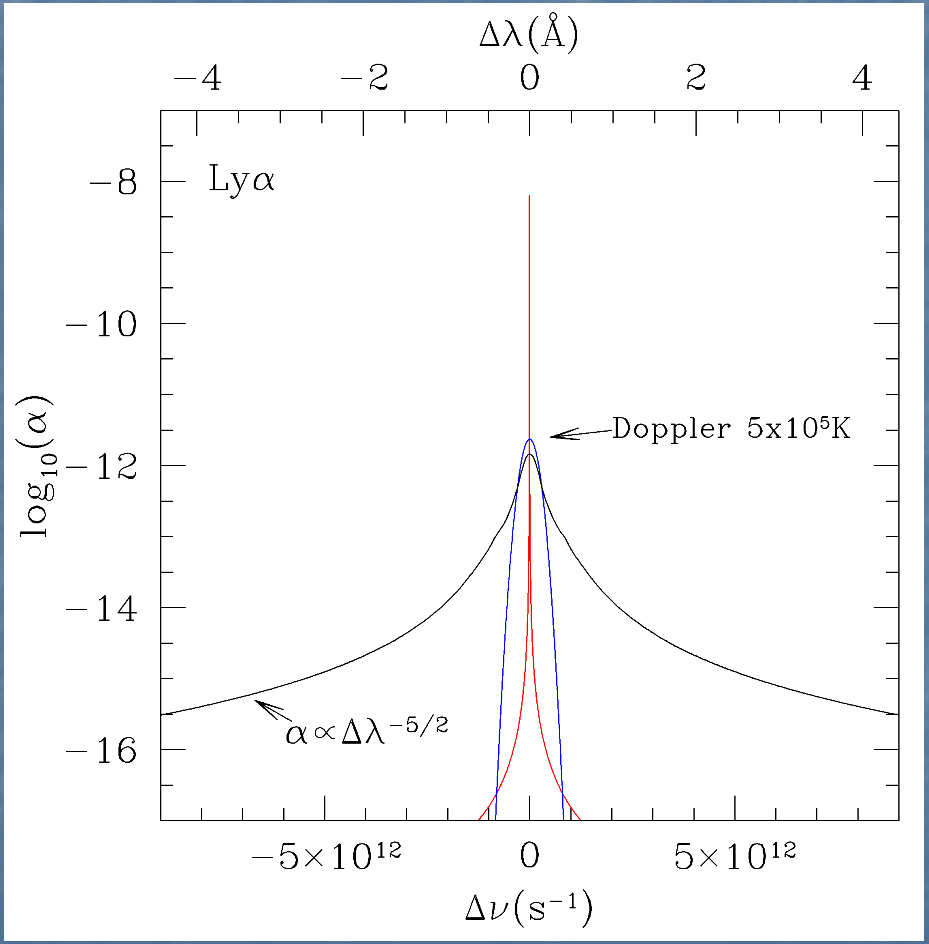
- R^{-n} describes the type of potential the particles are subjected to during an interaction.

Line Profiles

$$\frac{\Delta E}{h} = \Delta \nu = \frac{C_n}{R^n}$$

- n = 3: dipole-dipole interactions, when the neutral particles are of the same species (resonance).
- n = 6: if the neutral particles are of different species (van der Waals).
- n = 2: linear Stark effect, interactions between hydrogen atoms perturbed by protons and electrons.
- n = 4: quadratic Stark effect, interactions for most atoms perturbed by electrons.

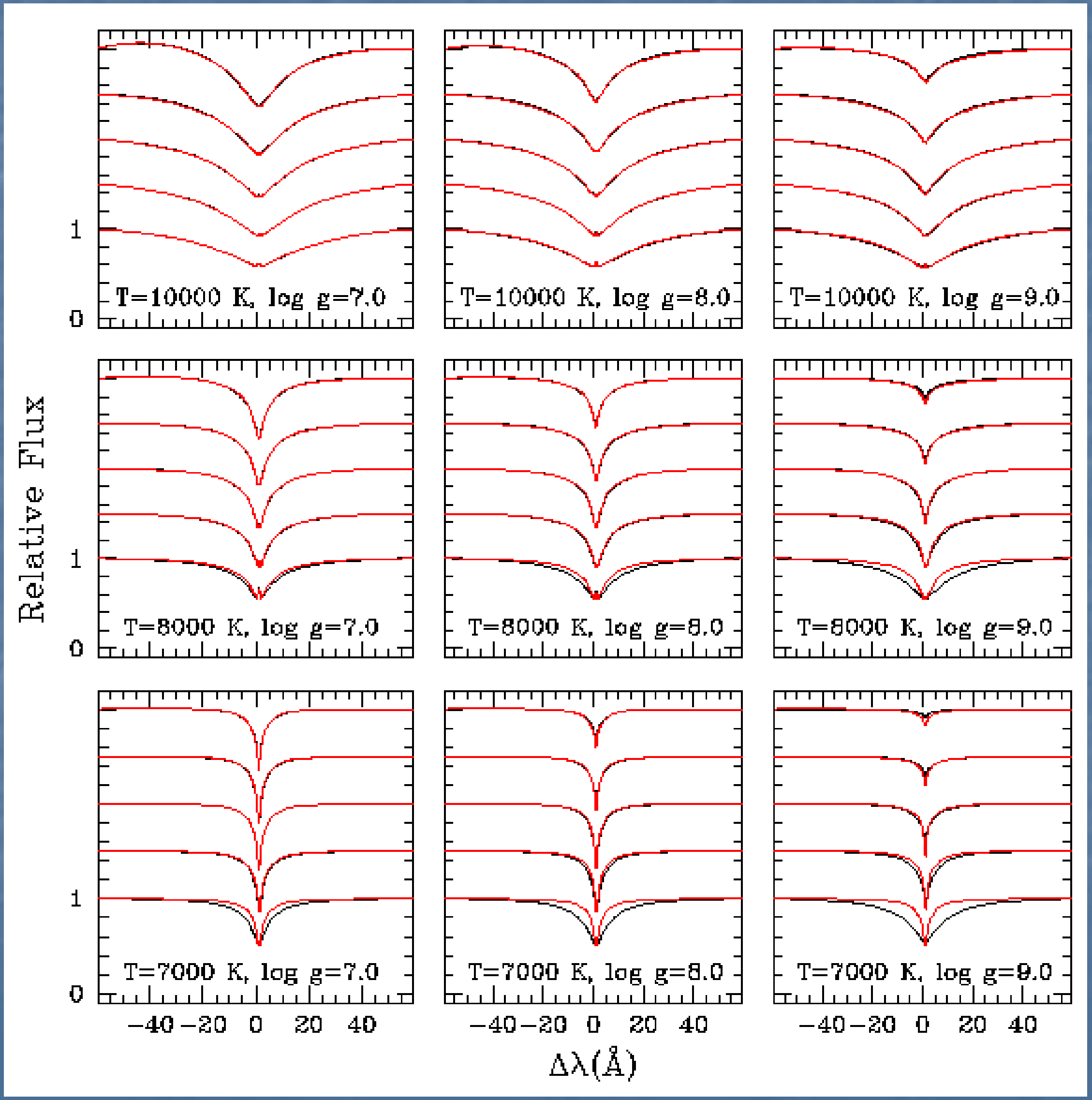
Line Profiles



- The final line profile is the convolution of the Stark, resonance and Doppler profiles.
- Lyman α broadening:
  - Doppler at T = 50 000 K
  - Stark effect – interaction with nearby e^- and p^+.
  - $n_e = n_p = 10^7$ cm^{-3}.
- Linear Stark broadening affects the H-lines and is dominant in white dwarfs with T_{eff} > 10 000 K, since hydrogen is mostly ionized.

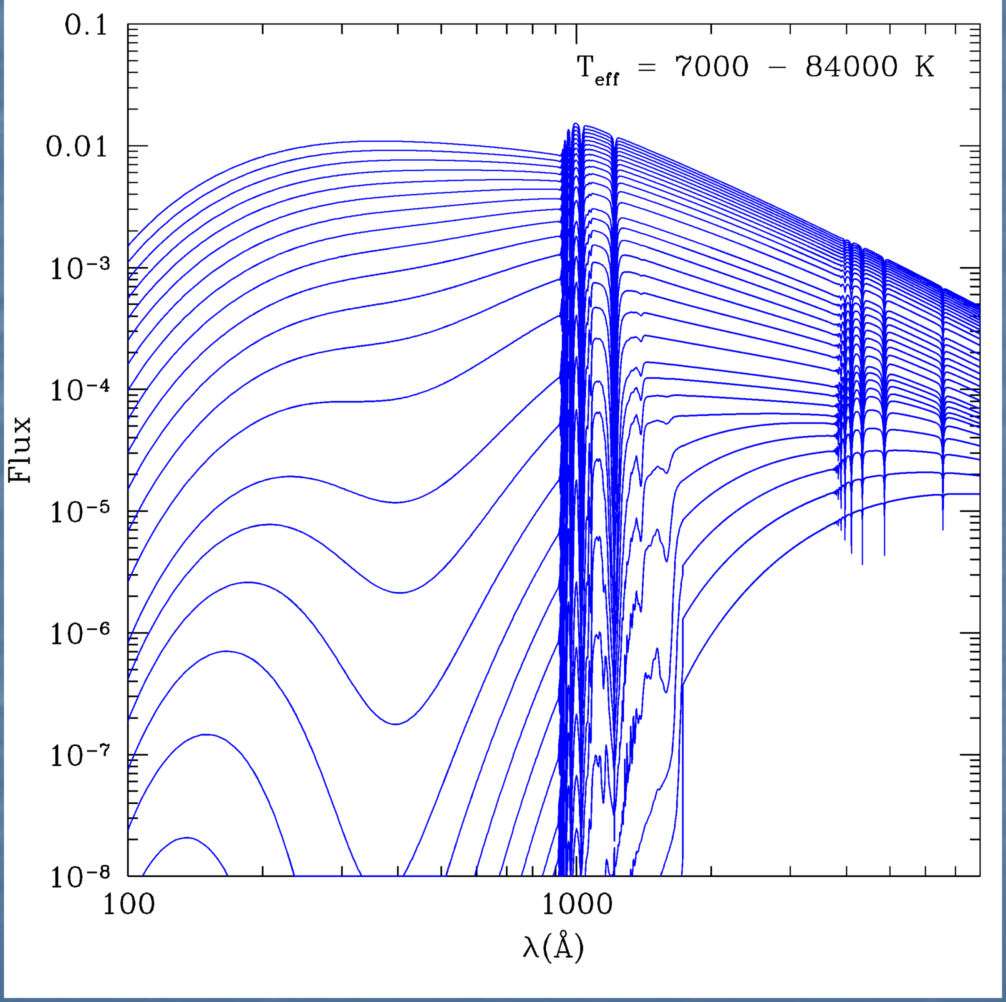
Line Profiles

- Resonance (self) broadening occurs as a result of interaction of radiation with the surrounding neutral atoms (hydrogen).
- Resonance broadening becomes important in cooler white dwarfs where hydrogen is mostly neutral.
- van der Waals broadening become important if there is significant He in the atmosphere.



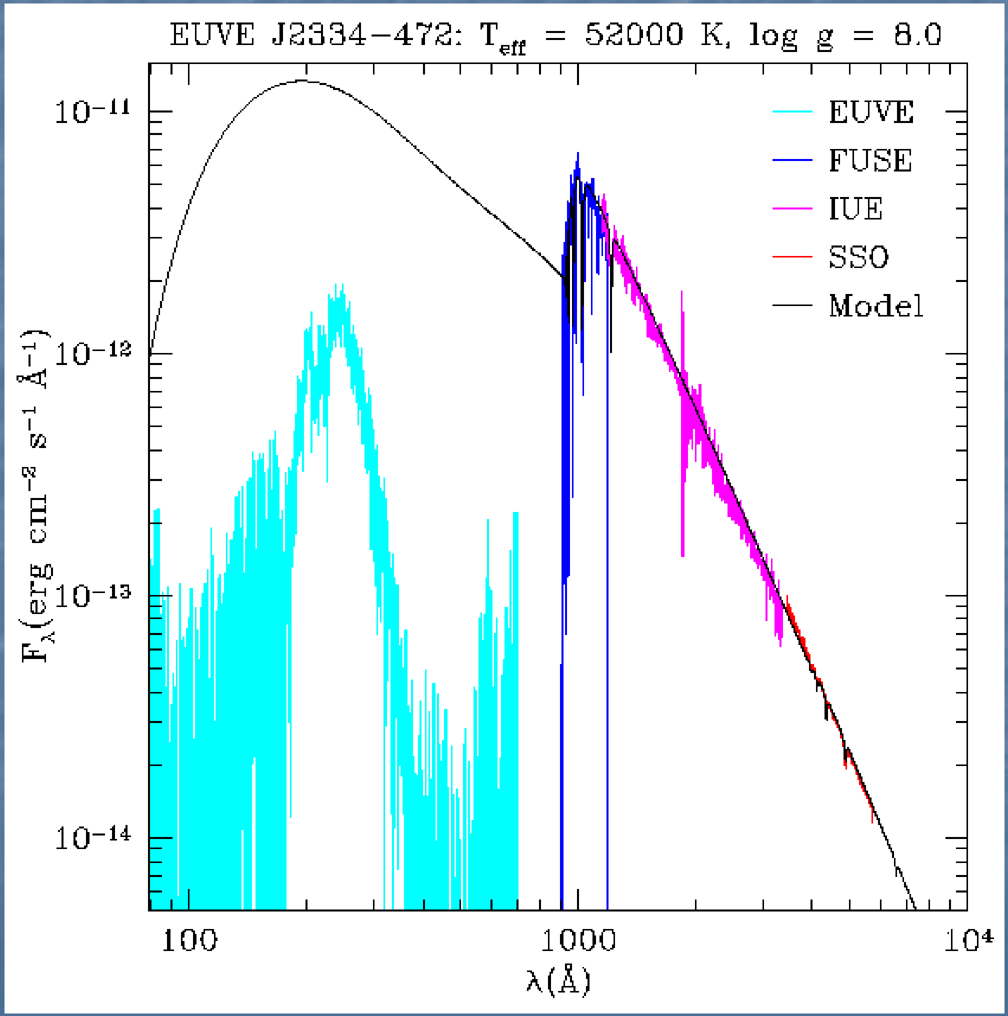
White Dwarf Spectra

- The maximum Balmer strength at 12 000 K.
- Note the decreasing Lyman strength and emerging EUV continuum.
- Area under each curve is $\sigma T^4/4\pi$.
- Lyman satellites – result of collisions between hydrogen atoms and neutral hydrogen or protons leading the formation of transitory molecules of H_2 and H_2^+.

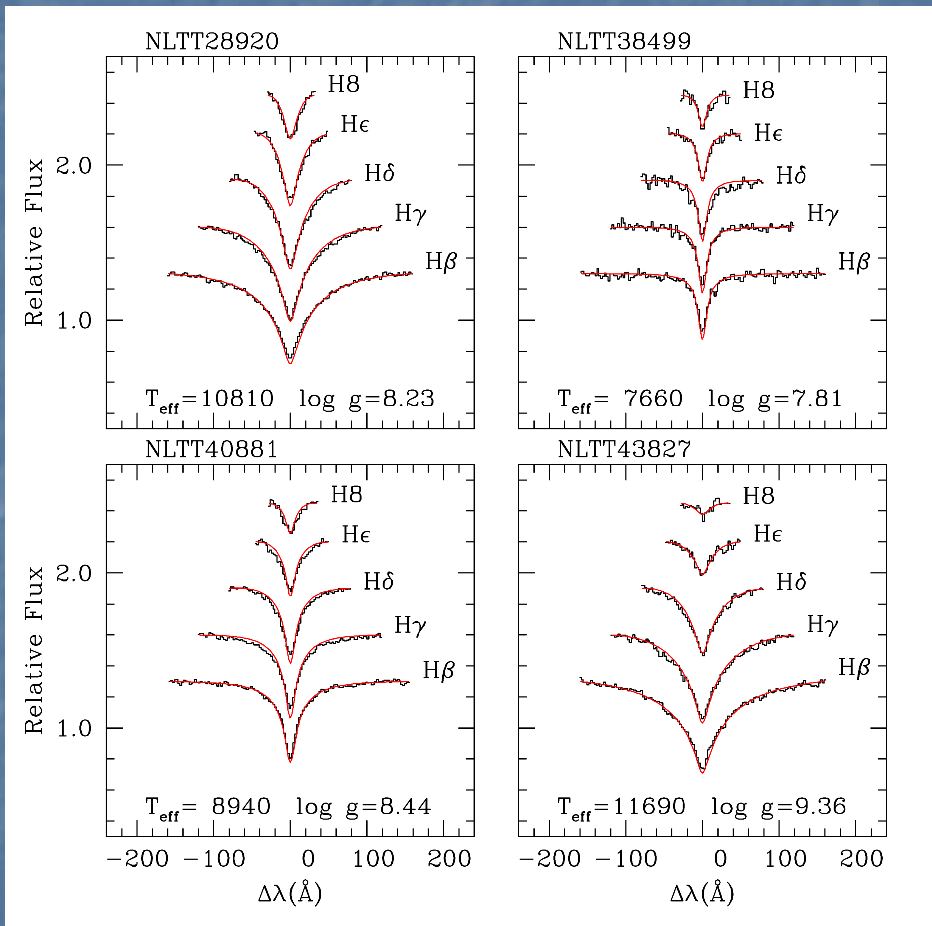


Observing White Dwarfs

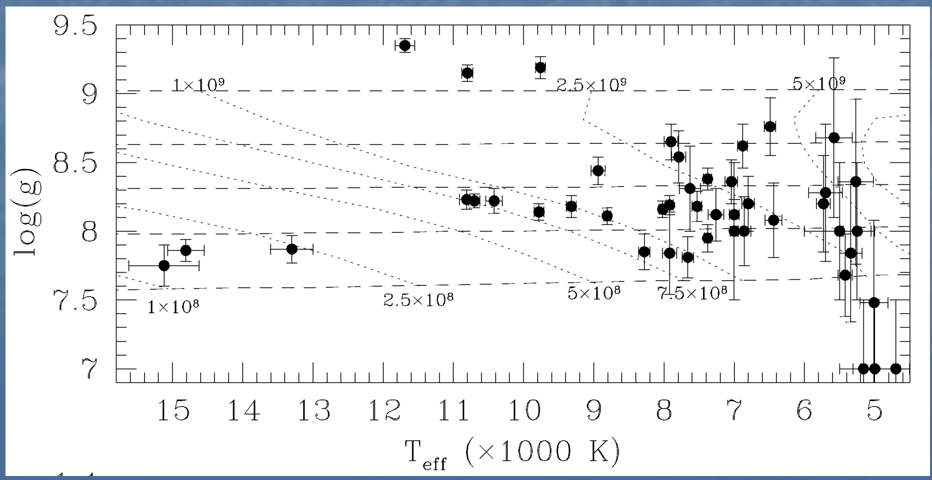
- Most observations of white dwarfs are done in the optical and ultraviolet.
- Ultraviolet observations are done using space-borne observatories.
- The EUV flux deficiency is due to:
  - Interstellar medium (ISM)
  - Fe opacities (Fe IV,V and VI).



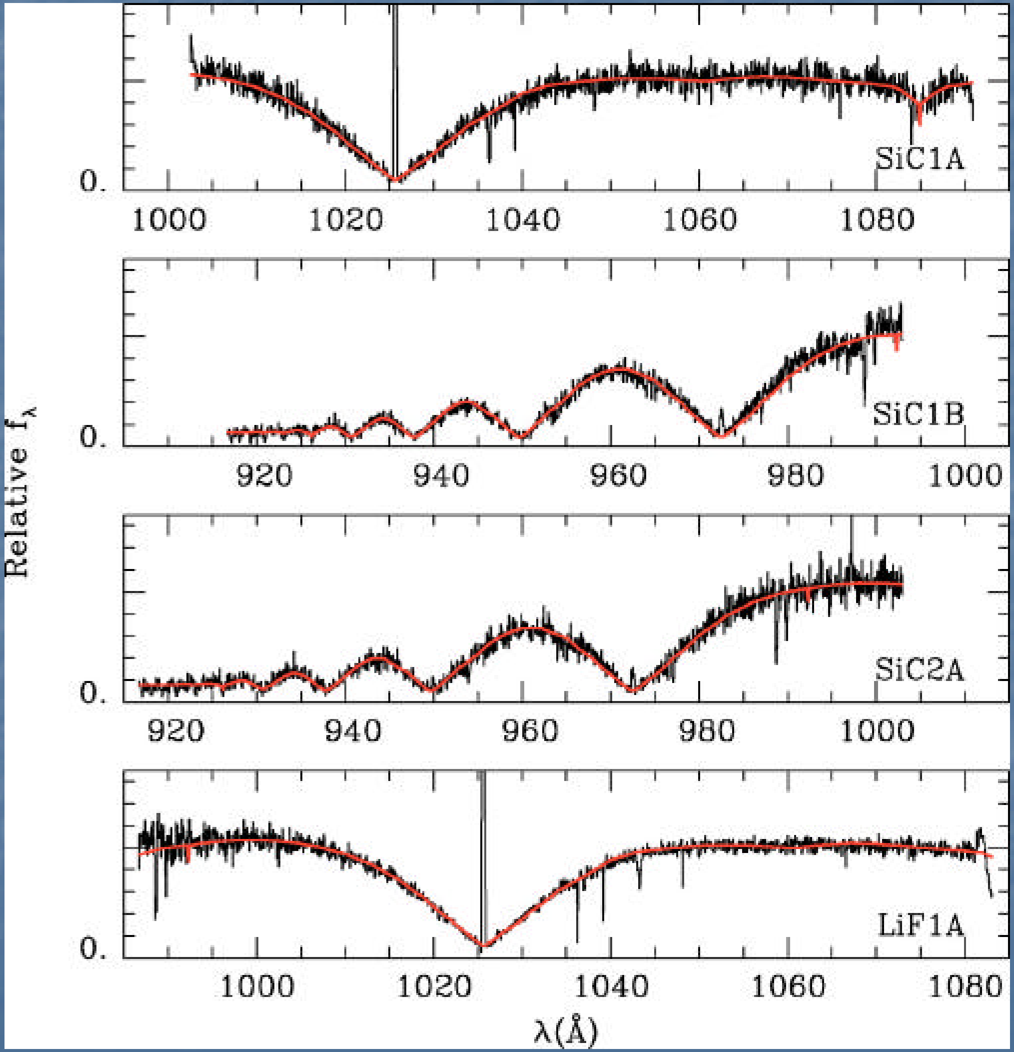
Observing White Dwarfs



- We can use the calculated model spectra to determine properties of white dwarfs.
- In the optical region, we can fit the Balmer lines to determine the effective temperature and surface gravity.



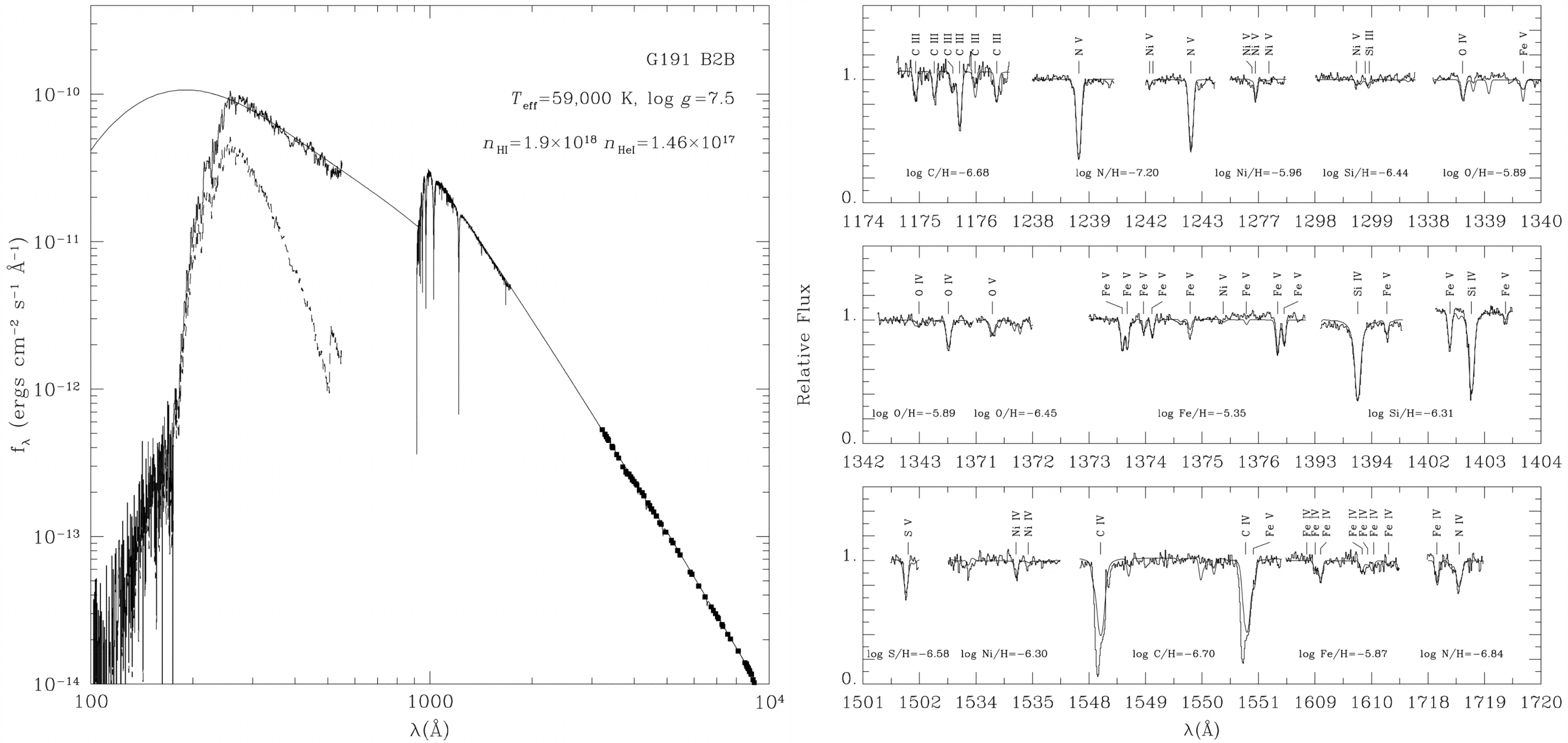
Observing White Dwarfs



(Vennes et al. 2004)

- *FUSE* (Far Ultraviolet Explorer) spectrum of PG 1603+432 showing the Lyman series.
  - T_{eff} = 35 075 ± 325 K,
  - log g = 7.96 ± 0.13,
  - M = 0.60 ±0.07 $M_\odot$.
- A rare DAB white dwarf that populates the *DB gap* (30 000 – 45 000 K).

Observing White Dwarfs

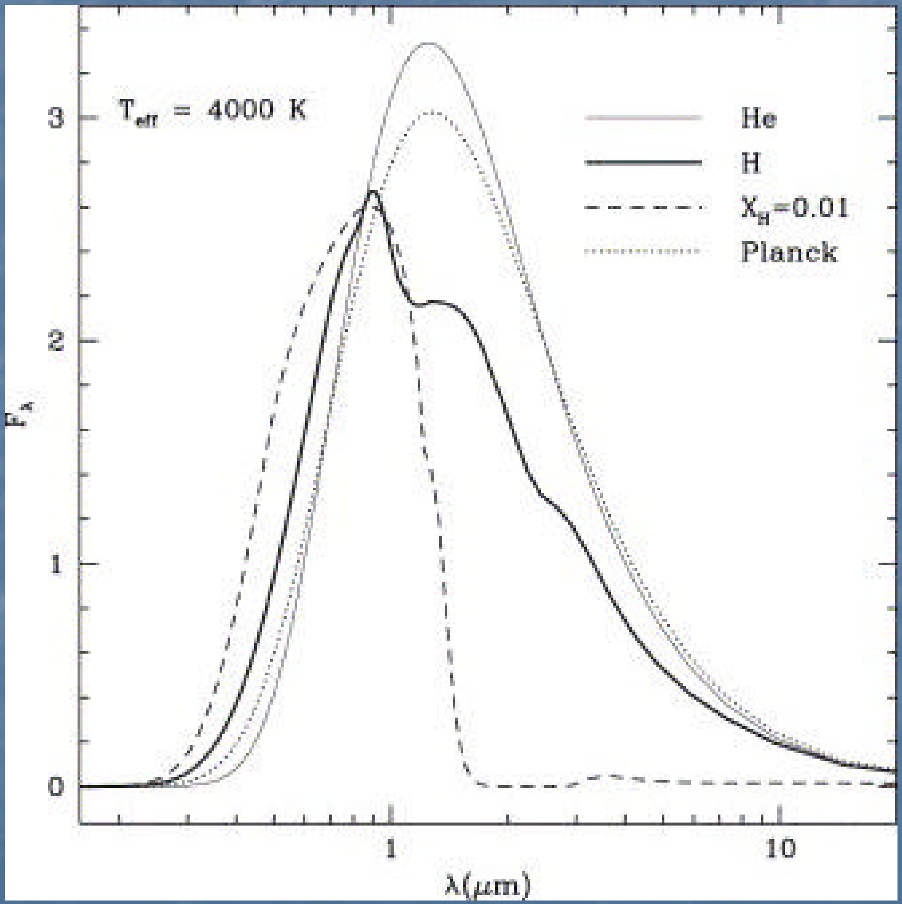


Vennes & Lanz 2001

- Traces of metals in the white dwarf atmosphere can be detected in the UV.

Observing White Dwarfs

- Infrared observations of white dwarfs are very useful in helping distinguish between very cool H- or He-rich atmospheres.
- H_2 molecule is the dominant opacity in very cool H-rich white dwarfs.
- In He-rich atmospheres, the dominant source of opacity are free-free absorption of He^- and Rayleigh scattering from neutral He atoms.



Hansen 2004

Summary

- The evolution of a white dwarf is primarily cooling – releasing thermal energy.
- The white dwarf will eventually crystallize.
- The thin atmosphere surrounding the core determines the rate of cooling.
- The atmosphere can be modeled by solving 5 equations:
  - Energy transfer
  - Energy conservation
  - Hydrostatic equilibrium
  - Equation of state – population levels
  - Charge and particle conservation
- In white dwarfs pressure broadening dominates over other broadening mechanisms.