# DIFFUSION AND RESISTIVITY

## Diffusion and mobility in weakly ionized plasma

Plasma is always spatialy limited, will have a density gradient and will tend to diffuse in the direction of lower density.

In controlled thermonuclear fusion, the magnetic field is used for the reducing this diffusion.

First we shall discuss the diffusion under following simplifications : 1) there is no magnetic field
2) the plasma is only weakly ionized
(charged particles collide primarily with neutral atoms than among themselves).

Weakly (partially) ionized plasma : ionospheric plasma, tokamak plasma close to the boundary.

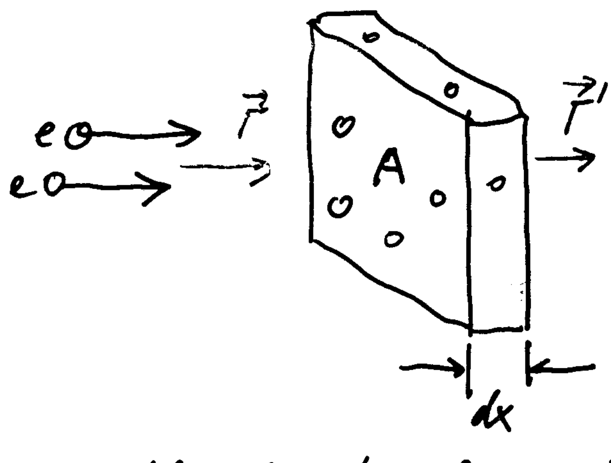
Collisions of charged particles with neutrals are dominant for $\frac{n_i}{n_n} \lesssim 10^{-3}$.

Collisions parameters : the mean free path $\lambda$

the mean frequency of collisions $\nu$.

When an electron collides with a neutral atom, it loses its momentum, partially or completely, depending on the angle of the interaction.

The probability of momentum loss is expressed by means of the equivalent cross section $\sigma$ of the atom, supposing that atoms are perfect absorbers of momentum.




let us consider that electrons are penetrating through a slab of area A and of thickness $dx$.

let the density of neutral atoms is $n_n$. let each atom is represented by a sphere of the cross-section area $\sigma$. let us suppose that whenever an electron comes within the area $\sigma$ (representing the atom) the electron loses all of its momentum.

The number of neutral atoms in a slab is:

$$dN_n = n_n A\, dx$$

The total area, which prevents electrons to penetrate:

$$n_n A\, dx\, \sigma .$$

The relative area (per $m^2$)

$$\frac{n_n A\, dx\, \sigma}{A} = n_n\, dx\, \sigma .$$

If a flux $\vec{\Gamma}$ of electrons is incident on the slab, the outgoing flux $\Gamma'$ is given as

$$\Gamma' = \Gamma(1 - n_n \sigma\, dx) .$$

The change of $\Gamma$ with distance is

$$\frac{d\Gamma}{dx} = -n_n \sigma \Gamma \Rightarrow \Gamma = \Gamma_0 e^{-n_n \sigma x} =$$

$$= \Gamma_0 e^{-\frac{x}{\lambda_m}}$$

$\lambda_m = \dfrac{1}{n_n \sigma}$ ... the mean free path.

The mean time $\tau$ between collisions of a particle with velocity $v$ is

$$\tau = \frac{\lambda_m}{v}$$

and the mean frequency of collisions, $\tau^{-1}$, is

$$\tau^{-1} = \frac{v}{\lambda_m} = n_n \sigma v .$$

The collision frequency $\nu$ is the average of $\tau^{-1}$ over particles with all velocities of a Maxwellian distribution

$$\nu = n_n \overline{\sigma v} .$$

Diffusion parameters : the mobility
the diffusion coefficient.

The fluid equation for the motion of the ion and electron component :

$$mn \frac{d\vec{v}}{dt} = mn \left[ \frac{\partial \vec{v}}{\partial t} + (\vec{v} \cdot \nabla) \vec{v} \right] = \pm en\vec{E} - \nabla p - \underline{mn\nu\vec{v}}$$

The stationary state: $\frac{\partial \vec{v}}{\partial t} = 0$. For sufficiently small $\vec{v}$, $\frac{d\vec{v}}{dt} = 0$ (a fluid element will not "see" between two collisions the change in $\vec{E}$, $\nabla p$). Then

$$\vec{v} = \frac{1}{mn\nu}\left(\pm en\vec{E} - KT\nabla n\right)$$

$$= \pm \frac{e}{m\nu}\vec{E} - \frac{KT}{m\nu}\frac{\nabla n}{n}.$$

$$\frac{|q|}{m\nu} = \mu \text{ .... mobility}$$

$$\frac{KT}{m\nu} = D \text{ .... diffusion coefficient.}$$

The Einstein relation: $\mu = |q| \frac{D}{KT}$.

The flux $\vec{\Gamma}_j$ of the $j$-th species

$$\vec{\Gamma}_j = n\vec{v}_j = \pm \mu_j n_j \vec{E} - D_j \nabla n.$$

Fick's law: $\vec{\Gamma} = -D\nabla n$.

## Decay of a Plasma by Diffusion

The equation of continuity (using the definition of fluxes $\vec{\Gamma}_j$)

$$\frac{\partial n_j}{\partial t} + \nabla \cdot \vec{\Gamma}_j = 0$$

Let $\vec{\Gamma}_i$, $\vec{\Gamma}_e$ are the fluxes of the ion and electron component. The lighter electrons have the tendency to leave the plasma first. But, as a result, the electric field $\vec{E}$ appears, accelerating the ion component. Therefore, the electric field makes equal both $\vec{\Gamma}_i$, $\vec{\Gamma}_e$

$$|\vec{\Gamma}_i| = |\vec{\Gamma}_e| = |\vec{\Gamma}|$$

$$\Rightarrow \quad \Gamma = \mu_i n \vec{E} - D_i \nabla n = -\mu_e n \vec{E} - D_e \nabla n$$

$$\Rightarrow \quad E = \frac{D_i - D_e}{\mu_i + \mu_e} \times \frac{\nabla n}{n}$$

$$\boxed{D_A = \frac{(T_e + T_i) D_i D_e}{T_i D_e + T_e D_i} \qquad T_e = T_i \Rightarrow D_A = 2 D_i}$$

$$\Rightarrow \quad \Gamma = \mu_i \frac{D_i - D_e}{\mu_i + \mu_e} \nabla n - D_i \nabla n = \frac{\mu_i D_i - \mu_i D_e - \mu_i D_i - \mu_e D_i}{\mu_i + \mu_e} \nabla n$$

$$\Rightarrow \quad \Gamma = -\frac{\mu_i D_e + \mu_e D_i}{\mu_i + \mu_e} \nabla n = -D_a \nabla n$$

$$\boxed{D_a = \frac{\mu_i D_e + \mu_e D_i}{\mu_i + \mu_e}}$$

$$\boxed{\frac{\partial n}{\partial t} = D_a \nabla^2 n} \quad \text{if } \nabla D = 0$$