turbulence [42], and also to the different modes of beat-wave acceleration of electrons by two laser beams.

In the foregoing, the linear interaction based on resonance $\omega - kv = 0$ has been discussed. Nonlinear Landau damping is based on resonance of the type

$$\omega_k - \omega_l - (k_k - k_l)v = 0, \tag{173}$$

where ω_k, k_k and ω_l, k_l are the frequencies and wave vectors of two waves (or of two components of the spectrum). The foregoing resonance is usually called the beat resonance, and the mechanism itself, due to analogy with linear Landau damping, is called nonlinear Landau damping.

The formalism describing nonlinear Landau damping is rather complicated. It follows directly from the discussion of the system of kinetic equations for waves, describing nonlinear wave-plasma interaction and determining resonant mode-coupling interaction. The mechanism of nonlinear Landau damping is then closest to the case of three- mode coupling.

The exact approach, describing the mutual mode coupling, forms an important, but rather extensive part of the plasma turbulence theory, and, therefore, exceeds the scope of this chapter. For a more thorough study, see, e.g., Kadomtsev [42], Davidson [43] or Cooc [20].

Due to this complexity, the exact formalism, describing nonlinear Landau damping, can, in the initial approach to this problem, somehow cover its interesting physical meaning. To describe the basic important effects we shall, therefore, use the instructive approach of Ott and Dum [44].

Let us assume two coherent electrostatic waves with amplitudes φ_1, φ_2, frequencies ω_1 , ω_2, and wave vectors $\mathbf{k_1}$, $\mathbf{k_2}$, $\mathbf{k_1} \parallel \mathbf{k_2}$. Let these waves propagate in a homogeneous plasma without a magnetic field.

Let the resonant interaction of two waves be given by resonance condition (173) and assume the linear resonance effect to be negligible. In this case, the following two conservation laws must be fulfilled:

$$\frac{\mathrm{d}U_1}{\mathrm{d}t} + \frac{\mathrm{d}U_2}{\mathrm{d}t} + \frac{\mathrm{d}T}{\mathrm{d}t} = 0 \qquad (energy) \tag{174}$$

$$\frac{\mathrm{d}P_1}{\mathrm{d}t} + \frac{\mathrm{d}P_2}{\mathrm{d}t} + \frac{\mathrm{d}P}{\mathrm{d}t} = 0 \qquad (momentum). \tag{175}$$

Here $U_{1,2}$ and $P_{1,2}$ are the total energy and momentum of the waves. The mutual coupling of both quantities is given by the following relation:

$$U_{1,2} = \omega_{1,2}N_{1,2}; \qquad P_{1,2} = k_{1,2}N_{1,2}. \tag{176}$$

Here, N_i is the action of the i-th wave. Action N_i is defined as [44]

$$N_i = \frac{\epsilon_0 k_i^2}{4}\frac{\partial \varepsilon^{(1)}}{\partial \omega_i}\varphi_i^2, \tag{177}$$

where $\varepsilon^{(1)}(k,\omega)$ is the linear dielectric constant.

The general discussion of the nonlinear waves-plasma interaction indicates (see, e.g [44] or [40]) that the total potential (with the background plasma response) is

$$\begin{aligned}\varphi(x,t) &= \varphi_{10}\cos(k_1 x-\omega_1 t-\theta_1)+\varphi_{20}\cos(k_2 x-\omega_2 t-\theta_2)- \\ &-\frac{1}{2}\varphi_{10}\varphi_{20}\, Re[\frac{\varepsilon^{(2)}}{\varepsilon^{(1)}}\exp[-i(\omega_1-\omega_2)t+i(k_1-k_2)x+i(\theta_1-\theta_2)]] \qquad (178)\end{aligned}$$

Here $\varepsilon^{(2)}$ is the nonlinear second-order dielectric constant, defined as

$$\begin{aligned}(k_1-k_2)^2\varepsilon^{(2)} &= \frac{e}{m}\omega_{pe}^2\int dv\frac{k_1k_2}{(\omega_1-\omega_2)-(k_1-k_2)v}\times \\ &\times\frac{\partial}{\partial v}[(\omega_2-k_2v)^{-1}-(\omega_1-k_1v)^{-1}]\frac{\partial F_0}{\partial v} \qquad (179)\end{aligned}$$

and F_0 is the unperturbed distribution function.

Let the resonant velocity v_R be defined as

$$v_R=\frac{\omega_1-\omega_2}{k_1-k_2} \qquad (180)$$

and let us use the identity

$$\omega=\omega_1-k_1-k_1v_R=\omega_2-k_2v_R. \qquad (181)$$

Using the transformation

$$y=x-v_Rt \qquad (182)$$

and splitting y into

$$y=y_0+y_1 \qquad (183)$$

where y_1 is the rapidly varying component of $y(t)$, we can obtain for y_0 the equation [44]

$$\frac{\mathrm{d}^2y_0}{\mathrm{d}t^2}=e\frac{\mathrm{d}\varphi_{eq}(y_0)}{\mathrm{d}y_0}. \qquad (184)$$

Here $\varphi_{eq}(y_0)$ is given as

$$\varphi_{eq}(y_0)=\varphi_0\cos[(k_1-k_2)y_0+\theta] \qquad (185)$$

with

$$\varphi_0=-\frac{\varphi_{10}\varphi_{20}}{2}[\frac{\varepsilon^{(2)}}{\varepsilon^{(1)}}+\frac{ek_1k_2}{m\omega^2}], \qquad (186)$$

where θ is the phase angle.

The foregoing equation describes the motion of a particle in a sinusoidal wave of amplitude φ_0 by analogy with the motion of a trapped particle in a wave, described in Section 3. Frequency ω_B of well trapped particles is

$$\omega_B=[\frac{e}{m}\,|k_1-k_2|\,\varphi_0]^{\frac{1}{2}}. \qquad (187)$$

For time t satisying $\omega_B t \ll 1$, the effect of these oscillations can be neglected. Following the results of O'Neil [8], the relation between $\frac{dT}{dt}$ and $\frac{dP}{dt}$ is then

$$\frac{dT}{dt} = v_R \frac{dP}{dt}. \tag{188}$$

Inserting this result into equations (174) and (175), yields

$$\frac{dU_1}{dt} = -\frac{\omega_1}{\omega_1 - \omega_2} \frac{dT}{dt} \tag{189}$$

$$\frac{dU_2}{dt} = \frac{\omega_2}{\omega_1 - \omega_2} \frac{dT}{dt}, \tag{190}$$

and, finally,

$$\frac{U_1}{\omega_1} + \frac{U_2}{\omega_2} = const. \tag{191}$$

For $\omega_B t \ll 1$, the total change of the kinetic energy of resonant particles is, therefore, according to [44], and using the analogy with [8],

$$\frac{dT}{dt} = -\frac{\pi}{2}(\omega_1 - \omega_2)\, \omega_{pe}^2 \, \frac{\varphi_0^2}{4\pi} \, sgn(k_1 - k_2) \, \frac{\partial F_0}{\partial v} / v_R. \tag{192}$$

Inserting (192) and (186) into (189), (190), we obtain the nonlinear Landau damping rate:

$$\begin{aligned} \gamma_{nL} \;=\; & (2U_1)^{-1} \frac{dU_1}{dt} = \frac{\omega_{pe}^2}{16} \, (\epsilon_0 k_1^2 \, \frac{\partial \varepsilon^{(1)}}{\partial \omega_1})^{-1} \, sgn(k_1 - k_2) \, \frac{\partial F_0}{\partial v} / v_R \times \\ & \times \left| \frac{\varepsilon^{(2)}}{\varepsilon^{(1)}} + \frac{e k_1 k_2}{m \omega^2} \right|^2 \varphi_{20}^2. \end{aligned} \tag{193}$$

Equation (186) can be regarded as a special form of the Manley-Rowe law, describing the mutual resonant interaction of three waves [41] (here, the effect of the third wave is replaced by the resonant particles). For a spectrum of Langmuir waves, the consequence of this law is the continuous flux of wave energy into the lower k part of the spectrum. This is the well- known result of Langmuir turbulence evolution [42].

The validity of criterion (191) is based on the assumptions used. It is of some interest to look for a regime under which these assumptions are violated. Such discussions appeared in [44] and in [45 - 47].

(According to the recent results of the weak turbulence theory (see e.g. [48, 49]), together with three-mode coupling and nonlinear Landau damping, the third interaction form has to be included, namely, the nonlinear mode-mode coupling between two kinds of plasma waves, ion sound waves and Langmuir waves. In this interaction, it is supposed [48] that the first type of waves satisfies the Landau resonant condition, and the second type does not obey either linear, or nonlinear

Landau resonance. The growth or damping of this nonresonant mode is given by the nonlinear interaction with resonant waves, caused solely by the nonlinearity of the dielectric function of Langmuir waves in the presence of ion-sound turbulence. Also in this case, the Manley-Rowe relation is fulfilled. This relation can, nevertheless, be violated, if the wave-particle system forms an open, nonconservative system, with external sources and sinks both for particles and waves).

The inhomogeneity of a plasma can change the energy of resonant particles. This effect was discussed in [50 - 52].

Recently, an interesting beat mechanism for particle acceleration was proposed by Tajima and Dawson [53]. The beat-wave acceleration of electrons by two laser beams is based on the interaction of two laser electromagnetic waves with frequencies $\omega_{1,2}$ and wave numbers $k_{1,2}$, propagating through a plasma. Their nonlinear interaction can generate on their beat a large amplitude plasma (Langmuir) wave, which accelerates resonant electrons up to large energies.

7. SATURATION OF BEAM-PLASMA INSTABILITY; TRAPPED PARTICLES SIDEBAND STUDIES

As follows from the foregoing, trapped particles play an important role in a plasma. In this chapter, we shall additionally mention two interesting effects, connected with particle trapping in a single wave. We shall shortly discuss the saturation of a beam-plasma instability by the trapping of beam particles by the generated wave, and the excitation of sidebands of a large amplitude wave, generated by trapped particles in this wave. These effects are important both for the theory and experiment.

Saturation of a beam-plasma instability

The simpliest case of beam-plasma interaction can be represented by an electron beam with velocity V, which propagates through a cold homogeneous plasma without magnetic field. In a 1-D model, the dispersion relation for electrostatic waves in such system reads [54]

$$1 - \frac{\omega_{pe}^2}{\omega^2} - \alpha \frac{\omega_{pe}^2}{(\omega - kV)^2} = 0. \tag{194}$$

Here,

$$\omega_{pe}^2 = \frac{ne^2}{\epsilon_0 m_e}; \qquad \alpha = \frac{n_1}{n_0}; \qquad \frac{n_1}{n_0} \ll 1. \tag{195}$$

where n_0 is the plasma density and n_1 is the density of the beam.

The system is unstable. The most unstable wave has parameters

$$\omega = \omega_{pe}(1 - \frac{\alpha^{\frac{1}{3}}}{2^{\frac{4}{3}}}); \qquad k \approx \frac{\omega_{pe}}{V}; \qquad \gamma = \omega_{pe}\frac{\sqrt{3}}{2^{\frac{4}{3}}}\alpha^{\frac{1}{3}} \tag{196}$$

where γ is the growth rate of this instability.

The saturation of this instability followed from the whole set of experiments. It was found that this saturation is caused by trapping beam particles in the generated wave; this trapping effect was found in [56], and experimentally verified, e.g., in [67].

The mechanism is simple. The phase velocity of the most unstable wave, v_{ph}, satisfies

$$v_{ph} = \frac{\omega}{k} < V. \tag{197}$$

The beam, therefore, runs ahead of the wave. For a small amplitude wave, the motion of beam particles is unaffected by this wave. For waves with larger amplitudes, the beam dynamics becomes affected by the electric field of the wave. At the threshold electric field amplitude [56],

$$E_{th} \approx \frac{m_e}{2ek}(\frac{\alpha}{2^{\frac{4}{3}}}\omega_{pe})^2 \tag{198}$$

the wave starts to trap beam particles. In this case, namely, in phase space, the wave separatrix just touches the beam (see Fig. 13).

This trapping leads to the destruction of the beam and to the saturation of the instability. The sequence of trapping of the beam is presented in Fig. 14 taken from [57]. The problem is treated as a spatial model of generation of a stationary wave. Coordinate η is the dimensionless spatial coordinate, coordinate $\beta - \varphi$ is the normalized phase of the beam relative to the wave, and coordinate $\frac{d\varphi}{d\eta}$ is the dimensionless velocity of beam particles. For $\eta = 9.2$, the beam is totally trapped and the instability is saturated. This can be seen in Fig. 15. Here F^2 is the dimensionless energy of the wave; the oscillations of F are caused by particle trapping.

Trapped particle sideband instability

Let us consider a homogeneous plasma without magnetic field and a large- amplitude electrostatic wave, propagating through this plasma. This wave will trap some plasma electrons. The oscillation of these trapped particles can generate an instability. Kruer et al. [58] found the dispersion relation for this instability to be

$$1 = \frac{\omega_T^2}{\Omega^2 - \omega_B^2}[\frac{1}{\varepsilon_L(k,\omega)} + \frac{1}{\varepsilon_L(k - 2k_0, \omega - 2\omega_0)}]; \qquad \varepsilon_L = 1 - \frac{\omega_{pe}^2}{\omega^2 - 3k^2v_T^2} \tag{199}$$

Here ω_0, k_0 are parameters of the large- amplitude wave, ω_T is the plasma frequency of the trapped particle, $\Omega = \omega - k\frac{\omega_0}{k_0}$, v_T is the thermal velocity, and ω_B is given as $\omega_B = \sqrt{\frac{ek_0^2E_0}{m}}$, where E_0 is the amplitude of the original wave. Is was found that the instability generates two sidebands with frequencies, symmetrically shifted relative to the frequency of the original wave. This shift $\Delta\omega$ is approximately

$$\Delta\,\omega \approx \omega_B \tag{200}$$

A similar symmetry was also observed for wave numbers.

The saturation is caused by detrapping. Trapped particles in the large- amplitude wave can be detrapped by sideband waves with sufficiently large amplitudes. For the 1-D case, this detrapping was discussed, e.g., in [59, 60]. The excitation of sidebands and its saturation seems to be very important in free-electrons lasers (FEL) and in gyrotrons. Here, of course, the waves are electromagnetic waves. Sideband excitation and saturation was discussed, e.g., in [61-63], see Fig. 16.

CONCLUSION

In the foregoing text, a short review of some basic parts of the plasma kinetic theory (or, rather, an introduction to it) has been presented. In many aspects, the plasma kinetic theory does not represent a closed system, but rather an extremely broad topic, which is still developing. This concerns especially the nonlinearity or turbulence problems. Moreover, we have seen that also even rather old solutions (like the quasilinear approximation), are still subject to some criticism. The more the reason for this brief review not providing an exhaustive account of the plasma kinetic theory. We have therefore concentrated only on problems, which a reader, who is not an expert in this branch of the plasma physics, or who is a beginner therein, might often come across. Readers interested in a more thorough study of the mechanisms mentiond, or in other effects, which we had to omit, either due to their complexity, or simply due to the limited extent of this contribution, are referred to our list of references, or to further lectures of this School.

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References

[1] Montgomery, D.C. and Tidman, D.A.: Plasma Kinetic Theory, MacGraw-Hill, New York 1964.

[2] Sanderson, J.J.: Kinetic Theory, in: Plasma Physics and Nuclear Fusion Research (Ed. R.D. Gill), Academic Press, London 1981, 119-154.

[3] Krall, N.A. and Trivelpiece, A.W.: Principles of Plasma Physics, MacGraw-Hill, New York 1973.

[4] Wesson, J.: Tokamaks, Oxford Science Publications, Oxford 1987.

[5] Vlasov, A.A.: J. Phys. (USSR), 8 (1938), 25.

[6] Landau, L.L.: J. Phys. (USSR), 10 (1946), 25.

[7] Goldstein, H.: Classical Mechanics, Addison-Wesley, Reading, Mass. 1951.

[8] O'Neil, T.M.: Collisionless damping of nonlinear plasma oscillations, Phys. Fluids, 8 (1965), 2255-2262.

[9] Rosenbluth, M.N.: Microinstabilities, in: Plasma Physics (Seminar, Trieste, 1964), IAEA, Vienna 1965, 485-513.

[10] Drummond, W.E. and Pines, D.: Nonlinear stability of plasma oscillations, Nucl. Fusion, Suppl. Pt. 3 (1962), 1049-1057.

[11] Vedenov, A.A., Velikhov, E.P. and Sagdeev, R.Z.: Nonlinear oscillations of rarified plasma, Nucl. Fusion, 1 (1961), 82-100.

[12] Tsunoda, T.I., Doveil, F. and Malmberg, J.H.: Experimental test of quasilinear theory, Phys. Fluids, B 3 (1991), 2747-2757.

[13] T.H.Stix, Waves in Plasmas, American Institute of Physics, New York 1992.

[14] Vedenov, A.A.: Introduction to the Theory of a Weak-Turbulent Plasma (in Russian) in: Voprosy teorii plasmy 3, Gosatomizdat, Moscow 1963, 203-244.

[15] Zaslavskii, G.M.: Stochasticity of Dynamical Systems (in Russian), Nauka, Moscow 1984.

[16] Cary, J.R., Doxas, I., Escande, D.F. and Verga, A.D.: Enhancement of the velocity diffusion in longitudinal plasma turbulence, Phys. Fluids, B 4, (1992), 2062-2069.

[17] Laval, G. and Pesme, D.: Inconsistency of quasilinear theory, Phys. Fluids, 26 (1983), 66-68.

[18] Laval, G. and Pesme, P.: Self-consistency effects in quasilinear theory: a model for turbulent trapping, Phys. Rev. Lett., 53 (1984), 270-273.

[19] Cary, J.M., Escande, D.F. and Verga, A.D.: Nonquasilinear diffusion far from the chaotic threshold, Phys. Rev. Lett., 65 (1990), 3132-3135.

[20] Cook, I.: Plasma Turbulence, in: Plasma Physics and Nuclear Fusion Research (Ed. R.D.Gill), Academic Press, London 1981, 293-304.

[21] Dupreé, T.H.: A perturbation theory for strong plasma turbulence, Phys. Fluids, 9 (1966), 1773-1782.

[22] Akchiezer, A.I.: Plasma Electrodynamics (in Russian), Nauka, Moscow 1974.

[23] Brambilla, M. and Cardinali, A.: Eikonal description of hf waves in toroidal plasmas, Plasma Physics, 24 (1982), 1187-1218.

[24] Wersinger, J.M., Ott, E. and Finn, J.M.: Ergodic behavior of lower hybrid decay wave ray trajectories in toroidal geometry, Phys. Fluids, 21 (1978), 2263-2267.

[25] Bonoli P.T. and Ott, E.: Toroidal and scattering effects in lower-hybrid wave propagation, Phys. Fluids, 25 (1982), 359-375.

[26] Fisch, N.J.: Confining a tokamak plasma with rf-driven currents, Phys. Rev. Lett., 41 (1978), 873-876.

[27] Pavlo, P., Krlín L. and Tlučhoř, Z.: Effects of magnetized alpha particles on lower hybrid heating and current drive in a reactor grade plasma, Nucl. Fusion, 31 (1991), 711-727; ITER-IL-Ph-6-9-S-23, Max-Planck-Institut fuer Plasma Physik, Garching, 1991.

[28] Klíma, R. and Longinov, A.V.: Excitation of a stationary current in a toroidal plasma by waves with a wide spectrum, Fiz. Plazmy, 5 (1979), 496-500.

[29] Barbato, E. and Santiti, F.: Quasi-linear absorption of lower hybrid waves by fusion generated alpha particles, Nucl. Fusion, 31 (1991), 673-685.

[30] Spada, M., Bornatici, M. and Engelmann, F.: Absorption of lower hybrid slow waves by fusion alpha particles, Nucl. Fusion, 31 (1991), 447-458.

[31] Chen, L., Vaclavik, J. and Hammet, G.W.: Ion radial transport induced by ICRF waves in tokamaks, Nucl. Fusion, 28 (1988), 389-398.

[32] Lichtenberg, A.J. and Lieberman, M.A.: Regular and Stochastic Motion, Springer, Berlin 1983.

[33] Balescu, R.: Equilibrium and Nonequilibrium Statistical Mechanics, Wiley, New York 1978.

[34] Whiteman, K.J.: Invariants and stability in classical mechanics, Rep. Progr. Phys., 40 (1977), 1033-1069.

[35] Krlín, L.: The intrinsic stochasticity of near-integrable Hamiltonian systems, Fortschr. Phys., 37 (1989), 735-760.

[36] Zaslavskii, G.M. and Chirikov, B.V.: The stochastic instability of non-linear oscillations, Uspekhi Fiz. Nauk, 105 (1971), 3-39.

[37] Whang, K.W. and Morales, G.J.: ICRF heating and its effects on single-particle confinement in tokamaks, Nucl. Fusion, 23 (1983), 481-497.

[38] Krlín, L., Pavlo, P., Tlučhoř, Z. and Gášek, Z.: On the stochastic interaction of monochromatic Alfén waves with toroidally trapped particles, Plasma Physics and Contr. Fusion, 29 (1987), 1653-1674.

[39] Gášek, Z., Krlín, L. and Tlučhoř, Z.: On the stochastic interaction of toroidally trapped alpha particles with lower hybrid waves in the tokamak reactor regime, Physics Letters, 135 (1989), 284-289.

[40] Sagdeev, R.Z. and Galeev, A.A.: Nonlinear Plasma Theory, Benjamin, New York 1969.

[41] Manley, J.M. and Rowe, H.E., Proc. IRE, 47 (1959), 2155.

[42] Kadomtsev, B.B.: Collective Phenomena in Plasmas (in Russian), Nauka, Moscow 1976.

[43] Davidson, R.C.: Methods in Nonlinear Plasma Theory, Academic Press, New York 1972.

[44] Ott, E. and Dum, C.T.: Nonlinear Landau damping and beat wave trapping, Phys. Fluids, 14 (1971), 959-961.

[45] Mima, K.: Modification of weak turbulent theory due to perturbed orbit effects. II. Nonlinear Landau damping of electron plasma waves, J. Phys. Soc. Jap., 35 (1973), 261-271.

[46] Krlín, L.: Application of the theory of mixing systems to nonlinear Landau damping, J. Plasma Phys., 12, part 3 (1974), 365-379.

[47] Krlín, L.: Effect of trapped particles in the beat of two waves on the wave dynamics, Czech. J. Phys. B, 31 (1981), 383-398.

[48] Nambu, M. and Hada, T.: Conservation relations and violation of the Manley-Rowe relation for plasma-maser instability, Phys. Fluids, B 3 (1993), 743-751.

[49] Nambu, M., Sarma, S.N. and Sarma, K.K.: Momentum source of the plasma maser, Phys. Rev., A 45 (1992), 7456-7462.

[50] Krlín, L.: On acceleration of particles trapped in the potential trough of the beat of two waves, Plasma Physics, 245 (1982), 775-781.

[51] Srivastava, S. and Morales, G.J.: Nonlinear Landau damping of resonantly excited plasma waves in a nonuniform plasma, Bull. Am. Phys. Soc., 34 (1989), 2025.

[52] Srivastava, S., Morales, G.J. and Maggs, J.E.: Nonlinear Landau damping of resonantly excited fields in nonuniform plasmas, Bull. Am. Phy. Soc., 37 (1992), 1359.

[53] Tajima, T. and Dawson, J.M.: Laser electron accelerator, Phys. Rev. Lett., 43 (1979), 267-270.

[54] Mikhailovskii, A.B.: Theory of Plasma Instabilities (in Russian), Atomizdat, Moscow 1970.

[55] O'Neil, T.M. and Winfrey, J.H.: Nonlinear interaction of a small cold beam and a plasma, part II, Phys. Fluids, 15 (1972), 1514-1522.

[56] Drummond, W.E., Malmberg, J.H., O'Neil, T.M. and Thompson, J.R.: Nonlinear development of the beam-plasma instability, Phys. Fluids, 13 (1970), 2422-2425.

[57] Jungwirth, K. and Krlín, L.: Generation of an intensive stationary wave in modulated beam-plasma systems, Plasma Physics, 17 (1975), 861-873.

[58] Kruer, W.L., Dawson, J.M. and Sudan, R.N.: Trapped-particle instability, Phys. Rev. Lett., 23 (1969), 838-841.

[59] Krlín, L.: On the instability of systems with trapped particles, Plasma Physics, 19 (1977), 109-115.

[60] Shoucri, M.: Destruction of trapping oscillations by sideband instability, Phys. Fluids, 23 (1980), 2030-2033.

[61] Riyopoulos, S. and Tang, C.M.: Chaotic electron motion caused by sidebands in free electron lasers, Phys. Fluids, 31 (1988), 3387-3402.

[62] Cohen, B.I. and Cohen, R.H.: An electromagnetic trapped-particle sideband instability, Preprint UCRL-98746, Lawrence Livermore National Laboratory, 1988.

[63] Guss, W.C., Basen, M.A., Kreischer, K.E., Temkin, R.J., Antonsen, T.A., Cai, Jr., S.Y., Saraph, C. and Levush, B.: Sideband mode competition in a gyrotron oscillator, Phys. Rev. Lett., 69 (1992), 3727-3730.

[64] Drummond, W.E.: Quasi-Linear Theory of Plasma Turbulence, in: Plasma Physics (Seminar, Trieste, 1964), IAEA, Vienna 1965, 527-541.

[65] Chen, F.F.: Introduction to Plasma Physics, Plenum Press, New York and London 1974.

[66] Fisch, N.J.: Theory of current drive in plasmas, Rev. Mod. Phys., 59 (1987), 175-234.

[67] Piffl, V., Šunka, P., Ullschmied, J., Jungwirth, K. and Krlín, L.: Some non-linear phenomena associated with high-frequency beam-plasma instabilities, Plasma Physics and Controlled Nuclear Fusion Research (1971), IAEA, Vienna 1972, 155-163.

Figure captions

Fig. 1. Orbit of an electron colliding with an ion. From [64].
Fig. 2. Landau countour. From [3].
Fig. 3. Maxwellian distribution function. From [3].
Fig. 4. Bump-in-tail distribution function. From [2].
Fig. 5. Phase space trajectories of trapped and untrapped particles.

Fig. 6. Motion of trapped particles in a potential trough of the electrostatic wave, and its influence on the wave amplitude. From [42].

Fig. 7. Evolution of the bump-in-tail distribution function $g(v)$ and of the generated wave spectrum $\epsilon(v)$. At the beginning of the instability, a peaked wave spectrum is generated. During the evolution of the instability, function $g(v)$ in the resonance regime tends to be flattened, whereas the spectrum $\epsilon(v)$ becomes broader. From [64].

Fig. 8. Velocity distribution function $F(w)$ (where w is the normalized velocity), initially Maxwellian, under the influence of an unidirectionally launched LHW. Fig. 8b is the enlargement of Fig. 8a. The flattening of the distribu - tion function causes the current drive. From [66].

Fig. 9. Baker's transform.

Fig. 10. Phase space trajectories close to the separatrix of a near-integrable (weakly non-integrable) Hamiltonian system.

Fig. 11. Stochastic layer.
Fig. 12. Examples of non-overlapping and just contacting separatrices.
Fig. 13. Trapping of beam particles into the generated wave. From [56].

Fig. 14. Trapping of beam particles during the beam-plasma instability. η is the spatial dimensionless coordinate, $\beta - \varphi$ is the normalized phase of the beam relative to the wave, and $\frac{d\varphi}{d\eta}$ is the dimensionless velocity of beam particles. From [57].

Fig. 15. Saturation of an instability in the trapping regime of beam particles. F^2 is the dimensionless wave energy. From [57].

Fig. 16. Growth rate γ of the trapped particle sideband instability, plotted as a function of wave number k of the instability. From [58].

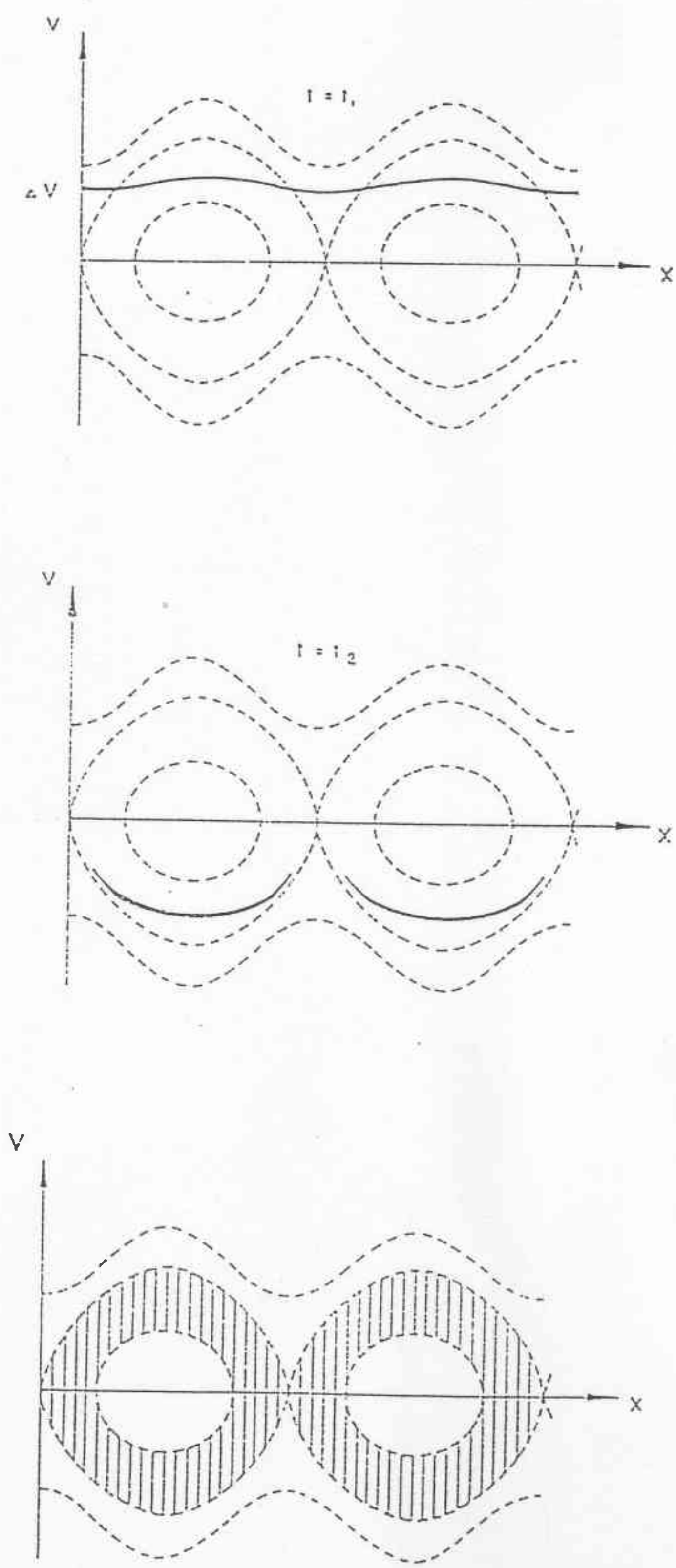


Fig. 13.

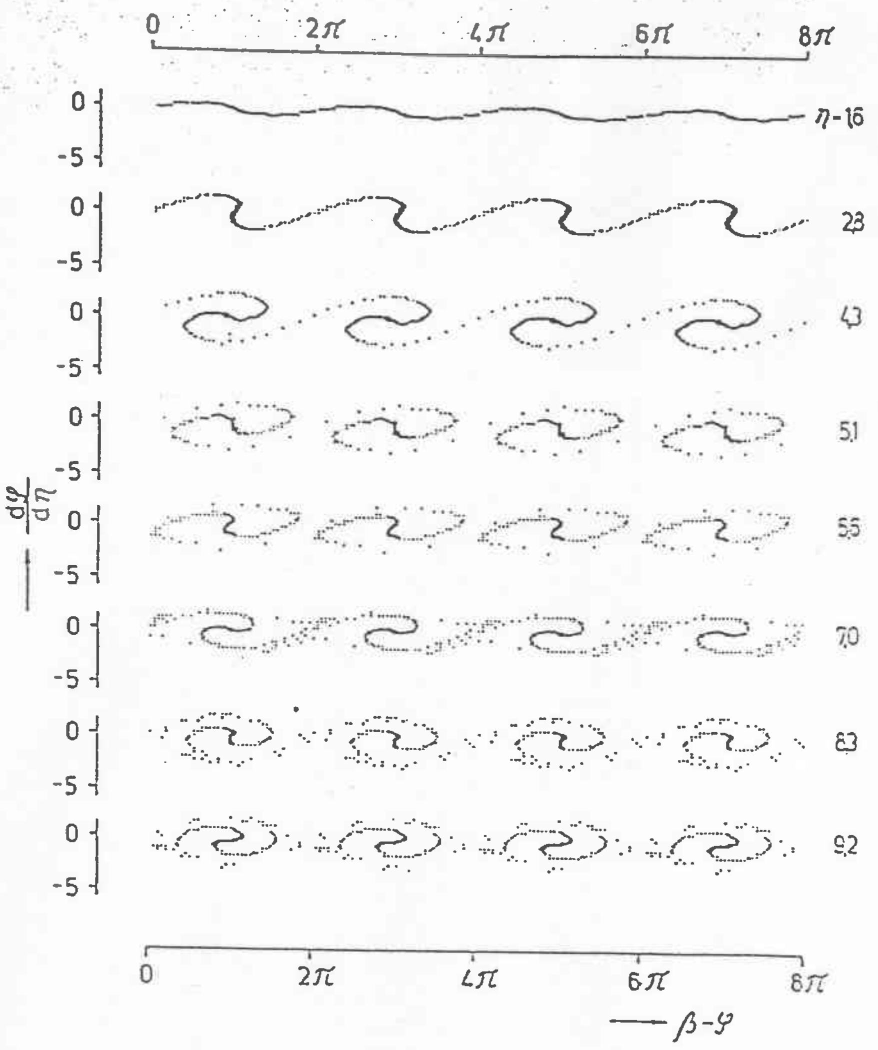


Fig. 14.

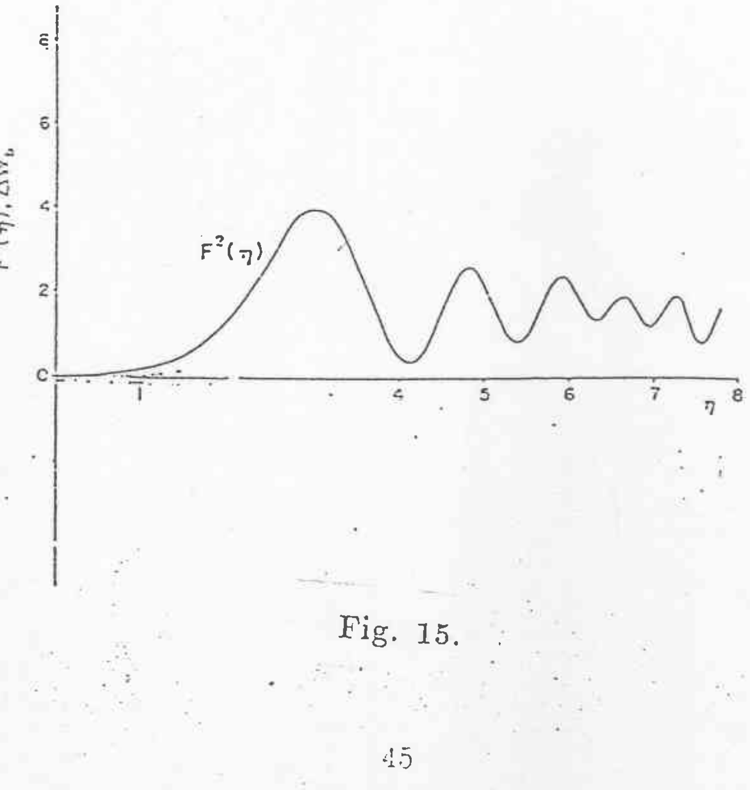
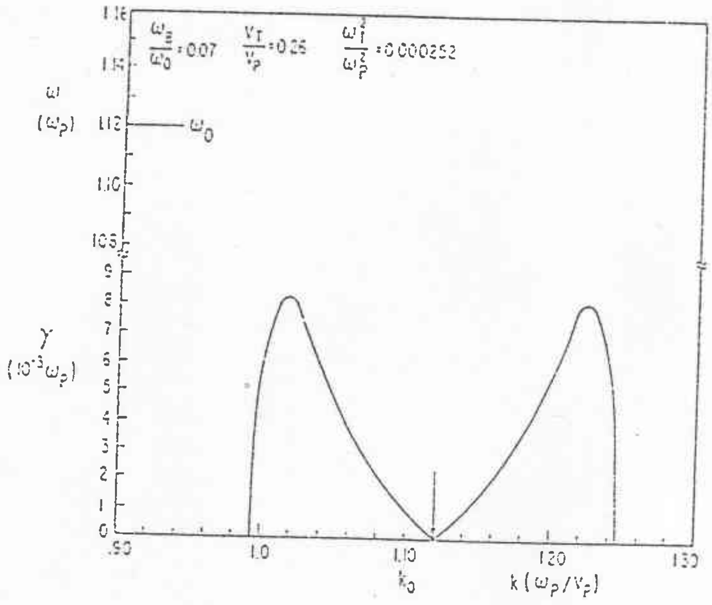


Fig. 15.



$\frac{\omega_s}{\omega_0} = 0.07$ $\frac{V_T}{V_p} = 0.26$ $\frac{\omega_i^2}{\omega_p^2} = 0.000252$
ω (ω_p)
ω_0
γ $(10^{-3}\omega_p)$
k_0
$k\,(\omega_p/V_p)$

Fig. 16.