Diffusion across a magnetic field

The rate of plasma loss by diffusion can be decreased by a magnetic field.

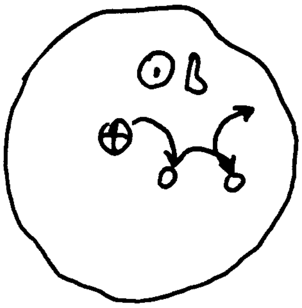
Consider again a weakly ionized plasma in a magnetic field.

There are two basically different diffusion motions:

1) the diffusion along the magnetic field; here the magnetic field has no effects - the flux along $\vec{B}$ has its usual form

$$\Gamma_z = \pm \mu n E_z - D \frac{\partial n}{\partial z} \quad ; \quad \vec{B} = B_0 \vec{z} .$$

2) the motion (and diffusion) across $\vec{B}$; here, the motion is strongly affected. When there are collisions, particle move across $\vec{B}$ by random-walk motion - the phase of cyclotron motion changes abruptly.



The guiding center of the particle changes its position and exhibits a random walk as well.

Whereas the step lenght in the random motion in a plasma without magnetic field was of the order λ_m, in a plasma with magnetic field is of the order r_L (Larmor radius).

To discuss the motion of a weakly ionized plasma in a magnetic field, we use the fluid equation of motion:

$$mn \frac{d\vec{v}_\perp}{dt} = \pm en\left(\vec{E} + \vec{v}\times\vec{B}\right) - KT\nabla n - mn\nu\vec{v} = 0$$

(supposing isothermal plasma, neglecting time-dependent terms and assuming that

$$\vec{E} = E_x\vec{x}_0 + E_y\vec{y}_0 + E_z\vec{z}_0$$

$$\vec{B} = B\vec{z}_0$$

$$\nabla n = \frac{\partial n}{\partial x}\vec{x}_0 + \frac{\partial n}{\partial y}\vec{y}_0 + \frac{\partial n}{\partial z}\vec{z}_0)$$

We shall follow the motion across the magnetic field, therefore in x, y direction.

The x and y components of the velocity $\vec{v}$ are:

$$m n \nu v_x = \pm e n E_x - KT \frac{\partial n}{\partial x} \pm e n v_y B$$

$$m n \nu v_y = \pm e n E_y - KT \frac{\partial n}{\partial y} \mp e n v_x B .$$

Let us use the expression for μ and D

$$\mu = \frac{|q|}{m\nu} \quad ; \quad D = \frac{KT}{m\nu} .$$

Then $$v_x = \pm \mu E_x - \frac{D}{n}\frac{\partial n}{\partial x} \pm \frac{\omega_c}{\nu} v_y$$

$$v_y = \pm \mu E_y - \frac{D}{n}\frac{\partial n}{\partial y} \mp \frac{\omega_c}{\nu} v_x .$$

Using $\tau = \nu^{-1}$, we obtain

$$v_y (1 + \omega_c^2 \tau^2) = \pm \mu E_y - \frac{D}{n}\frac{\partial n}{\partial y} - \omega_c^2 \tau^2 \frac{E_x}{B} \pm$$

$$\pm \omega_c^2 \tau^2 \frac{KT}{eB} \frac{1}{n} \frac{\partial n}{\partial x}$$

$$v_x (1 + \omega_c^2 \tau^2) = \pm \mu E_x - \frac{D}{n}\frac{\partial n}{\partial x} + \omega_c^2 \tau^2 \frac{E_y}{B} \mp$$

$$\mp \omega_c^2 \tau^2 \frac{KT}{eB} \frac{1}{n} \frac{\partial n}{\partial y} .$$

The terms with E_x, E_y ; $\frac{\partial n}{\partial x}, \frac{\partial n}{\partial y}$ are components of the drift $\vec{E}\times\vec{B}$ and of the diamagnetic drift, respectively:

$$\vec{v}_E = \frac{\vec{E}\times\vec{B}}{B^2} \quad ; \quad \vec{v}_D = -\frac{\nabla p \times \vec{B}}{q\,n\,B^2} .$$

Using further the definitions for perpendicular mobility $\mu_\perp$ and perpendicular diffusion coefficient $D_\perp$

$$\mu_\perp = \frac{\mu}{1+\omega_c^2\tau^2} \quad ; \quad D_\perp = \frac{D}{1+\omega_c^2\tau^2}$$

we can express the equation for the perpendicular velocity $\vec{v}_\perp$ in a more compact form:

$$\vec{v}_\perp = \pm\mu_\perp\vec{E} - D_\perp\frac{\nabla n}{n} + \frac{\vec{v}_E + \vec{v}_D}{1+\frac{\nu^2}{\omega_c^2}} .$$

From this follows:

1) the usual drifts $\vec{v}_E, \vec{v}_D$ are slowed down due to the factor $1+\frac{\nu^2}{\omega_c^2}$.

2) The mobility and diffusion drifts parallel to the electric field $\vec{E} \perp \vec{B}_0$ and to the gradient $\nabla n \perp \vec{B}_0$ are reduced by the factor $1+\omega_c^2\tau^2$.

For $\omega_c^2\tau^2 \gg 1$, $D_\perp = \frac{KT}{m\nu}\frac{1}{\omega_c^2\tau^2} = \frac{KT\nu}{m\omega_c^2} \Rightarrow$

$\Rightarrow$ $D_\perp$ decreases with the increasing of B.

The comparison of D (diffusion for $B=0$) with $D_\perp$ (perpendicular diffusion with $B \neq 0$).

$$D = \frac{KT}{m\nu} \cong v_{th}^2\tau \sim \frac{\lambda_m^2}{\tau} \qquad \left(\tau \sim \frac{\lambda_m}{v} \Rightarrow v = v_{th} = \frac{\lambda_m}{\tau}\right)$$

$$D_\perp = \frac{KT\nu}{m\omega_c^2} \sim v_{th}^2 \frac{r_L^2}{v_{th}^2}\nu \sim \frac{r_L^2}{\tau} \qquad \left(r_L = \frac{v_\perp}{\omega_c} \sim \frac{v_{th}}{\omega_c}\right)$$

Whereas the diffusion in a plasma without magnetic field has basic step length (λ_m), the diffusion in a magnetic field has basic step length (r_L).