

can be simply presented by Fig. 4. It is well known that this system is unstable, that a spectrum of Langmuir waves is excited, and that the system tends to saturate. We have an example of a typical turbulent regime.

We shall consider the Vlasov equation for the electron distribution function  $f(x, v, z)$

$$\frac{\partial f}{\partial t} + v \frac{\partial f}{\partial x} - \frac{e}{m} E \frac{\partial f}{\partial v} = 0 \quad (131)$$

where  $E$  represents the electric field of longitudinal Langmuir waves. Ions form an immovable homogeneous background.

It is convenient to express the electric field  $E(x, t)$  of the spectrum in the form of a discrete spectrum,

$$E = \sum_k E_k \exp[ikx - i\omega(k)t], \quad (132)$$

where

$$\omega = \omega(k) \quad (133)$$

is the dispersion relation, given by the linear theory. The complex amplitudes  $E_k$  are assumed to vary slowly with time.

We shall further suppose that electron distribution function  $F(x, v, t)$  represents a perturbation of a stationary, spatially independent distribution function  $F_0(v)$ . We shall express this function also in a form of discrete modes,

$$f(x, v, t) = f_0(v) + \sum_k f_k \exp[ikx - i\omega(k)t]. \quad (134)$$

Let us now summarize the basic assumptions, used in the derivation of the quasilinear theory. First, let us assume that perturbations  $f_k$  form only a weak perturbation of the unperturbed part, i.e.

$$f_0 \gg f_k. \quad (135)$$

Let us further suppose that the direct interaction of modes  $E_k, E_q$  is negligible, i.e., that, e.g., mode  $f_k$  is unaffected by the possible resonant interaction of modes  $E(k - q), f_q$ . Let us further assume that the effect of the excited spectrum consists only in the change of the space-independent part of the distribution function,  $f_0$ , and that the changes of  $E_k, f_k$  are given only by the linear Landau theory. Let us further assume that the discrete modes  $E_k$  have random phases, and, in connection with that, there is no trapped particle effect.

Since the behaviour of the modes is given only by the linear theory, the expression for  $f_k$  can be obtained from the Vlasov equation

$$\frac{\partial f_k}{\partial t} + v \frac{\partial f_k}{\partial x} - \frac{e E_k}{m} \frac{\partial f_0}{\partial v} = 0. \quad (136)$$

Considering mode  $f_k$  in the form

$$f_k = f_{k0} e^{ikx - i\omega t}, \quad (137)$$

the solution of the foregoing equation is the same as in the Landau damping approach

$$f_k = i \frac{e}{m} \frac{E_k}{\omega - kv} \frac{\partial f_0}{\partial v}. \quad (138)$$

Inserting this result into the Vlasov equation for the unperturbed part  $f_0$ , and retaining only the nonlinear term, for  $f_0$  we obtain the expression

$$\frac{\partial f_0}{\partial t} = \frac{e}{m} \sum_k E_{-k} \frac{\partial f_k}{\partial v} = \frac{e^2}{m^2} i \sum_k E_k E_{-k} \frac{\partial}{\partial v} \frac{1}{\omega - kv} \frac{\partial f_0}{\partial v}. \quad (139)$$

Let us express  $E_k E_{-k}$  as

$$E_k E_{-k} = |E_k|^2 \quad (140)$$

and let us use the Plemelj formula (see e.g. [13])

$$\lim_{\nu \rightarrow 0} \frac{1}{\omega_k + i\nu - kv} = P\left(\frac{1}{\omega_k - kv}\right) - i\pi\delta(\omega_k - kv). \quad (141)$$

where  $P$  means the Cauchy principal value of the integral taken at singularity  $\omega_k = kv$ . The equation for  $f_0$  then reads

$$\frac{\partial f_0}{\partial t} = \frac{\partial}{\partial v} D \frac{\partial f_0}{\partial v} \quad (142)$$

where

$$D = \sum_k \frac{e^2}{m^2} \pi |E_k|^2 \delta(\omega_k - kv). \quad (143)$$

(The term, proportional to principal value  $P$  can be neglected [13]. According to [13], this term takes into account the interaction of waves with nonresonant particles. It plays a role in the discussion of the global energy and momentum balance. It can be proved that the energy, absorbed by resonant particles, is fed by the coherent wave motion of nonresonant particles and by the electric field).

According to one from our assumptions, the interaction of waves with the plasma is governed by the linear Landau mechanism (yielding either excitation, or damping), where function  $f_0$  takes the role of function  $f_M$  in the discussion of the linear Landau description of the interaction. Consequently, the diffusion equation for  $f_0$  must be supplemented with the equation for wave amplitudes,

$$\frac{\partial |E_k|^2}{\partial t} = 2\gamma_k |E_k|^2. \quad (144)$$

It is possible to prove that conservation of energy and of the momentum is fulfilled for this system (given by equations (142) and (144)) [14], [13].

The mutual interplay between waves and the distribution function of particles  $f_0$  is often presented just for the case of the bump-on-tail instability (Fig. 4). The second maximum of the distribution function (which can be created by a warm electron beam)

is the source of the instability. Waves with phase velocities  $\approx \frac{\omega}{k}$  close to the velocity of beam particles, forming just the second maximum, but slightly shifted to the region of the positive slope of the distribution function, are generated. Due to the diffusion, which is given by the quasilinear diffusion equation (142), these waves act reversely on the distribution function, flattening the second maximum and creating so-called plateau. The decrease of the positive slope of the distribution function causes the decrease of the instability growth rate. Consequently, the instability saturates - see Figs. 7 a,b,c,d. (This type of saturation is not unique. At the end of this chapter, we shall briefly discuss the saturation of the instability of the cold plasma - cold beam system. In this case, the instability saturates due to the trapping of the beam particles in the generated wave).

One of the basic assumptions, required for the quasilinear approach to be valid, requires the phases of the waves to be chaotic; any coherence between modes must be destroyed by phase mixing [14]. Nevertheless, according to the results of the theory of deterministic chaos in non-integrable Hamiltonian systems (we shall briefly mention this interesting phenomenon in separate section), this assumption can be replaced by the requirement of sufficient nonlinearity of the wave-particle interaction. This requirement is fulfilled in the regime of overlapping of resonances [15], [16]. This regime requires the fulfilment of the following relation (the overlap criterion)

$$A = 4\pi^2(eE_n \frac{1}{mk_n \delta v_{\varphi n}^2}) \geq 1. \quad (145)$$

Here  $A$  is the overlap parameter,  $E_n$  is the amplitude and  $k_n$  the wave number of the  $n$ -th mode of the applied spectrum, and  $\delta v_{\varphi n}$  is the difference between the phase velocities of the neighbouring modes of the spectrum. For the continuous-spectrum limit, the overlap criterion is always fulfilled; this fact is often used to justify the validity of the quasilinear theory (QLT) in this case [16].

In spite of this important (and now, well- founded) support of the validity of the QLT for the instability regime, the discussions of the global validity of the quasilinear approach are still frequent and form an evergreen of the plasma physics theory. The reader is referred to the excellent paper by Cary et al. [16], and also papers [12], and [17-19], to mention but a few. Earlier this was criticized (see, e.g., [1] or [20]), and gave rise to other approaches (as, e.g., the Dupréé theory [21], [20]). It can be said that the theory is still not closed (as the theory of the linear Landau damping). Recent more thorough analytical and numerical studies (for references, see Cary[16]), discussed, inter alia, different (i.e. larger) values of the diffusion coefficient, in comparison to the quasilinear approach. It seems that two effects have important consequences - the neglect of mode coupling (this coupling probably leads to larger discretization of the turbulent spectrum), and the difference in the interaction character for the Gaussian or non-Gaussian form of the wave spectrum.

On the other hand, it is not surprising that the problem of plasma turbulence - even in its simple form - is still open; the general problem of turbulence and chaos

now represents one of the key problems of physics.

In the foregoing, the plasma without magnetic field has been considered. The form of the quasilinear diffusion for a magnetized plasma (i.e., for the plasma in an external magnetic field) is rather complicated. Analogously to the case of the Landau damping, the set of possible resonances is now given by the resonant condition

$$\omega - k_{\parallel}v_{\parallel} - n\omega_c = 0. \quad (146)$$

The quasilinear diffusion coefficient must be in this case replaced by the diffusion tensor. For the general form of the diffusion tensor, see e.g. [22].

Let us now turn to the second important application of the quasilinear theory - to RF quasilinear plasma heating and RF current drive. The basic difference between the discussion of the selfconsistent model of the quasilinear description of saturation of the kinetic instability regime, and the RF heating and RF current drive problems consists in the fact that, whereas under the instability regime waves and particles form a closed system, the RF heating problems represent an open system, in which the steady-state source [13] feeds the system with RF power.

To describe quasilinear RF heating and current drive, the best point of departure it is to choose a special type of RF field. The discussion loses its general character, but is also less formal. We shall consider a very frequently used type of RF field - so called lower hybrid waves (LHW). LHW heating now seems to be a well understood type of RF heating, theoretically as well as experimentally. Its discussion will enable us to become acquainted with almost all problems of RF quasilinear heating and current drive. Some specificities, which appear in the RF-plasma interaction for other types of waves will be mentioned later on.

The frequencies of LHW are in the range of  $1 - 10 GHz$ , and their wavelength in the  $10^{-2}m$  range. These waves are launched from outside the tokamak plasma by a special system of waveguides - the so-called grill. The waves, propagating from the grill do not form the discrete spectrum, as assumed in the foregoing simple model. The grill radiates a spectrum, continuous in wave numbers  $k$ , with some effective width  $\Delta k$ .

Since a tokamak plasma is strongly inhomogeneous, the propagation of LHW must be treated carefully. LHW belong to the type of waves, the propagation of which is governed by the eikonal approximation, see e.g. [23]. Whereas frequency  $\omega$  of LHW is usually prescribed, wave vector  $k$  varies according to the eikonal equations (which are formally identical with Hamiltonian canonical equations). The position  $r$  of a ray and wave vector  $k$  are given by the equations

$$\frac{dr}{d\tau} = \frac{\partial D}{\partial k}; \quad \frac{dk}{d\tau} = -\frac{\partial D}{\partial r}; \quad \frac{dt}{d\tau} = -\frac{\partial D}{\partial \omega} \quad (147)$$

where  $D = 0$  is the dispersion equation, which takes into account the space inhomogeneity of the plasma,  $r$  is the spatial vector, describing the position of the ray, and  $\tau$  is the parameter, whose connection with time  $t$  is given by the last equation in

(147). (The formal coincidence of ray dynamics (147) with the Hamiltonian **canonical** equations may lead - as for all nonintegrable Hamiltonian systems - to the **stochasticity** of LHW rays, as has been shown by Wersinger et al. [24] and by Bonoli et al. [25]).

RF heating (and RF current drive) in a hot plasma is a net effect, given by two inversely working effects; these effects are represented by Coulomb collisions and by quasilinear diffusion. Whereas the former effect has a relaxing influence, which drives the distribution function to Maxwellian form, the quasilinear effect tends to flatten the distribution function in the resonant region into a plateau. For a stationary case, the balance of these two effects gives the resulting form of the distribution function.

To show the procedure, which leads to this stationary case, we shall again consider the simplest 1D case of the distribution, namely the evolution of the distribution function  $f(v, t)$ . This evolution will be now given by the Fokker-Planck equation (in the Lenard-Bernstein model, approximating the Fokker-Planck term (40)), supplemented by the quasilinear term with diffusion coefficient  $D_{QL}$ , namely [26], [13]

$$\frac{\partial f}{\partial t} = \frac{\partial}{\partial v} \left[ \nu(vf + \frac{KT_e}{m} \frac{\partial f}{\partial v}) \right] + \frac{\partial}{\partial v} D_{QL} \frac{\partial f}{\partial v}. \quad (148)$$

Here, function  $f$  corresponds to function  $f_0$  in the quasilinear equation (142) and, therefore, to the slowly varying part of the exact distribution function,  $\nu$  is the collision frequency,  $K$  is the Boltzmann constant, and  $D_{QL}$  is the diffusion coefficient, which must be expressed for the lower hybrid wave spectrum. For usual tokamak parameters lower hybrid waves can be considered as electrostatic waves, propagating with some vector  $\mathbf{k}$  into the plasma. Plasma electrons interact resonantly with the component of the wave spectrum, which is parallel to the tokamak magnetic field. Let  $W_k$  be the spectral energy density of the LHW spectrum, given approximately as

$$W_k \approx \frac{P(k_{||})}{v_{gr}}, \quad (149)$$

where  $P(k_{||})$  is the spectral power density for the parallel component and  $v_{gr}$  is the group velocity (see e.g. [27]). The diffusion coefficient can be expressed as (see [22] and e.g. [27])

$$D_{QL} = \frac{\pi e^2}{\epsilon_0 m_e^2} \frac{\omega^2}{\omega_p^2} \frac{1}{|v_{||}|} W_k / k_{||} v_{||} = \omega. \quad (150)$$

The stationary solution of the quasilinear diffusion equation (134) can be found in the form [26], [13] (see Fig. 8)

$$f(v) = \text{const} \cdot \exp \left[ - \int \frac{mv}{KT_{eff}(v)} dv \right]. \quad (151)$$

Here,  $KT_{eff}$  is defined as

$$KT_{eff} = KT_e + \frac{m D_{QL}(v)}{\nu(v)}. \quad (152)$$

The two last equations obviously indicate that the slope of the distribution function decreases with increase of  $D_{QL}$ . Consequently, the Landau decrement decreases, as the input power increases, as was expected.

The power  $P_d$ , absorbed in the plasma, can be estimated as [13]

$$P_d = \frac{1}{2} \int m v^2 \frac{\partial}{\partial v} D_{QL} \frac{\partial f}{\partial v}. \quad (153)$$

Let us suppose that lower hybrid waves propagate into the tokamak plasma with some asymmetry, favouring one toroidal direction over the other [26], [28]. In this case, the quasilinear deformation of the distribution function in velocity space will also be asymmetrical (see Fig. 8). Defining the current density as

$$i = -e \int_{-\infty}^{+\infty} f(v) v dv \quad (154)$$

we obviously obtain a net current density  $i \neq 0$ . This is the basic mechanism for generating the driven current.

This driven current, generated by the absorption of LHW in the plasma, can be generated with surprisingly large magnitudes. For tokamak reactor plasma, currents of  $\approx 10 MA$  or even larger can be achieved. The necessary input power is, of course, also impressive,  $\approx 100 MW$ .

LHW can interact not only with electrons, but also with thermonuclear alpha particles. Alpha particles can interact resonantly at the alpha particle cyclotron harmonics

$$\omega - k_{\parallel} v_{\parallel} - n \omega_{c\alpha} = 0 \quad (155)$$

where  $\omega_{c\alpha}$  is the cyclotron frequency of alpha particles, and  $n$  is the number of the resonant harmonics. The interaction of alpha particles with LHW also requires the quasilinear description. It is necessary to solve the following form of the quasilinear equation for the alpha particle distribution function  $f_{\alpha}$  (see e.g. [27])

$$\frac{\partial f_{\alpha}}{\partial t} = \sum_{\beta \neq \alpha} L_{\alpha\beta}[f_{\alpha}] + L_{QL}[f_{\alpha}] + p_{\alpha} \delta(v - v_{\alpha}) - \nu f_{\alpha}. \quad (156)$$

Here,  $L_{\alpha\beta}$  is the collision operator between alpha particles and other plasma particles, and  $L_{QL}$  is the quasilinear operator, respecting either the fact that the resonant interaction has the form (141), or the fact that it is possible to simplify the problem, considering the alpha particles to be unmagnetized, stochastically interacting with LHW in perpendicular velocities [29], [30]. The term  $p_{\alpha} \delta(v - v_{\alpha})$  represents the source of thermonuclearly generated alpha particles, and the last term is the sink, modelling the escape of alpha particles. In this regime, LHW is absorbed by alpha particles. This absorption can have a negative effect on the efficiency of the current drive.

Besides LHW, also other types of waves are used for plasma heating or current drive (e.g. ion cyclotron waves or electron cyclotron waves). Here, the interaction with particles takes place at the cyclotron resonance. Together with the quasilinear effects, also the intrinsic stochasticity of the interaction has here an important role (see e.g. [31]).

## 5. INTERACTION OF PARTICLES WITH WAVES IN THE INTRINSIC STOCHASTICITY REGIME

In discussing the validity of the quasilinear theory, we have touched on the problem of the deterministic chaos of the non-integrable Hamiltonian systems. The overlap criterion, enabling the requirement of chaotic phases of the waves to be avoided, has been used there. The overlap criterion is among the results, obtained from the study of near-integrable Hamiltonian systems. In this section, we shall briefly mention the facts, which lead to this criterion. For a more thorough acquaintance with this problem, we recommend the excellent monography of Lichtenberg and Lieberman [32]. Short reviews have appeared, e.g., in [33-35].

The deterministic chaos in non-integrable Hamiltonian systems is closely related to the modern ergodic theory, especially to transformations, which possess the mixing property. An example of this transformation is the baker's transformation, which is presented in Fig. 9. If we were to follow a point through a set of these transformations, we would soon be lost. It was recognized this type of transformation is suitable for modelling chaos. The theory of ergodic systems is well-founded, and the loss of correlation during these transformations, which is sign for the origin of chaos, has been proved mathematically.

Poincaré studied the dynamics, given by the following Hamiltonian

$$H = H_0 + H_1; \quad H_0 \gg H_1, \quad (157)$$

where the Hamiltonian represents the nonlinear oscillator  $H_0$ , which is perturbed by a small perturbation  $H_1$ . The coordinate system action  $J$  - angle  $w$  is usually used (for the symbolics, see again e.g. [7]). The foregoing Hamiltonian can, therefore, be written in the following form

$$H = H_0(J) + H_1(J, w), \quad (158)$$

considering only the one-dimensional problem. Hamiltonian  $H(J, w)$  (158) is integrable, if there exists a generating function that enables (144) to be expressed in a cyclic form  $H(J)$ . If there is no such generating function, the Hamiltonian is assumed to be non-integrable. For small perturbations  $H_1$ , the Hamiltonian is called near-integrable, or weakly non-integrable.



Let us now consider that Hamiltonian (158) is weakly non-integrable, and let us monitor the phase trajectories close to the separatrix of Hamiltonian  $H_0$ . Already Poincaré found the behaviour of these trajectories to be extremely complicated (see Fig. 10). Close to the separatrix, frequency  $\omega_{osc}$  of the unperturbed Hamiltonian  $H_0$

$$\omega_{osc} = \frac{\partial H_0}{\partial J} \quad (159)$$

is strongly nonlinear. Perturbation  $H_1$  yields a set of nonlinear resonances. System (158), close to the separatrix, passes through these resonances, it is strongly unstable there, and due to its nonlinearity behaves quite erratically. The intensive analytical study of these weak non-integrable systems have brought very interesting conclusions. Close to the separatrix, an infinite number of resonances exists. It can be proved that the Hamiltonian is really non-integrable in this region. The dynamics, represented by some difference mapping for discrete time elements is identical with the dynamics of the systems with mixing, and, consequently, the dynamics is chaotic in this region in the same sense as the chaos of the systems with mixing.

Let us now express the above mentioned dynamics in a model, which is very close to plasma physics. Let us consider a homogeneous plasma without any magnetic field, through which two electrostatic (Langmuir) waves with potentials  $\varphi^{(0)}$  and  $\varphi^{(1)}$  propagate:

$$\varphi^{(0)} = \varphi_0 \cos(k_0 x - \omega_0 t) \quad (160)$$

$$\varphi^{(1)} = \varphi_1 \cos(k_1 x - \omega_1 t) \quad (161)$$

(we have again used the one-dimensional system of coordinate  $x$  and momentum  $p_x$ ).

Let us further suppose that  $\varphi_0 \gg \varphi_1$  and let consider the following Hamiltonian

$$H_0 = \frac{1}{2m} p_x^2 + e\varphi_0 \cos(k_0 x - \omega_0 t). \quad (162)$$

The total Hamiltonian, describing the behaviour of particles in these two waves can be expressed as

$$H = H_0 + e\varphi_1 \cos(k_1 x - \omega_1 t) = H_0 + H_1. \quad (163)$$

Hamiltonian (163) is identical with the Hamiltonian  $H_x$  (105), which describes the dynamics of trapped particles. The discussion of  $H_x$  disclosed that particles, trapped in the wave, oscillate with a strongly nonlinear frequency. It is well known that the dynamics of these particles can be expressed in the action-angle representation (see e.g. [32]), where  $J$  and  $w$  for trapped particles are

$$J = R \frac{8}{\pi} [E(k) - (1 - k^2)K(k)] \quad (164)$$

$$w = \frac{\pi}{2} K(k^{-1}) F(\eta, k). \quad (165)$$



Here,

$$R^2 = \frac{2me\varphi_0}{k_0^2}; \quad k \sin \eta = \sin \frac{Q}{2}, \quad (166)$$

and  $K(k)$ ,  $E(k)$  are total elliptic integrals of the first and second kind,  $F$  is the incomplete form of  $K$ , and  $Q = k_0 x$ .

Using this transformation and excluding the time dependence by means of extended phase space [32], we can finally express our Hamiltonian (163) in the form

$$\bar{H} = \bar{H}_0(J, J_1) + \bar{H}_1(J, J_1, w, w_1), \quad (167)$$

where  $J_1 = -H$ ,  $w_1 = t$ . This is already the form of Hamiltonian (158), for which the dynamics of the phase trajectories close to the separatrix has been discussed.

The behaviour of phase trajectories in the separatrix region is usually depicted in the form of stochastic layer. Within this layer, the thickness of which is proportional to the magnitude of the perturbation  $H_1$ , the representative points undergo a motion which can be identified with diffusion. For the original coordinate  $x$ ,  $p_x$ , and for the system moving at velocity  $\frac{\omega}{k_0}$ , i.e., at the phase velocity of the wave with potential  $\varphi_0$ , the stochastic layer is depicted in Fig. 11.

We do not usually deal with a pair of waves of form (163), but with a spectrum (for simplicity, with a discrete spectrum; the generalization for a continuous spectrum presents no problems). In this case, perturbation methods cannot be applied. Zaslavskii and Chirikov [36] found a suitable phenomenological approach for this case. They expected the diffusion to appear in a broader region of phase space, if the separatrices of the neighbouring waves make contact (see Fig. 12); in this case, the resonance conditions of the neighbouring waves will overlap. This idea has been verified in a set of numerical experiments. (The analytical approach, which uses the perturbation analysis, can describe the chaotic behaviour of the phase trajectories only in the region close to the separatrix. To study the chaotic motion in a large-scale, the numerical simulation on computers is unavoidable).

According to them, the condition under which the diffusion will appear in a discrete spectrum can really be roughly identified with the condition of contact of neighbouring separatrices. And this is indeed expressed by formula (145). A more exact study nevertheless shows that the generated chaos is strongly inhomogeneous in the phase space. This is perhaps one of the effects, which complicates the quasilinear description. On the other hand, the extreme complexity of the dynamics shows that the theory of deterministic chaos is still open to further discussions.

We have so far discussed only the simplest dynamics of particles, moving in a spectrum of waves with no magnetic field. The existence of an external magnetic field generates new effects. A considerable amount of work has already been done with homogeneous and mirror fields (see references in [32]). For a tokamak magnetic field, new interesting effects appear in the interaction of the RF field with toroidally trapped particles (bananas). Here, stochasticity even induces the space (radial) diffusion. For this, see e.g. [31], [37-39].

## 6. NONLINEAR LANDAU DAMPING

The mechanism of nonlinear Landau damping is usually discussed **together with** the description of resonant wave-mode coupling; it thus belongs to a **broader set of** different types of nonlinear wave-plasma interaction, commonly referred to as **weak plasma turbulence**.

As already mentioned, the quasilinear theory neglects resonant mode coupling. Without this neglect, a new set of phenomena appears.

The full expression for the evolution of the mode of the distribution function,  $f_k$ , reads

$$\frac{\partial f_k}{\partial t} + ikvf_k = \frac{e}{m} \sum_q E_{k-q} \frac{\partial f_q}{\partial v}, \quad (168)$$

where, obviously, the right-hand side contains the resonant interaction of mode  $E_{k-q}$  and of mode  $f_q$  with mode  $f_k$ . For  $k = 0$ , we obtain the usual form of the quasilinear approach, the evolution of  $f_0$ .

For, e.g.,

$$\omega_k = \omega_{k-q} + \omega_q; \quad k_k = k_{k-q} + k_q, \quad (169)$$

the three modes may interact in resonance. The mutual interaction of this wave system is described by the closed system of kinetic equations for the waves, derived from the kinetic equations for modes of the particle distribution function,  $f_k$ , and from the Maxwell equations (see, e.g. [22]). (As an example of such coupling, the interaction of two Langmuir waves with different  $\omega$ ,  $k$ , and of the ion sound wave can be mentioned (see, e.g. [22]). As a consequence of the resonant conditions and of the form of the kinetic equations, the following interesting conservation law for the energies of waves,  $U_i$ , can be derived (see, e.g., [40])

$$\frac{U_1}{\omega_1} + \frac{U_2}{\omega_2} = \text{const.} \quad (170)$$

$$\frac{U_1}{\omega_1} + \frac{U_3}{\omega_3} = \text{const.} \quad (171)$$

$$\frac{U_2}{\omega_2} - \frac{U_3}{\omega_3} = \text{const.} \quad (172)$$

(This conservation law, called the Manley-Rowe relation, was originally derived by Manley and Rowe [41] for parametric amplifiers). This law determines the way in which wave energy is transformed.

The nonlinear Landau damping interaction describes the interaction of two modes; the resonant interaction is mediated by resonant particles (unlike the case of resonant three-mode coupling, when the interaction is given only by the resonant coupling of three modes).

The mechanism of nonlinear Landau interaction is closely connected with the phenomenon of weak Langmuir turbulence, which can result in strong Langmuir

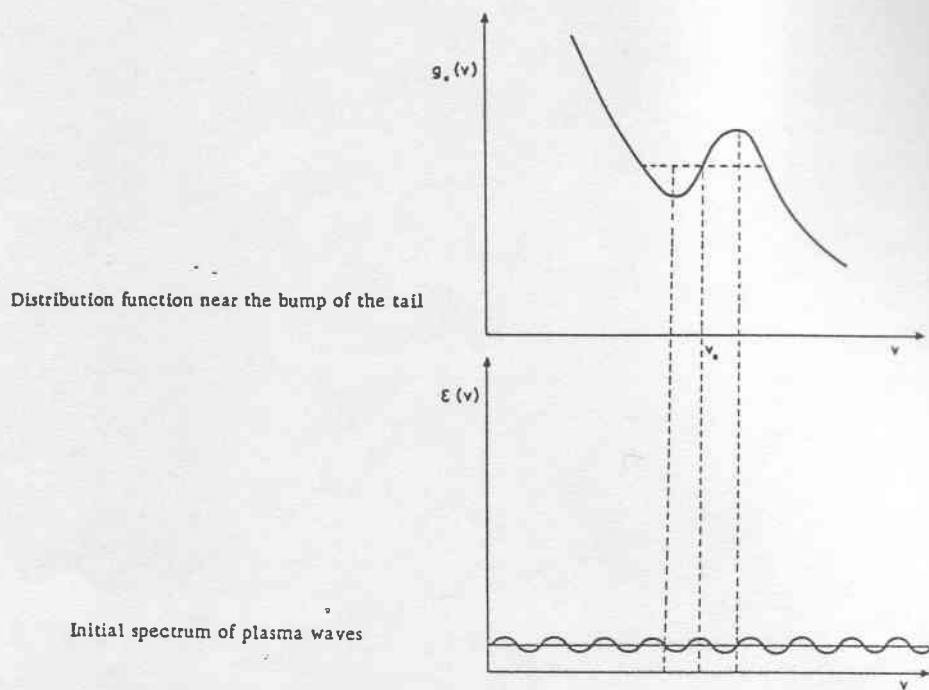


Fig. 7a.

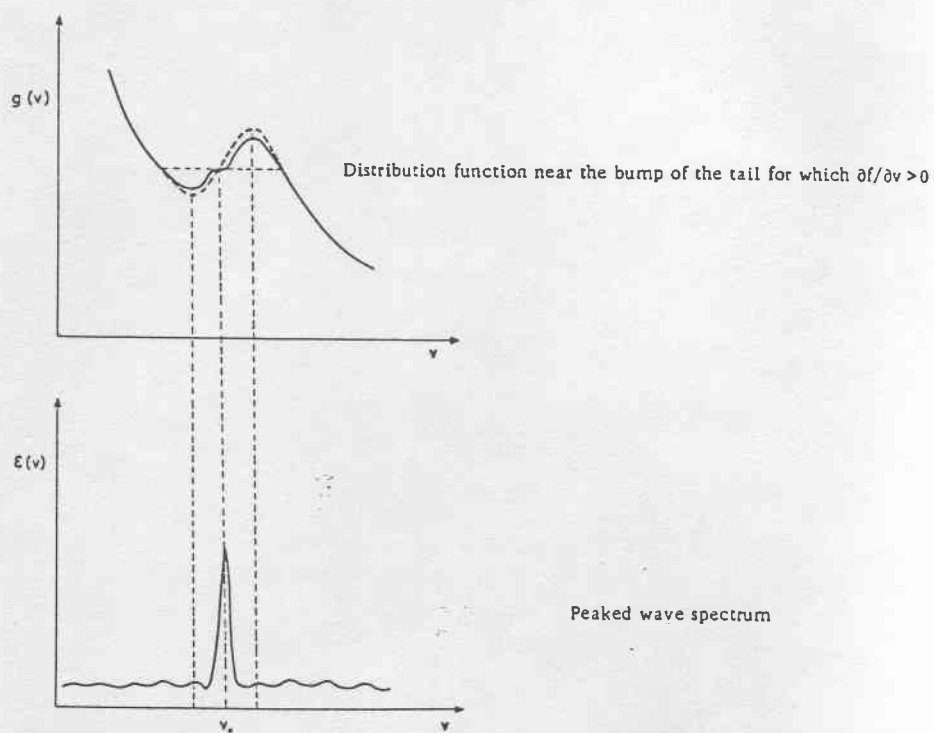
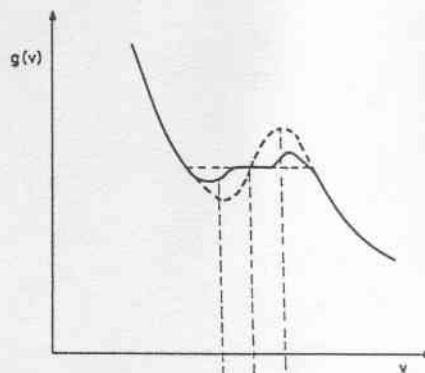


Fig. 7b.

Effects of the diffusion on the distribution



Widening of the spectrum

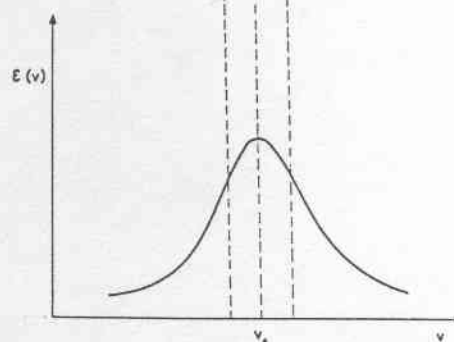
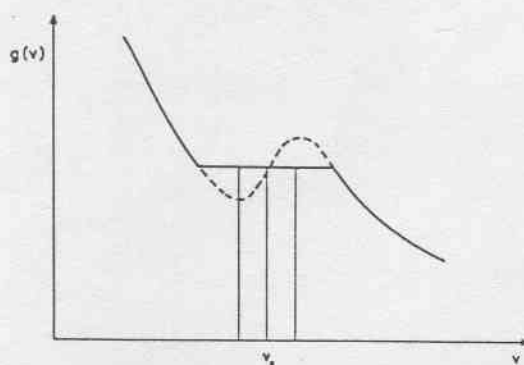
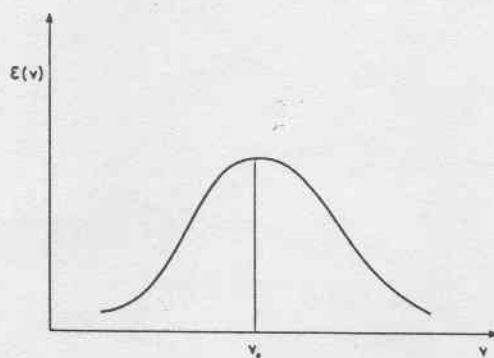


Fig. 7c.



Flattening of the distribution function



Broadening of the wave spectrum

Fig. 7d.

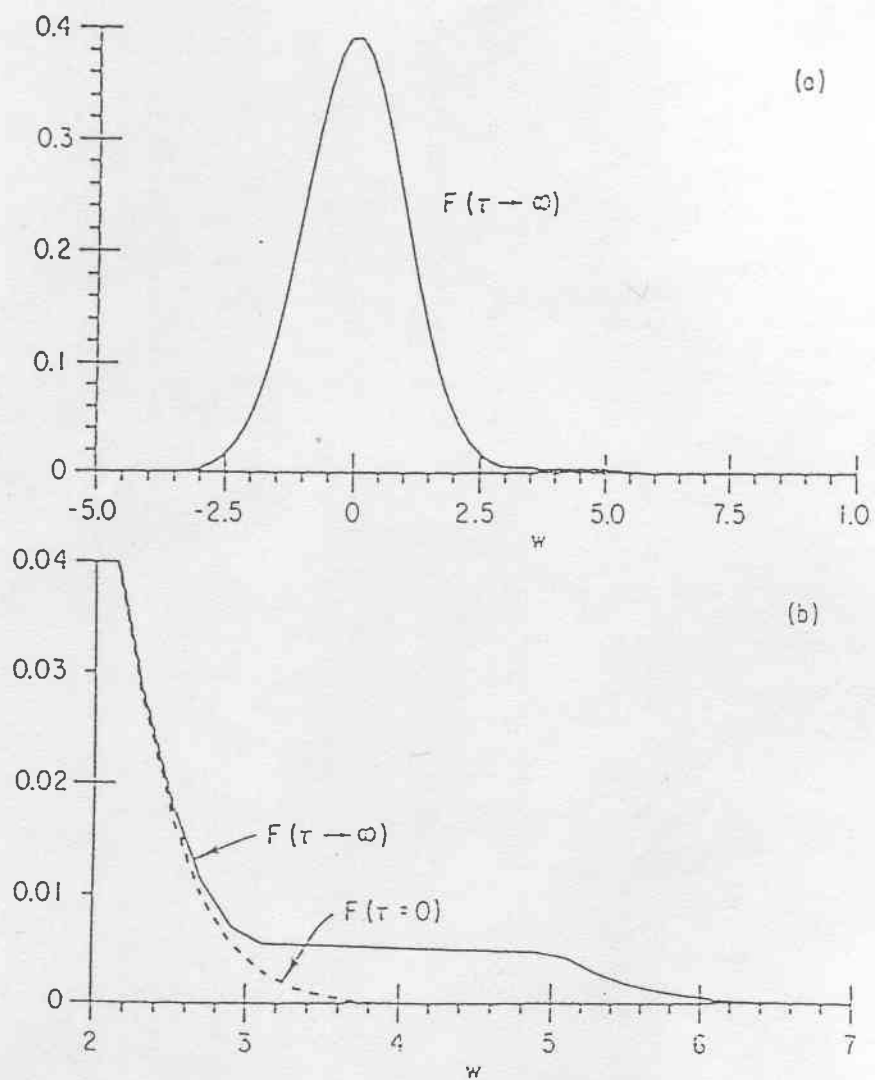


Fig. 8.

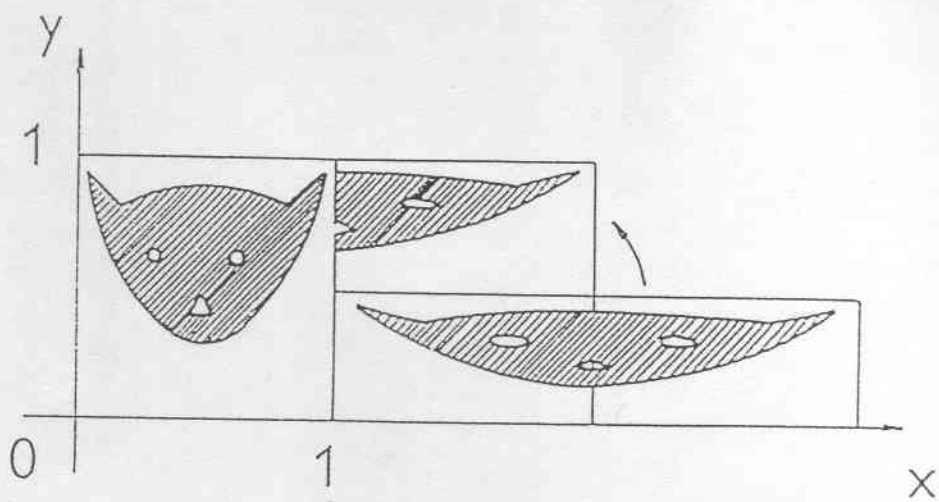


Fig. 9.

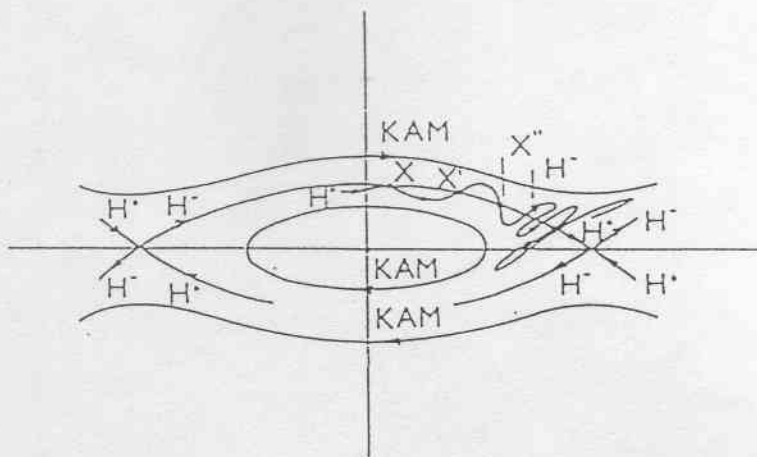


Fig. 10.

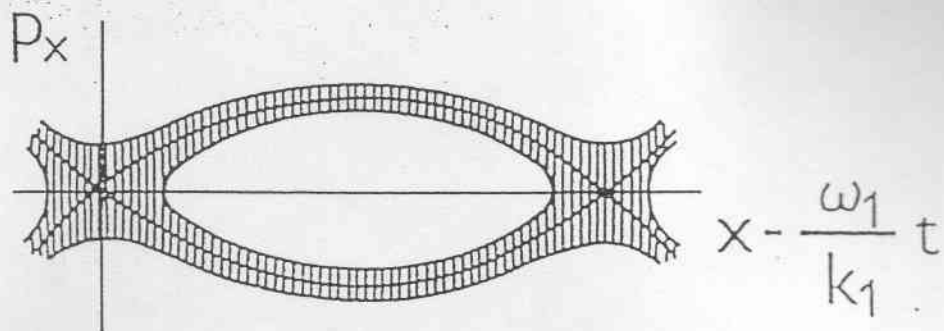


Fig. 11.

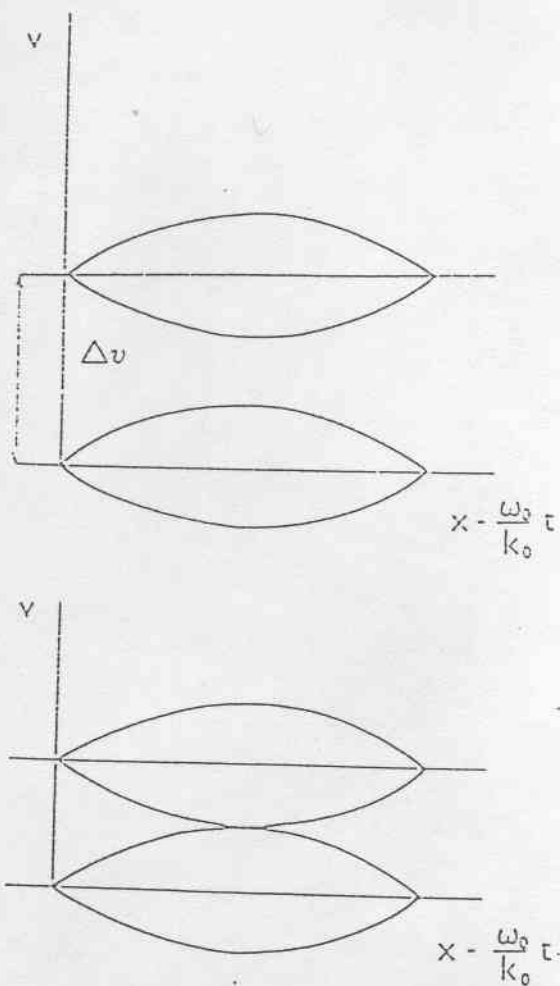


Fig. 12.