

INTRODUCTION

1. Occurrence of plasmas in nature
2. Definition of plasma
3. Concept of temperature
4. Debye shielding
5. Criterion for plasmas

1. Occurrence of plasmas in nature

99% of the matter of the universe is in the plasma state. In our neighborhood, one encounters the plasma comprising Van Allen radiation belts and the solar wind. On the other hand, in our everyday lives encounters with plasma are limited to a few examples. The flash of a lightning bolt, the conducting gas inside a fluorescent tube or neon sign, etc. The reason for that can be seen from the Saha equation, which tells us the amount of ionization in a gas in thermal equilibrium, assuming ionizations and recombinations (Chen)

$$\frac{n_i}{n_n} \approx 2.4 \times 10^{15} \frac{T^{\frac{3}{2}}}{n_i} e^{-\frac{U_i}{KT}}.$$

Here n_i and n_n are, respectively, the density (number of particles per m^3) of ionized atoms and of neutrals atoms, T is the gas temperature in degree of kelvins, K is Boltzmann's constant and U_i is the ionization energy requiring for removing the outermost electron from the atom. For ordinary air at room temperature, we may take $n_n = 3 \times 10^{25} m^{-3}$, $T = 300^\circ K$, $U_i = 14.5 eV$ (for nitrogen). Then

$$\frac{n_i}{n_n} \approx 10^{-122}.$$

For thermonuclear temperature $10 keV$, plasma is almost fully ionized.

2. Definition of plasma

A plasma is a quasineutral gas of charged and neutral particles which exhibits collective behavior.

3. Concept of temperature

Let us suppose the one-dimensional Maxwellian distribution of the plasma

$$f(u) = A \exp\left(-\frac{1}{2}mu^2 / KT\right)$$

where u is the velocity. The density n is given as

$$n = \int_{-\infty}^{+\infty} f(u) du.$$

The constant A is related to the density n by

$$A = n \left(\frac{m}{2\pi KT} \right)^{\frac{1}{2}}.$$

The width of the distribution is characterized by the constant T , which we shall call the temperature. Let us compute the averaged kinetic energy of particles in this distribution

$$E_{av} = \frac{\int_{-\infty}^{+\infty} \frac{1}{2} mu^2 f(u) du}{\int_{-\infty}^{+\infty} f(u) du}.$$

Using the form of our distribution function, we obtain after some algebra

$$E_{av} = \frac{1}{2} KT.$$

Thus the averaged kinetic energy is $\frac{1}{2} KT$.

For the 3D distribution, we obtain that $E_{av} = \frac{3}{2} KT$.

4.. Debye shielding

A fundamental characteristic of the behavior of a plasma is its ability to shield out electric potentials that are applied to it. Suppose we tried to put an electric field inside a plasma by inserting two charged balls connected to a battery (see Fig.1). The balls would attract particles of the opposite charge, and almost immediately a cloud of ions would surround the negative ball and a cloud of electrons would surround the positive ball. If the plasma were cold and there were no thermal motion, the shielding would be perfect. On the contrary, if the temperature of plasma is finite, also the cloud will be finite. We shall approximately estimate thickness of a charge cloud. Imaging that the potential ϕ on the plane $x = 0$ is held by a perfectly transparent grid (Fig. 2). For that we shall suppose that ions do not move and form uniform background of positive charge. We shall estimate the potential $\phi(x)$. Poisson's equation in one dimension reads:

$$\nabla^2 \phi = \frac{d^2 \phi}{dx^2} = -\frac{e(n_i - n_e)}{\epsilon_0} \quad (Z = 1)$$

In the presence of a potential energy $q\phi$, the electron distribution function is

$$f(u) = A \exp \left[-\left(\frac{1}{2} m u^2 + q\phi \right) (KT)^{-1} \right].$$

Integrating over u , and since $n_e(\phi \rightarrow 0) = n_\infty$, we obtain the electron distribution function in the form

$$n_e = n_\infty \exp \left(\frac{e\phi}{KT_e} \right).$$

Inserting it into the Poisson equation and using for ions $n_i = n_\infty$, we obtain

$$\frac{d^2 \phi}{dx^2} = \frac{e}{\epsilon_0} n_\infty \left\{ \exp \left(\frac{e\phi}{KT_e} \right) - 1 \right\}.$$

Assuming $\frac{e\phi}{KT_e} \ll 1$, we can expand the foregoing into a Taylor series

$$\frac{d^2 \phi}{dx^2} = \frac{e}{\epsilon_0} \left[\frac{e\phi}{KT_e} + \frac{1}{2} \left(\frac{e\phi}{KT_e} \right)^2 + \dots \right].$$

Taking only the linear part, we obtain

$$\frac{d^2\phi}{dx^2} = \frac{n_{\infty}e^2}{\epsilon_0 K T_e} \phi.$$

Defining

$$\lambda_D = \left(\frac{\epsilon_0 K T_e}{n e^2} \right)^{1/2},$$

the solution of the foregoing equation is

$$\phi = \phi_0 \exp\left(-\frac{|x|}{\lambda_D}\right).$$

The quantity λ_D is called the Debye length and is a measure of the shielding distance. The expression

$$N_D = n \frac{4}{3} \pi \lambda_D^3$$

gives the number of N_D particles in a „Debye sphere“.

5. Criteria for plasmas

$$\lambda_D \ll L$$

$$N_D \gg 1$$

$$\omega\tau > 1.$$

Here, L is a characteristic dimension of a plasma, and ω, τ^{-1} are the frequency of typical plasma oscillations and the collision frequency.

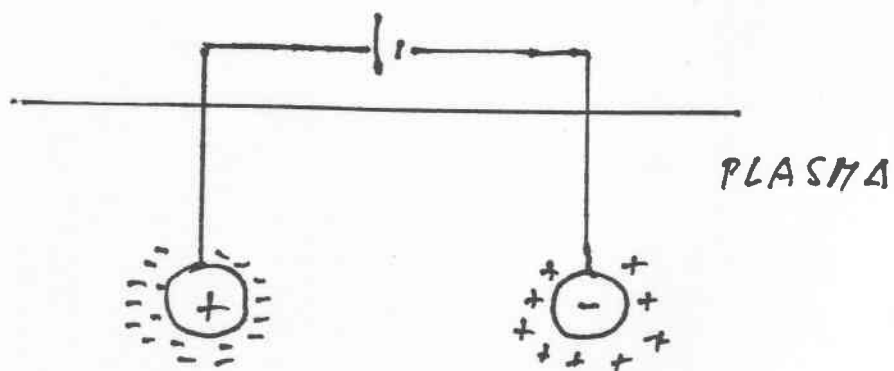


Fig. 1.

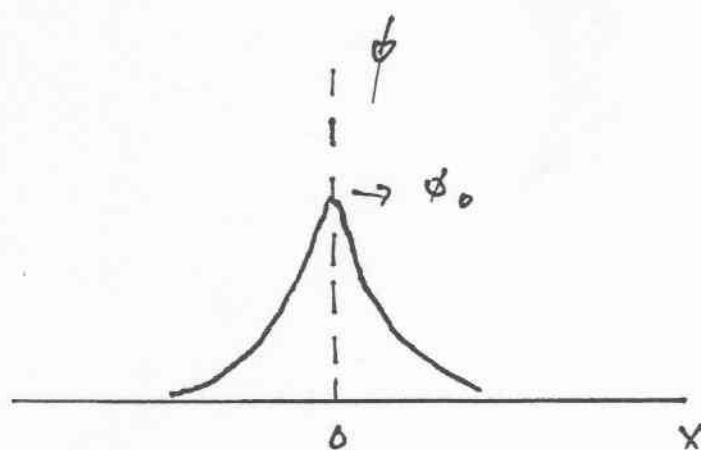


Fig. 2.