

Supplementary materials to “Eclipse timing variations in coplanar triple systems” by Sławomir Breiter and David Vokrouhlický

The purpose of this supplement is (i) to display results which were not included in Sec. 3 of the main text, and (ii) briefly discuss results of additional simulations, whose goal was to explore dependence on relevant parameters. There are three main groups of the latter, namely (i) the ratio $\pi = P_1/P_2$ of the orbital periods (an expression of the system’s compactness), (ii) the ratio $\mu = m_2/M_2$ of the mass of the third star m_2 and the total mass of the system M_2 (an expression of the strength of perturbation of the binary orbit), and (iii) the eccentricities e_1 and e_2 of the inner and outer orbits. As in the main text, we assume coplanarity of the triple system and prograde motion of the third component, namely $i_1 = i_2$. In addition, we illustrate results for a limited number of retrograde coplanar configurations, namely cases 1 to 3 from the main text of the paper. We also perform simulations for both equal-mass binaries with $m_0 = m_1$, and non-equal-mass cases with $m_0 \neq m_1$ (in order to probe the role of the octupole coupling).

Overview of the material and simulations presented in this Supplementary materials is given in Table 1.

1 Additional figures to Sec. 3 in the main text

Here we provide information not shown in Sec. 3 of the main text. Figure 1 shows osculating (gray) and mean (black) non-singular eccentricity variables $\mathbf{z}_1 = k_1 + \iota h_1$ (left panels) and $\mathbf{z}_2 = k_2 + \iota h_2$ (right panels) of the case 2 system discussed in Sec. 3.1. Components of the eclipsing binary have the same mass $m_0 = m_1 = 3.2 M_\odot$ (with also $m_2 = 1.6 M_\odot$, $P_1 = 4$ days, $P_2 = 42$ days, $e_1 = 0.03$ and $e_2 = 0.2$), which results in a simple, single-component secular behavior. The proper periods of \mathbf{z}_1 and \mathbf{z}_2 are 5.55 and 15.95 yr, respectively. Figure 2 shows the same, but now for a non-equal-mass binary presented in Sec. 3.2 with $m_0 = 3 M_\odot$, $m_1 = 1 M_\odot$, $m_2 = 4 M_\odot$, $P_1 = 4$ days, $P_2 = 42$ days, $e_1 = 0.03$ and $e_2 = 0.2$. The outer orbit \mathbf{z}_2 vector has a proper secular period of 34.2 yr, which is recovered also in \mathbf{z}_1 as a forced secular period via octupole coupling. The proper secular period of \mathbf{z}_1 is now 1.6 yr. Figure 3 complements information shown in Fig. 5 of the main text by providing series of ETVs for secondary eclipses.

Table 1: Overview of results presented in the Supplementary materials.

Simulations with equal-mass eclipsing binaries: osculating vs mean elements of the second system analysed in Sec. 3.1 (eclipses shown in Fig. 3)							
Orbital elements				Masses			Figures
P_1	P_2	e_1	e_2	m_0	m_1	m_2	
4	42	0.03	0.20	3.2	3.2	1.6	Fig. 1
Simulations with nonequal-mass eclipsing binaries: osculating vs mean elements, and secondary eclipses of the system analysed in Sec. 3.2 (in parallel to Fig. 5)							
Orbital elements				Masses			Figures
P_1	P_2	e_1	e_2	m_0	m_1	m_2	
4	42	0.03	0.20	3	1	4	Figs. 2-3
Simulations with equal-mass eclipsing binaries. Bold items highlight the difference with respect to simulations shown in the main text							
Orbital elements				Masses			Figures
P_1	P_2	e_1	e_2	m_0	m_1	m_2	
4	85	0.03	0.20	2	2	4	Figs. 4-5
4	85	0.03	0.45	2	2	4	Figs. 6-7
4	85	0.15	0.20	2	2	4	Figs. 8-9
4	35	0.03	0.20	3.2	3.2	1.6	Figs. 10-11
4	42	0.15	0.05	2	2	4	Figs. 12-13
4	42	0.15	0.05	3.2	3.2	1.6	Figs. 14-15
4	35	0.01	0.01	1	1	4	Figs. 16-17
Simulations with nonequal-mass eclipsing binaries. Bold items highlight the difference with respect to simulations shown in the main text							
Orbital elements				Masses			Figures
P_1	P_2	e_1	e_2	m_0	m_1	m_2	
4	42	0.03	0.20	3.6	0.4	4	Figs. 18-20
4	85	0.03	0.45	3.6	0.4	4	Figs. 21-23
4	42	0.03	0.20	3.6	0.4	1	Figs. 24-26
4	42	0.15	0.05	3.6	0.4	1	Figs. 27-29
4	35	0.01	0.01	0.8	0.2	4	Figs. 30-32
Simulations with retrograde orbits of the third component (cases 1-3)							
Orbital elements				Masses			Figures
P_1	P_2	e_1	e_2	m_0	m_1	m_2	
4	42	0.03	0.20	2	2	4	Figs. 33-34 (case 1)
4	42	0.03	0.20	3.2	3.2	1.6	Figs. 35-36 (case 2)
4	42	0.03	0.20	3	1	4	Figs. 37-39 (case 3)

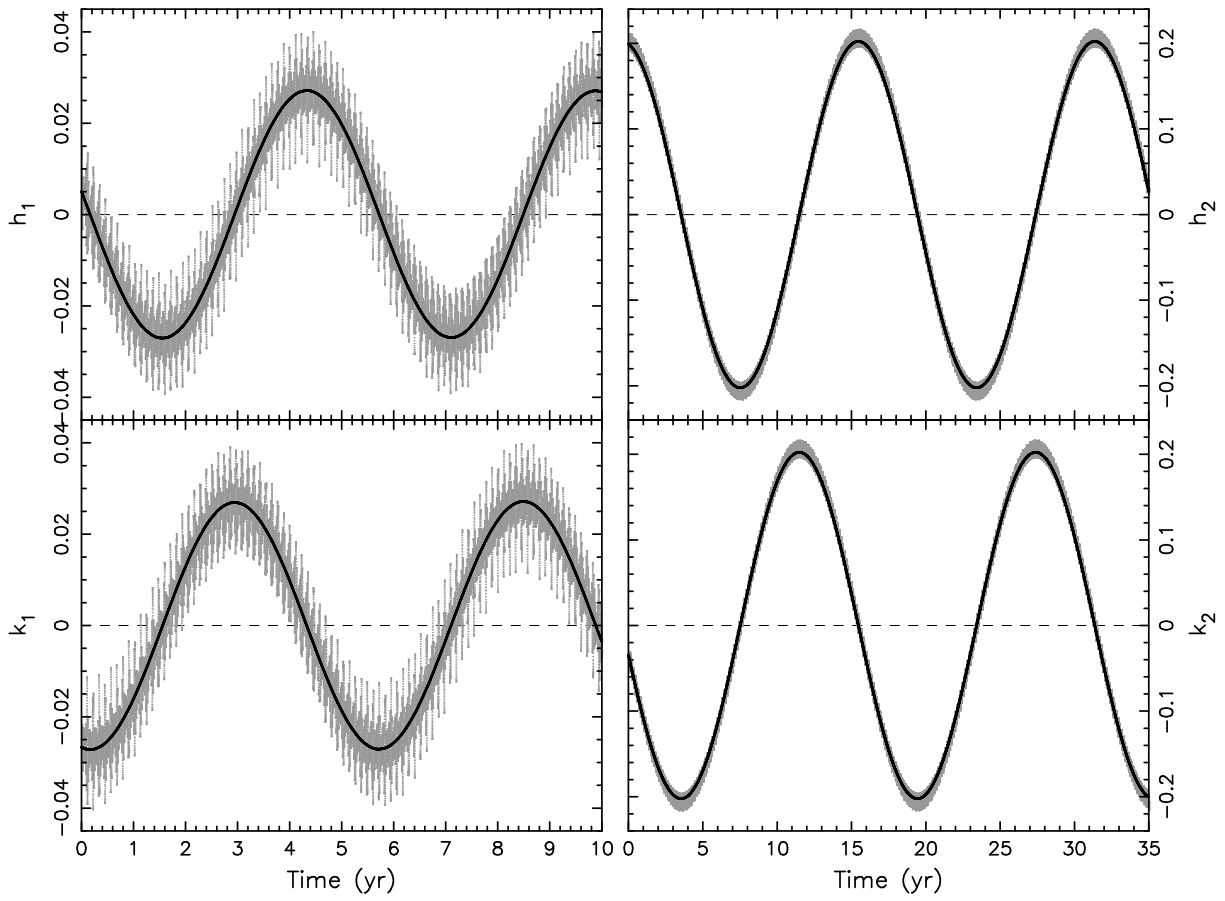


Figure 1: Osculating vs mean orbital elements for the second system (case 2) discussed in Sec. 3.1 of the main text (eclipses shown in Fig. 3).

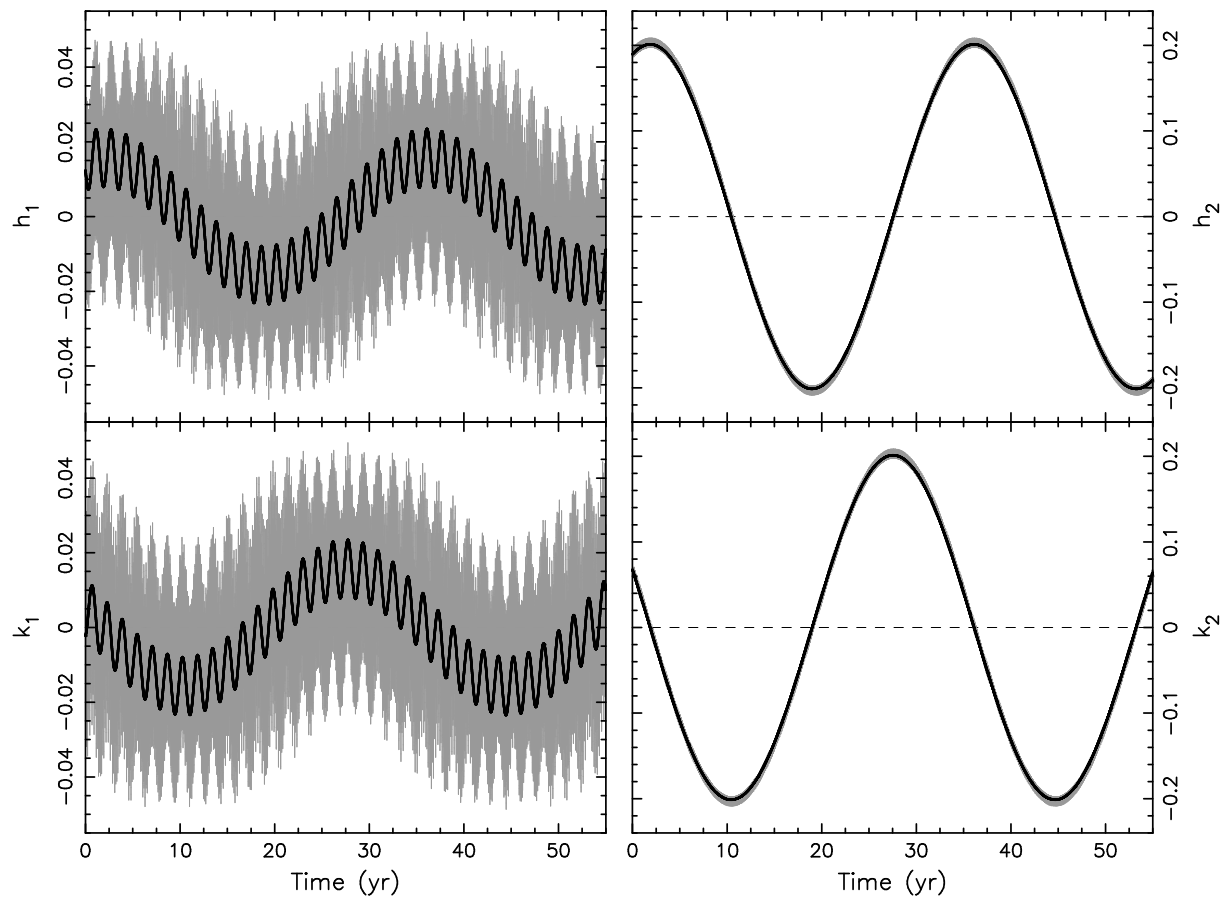


Figure 2: Osculating vs mean orbital elements for the system discussed in Sec. 3.2 of the main text (primary eclipses shown in Fig. 5).

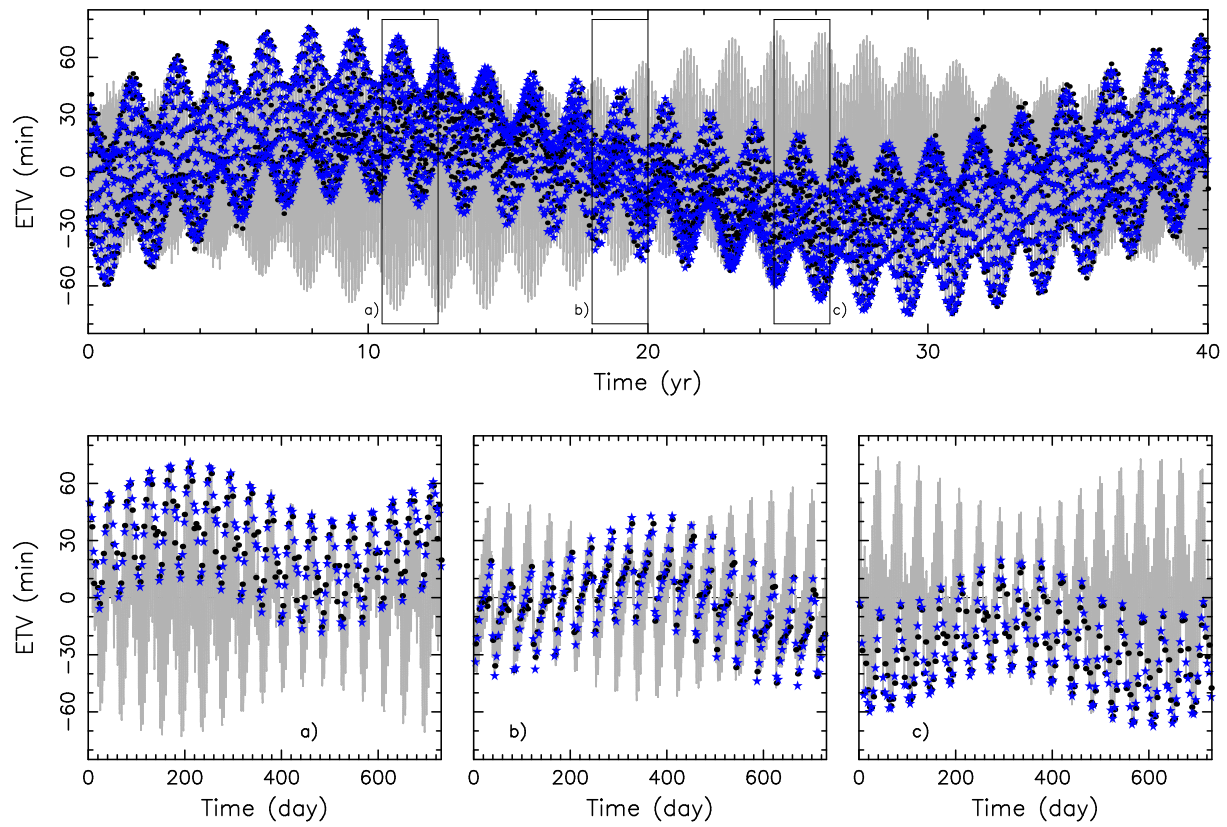


Figure 3: ETV series for secondary eclipses of the system discussed in Sec. 3.2 of the main text (primary eclipses shown in Fig. 5).

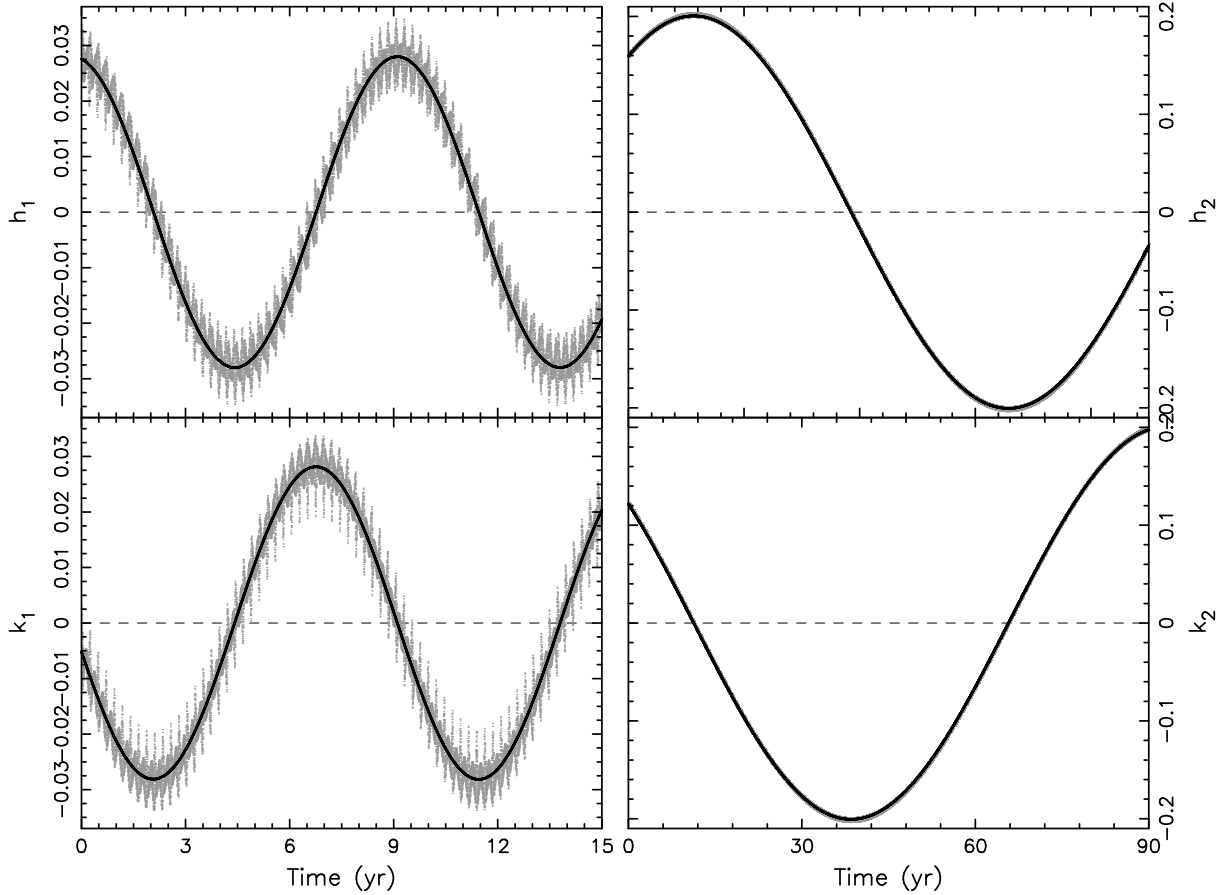


Figure 4: A comparison of the osculating (gray dots) and mean (black line) values of the nonsingular eccentricity elements (k, h) for the inner (left panels) and outer (right panels) orbits. We assumed an equal-mass binary with $m_0 = m_1 = 2 M_\odot$ and third star with $m_2 = 4 M_\odot$. Initial osculating values of the inner and outer periods were $P_1 = 4$ days and $P_2 = 85$ days, inner and outer eccentricities $e_1 = 0.03$ and $e_2 = 0.2$, and inclination $i_1 = i_2 = 80^\circ$. For definiteness, we also set initial osculating values of the argument of pericenter $\omega_1 = 0^\circ$ and $\omega_2 = 45^\circ$, and osculating mean anomalies $\ell_1 = 0^\circ$ and $\ell_2 = 90^\circ$. The mean elements we determined by online digital filtering procedure described in the text. Time at the abscissa has been shifted by 2.5 yr since the initial epoch of the simulation. This is because the architecture of the digital filter provides the first mean elements value at 2.4334 yr after the origin.

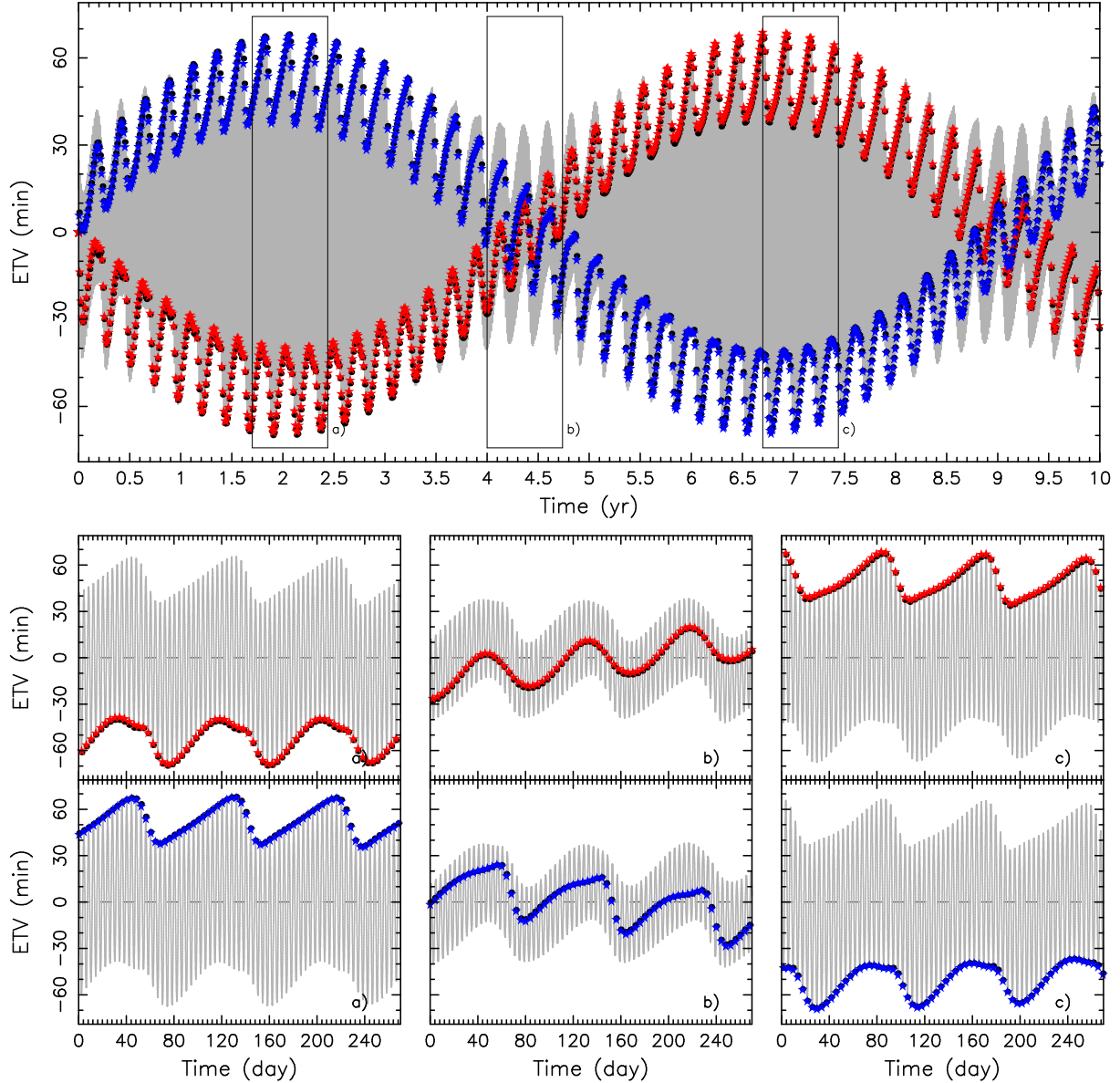


Figure 5: ETV series for a coplanar triple system with masses $m_0 = m_1 = 2 M_\odot$ and $m_2 = 4 M_\odot$, inner and outer periods $P_1 = 4$ days and $P_2 = 85$ days, inner and outer eccentricities $e_1 = 0.03$ and $e_2 = 0.2$, and inclination $i_1 = i_2 = 80^\circ$. The gray line is the periodic part of the target function and black circles are epochs of the primary and secondary eclipses, both determined numerically. The red and blue stars are ETVs of primary and secondary eclipses from our formulae. The upper panel shows a continuous signal over 3.5 yr, and the bottom panels zoom into 250-day windows shown by rectangles in the upper panel. The shortest period of the gray line is P_1 of the binary.

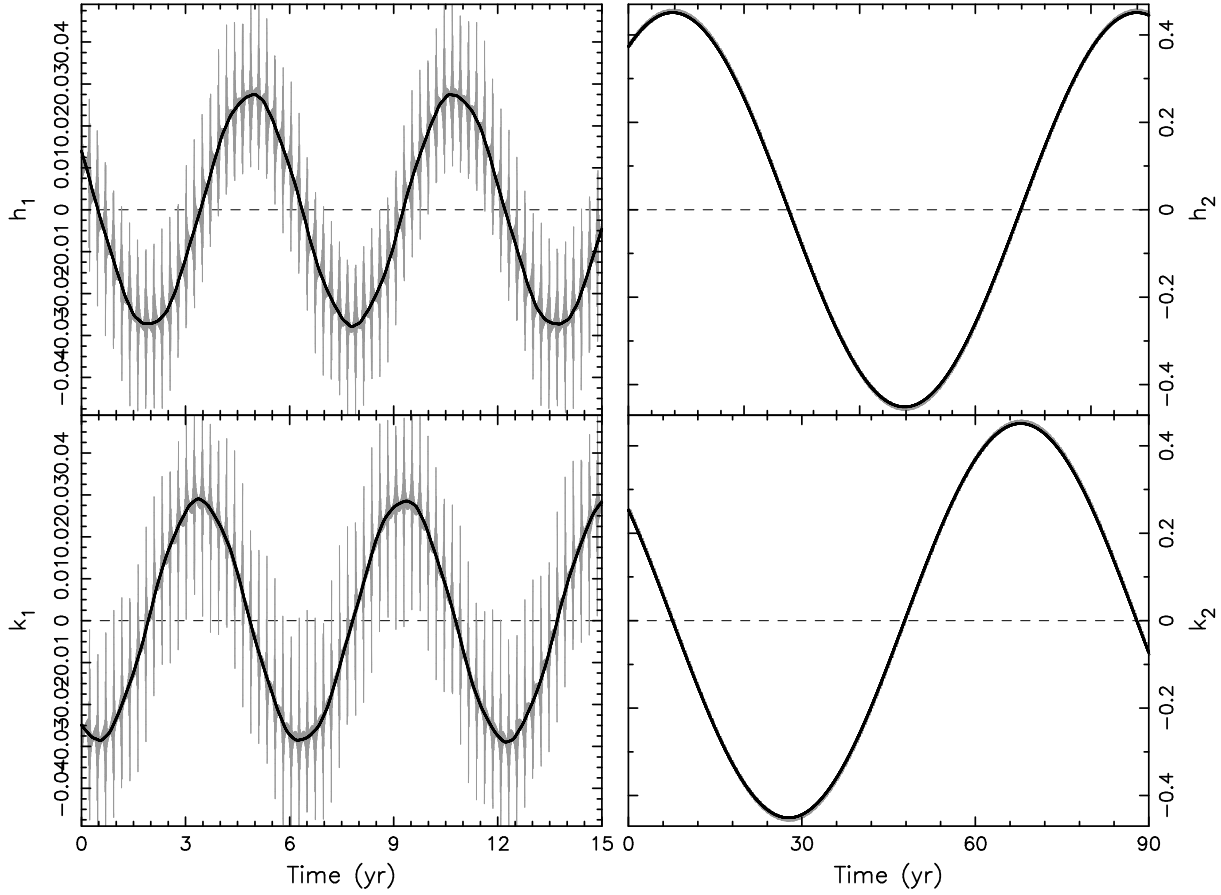


Figure 6: A comparison of the osculating (gray dots) and mean (black line) values of the nonsingular eccentricity elements (k, h) for the inner (left panels) and outer (right panels) orbits. We assumed an equal-mass binary with $m_0 = m_1 = 2 M_\odot$ and third star with $m_2 = 4 M_\odot$. Initial osculating values of the inner and outer periods were $P_1 = 4$ days and $P_2 = 85$ days, inner and outer eccentricities $e_1 = 0.03$ and $e_2 = 0.45$, and inclination $i_1 = i_2 = 80^\circ$. For definiteness, we also set initial osculating values of the argument of pericenter $\omega_1 = 0^\circ$ and $\omega_2 = 45^\circ$, and osculating mean anomalies $\ell_1 = 0^\circ$ and $\ell_2 = 90^\circ$. The mean elements we determined by online digital filtering procedure described in the text. Time at the abscissa has been shifted by 2.5 yr since the initial epoch of the simulation. This is because the architecture of the digital filter provides the first mean elements value at 2.4334 yr after the origin.

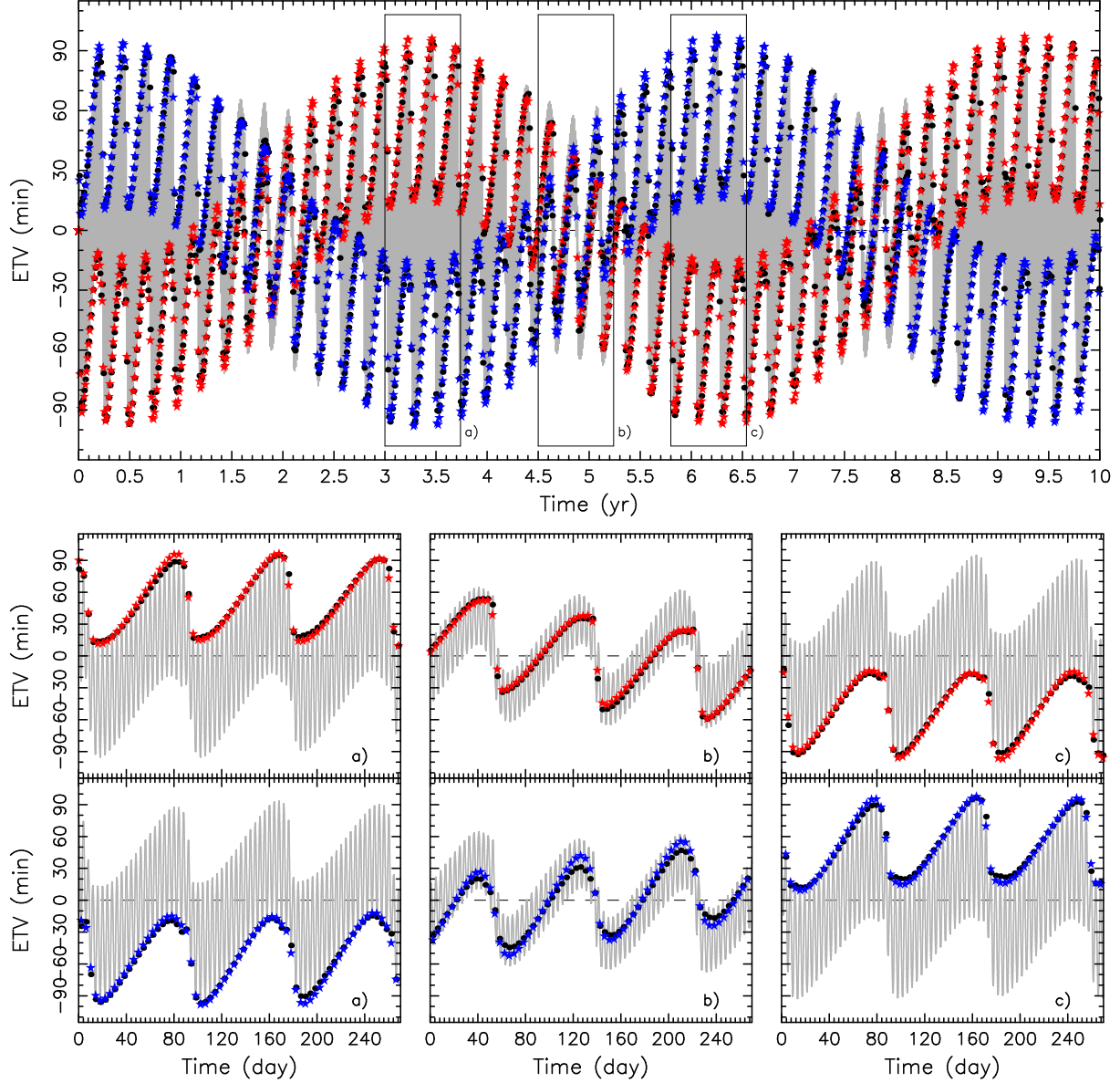


Figure 7: ETV series for a coplanar triple system with masses $m_0 = m_1 = 2 M_\odot$ and $m_2 = 4 M_\odot$, inner and outer periods $P_1 = 4$ and $P_2 = 85$ days, inner and outer eccentricities $e_1 = 0.03$ and $e_2 = 0.45$, and inclination $i_1 = i_2 = 80^\circ$. The gray line is the periodic part of the target function and black circles are epochs of the primary and secondary eclipses, both determined numerically. The red and blue stars are ETVs of primary and secondary eclipses from our formulae. The upper panel shows a continuous signal over 3.5 yr, and the bottom panels zoom into 250-day windows shown by rectangles in the upper panel. The shortest period of the gray line is P_1 of the binary.

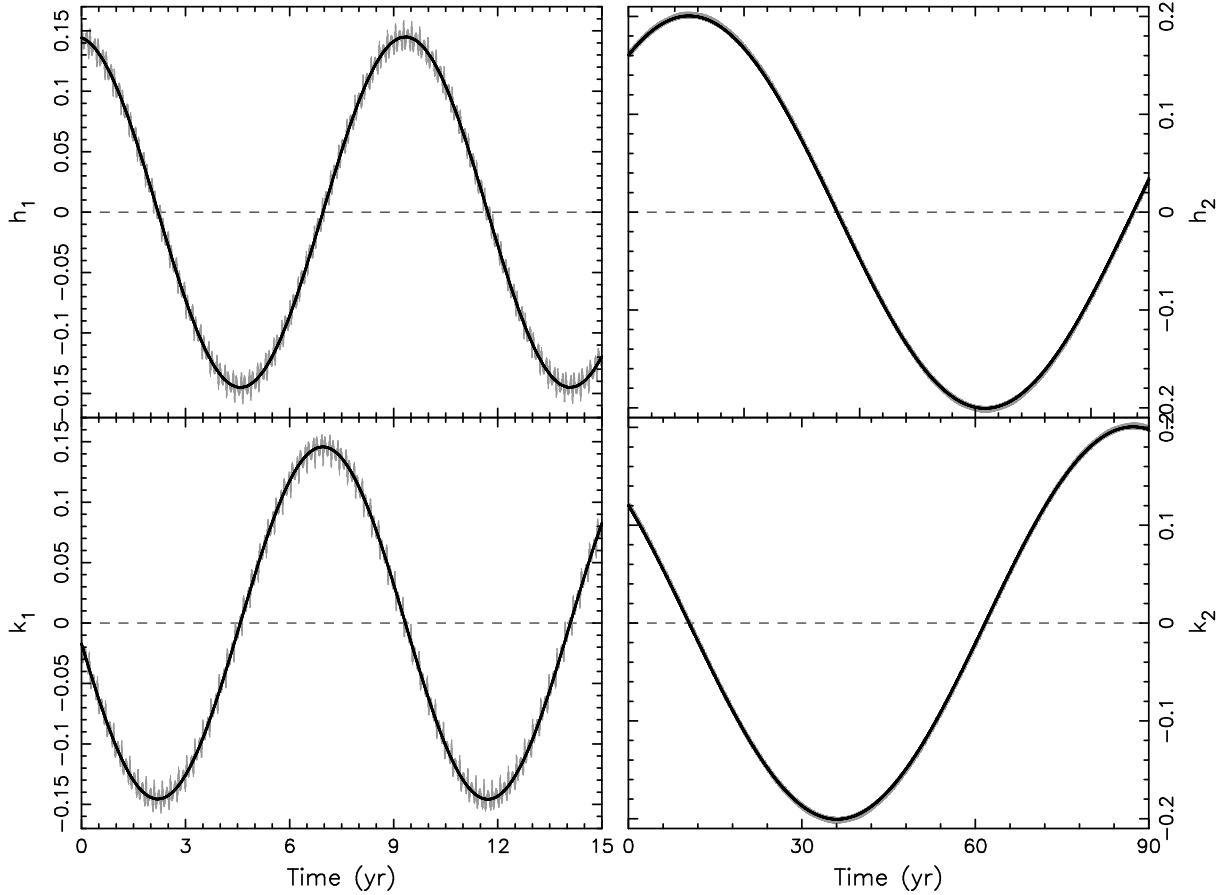


Figure 8: A comparison of the osculating (gray dots) and mean (black line) values of the nonsingular eccentricity elements (k, h) for the inner (left panels) and outer (right panels) orbits. We assumed an equal-mass binary with $m_0 = m_1 = 2 M_\odot$ and third star with $m_2 = 4 M_\odot$. Initial osculating values of the inner and outer periods were $P_1 = 4$ days and $P_2 = 85$ days, inner and outer eccentricities $e_1 = 0.15$ and $e_2 = 0.2$, and inclination $i_1 = i_2 = 80^\circ$. For definiteness, we also set initial osculating values of the argument of pericenter $\omega_1 = 0^\circ$ and $\omega_2 = 45^\circ$, and osculating mean anomalies $\ell_1 = 0^\circ$ and $\ell_2 = 90^\circ$. The mean elements we determined by online digital filtering procedure described in the text. Time at the abscissa has been shifted by 2.5 yr since the initial epoch of the simulation. This is because the architecture of the digital filter provides the first mean elements value at 2.4334 yr after the origin.

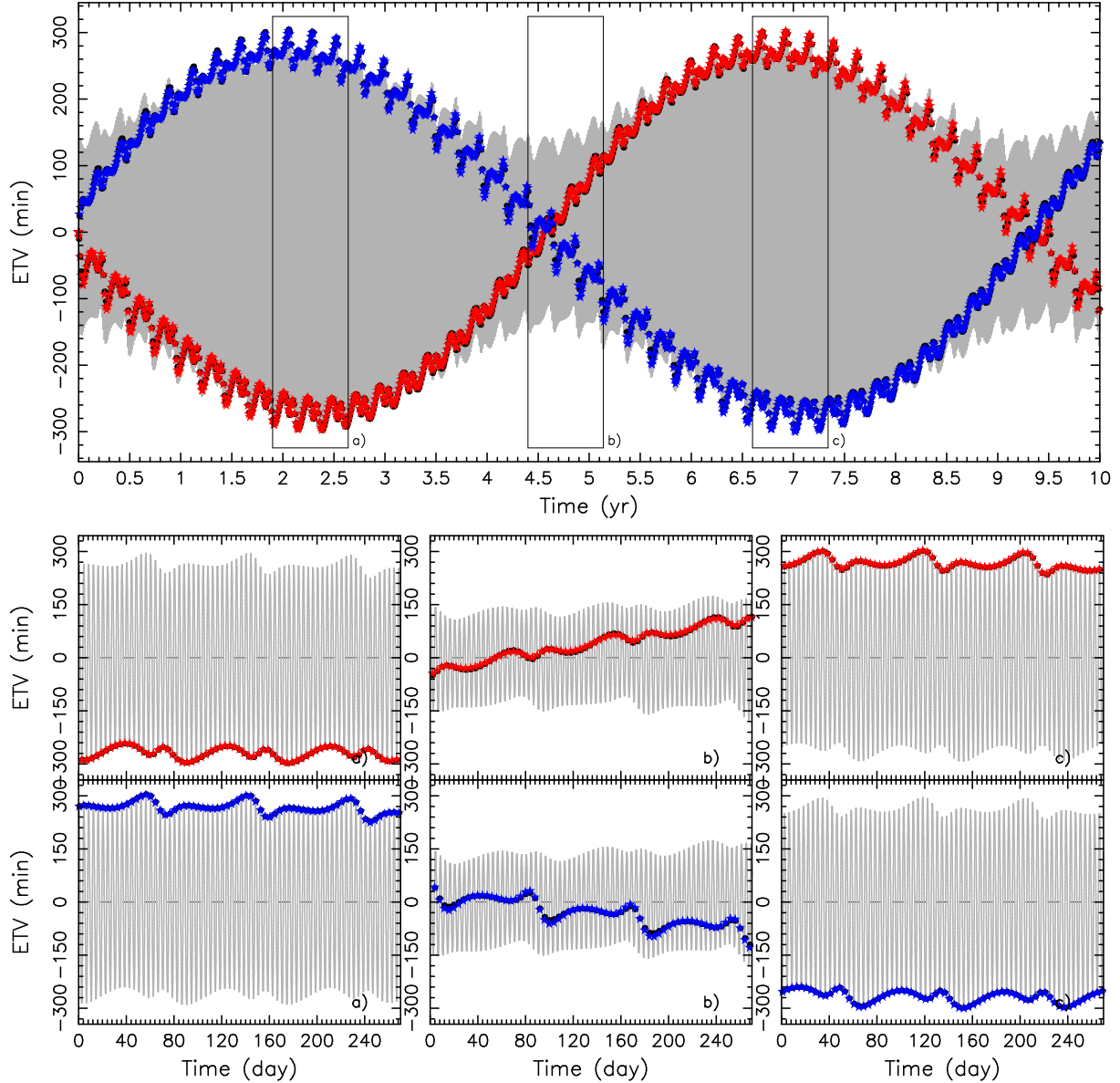


Figure 9: ETV series for a coplanar triple system with masses $m_0 = m_1 = 2 M_\odot$ and $m_2 = 4 M_\odot$, inner and outer periods $P_1 = 4$ days and $P_2 = 85$ days, inner and outer eccentricities $e_1 = 0.15$ and $e_2 = 0.2$, and inclination $i_1 = i_2 = 80^\circ$. The gray line is the periodic part of the target function and black circles are epochs of the primary and secondary eclipses, both determined numerically. The red and blue stars are ETVs of primary and secondary eclipses from our formulae. The upper panel shows a continuous signal over 3.5 yr, and the bottom panels zoom into 250-day windows shown by rectangles in the upper panel. The shortest period of the gray line is P_1 of the binary.

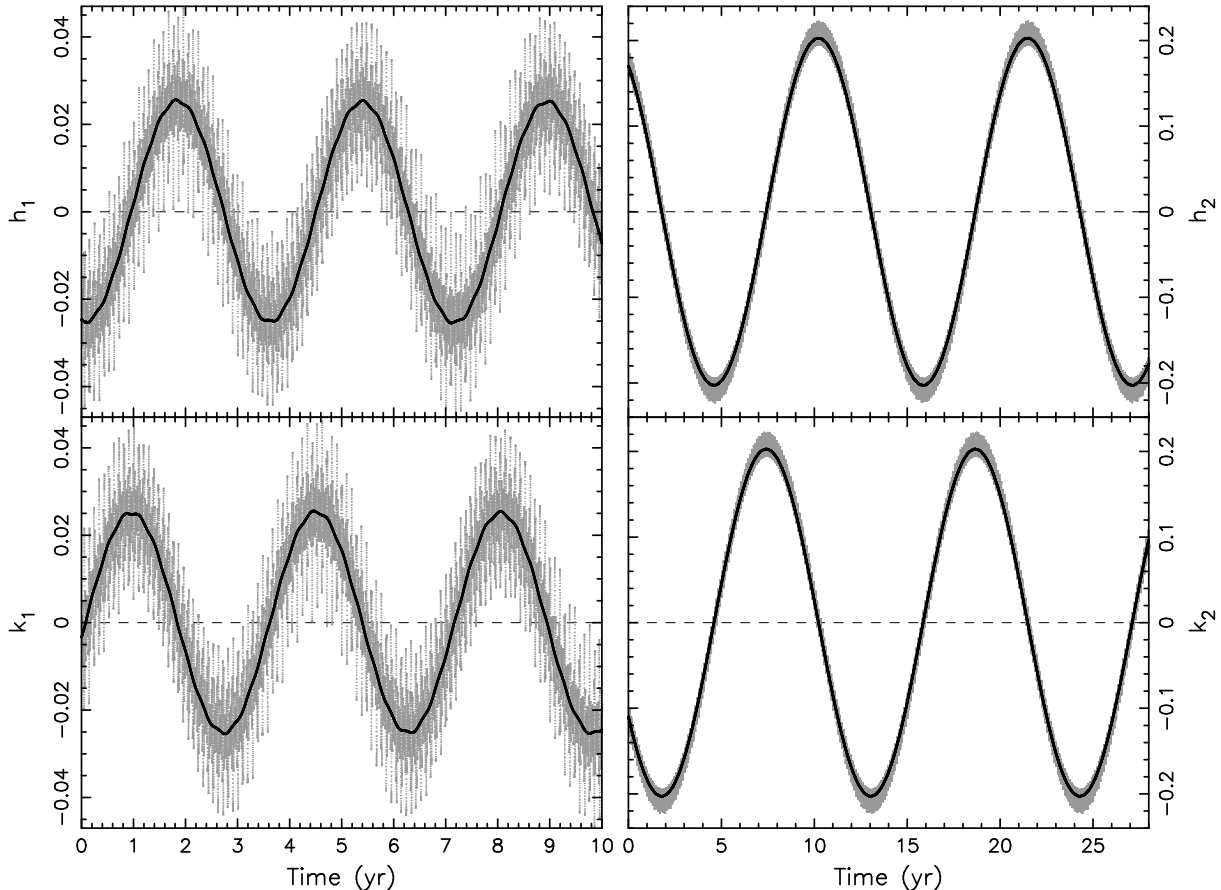


Figure 10: A comparison of the osculating (gray dots) and mean (black line) values of the nonsingular eccentricity elements (k, h) for the inner (left panels) and outer (right panels) orbits. We assumed an equal-mass binary with $m_0 = m_1 = 3.2 M_\odot$ and third star with $m_2 = 1.6 M_\odot$. Initial osculating values of the inner and outer periods were $P_1 = 4$ days and $P_2 = 35$ days, inner and outer eccentricities $e_1 = 0.03$ and $e_2 = 0.2$, and inclination $i_1 = i_2 = 80^\circ$. For definiteness, we also set initial osculating values of the argument of pericenter $\omega_1 = 0^\circ$ and $\omega_2 = 45^\circ$, and osculating mean anomalies $\ell_1 = 0^\circ$ and $\ell_2 = 90^\circ$. The mean elements we determined by online digital filtering procedure described in the text. Time at the abscissa has been shifted by 2.5 yr since the initial epoch of the simulation. This is because the architecture of the digital filter provides the first mean elements value at 2.4334 yr after the origin.

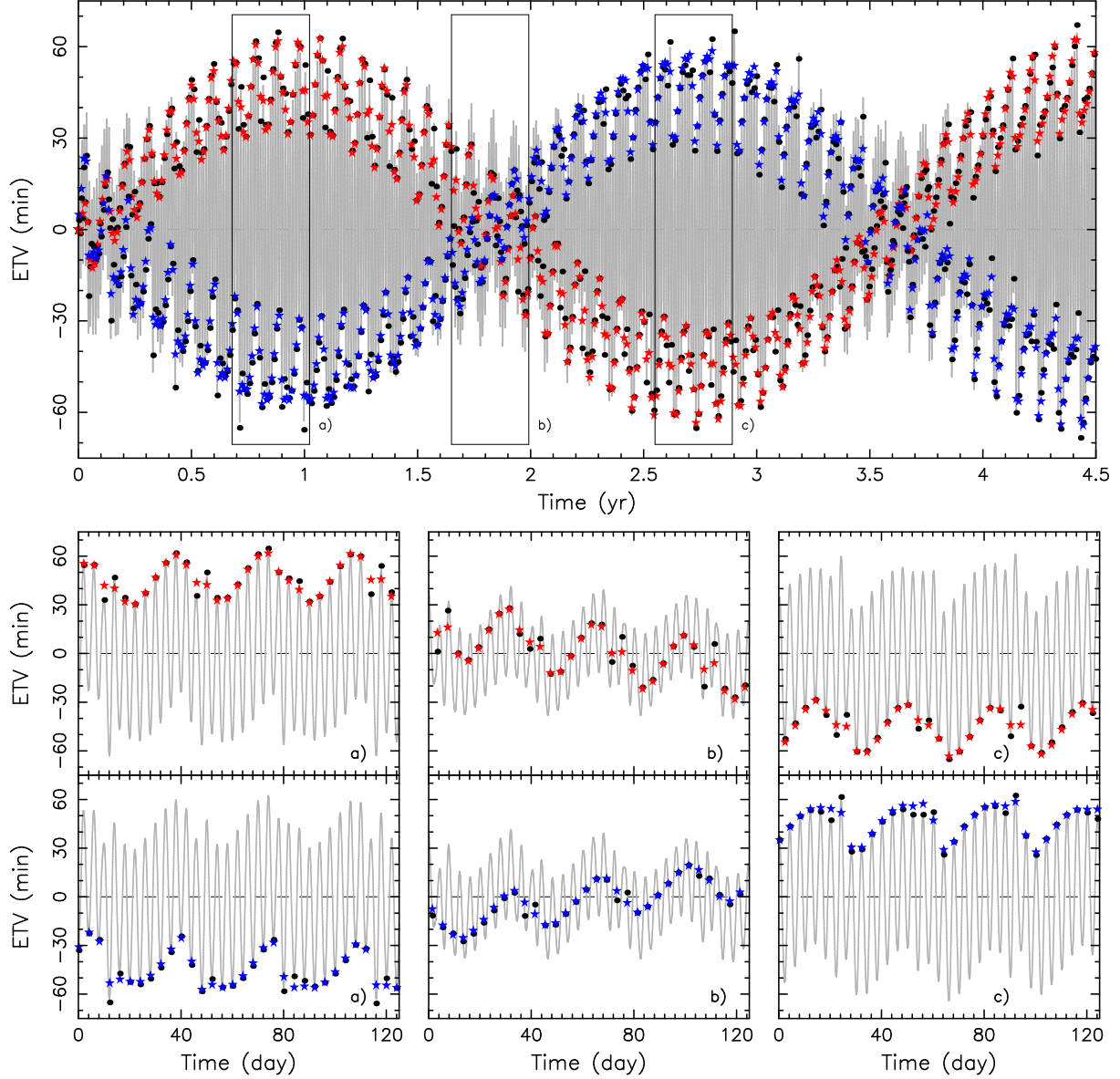


Figure 11: ETV series for a coplanar triple system with masses $m_0 = m_1 = 3.2 M_\odot$ and $m_2 = 1.6 M_\odot$, inner and outer periods $P_1 = 4$ days and $P_2 = 35$ days, inner and outer eccentricities $e_1 = 0.03$ and $e_2 = 0.2$, and inclination $i_1 = i_2 = 80^\circ$. The gray line is the periodic part of the target function and black circles are epochs of the primary and secondary eclipses, both determined numerically. The red and blue stars are ETVs of primary and secondary eclipses from our formulae. The upper panel shows a continuous signal over 3.5 yr, and the bottom panels zoom into 250-day windows shown by rectangles in the upper panel. The shortest period of the gray line is P_1 of the binary.

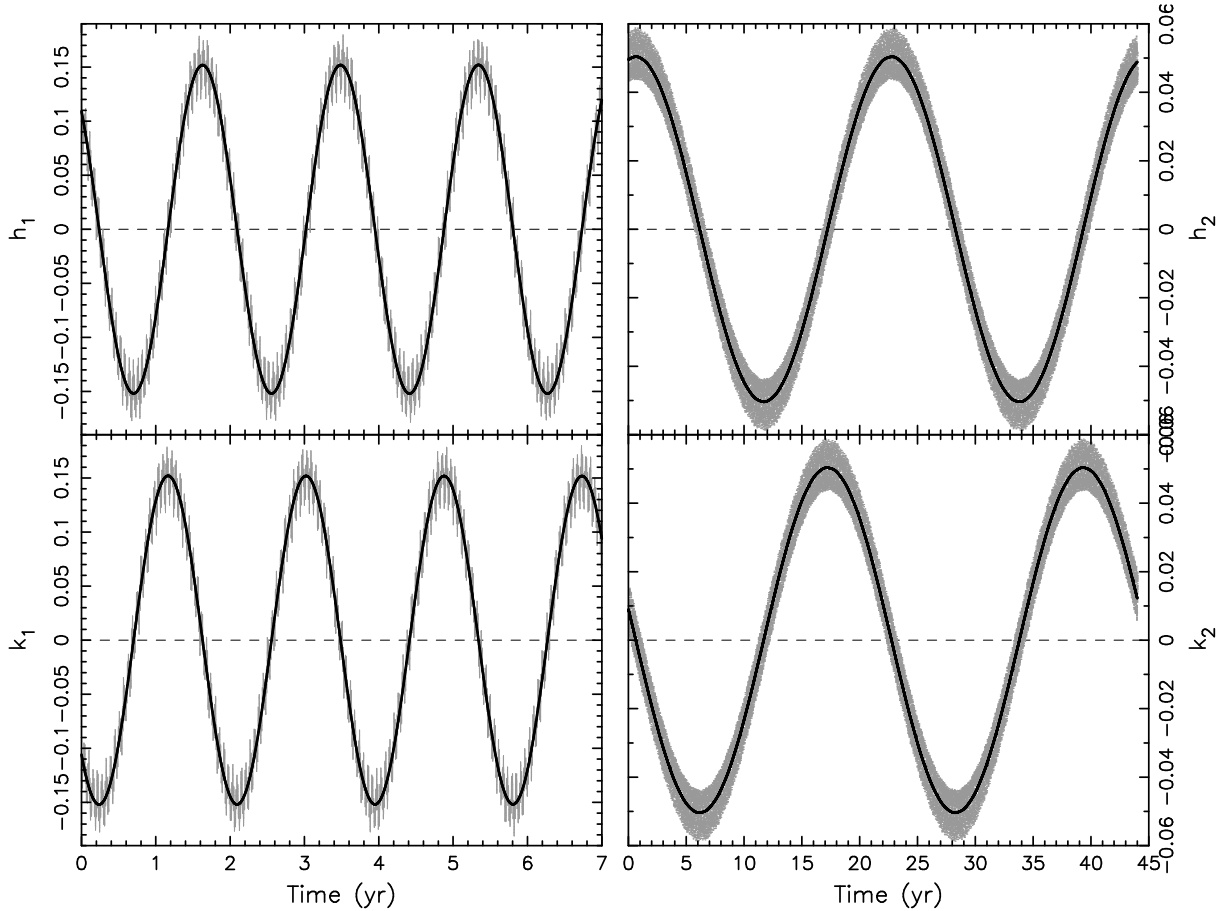


Figure 12: A comparison of the osculating (gray dots) and mean (black line) values of the nonsingular eccentricity elements (k, h) for the inner (left panels) and outer (right panels) orbits. We assumed an equal-mass binary with $m_0 = m_1 = 2 M_\odot$ and third star with $m_2 = 4 M_\odot$. Initial osculating values of the inner and outer periods were $P_1 = 4$ days and $P_2 = 42$ days, inner and outer eccentricities $e_1 = 0.15$ and $e_2 = 0.05$, and inclination $i_1 = i_2 = 80^\circ$. For definiteness, we also set initial osculating values of the argument of pericenter $\omega_1 = 0^\circ$ and $\omega_2 = 45^\circ$, and osculating mean anomalies $\ell_1 = 0^\circ$ and $\ell_2 = 90^\circ$. The mean elements we determined by online digital filtering procedure described in the text. Time at the abscissa has been shifted by 2.5 yr since the initial epoch of the simulation. This is because the architecture of the digital filter provides the first mean elements value at 2.4334 yr after the origin.

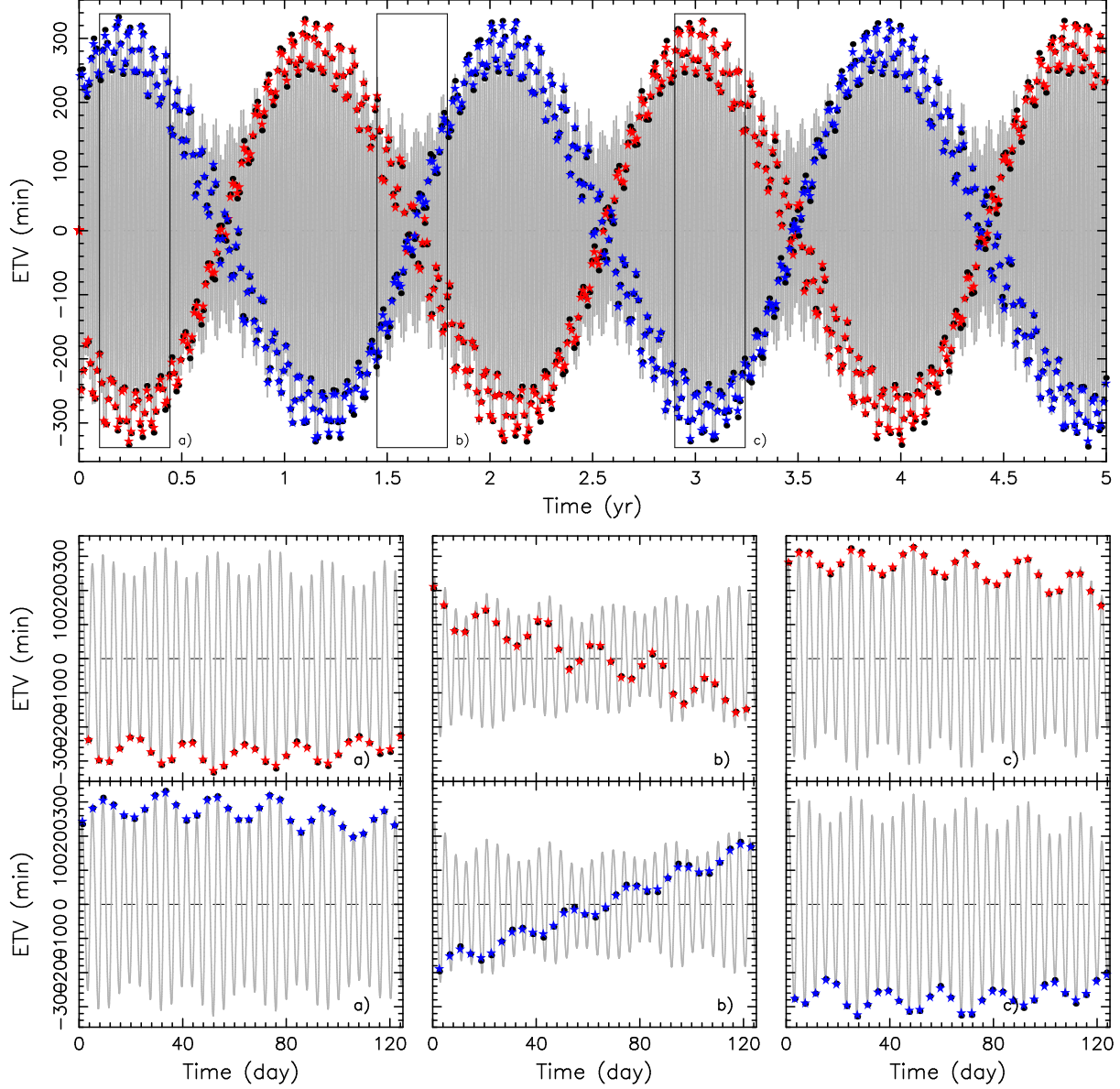


Figure 13: ETV series for a coplanar triple system with masses $m_0 = m_1 = 2 M_\odot$ and $m_2 = 4 M_\odot$, inner and outer periods $P_1 = 4$ days and $P_2 = 42$ days, inner and outer eccentricities $e_1 = 0.15$ and $e_2 = 0.05$, and inclination $i_1 = i_2 = 80^\circ$. The gray line is the periodic part of the target function and black circles are epochs of the primary and secondary eclipses, both determined numerically. The red and blue stars are ETVs of primary and secondary eclipses from our formulae. The upper panel shows a continuous signal over 5 yr, and the bottom panels zoom into 125-day windows shown by rectangles in the upper panel. The shortest period of the gray line is P_1 of the binary.

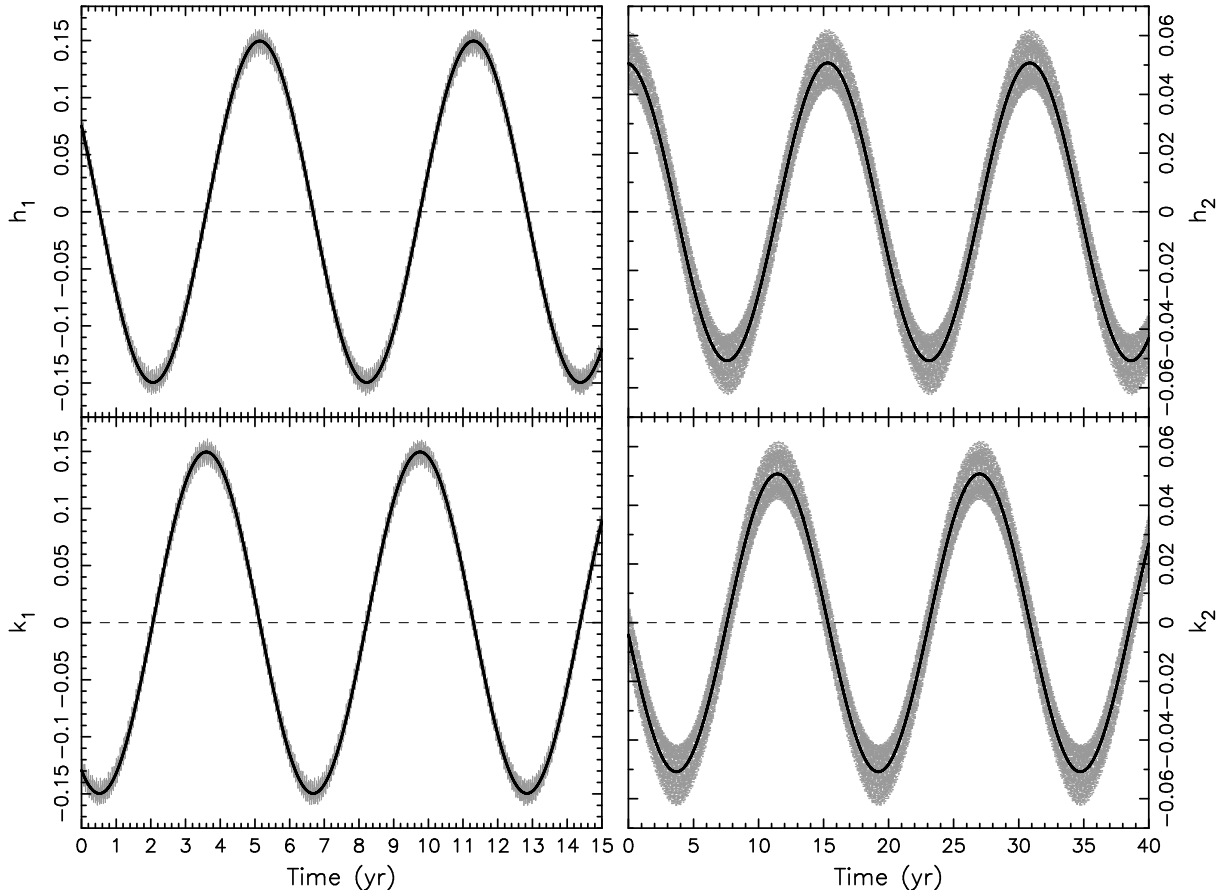


Figure 14: A comparison of the osculating (gray dots) and mean (black line) values of the nonsingular eccentricity elements (k, h) for the inner (left panels) and outer (right panels) orbits. We assumed an equal-mass binary with $m_0 = m_1 = 3.2 M_\odot$ and third star with $m_2 = 1.6 M_\odot$. Initial osculating values of the inner and outer periods were $P_1 = 4$ days and $P_2 = 42$ days, inner and outer eccentricities $e_1 = 0.15$ and $e_2 = 0.05$, and inclination $i_1 = i_2 = 80^\circ$. For definiteness, we also set initial osculating values of the argument of pericenter $\omega_1 = 0^\circ$ and $\omega_2 = 45^\circ$, and osculating mean anomalies $\ell_1 = 0^\circ$ and $\ell_2 = 90^\circ$. The mean elements we determined by online digital filtering procedure described in the text. Time at the abscissa has been shifted by 2.5 yr since the initial epoch of the simulation. This is because the architecture of the digital filter provides the first mean elements value at 2.4334 yr after the origin.

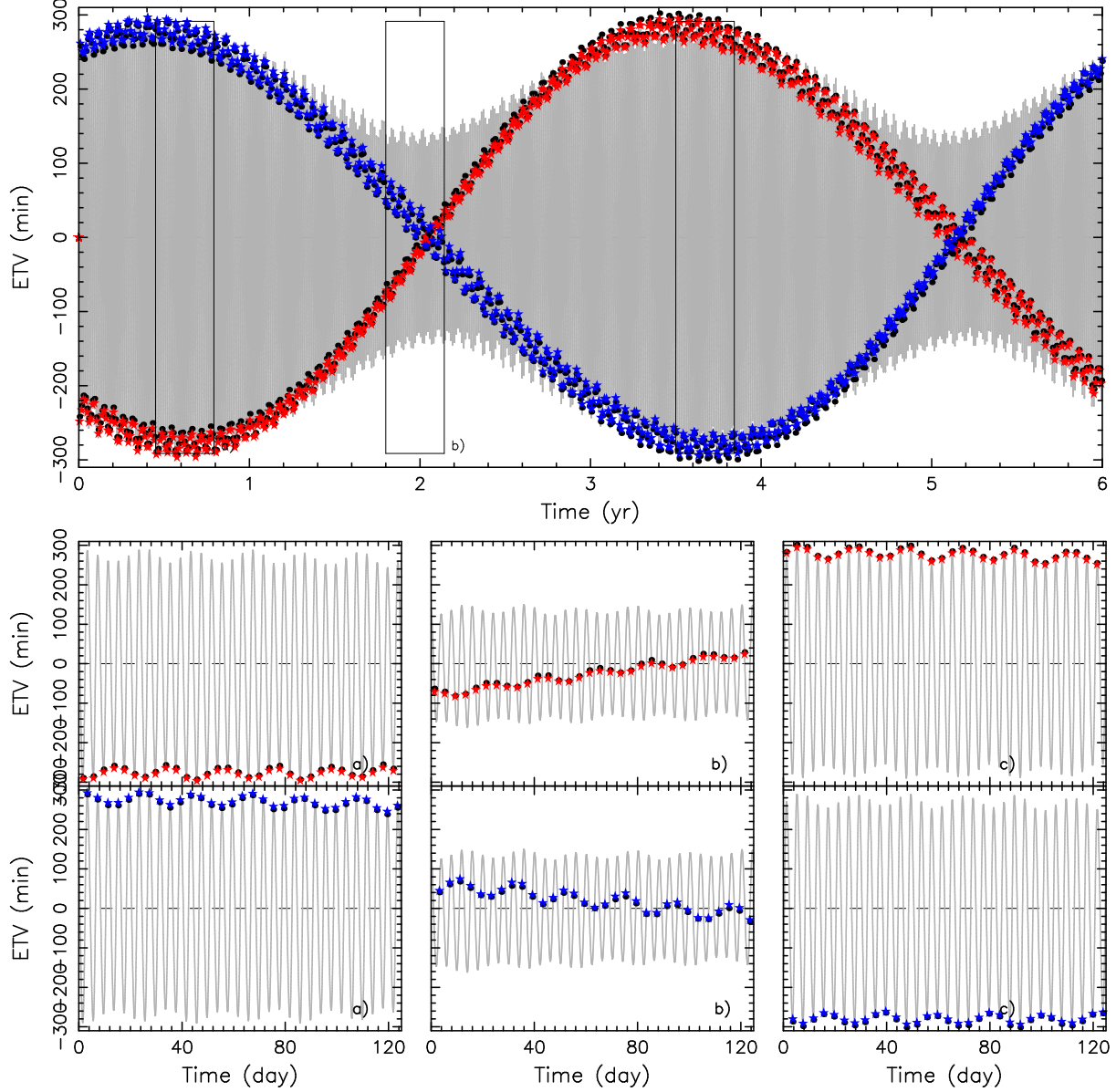


Figure 15: ETV series for a coplanar triple system with masses $m_0 = m_1 = 3.2 M_\odot$ and $m_2 = 1.6 M_\odot$, inner and outer periods $P_1 = 4$ days and $P_2 = 42$ days, inner and outer eccentricities $e_1 = 0.15$ and $e_2 = 0.05$, and inclination $i_1 = i_2 = 80^\circ$. The gray line is the periodic part of the target function and black circles are epochs of the primary and secondary eclipses, both determined numerically. The red and blue stars are ETVs of primary and secondary eclipses from our formulae. The upper panel shows a continuous signal over 6 yr, and the bottom panels zoom into 125-day windows shown by rectangles in the upper panel. The shortest period of the gray line is P_1 of the binary.

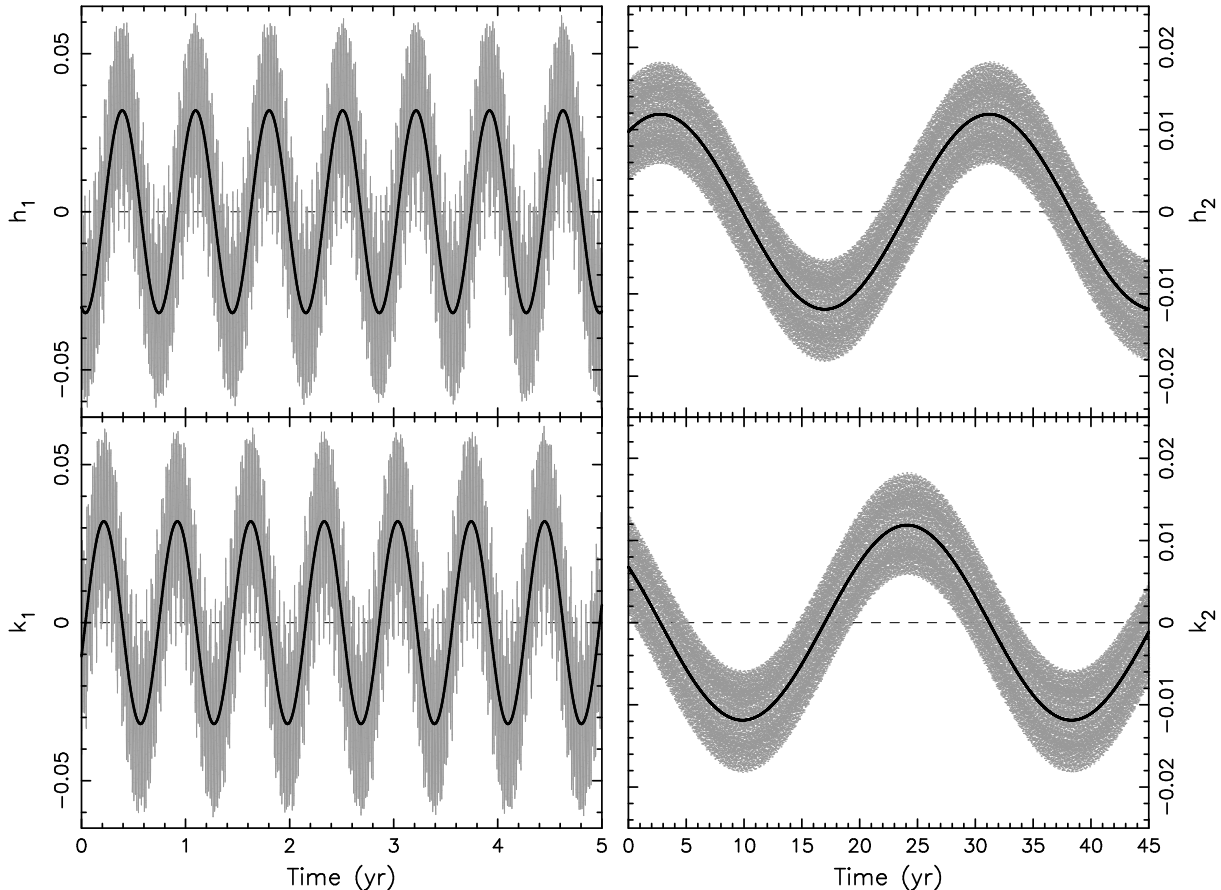


Figure 16: A comparison of the osculating (gray dots) and mean (black line) values of the nonsingular eccentricity elements (k, h) for the inner (left panels) and outer (right panels) orbits. We assumed an equal-mass binary with $m_0 = m_1 = 1 M_\odot$ and third star with $m_2 = 4 M_\odot$. Initial osculating values of the inner and outer periods were $P_1 = 4$ days and $P_2 = 35$ days, inner and outer eccentricities $e_1 = 0.01$ and $e_2 = 0.01$, and inclination $i_1 = i_2 = 85^\circ$. For definiteness, we also set initial osculating values of the argument of pericenter $\omega_1 = 0^\circ$ and $\omega_2 = 45^\circ$, and osculating mean anomalies $\ell_1 = 0^\circ$ and $\ell_2 = 90^\circ$. The mean elements we determined by online digital filtering procedure described in the text. Time at the abscissa has been shifted by 2.5 yr since the initial epoch of the simulation. This is because the architecture of the digital filter provides the first mean elements value at 2.4334 yr after the origin.

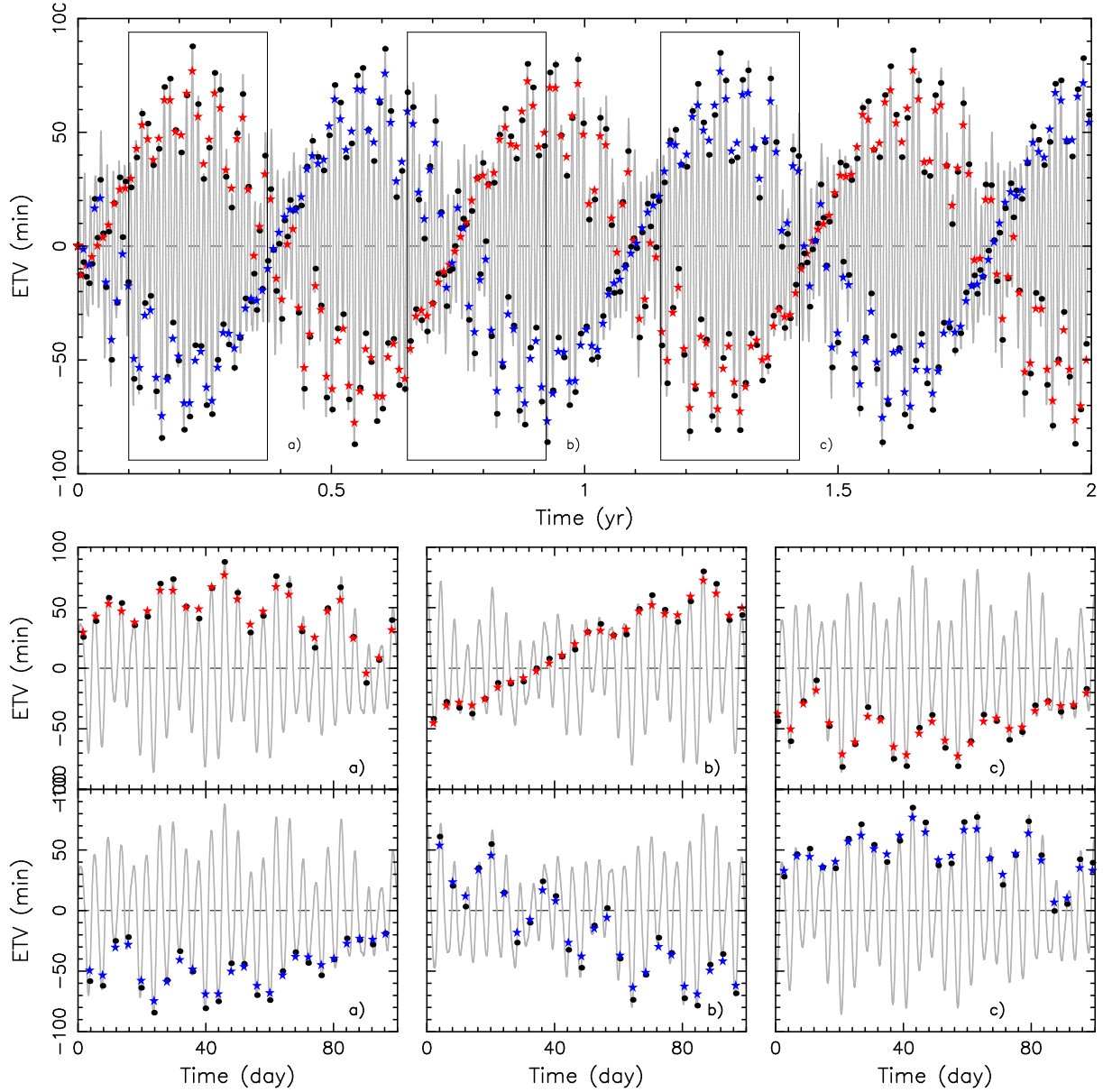


Figure 17: ETV series for a coplanar triple system with masses $m_0 = m_1 = 1 M_\odot$ and $m_2 = 4 M_\odot$, inner and outer periods $P_1 = 4$ days and $P_2 = 35$ days, inner and outer eccentricities $e_1 = 0.15$ and $e_2 = 0.05$, and inclination $i_1 = i_2 = 85^\circ$. The gray line is the periodic part of the target function and black circles are epochs of the primary and secondary eclipses, both determined numerically. The red and blue stars are ETVs of primary and secondary eclipses from our formulae. The upper panel shows a continuous signal over 2 yr, and the bottom panels zoom into 100-day windows shown by rectangles in the upper panel. The shortest period of the gray line is P_1 of the binary.

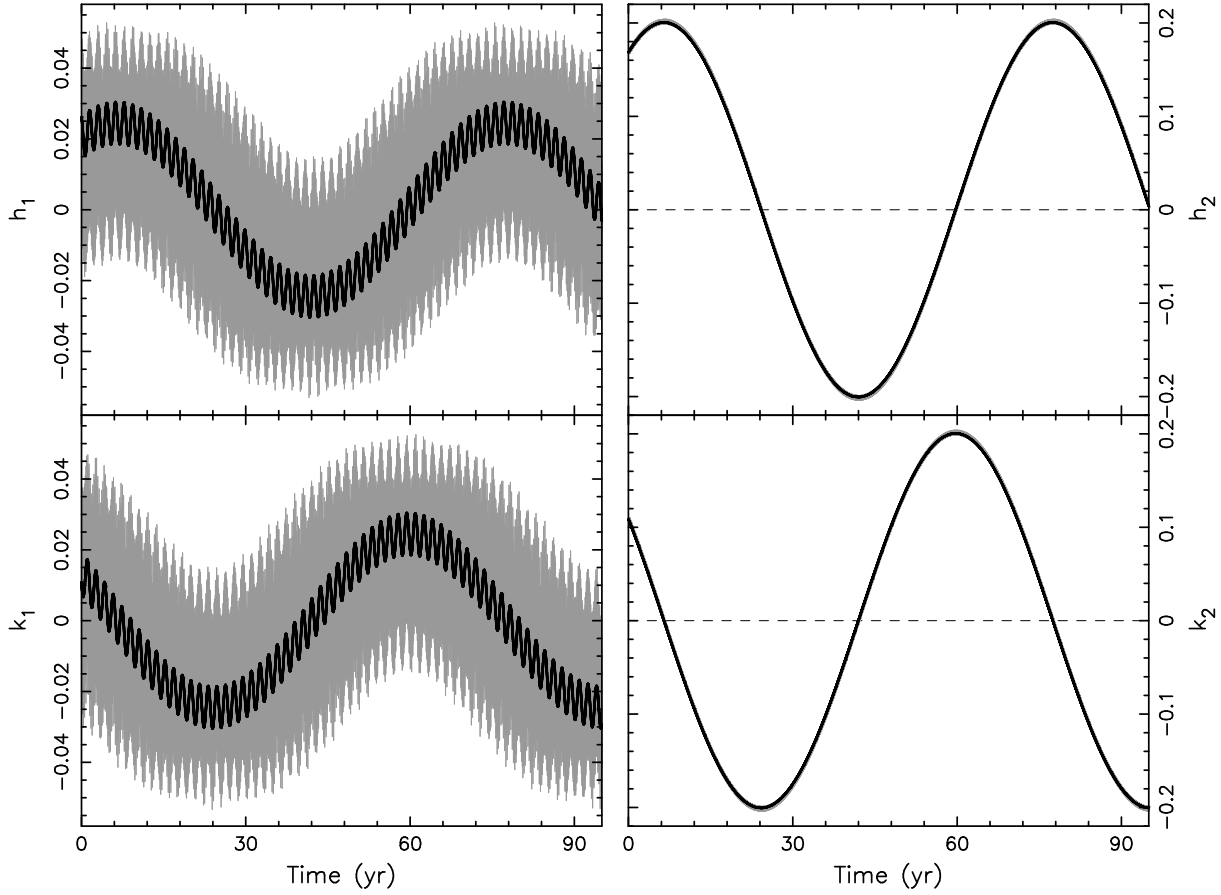


Figure 18: A comparison of the osculating (gray dots) and mean (black line) values of the nonsingular eccentricity elements (k, h) for the inner (left panels) and outer (right panels) orbits. We assumed a nonequal-mass binary with $m_0 = 3.6 M_\odot$, $m_1 = 0.4 M_\odot$ and third star with $m_2 = 4 M_\odot$. Initial osculating values of the inner and outer periods were $P_1 = 4$ days and $P_2 = 42$ days, inner and outer eccentricities $e_1 = 0.03$ and $e_2 = 0.2$, and inclination $i_1 = i_2 = 80^\circ$. For definiteness, we also set initial osculating values of the argument of pericenter $\omega_1 = 0^\circ$ and $\omega_2 = 45^\circ$, and osculating mean anomalies $\ell_1 = 0^\circ$ and $\ell_2 = 90^\circ$. The mean elements we determined by online digital filtering procedure described in the text. Time at the abscissa has been shifted by 2.5 yr since the initial epoch of the simulation. This is because the architecture of the digital filter provides the first mean elements value at 2.4334 yr after the origin.

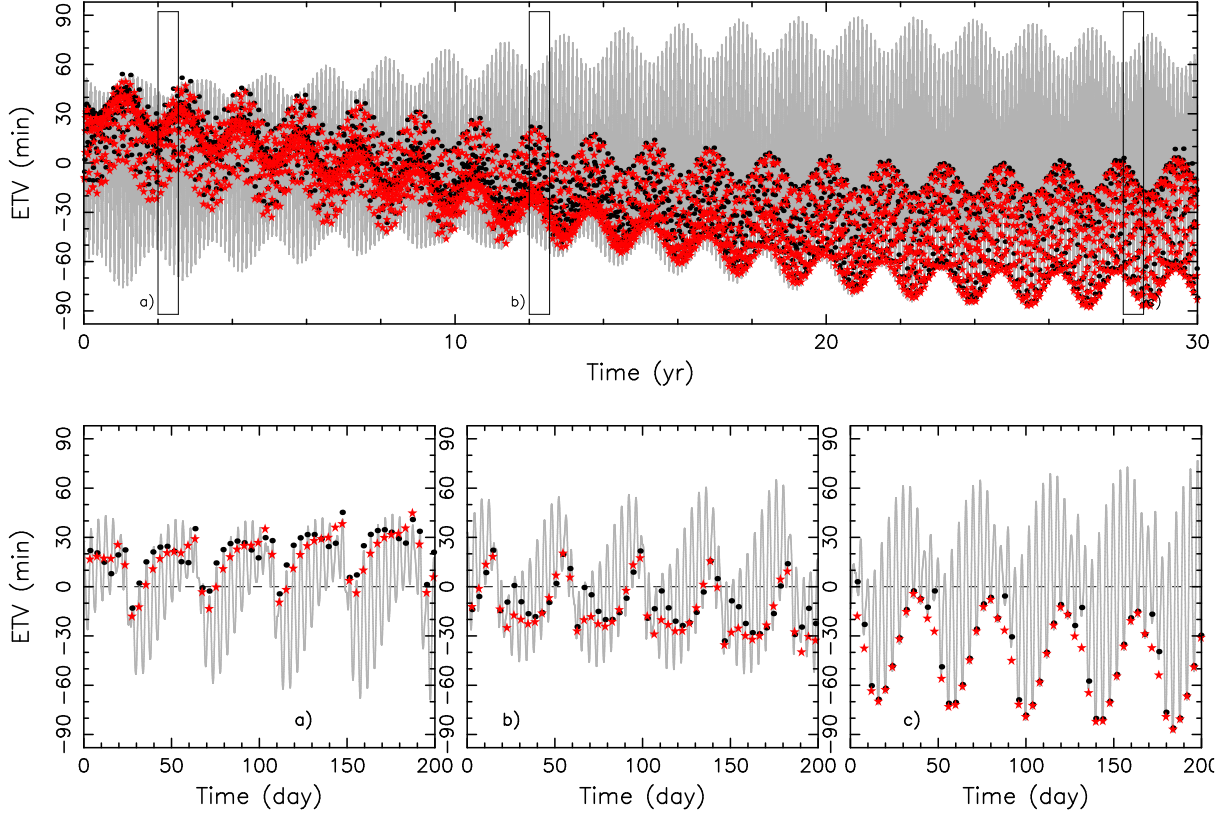


Figure 19: ETV series for a coplanar triple system with masses $m_0 = 3.6 M_\odot$, $m_1 = 0.4 M_\odot$ and $m_2 = 4 M_\odot$, inner and outer periods $P_1 = 4$ days and $P_2 = 42$ days, inner and outer eccentricities $e_1 = 0.03$ and $e_2 = 0.2$, and inclination $i_1 = i_2 = 80^\circ$. The gray line is the periodic part of the target function and black circles are epochs of the primary eclipses, both determined numerically. The red stars are ETVs of primary eclipses from our formulae. The upper panel shows a continuous signal over 30 yr, and the bottom panels zoom into 200-day windows shown by rectangles in the upper panel. The shortest period of the gray line is P_1 of the binary.

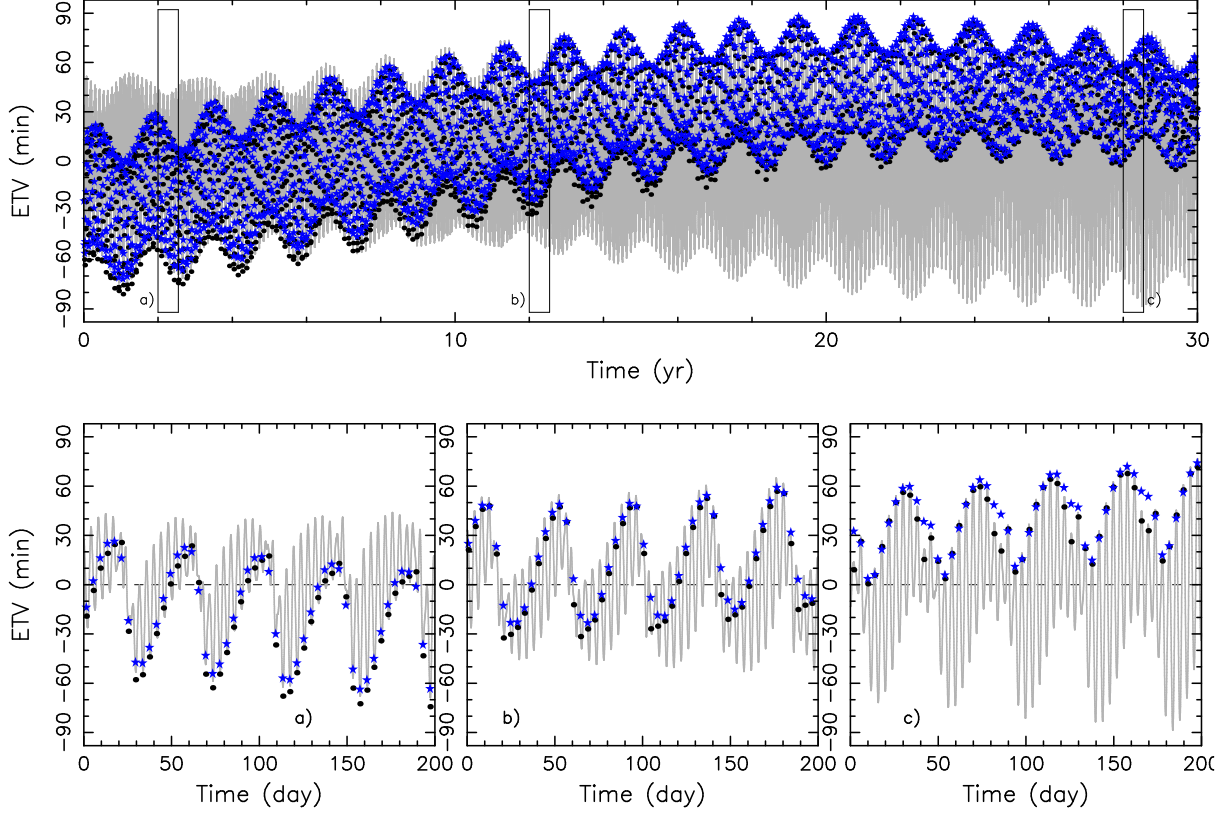


Figure 20: ETV series for a coplanar triple system with masses $m_0 = 3.6 M_\odot$, $m_1 = 0.4 M_\odot$ and $m_2 = 4 M_\odot$, inner and outer periods $P_1 = 4$ days and $P_2 = 42$ days, inner and outer eccentricities $e_1 = 0.03$ and $e_2 = 0.2$, and inclination $i_1 = i_2 = 80^\circ$. The gray line is the periodic part of the target function and black circles are epochs of the secondary eclipses, both determined numerically. The red stars are ETVs of secondary eclipses from our formulae. The upper panel shows a continuous signal over 30 yr, and the bottom panels zoom into 200-day windows shown by rectangles in the upper panel. The shortest period of the gray line is P_1 of the binary.

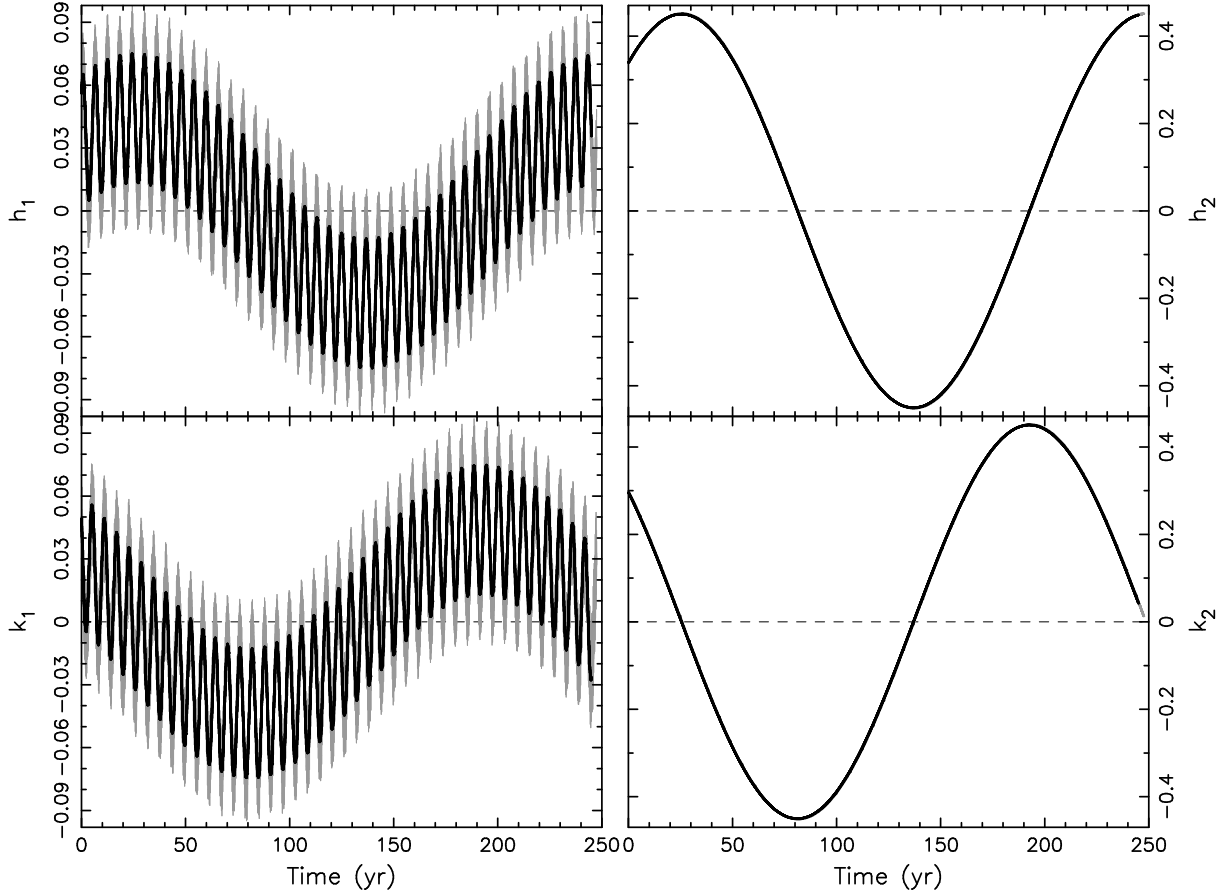


Figure 21: A comparison of the osculating (gray dots) and mean (black line) values of the nonsingular eccentricity elements (k, h) for the inner (left panels) and outer (right panels) orbits. We assumed a nonequal-mass binary with $m_0 = 3.6 M_\odot$, $m_1 = 0.4 M_\odot$ and third star with $m_2 = 4 M_\odot$. Initial osculating values of the inner and outer periods were $P_1 = 4$ days and $P_2 = 42$ days, inner and outer eccentricities $e_1 = 0.03$ and $e_2 = 0.45$, and inclination $i_1 = i_2 = 80^\circ$. For definiteness, we also set initial osculating values of the argument of pericenter $\omega_1 = 0^\circ$ and $\omega_2 = 45^\circ$, and osculating mean anomalies $\ell_1 = 0^\circ$ and $\ell_2 = 90^\circ$. The mean elements we determined by online digital filtering procedure described in the text. Time at the abscissa has been shifted by 2.5 yr since the initial epoch of the simulation. This is because the architecture of the digital filter provides the first mean elements value at 2.4334 yr after the origin.

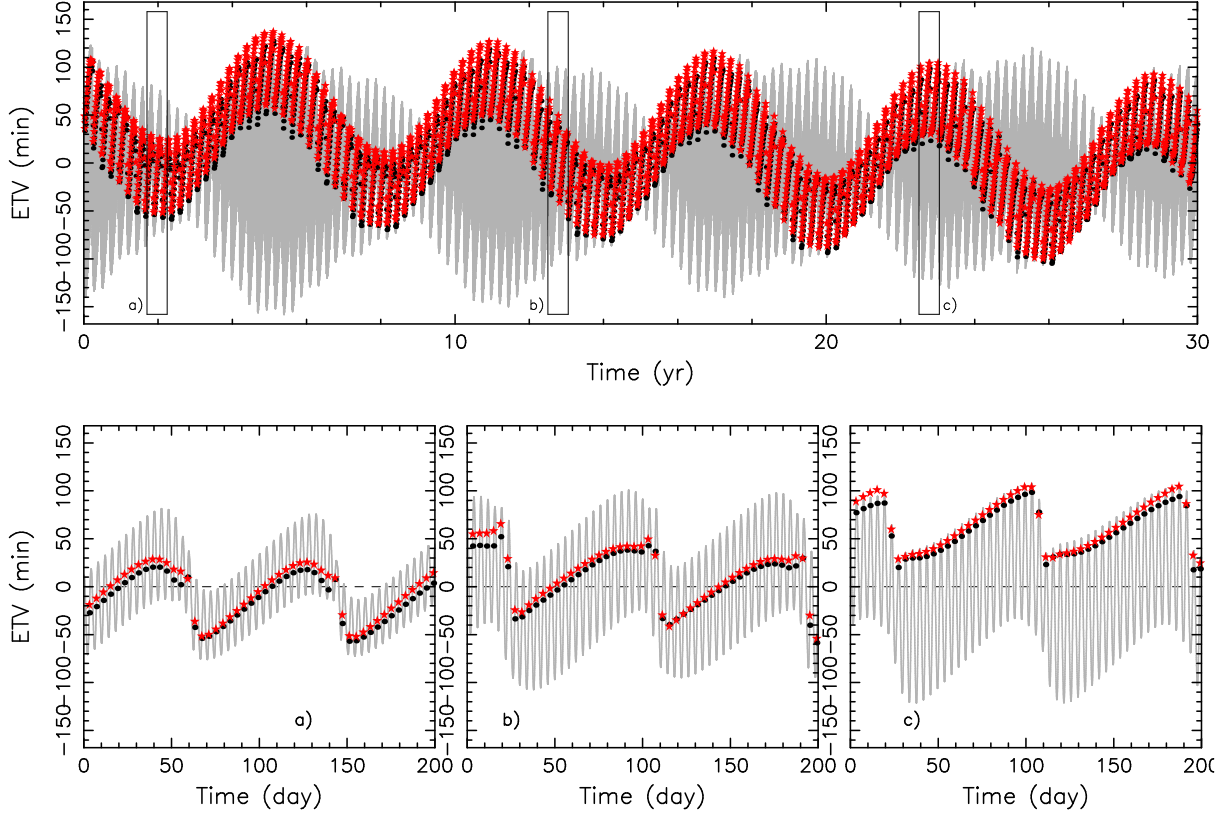


Figure 22: ETV series for a coplanar triple system with masses $m_0 = 3.6 M_\odot$, $m_1 = 0.4 M_\odot$ and $m_2 = 4 M_\odot$, inner and outer periods $P_1 = 4$ days and $P_2 = 42$ days, inner and outer eccentricities $e_1 = 0.03$ and $e_2 = 0.45$, and inclination $i_1 = i_2 = 80^\circ$. The gray line is the periodic part of the target function and black circles are epochs of the primary eclipses, both determined numerically. The red stars are ETVs of primary eclipses from our formulae. The upper panel shows a continuous signal over 30 yr, and the bottom panels zoom into 200-day windows shown by rectangles in the upper panel. The shortest period of the gray line is P_1 of the binary.

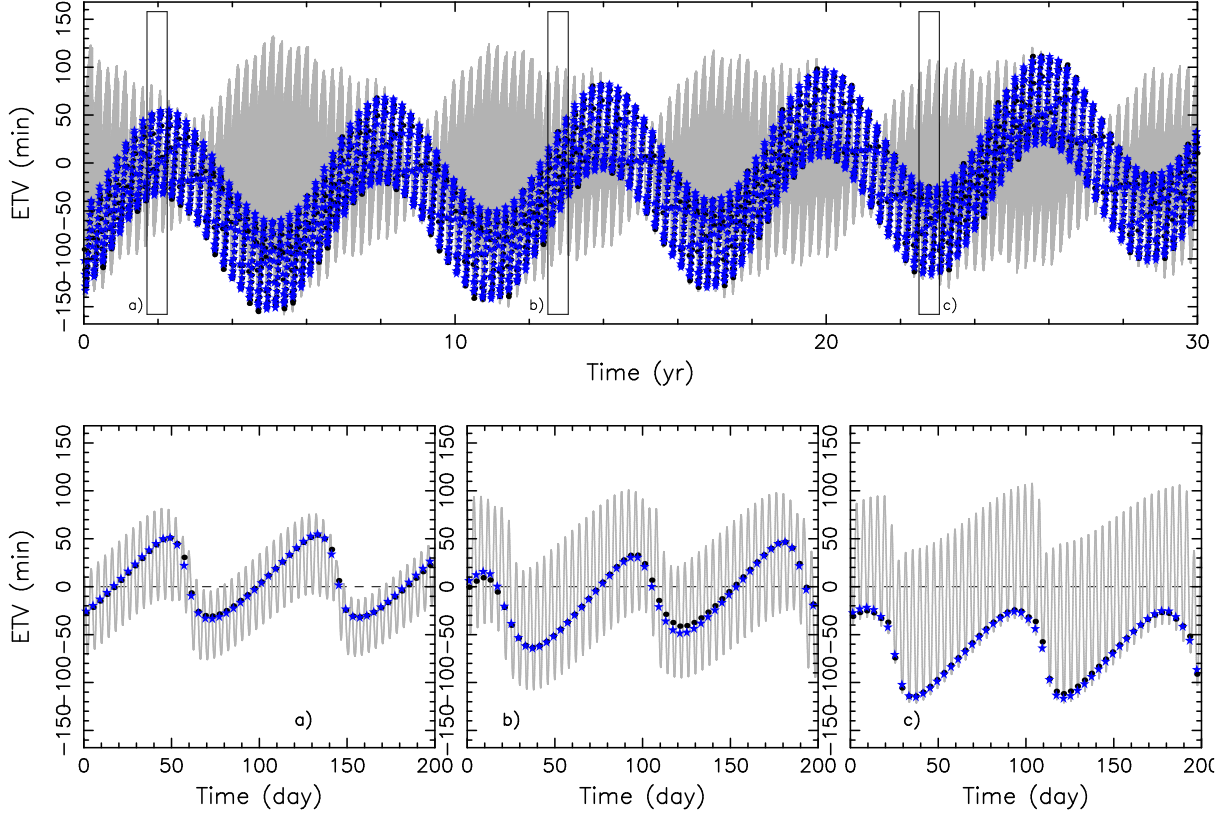


Figure 23: ETV series for a coplanar triple system with masses $m_0 = 3.6 M_\odot$, $m_1 = 0.4 M_\odot$ and $m_2 = 4 M_\odot$, inner and outer periods $P_1 = 4$ days and $P_2 = 42$ days, inner and outer eccentricities $e_1 = 0.03$ and $e_2 = 0.45$, and inclination $i_1 = i_2 = 80^\circ$. The gray line is the periodic part of the target function and black circles are epochs of the secondary eclipses, both determined numerically. The red stars are ETVs of secondary eclipses from our formulae. The upper panel shows a continuous signal over 30 yr, and the bottom panels zoom into 200-day windows shown by rectangles in the upper panel. The shortest period of the gray line is P_1 of the binary.

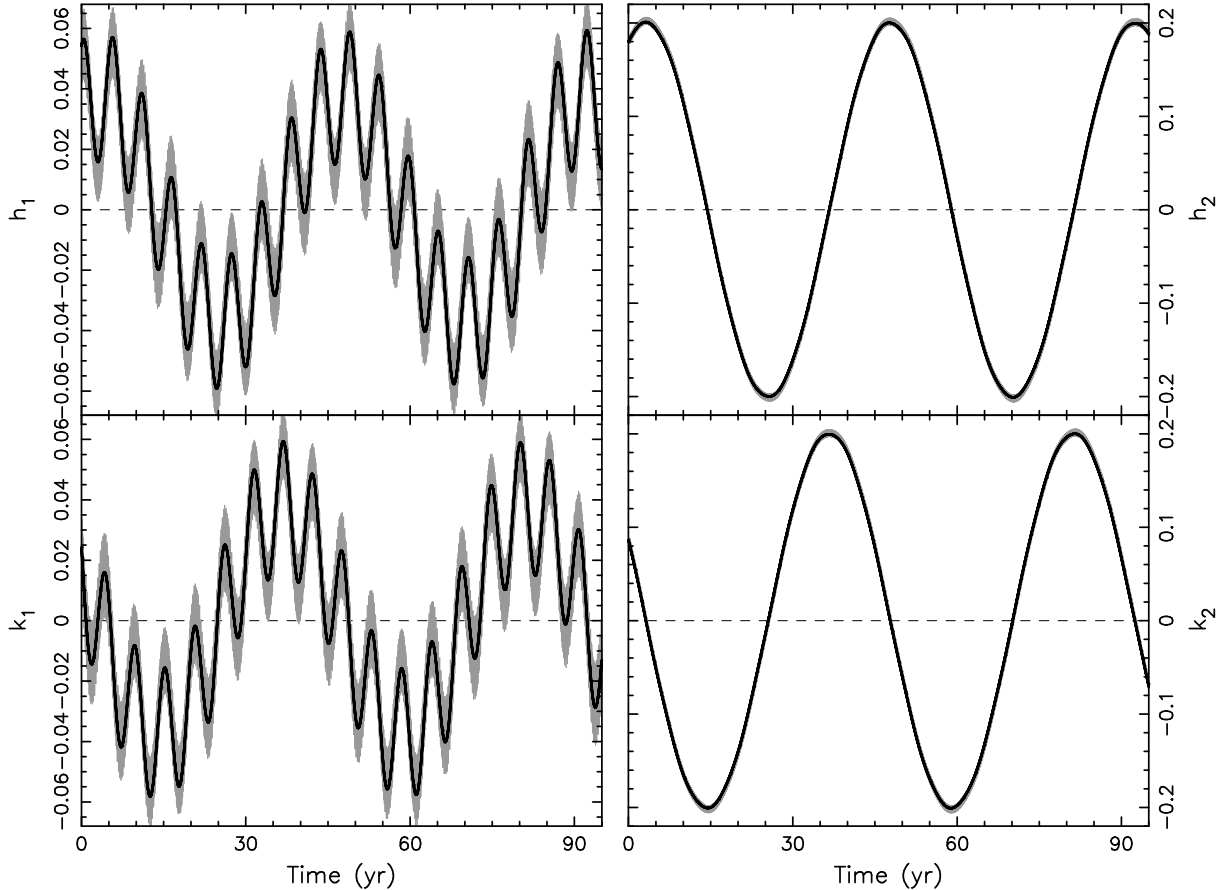


Figure 24: A comparison of the osculating (gray dots) and mean (black line) values of the nonsingular eccentricity elements (k, h) for the inner (left panels) and outer (right panels) orbits. We assumed a nonequal-mass binary with $m_0 = 3.6 M_\odot$ $m_1 = 0.4 M_\odot$ and third star with $m_2 = 1 M_\odot$. Initial osculating values of the inner and outer periods were $P_1 = 4$ days and $P_2 = 42$ days, inner and outer eccentricities $e_1 = 0.03$ and $e_2 = 0.2$, and inclination $i_1 = i_2 = 80^\circ$. For definiteness, we also set initial osculating values of the argument of pericenter $\omega_1 = 0^\circ$ and $\omega_2 = 45^\circ$, and osculating mean anomalies $\ell_1 = 0^\circ$ and $\ell_2 = 90^\circ$. The mean elements we determined by online digital filtering procedure described in the text. Time at the abscissa has been shifted by 2.5 yr since the initial epoch of the simulation. This is because the architecture of the digital filter provides the first mean elements value at 2.4334 yr after the origin.

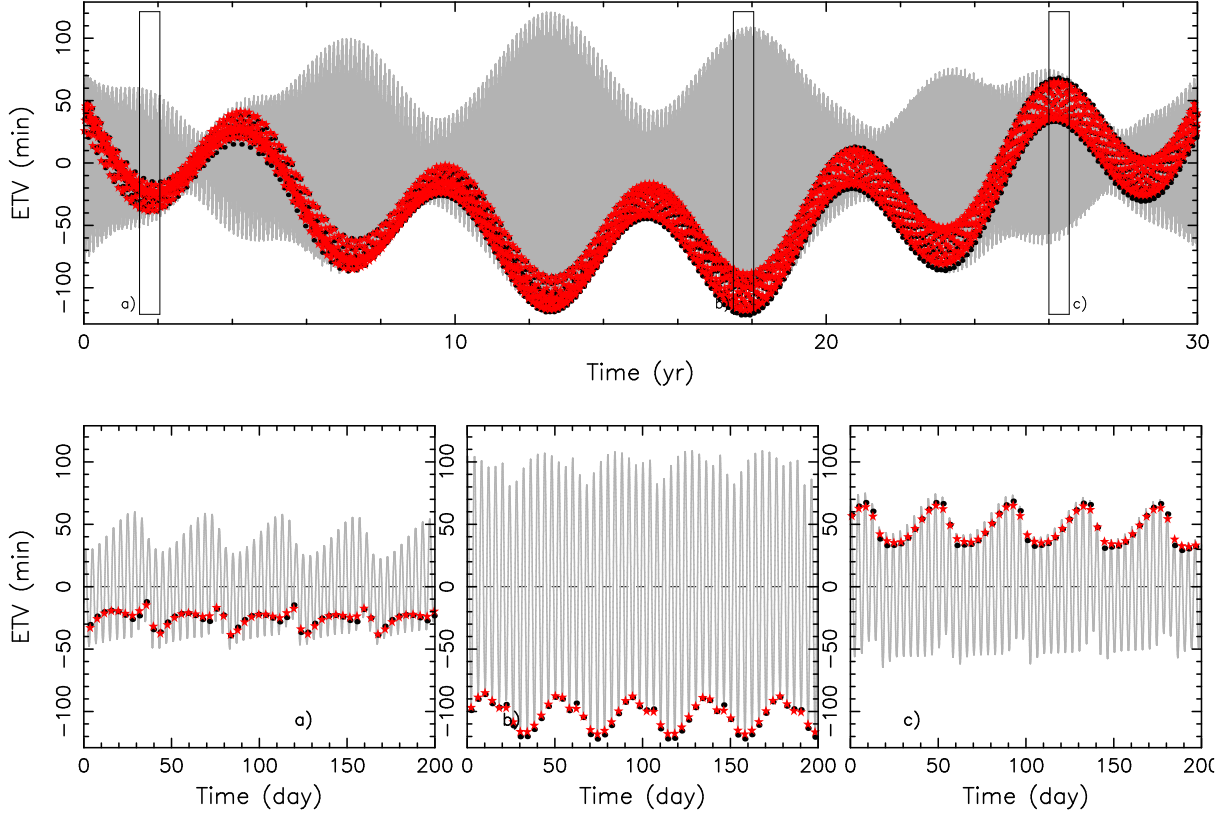


Figure 25: ETV series for a coplanar triple system with masses $m_0 = 3.6 M_\odot$, $m_1 = 0.4 M_\odot$ and $m_2 = 1 M_\odot$, inner and outer periods $P_1 = 4$ days and $P_2 = 42$ days, inner and outer eccentricities $e_1 = 0.03$ and $e_2 = 0.2$, and inclination $i_1 = i_2 = 80^\circ$. The gray line is the periodic part of the target function and black circles are epochs of the primary eclipses, both determined numerically. The red stars are ETVs of primary eclipses from our formulae. The upper panel shows a continuous signal over 30 yr, and the bottom panels zoom into 200-day windows shown by rectangles in the upper panel. The shortest period of the gray line is P_1 of the binary.

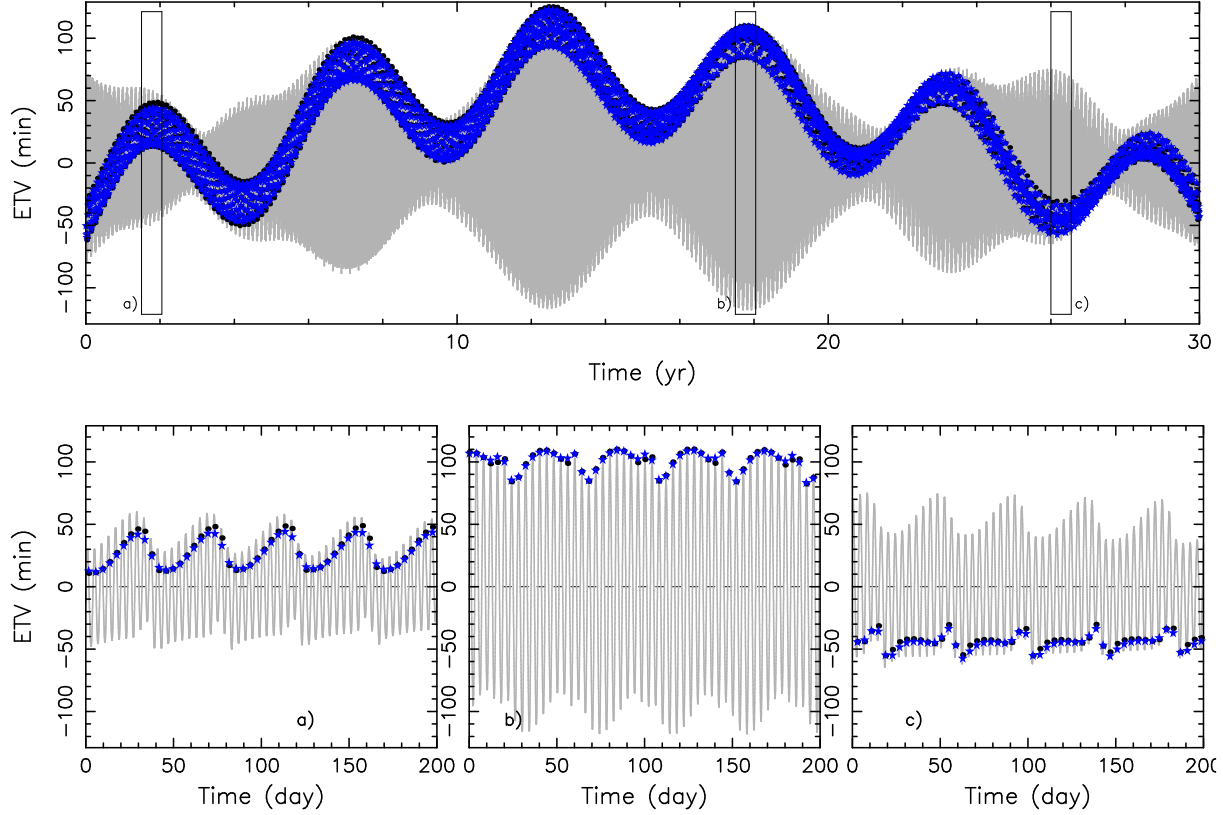


Figure 26: ETV series for a coplanar triple system with masses $m_0 = 3.6 M_\odot$, $m_1 = 0.4 M_\odot$ and $m_2 = 1 M_\odot$, inner and outer periods $P_1 = 4$ days and $P_2 = 42$ days, inner and outer eccentricities $e_1 = 0.03$ and $e_2 = 0.2$, and inclination $i_1 = i_2 = 80^\circ$. The gray line is the periodic part of the target function and black circles are epochs of the secondary eclipses, both determined numerically. The red stars are ETVs of secondary eclipses from our formulae. The upper panel shows a continuous signal over 30 yr, and the bottom panels zoom into 200-day windows shown by rectangles in the upper panel. The shortest period of the gray line is P_1 of the binary.

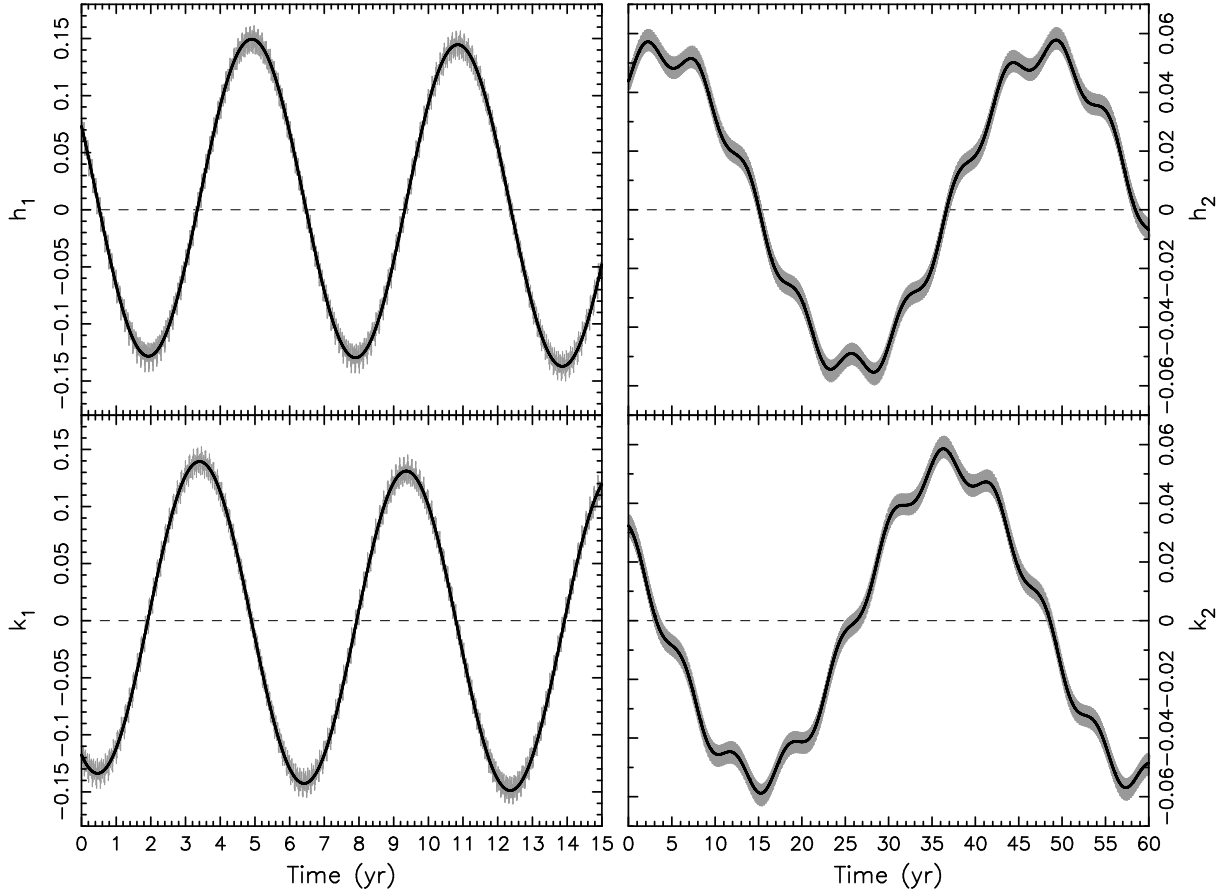


Figure 27: A comparison of the osculating (gray dots) and mean (black line) values of the nonsingular eccentricity elements (k, h) for the inner (left panels) and outer (right panels) orbits. We assumed a nonequal-mass binary with $m_0 = 3.6 M_\odot$, $m_1 = 0.4 M_\odot$ and third star with $m_2 = 1 M_\odot$. Initial osculating values of the inner and outer periods were $P_1 = 4$ days and $P_2 = 42$ days, inner and outer eccentricities $e_1 = 0.15$ and $e_2 = 0.05$, and inclination $i_1 = i_2 = 80^\circ$. For definiteness, we also set initial osculating values of the argument of pericenter $\omega_1 = 0^\circ$ and $\omega_2 = 45^\circ$, and osculating mean anomalies $\ell_1 = 0^\circ$ and $\ell_2 = 90^\circ$. The mean elements we determined by online digital filtering procedure described in the text. Time at the abscissa has been shifted by 2.5 yr since the initial epoch of the simulation. This is because the architecture of the digital filter provides the first mean elements value at 2.4334 yr after the origin.

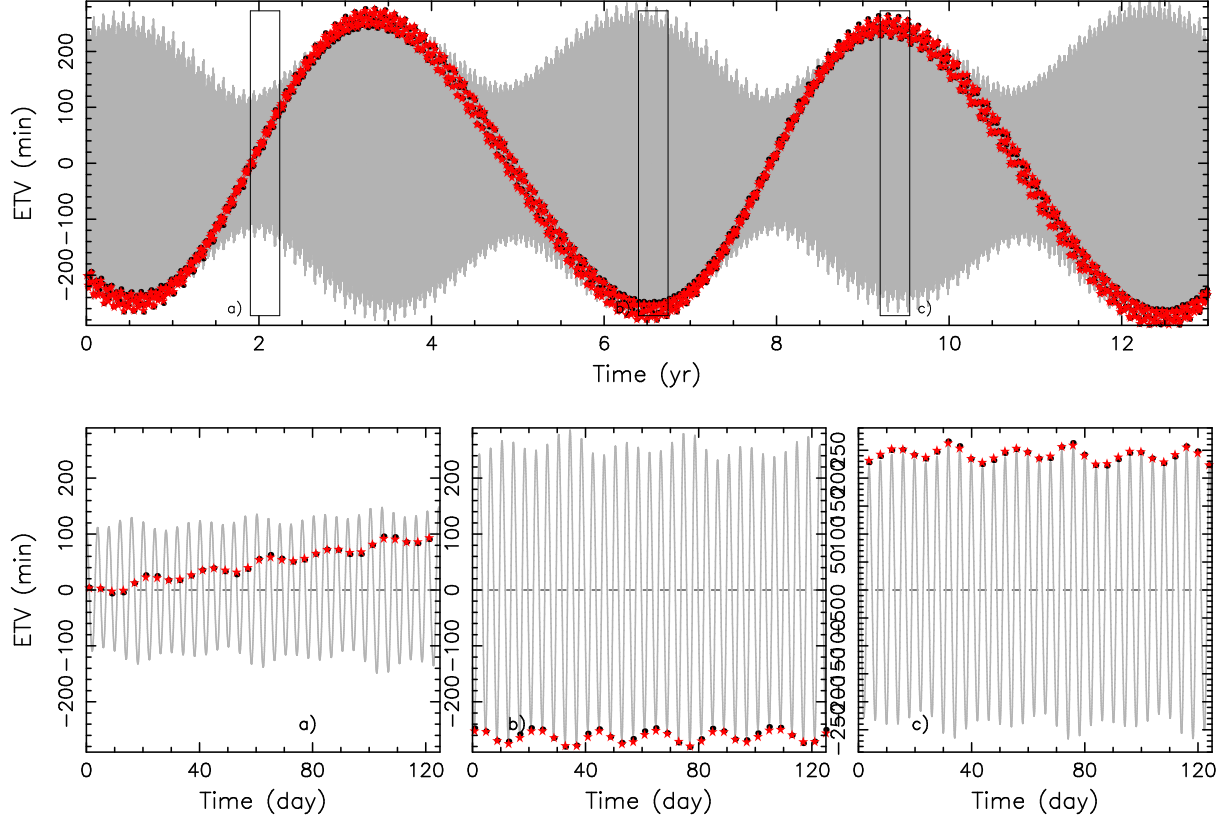


Figure 28: ETV series for a coplanar triple system with masses $m_0 = 3.6 M_\odot$, $m_1 = 0.4 M_\odot$ and $m_2 = 1 M_\odot$, inner and outer periods $P_1 = 4$ days and $P_2 = 42$ days, inner and outer eccentricities $e_1 = 0.15$ and $e_2 = 0.05$, and inclination $i_1 = i_2 = 80^\circ$. The gray line is the periodic part of the target function and black circles are epochs of the primary eclipses, both determined numerically. The red stars are ETVs of primary eclipses from our formulae. The upper panel shows a continuous signal over 13 yr, and the bottom panels zoom into 125-day windows shown by rectangles in the upper panel. The shortest period of the gray line is P_1 of the binary.

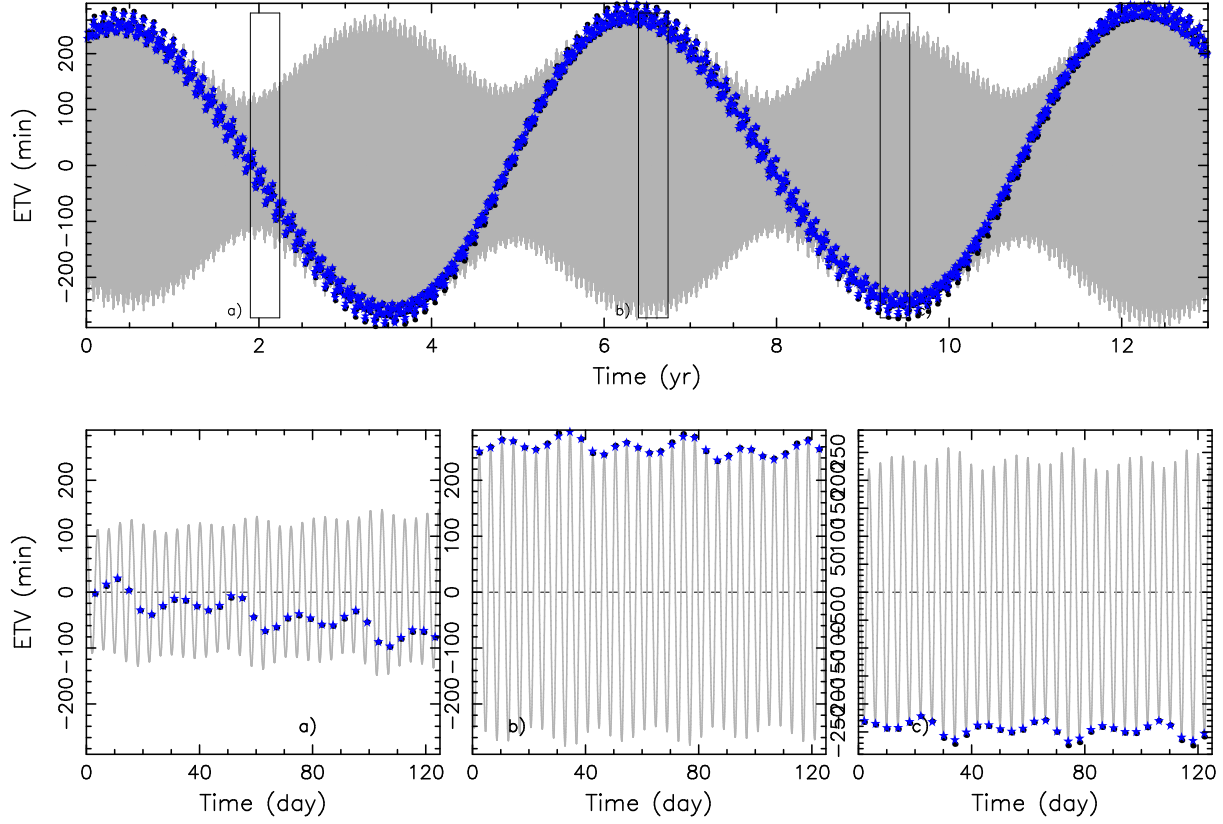


Figure 29: ETV series for a coplanar triple system with masses $m_0 = 3.6 M_\odot$, $m_1 = 0.4 M_\odot$ and $m_2 = 1 M_\odot$, inner and outer periods $P_1 = 4$ days and $P_2 = 42$ days, inner and outer eccentricities $e_1 = 0.15$ and $e_2 = 0.05$, and inclination $i_1 = i_2 = 80^\circ$. The gray line is the periodic part of the target function and black circles are epochs of the secondary eclipses, both determined numerically. The red stars are ETVs of secondary eclipses from our formulae. The upper panel shows a continuous signal over 13 yr, and the bottom panels zoom into 125-day windows shown by rectangles in the upper panel. The shortest period of the gray line is P_1 of the binary.

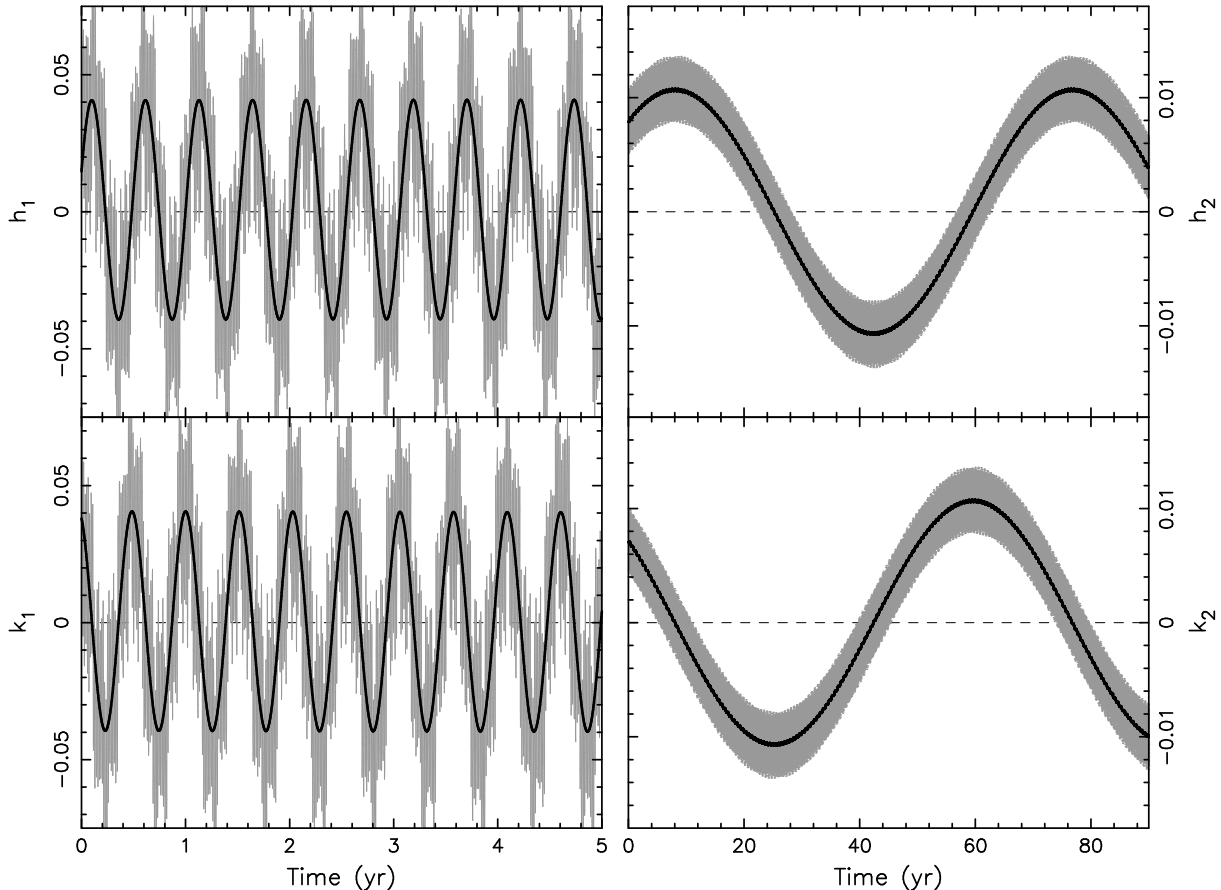


Figure 30: A comparison of the osculating (gray dots) and mean (black line) values of the nonsingular eccentricity elements (k, h) for the inner (left panels) and outer (right panels) orbits. We assumed a nonequal-mass binary with $m_0 = 0.8 M_\odot$ $m_1 = 0.2 M_\odot$ and third star with $m_2 = 4 M_\odot$. Initial osculating values of the inner and outer periods were $P_1 = 4$ days and $P_2 = 35$ days, inner and outer eccentricities $e_1 = 0.01$ and $e_2 = 0.01$, and inclination $i_1 = i_2 = 85^\circ$. For definiteness, we also set initial osculating values of the argument of pericenter $\omega_1 = 0^\circ$ and $\omega_2 = 45^\circ$, and osculating mean anomalies $\ell_1 = 0^\circ$ and $\ell_2 = 90^\circ$. The mean elements we determined by online digital filtering procedure described in the text. Time at the abscissa has been shifted by 2.5 yr since the initial epoch of the simulation. This is because the architecture of the digital filter provides the first mean elements value at 2.4334 yr after the origin.

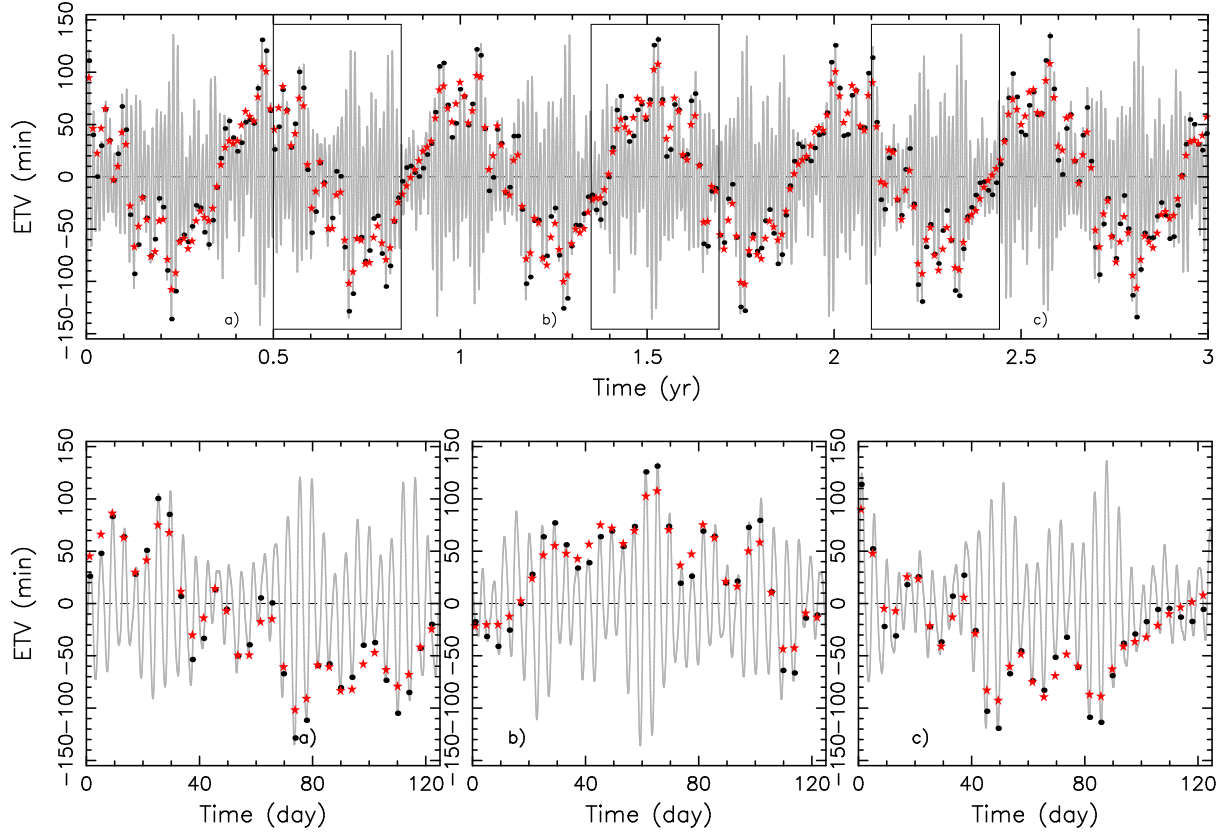


Figure 31: ETV series for a coplanar triple system with masses $m_0 = 0.8 M_\odot$, $m_1 = 0.2 M_\odot$ and $m_2 = 4 M_\odot$, inner and outer periods $P_1 = 4$ days and $P_2 = 35$ days, inner and outer eccentricities $e_1 = 0.01$ and $e_2 = 0.01$, and inclination $i_1 = i_2 = 85^\circ$. The gray line is the periodic part of the target function and black circles are epochs of the primary eclipses, both determined numerically. The red stars are ETVs of primary eclipses from our formulae. The upper panel shows a continuous signal over 3 yr, and the bottom panels zoom into 155-day windows shown by rectangles in the upper panel. The shortest period of the gray line is P_1 of the binary.

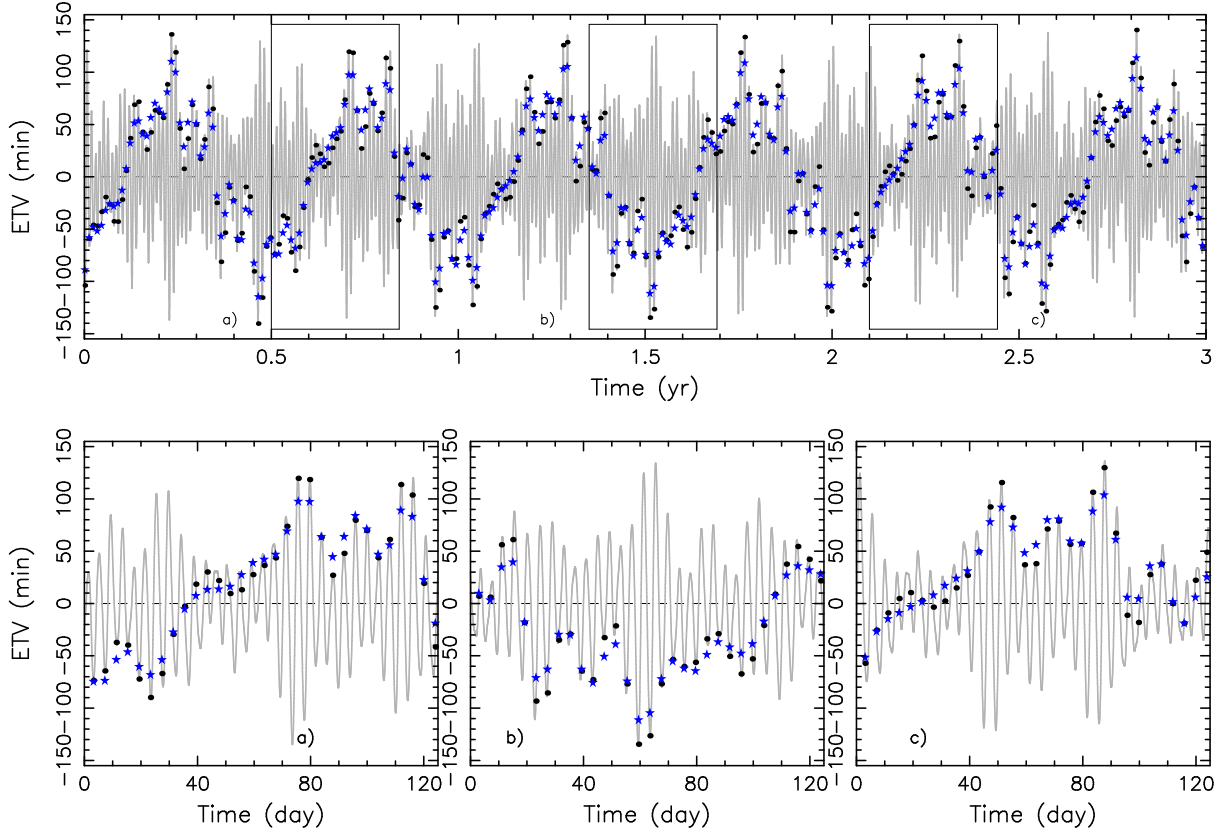


Figure 32: ETV series for a coplanar triple system with masses $m_0 = 0.8 M_\odot$, $m_1 = 0.2 M_\odot$ and $m_2 = 4 M_\odot$, inner and outer periods $P_1 = 4$ days and $P_2 = 35$ days, inner and outer eccentricities $e_1 = 0.01$ and $e_2 = 0.01$, and inclination $i_1 = i_2 = 85^\circ$. The gray line is the periodic part of the target function and black circles are epochs of the secondary eclipses, both determined numerically. The red stars are ETVs of secondary eclipses from our formulae. The upper panel shows a continuous signal over 3 yr, and the bottom panels zoom into 155-day windows shown by rectangles in the upper panel. The shortest period of the gray line is P_1 of the binary.

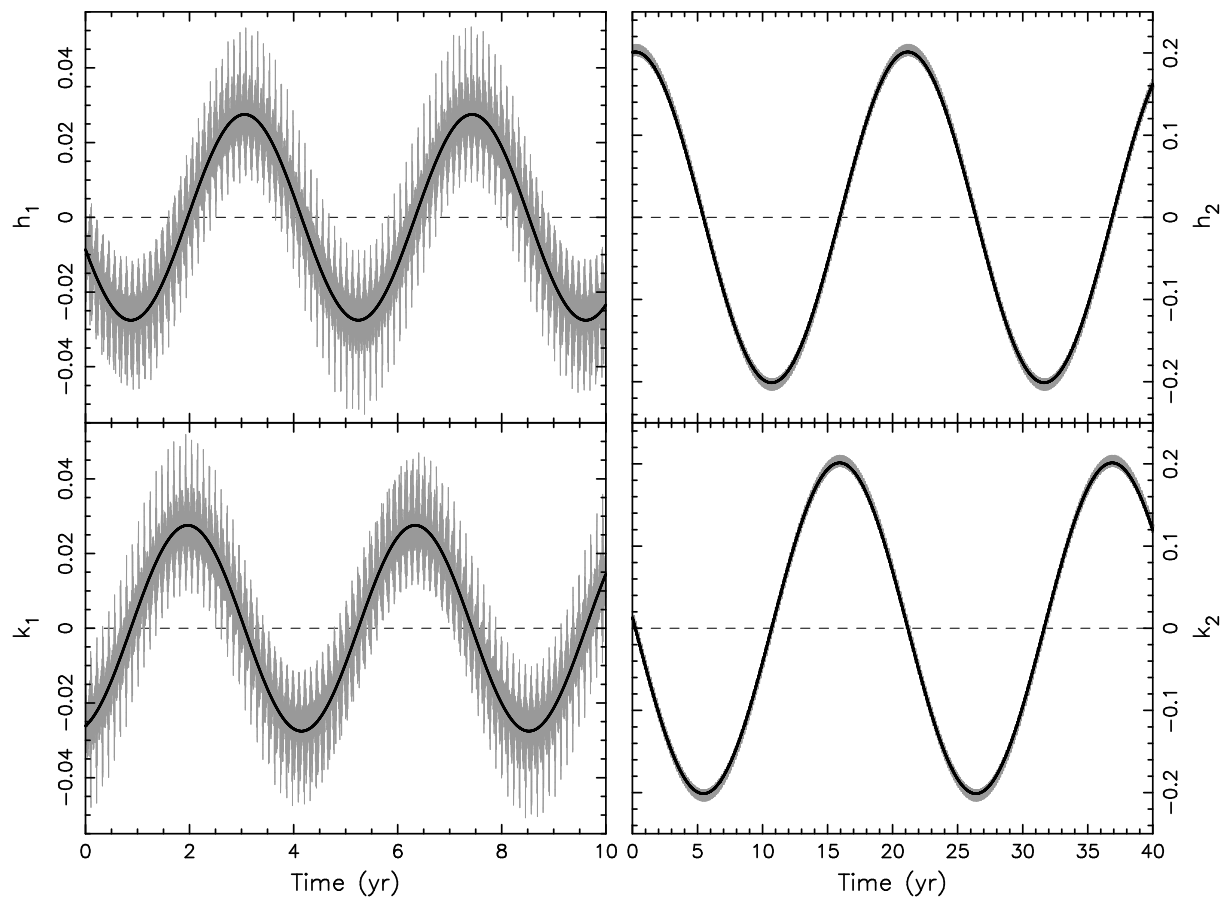


Figure 33: Osculating vs mean orbital elements for the first system (case 1) discussed in Sec. 3.1 of the main text, but now with the third star orbiting the inner binary in a retrograde sense.

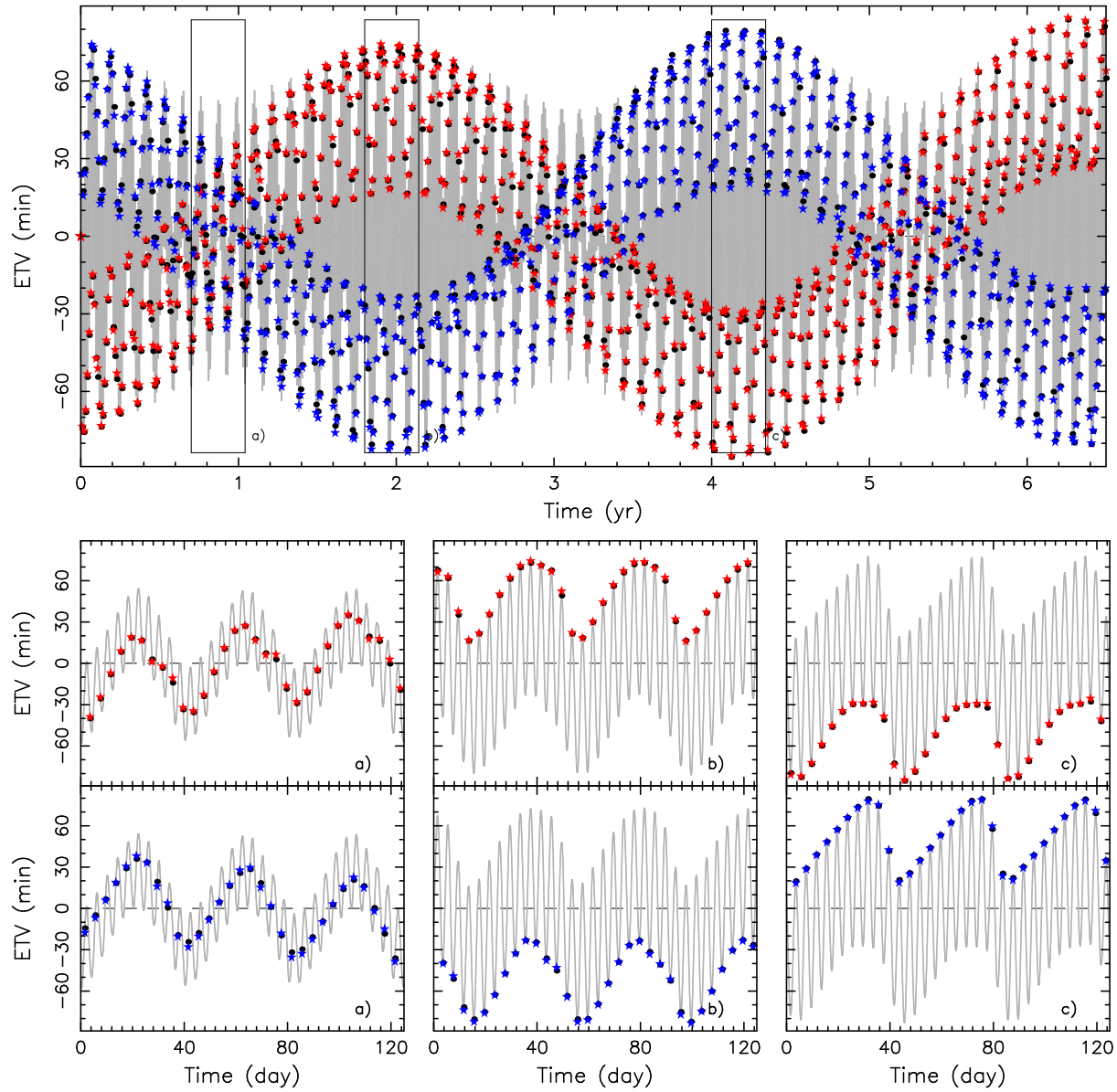


Figure 34: ETV series for the first system (case 1) discussed in Sec. 3.1 of the main text, but now with the third star orbiting the inner binary in a retrograde sense.

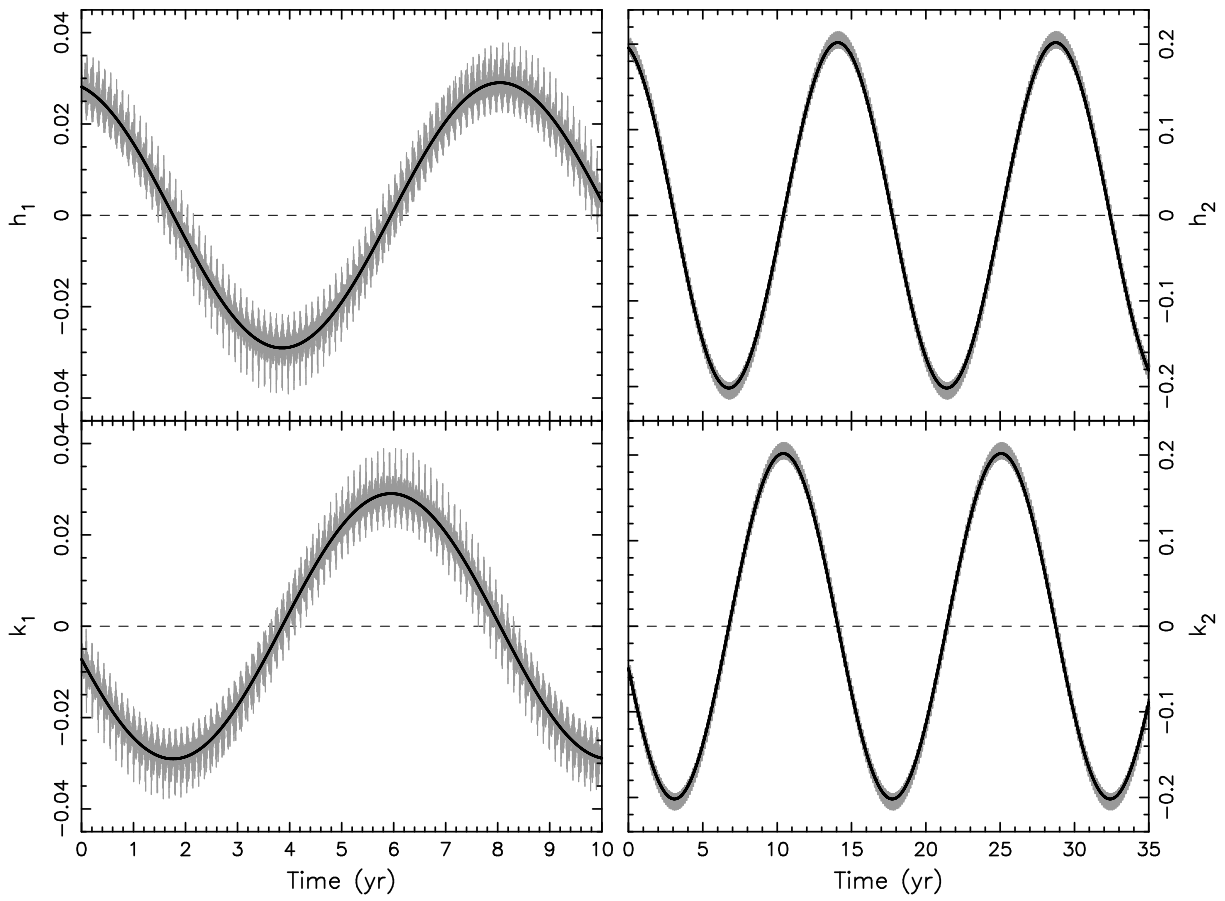


Figure 35: Osculating vs mean orbital elements for the second system (case 2) discussed in Sec. 3.1 of the main text, but now with the third star orbiting the inner binary in a retrograde sense.

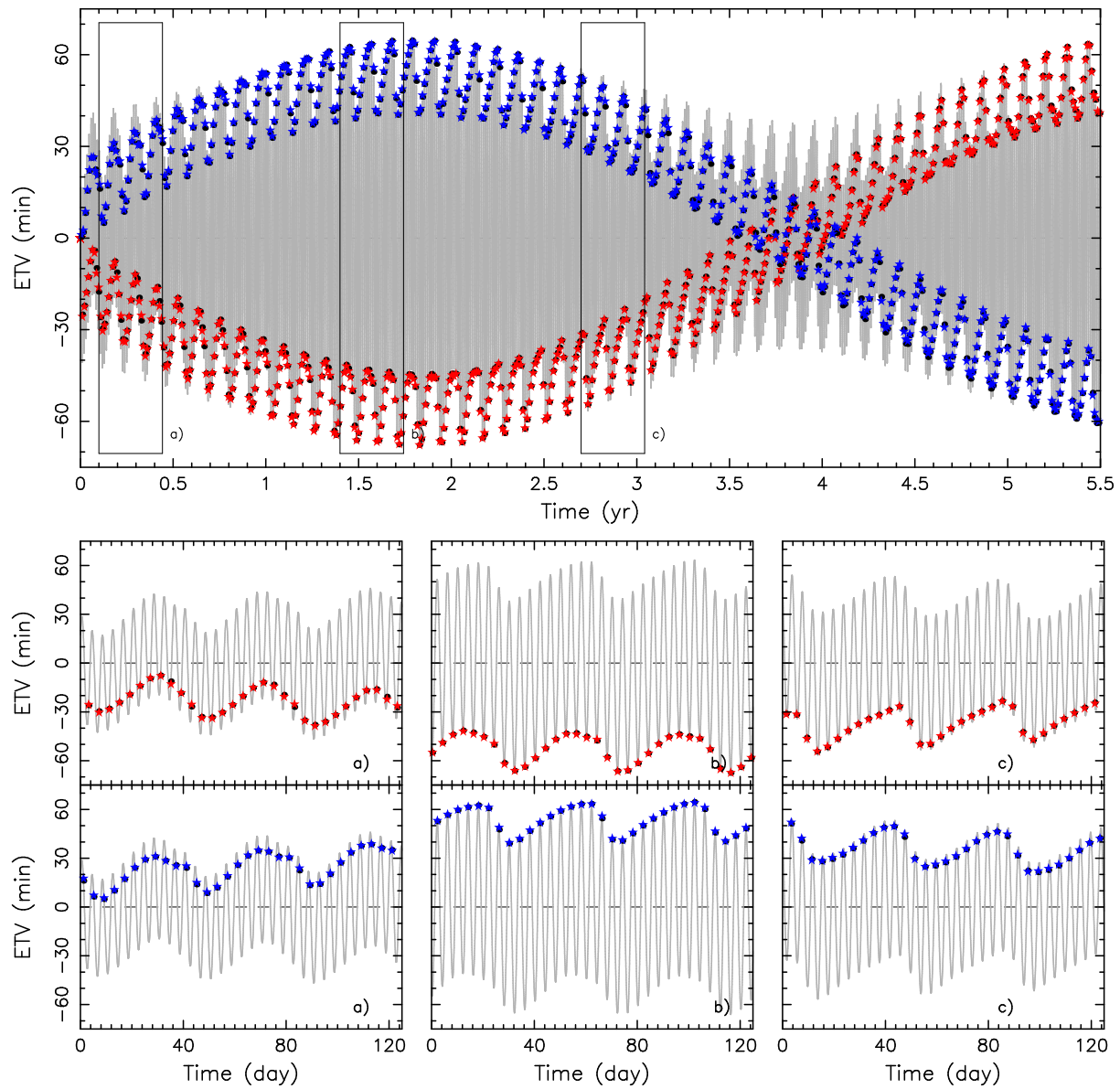


Figure 36: ETV series for the second system (case 2) discussed in Sec. 3.1 of the main text, but now with the third star orbiting the inner binary in a retrograde sense.

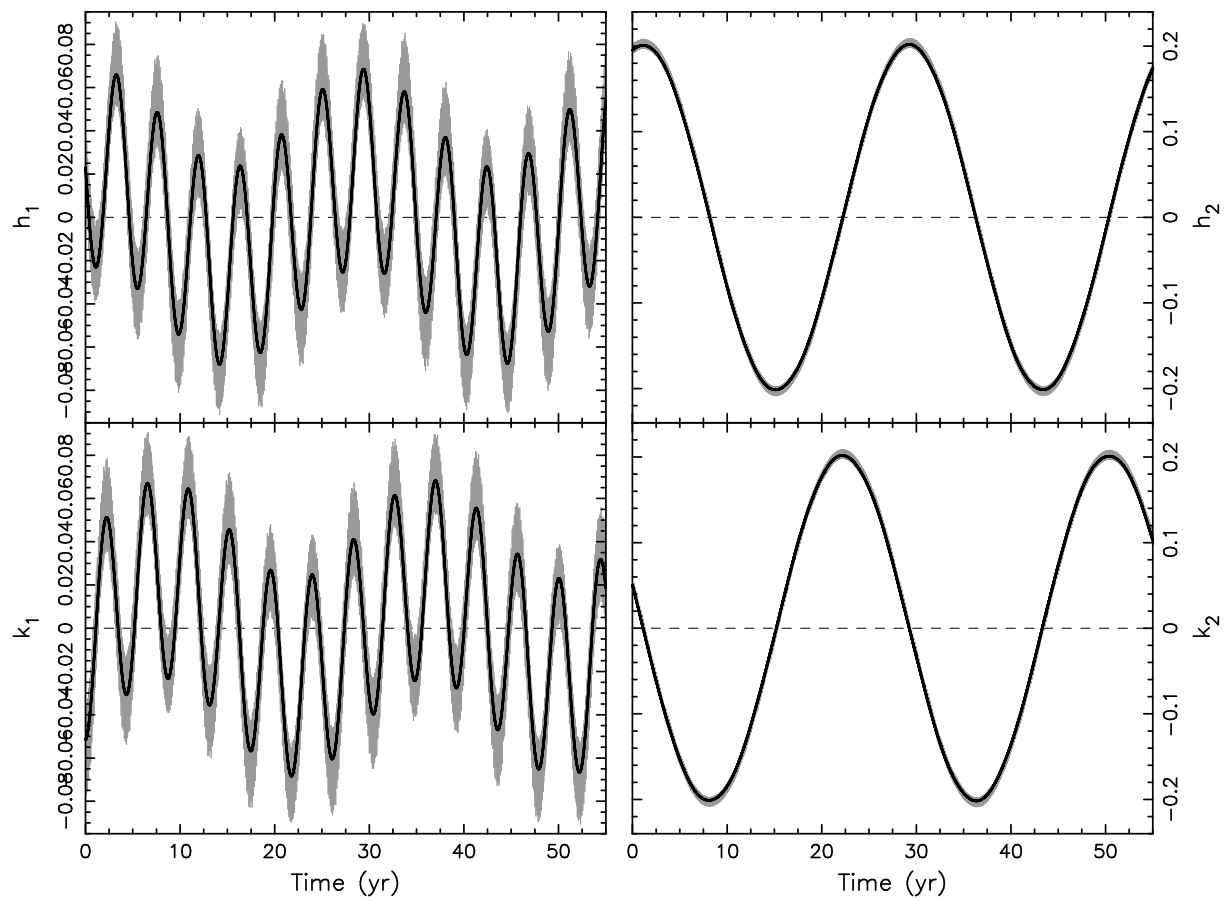


Figure 37: Osculating vs mean orbital elements for the system discussed in Sec. 3.2 of the main text, but now with the third star orbiting the inner binary in a retrograde sense.

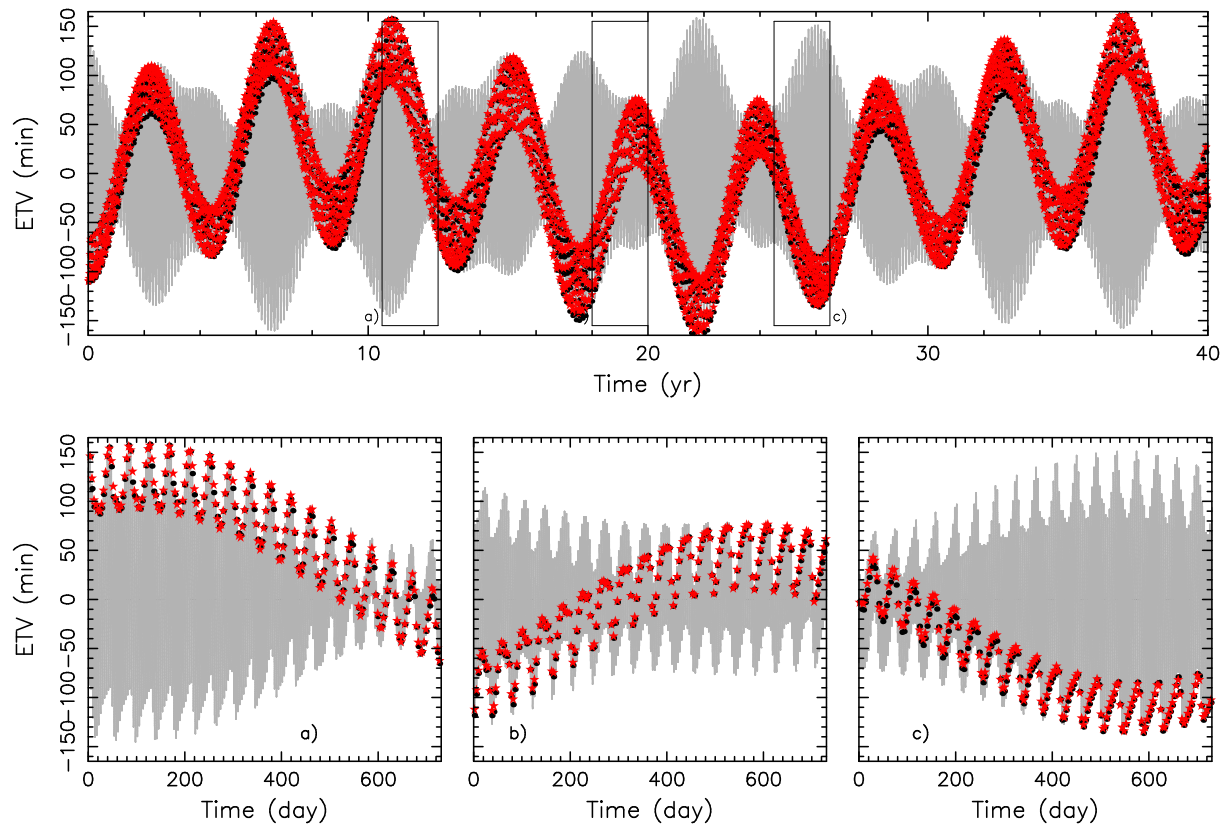


Figure 38: ETV series for primary eclipses of the system discussed in Sec. 3.2 of the main text, but now with the third star orbiting the inner binary in a retrograde sense.

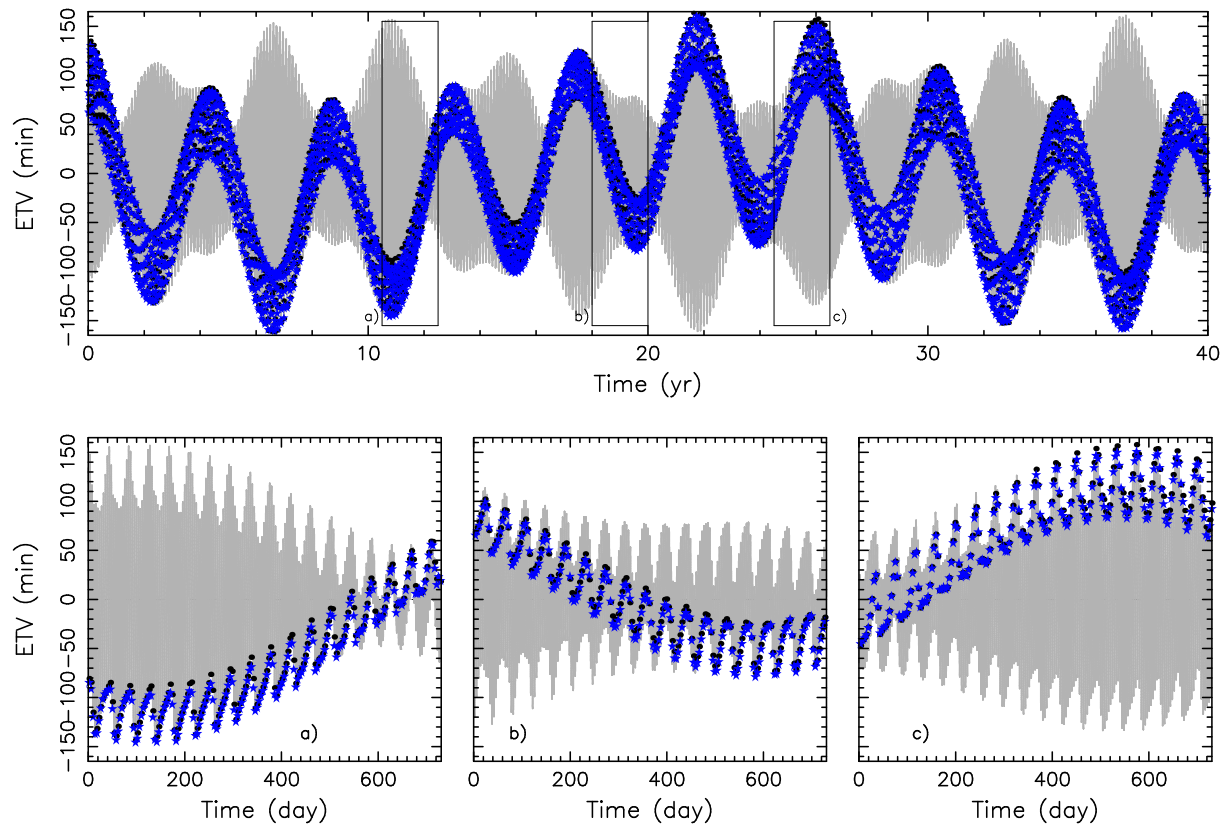


Figure 39: ETV series for secondary eclipses of the system discussed in Sec. 3.2 of the main text, but now with the third star orbiting the inner binary in a retrograde sense.