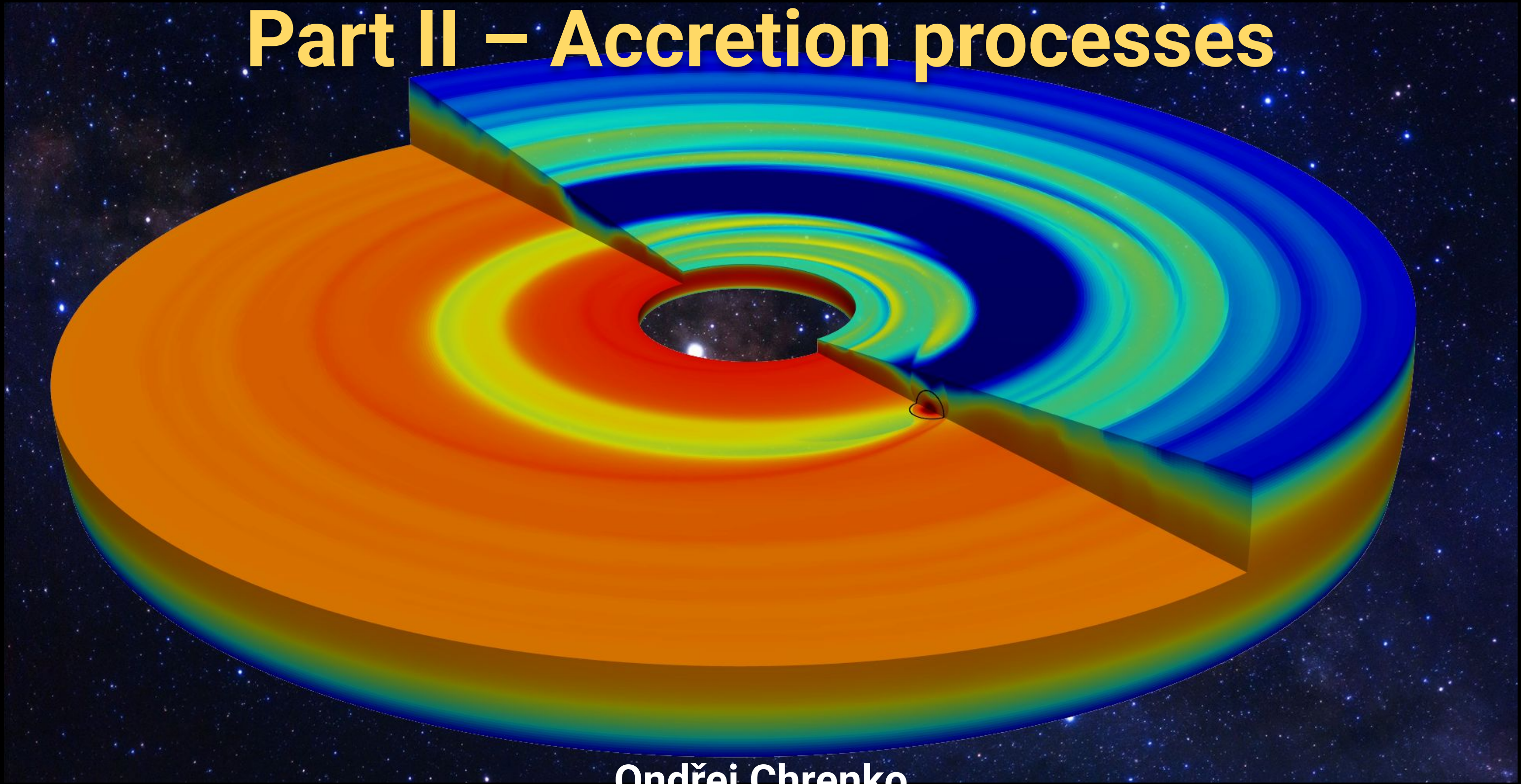
# Exoplanets II – Planet formation
# Part II – Accretion processes



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**2026**

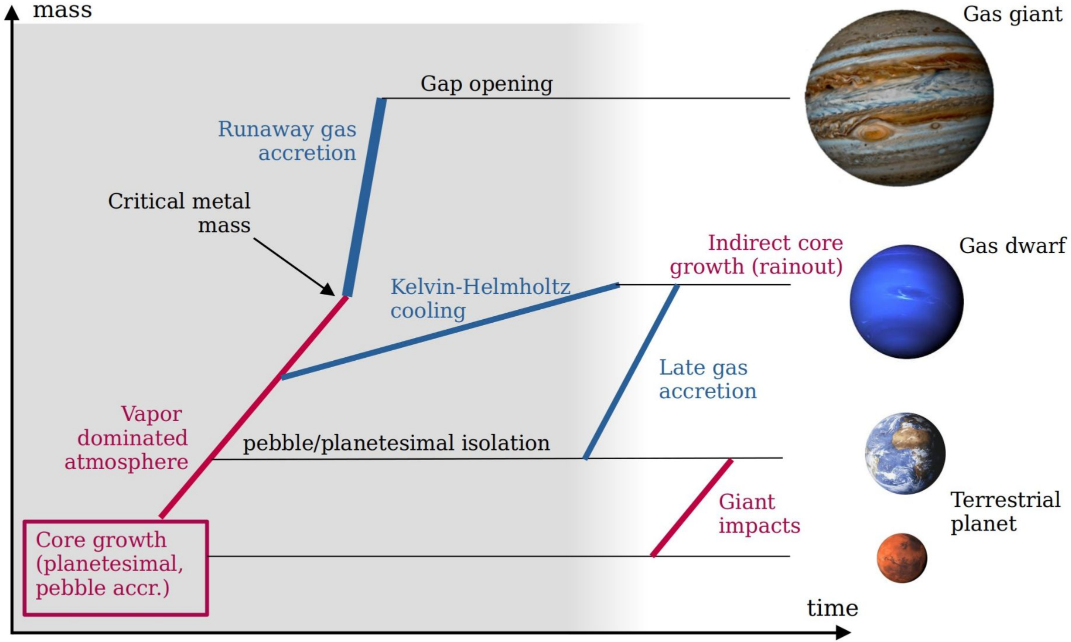
# Building planets

- **Scenario 1: Disk instability –** Fragmentation of a self-gravitating disk followed by the collapse of fragments

  Kuiper (1951), Toomre (1964), Cameron (1978), Boss (1997), Gammie (2001), …

- **Scenario 2: Core accretion –** Bottom-up process, solid core first, gaseous envelope next (requires planetesimal formation as a pre-step)

  Safronov (1969), Goldreich & Ward (1973), Mizuno (1980), Pollack et al. (1996), …




Ormel (2026)

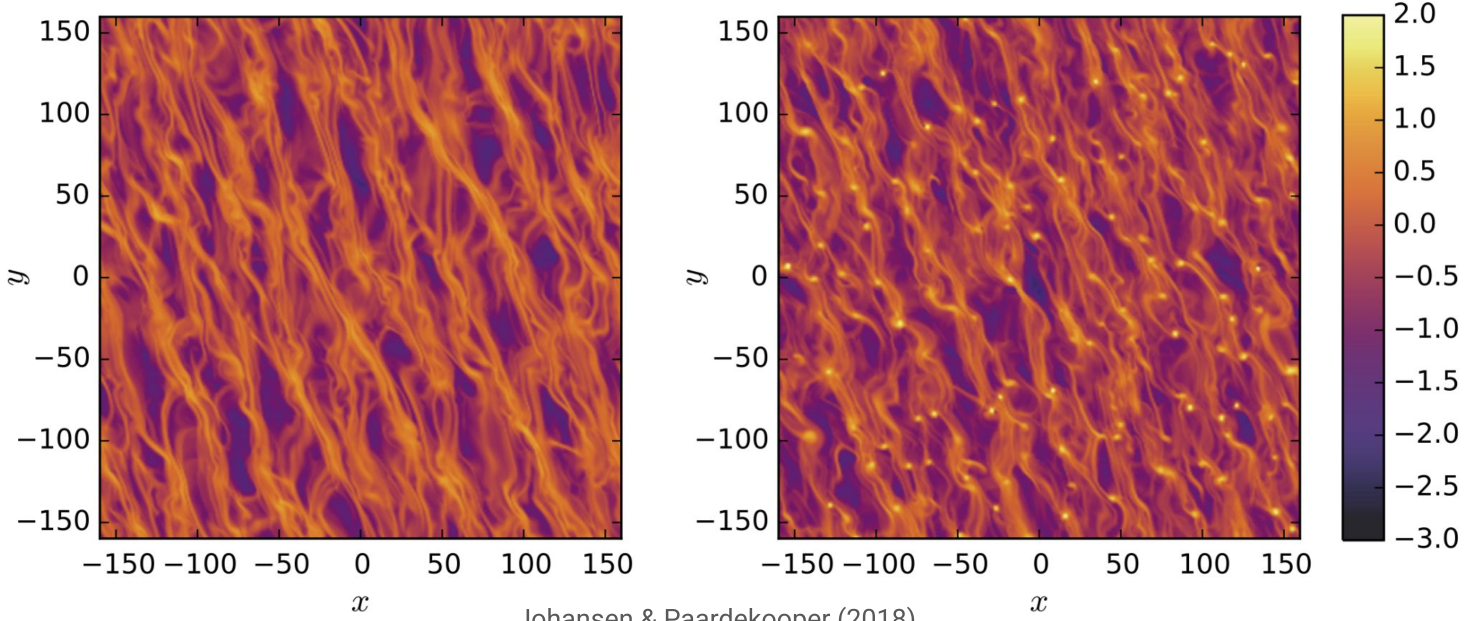
# Disk instability

- Stability analysis of a thin 2D self-gravitating disk leads to the dispersion relation for the density waves: $$\omega^2 = \kappa^2 - 2\pi G \Sigma |k| + c_s^2 k^2$$
- Stabilizing rotation VS destabilizing gravity VS stabilizing pressure
- Toomre (1964) criterion $$Q = \frac{\kappa c_s}{\pi G \Sigma}$$
- Instability (collapse) requires: $Q < 1$
- Non-axisymmetric setups:
  - The instability seems to start already at a bit larger $Q$ …
  - … but spiral wave launching and dissipation → disk heating → $Q$ moves away from instability (via $c_s$)

# Disk instability

- The role of the cooling rate: collapsing sub-structures → heat generation → increased pressure → cooling must be fast to allow further collapse (Gammie 2001): $t_{\mathrm{cool}}\Omega \lesssim \mathrm{a\ few}$

- Gravito-turbulent quasi-steady state (left) vs fragmenting state (right)




Johansen & Paardekooper (2018)

# Disk instability

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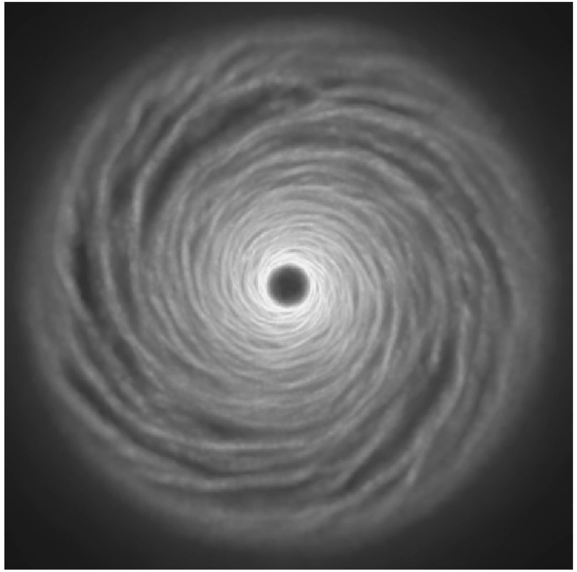
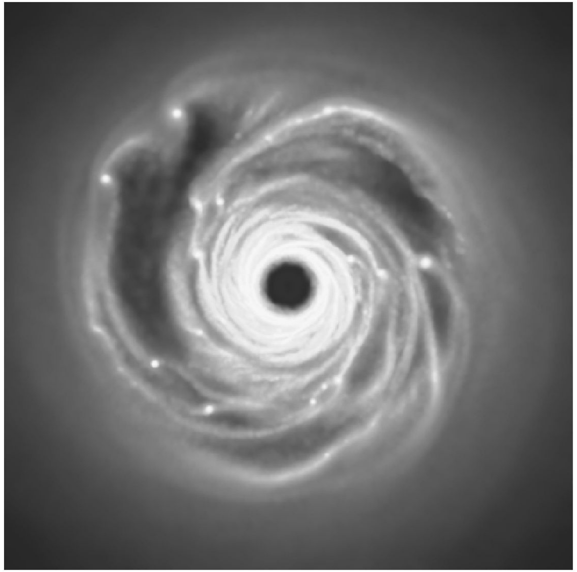




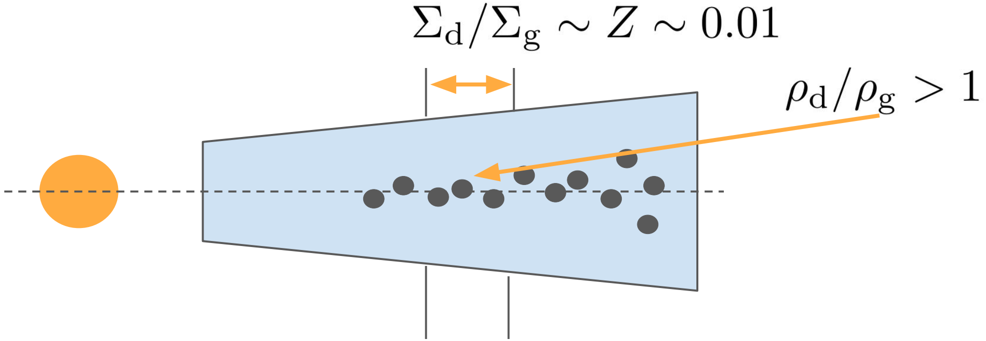
Rice et al. (2003)

# Disk instability

- Pros: it can happen fast and early, skipping the cumbersome growth from dust to planetary cores
- Cons:
  - disk must be massive (many disks probably not self-gravitating)
  - disk must cool fast (only fulfilled in the outermost disk regions)
  - even if the disk fragments, contracting the fragments to planetary bodies is not easy (again due to cooling)
  - the instability has a tendency to overshoot giant-planet masses, forming e.g. brown dwarfs (Kratter et al. 2010)

# Planetesimal formation

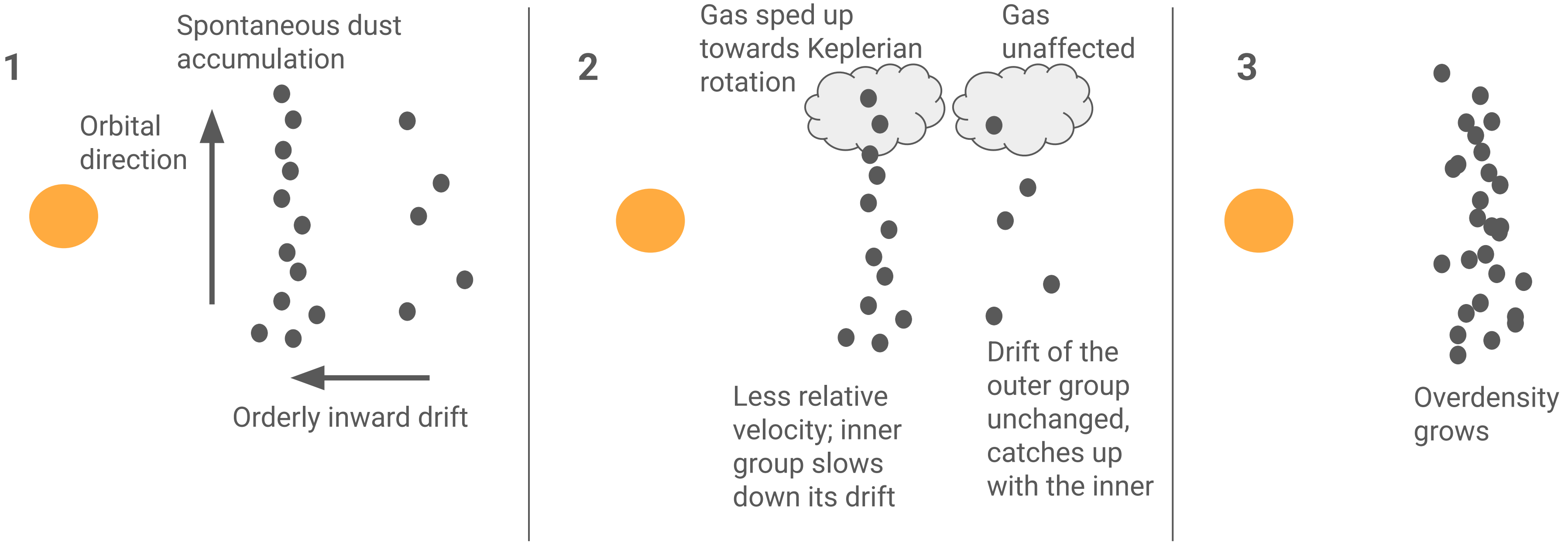
- Necessary step towards core accretion
- Planetesimals = first gravitationally interacting solid bodies in the disk, typically ~$10^0$–$10^2$ km in size
- Goldreich & Ward (1973): when particle settling to midplane is unopposed, an ultra dense layer forms, assembling self-gravitating clumps of solids of size scale $\lambda = \frac{4\pi^2 G\Sigma_\mathrm{d}}{\Omega^2}$ (=> 10km bodies at 1 au)
- Weidenschilling (1980): if most dust in midplane → gas sped up to Keplerian rotation → higher altitudes still sub-Keplerian → Kelvin-Helmholtz turbulence → dust stirred back up




- Emerging paradigm: what if some turbulent state creates local overdensities in a sedimented midplane?

# Streaming instability

- Drag is a two-way force
- Assume the *midplane* dust-to-gas ratio can grow (due to settling) so that the back-reaction on gas can no longer be neglected
- Positive feedback between spontaneous dust grain accumulation and slowdown of their drift



# Streaming instability

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- Positive feedback bet ... n acc ... eir drift

# Streaming instability

- How to get it?
- Set of equations from Youdin & Goodman (2005):

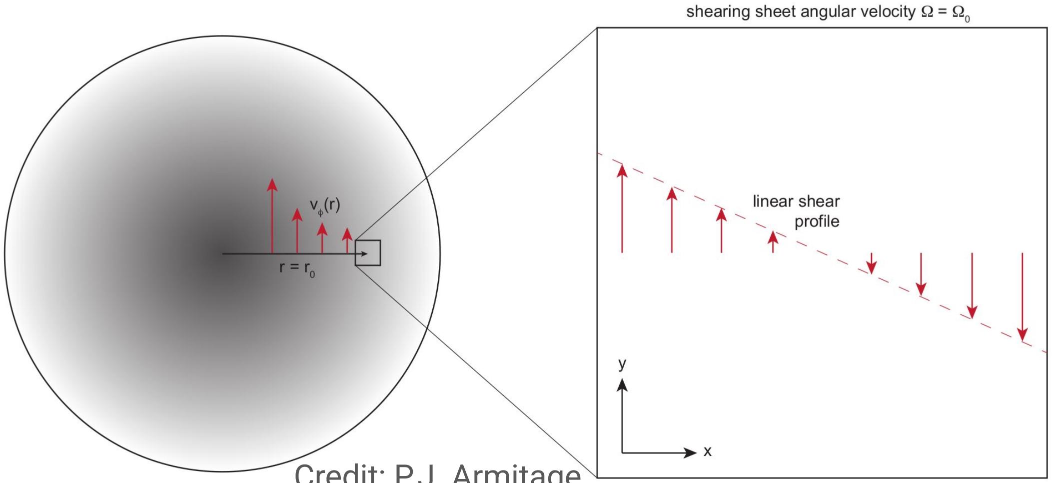
$$\frac{\partial \rho_p}{\partial t} + \nabla \cdot (\rho_p \boldsymbol{V}_p) = 0, \qquad (1)$$

$$\nabla \cdot \boldsymbol{V}_g = 0, \qquad (2)$$

$$\frac{\partial \boldsymbol{V}_p}{\partial t} + \boldsymbol{V}_p \cdot \nabla \boldsymbol{V}_p = -\Omega_{\mathrm{K}}^2 \boldsymbol{r} - \frac{\boldsymbol{V}_p - \boldsymbol{V}_g}{t_{\mathrm{stop}}}, \qquad (3)$$

$$\frac{\partial \boldsymbol{V}_g}{\partial t} + \boldsymbol{V}_g \cdot \nabla \boldsymbol{V}_g = -\Omega_{\mathrm{K}}^2 \boldsymbol{r} + \frac{\rho_p}{\rho_g} \frac{\boldsymbol{V}_p - \boldsymbol{V}_g}{t_{\mathrm{stop}}} - \frac{\nabla P}{\rho_g}, \qquad (4)$$

- Numerical simulations:
  - commonly a shearing sheet (2D) or a box (3D)
  - gas as a fluid (Eulerian approach), dust as particles (Lagrangian approach)

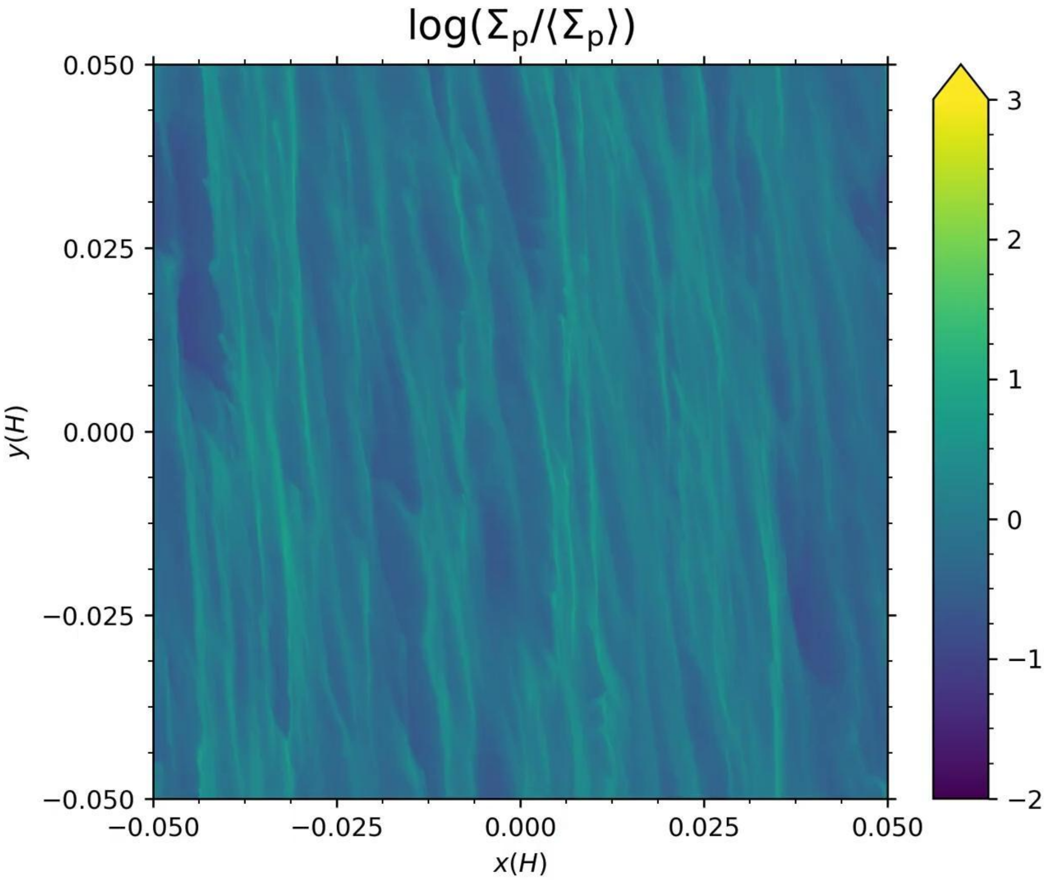



Credit: P.J. Armitage

# Streaming instability

Inertial orbital direction

Towards the star




Nesvorný et al. (2019)

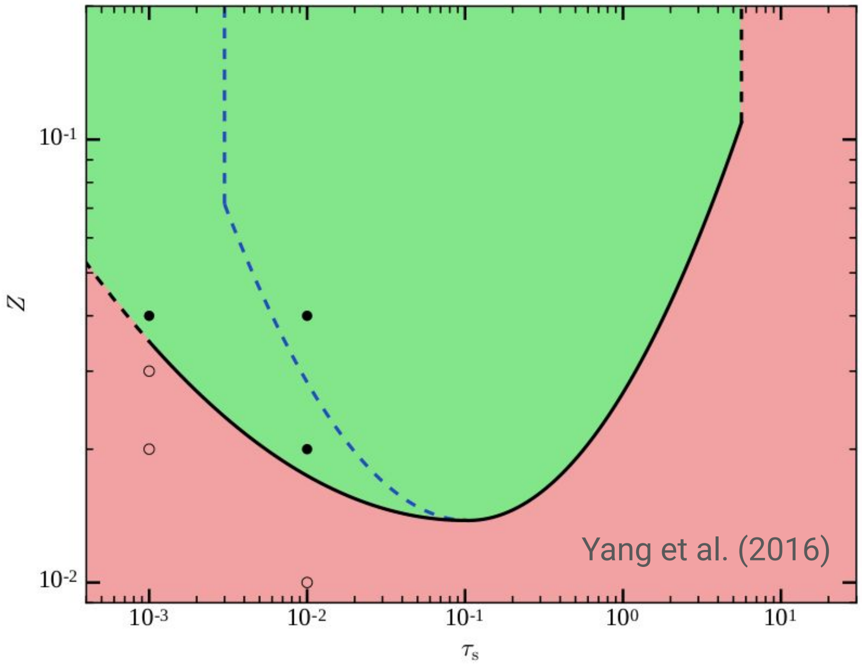
- Particle overdensities can become self-gravitating (exceeding the Roche density):

$$\rho_{\mathrm{Roche}} = \frac{9}{4\pi}\frac{\Omega^2}{G}$$

- If followed with a self-gravitating N-body code, they collapse into planetesimals

# Streaming instability – criteria

- With the canonical dust-to-gas ratio 1%, the SI not so easy to launch
- Either it requires optimal Stokes number ~0.1 (strong settling; good at exchanging momentum with gas), or increased dust-to-gas ratio 4–10%




- More up-to-date criteria (Lim et al. 2024, 2025; Liu et al. 2020, Lorek & Johansen 2022):

3D dust-to-gas:

$$\log(\epsilon_{\rm crit}) \approx 0.42(\log {\rm St})^2 + 0.72\log {\rm St} + 0.37$$

2D dust-to-gas:

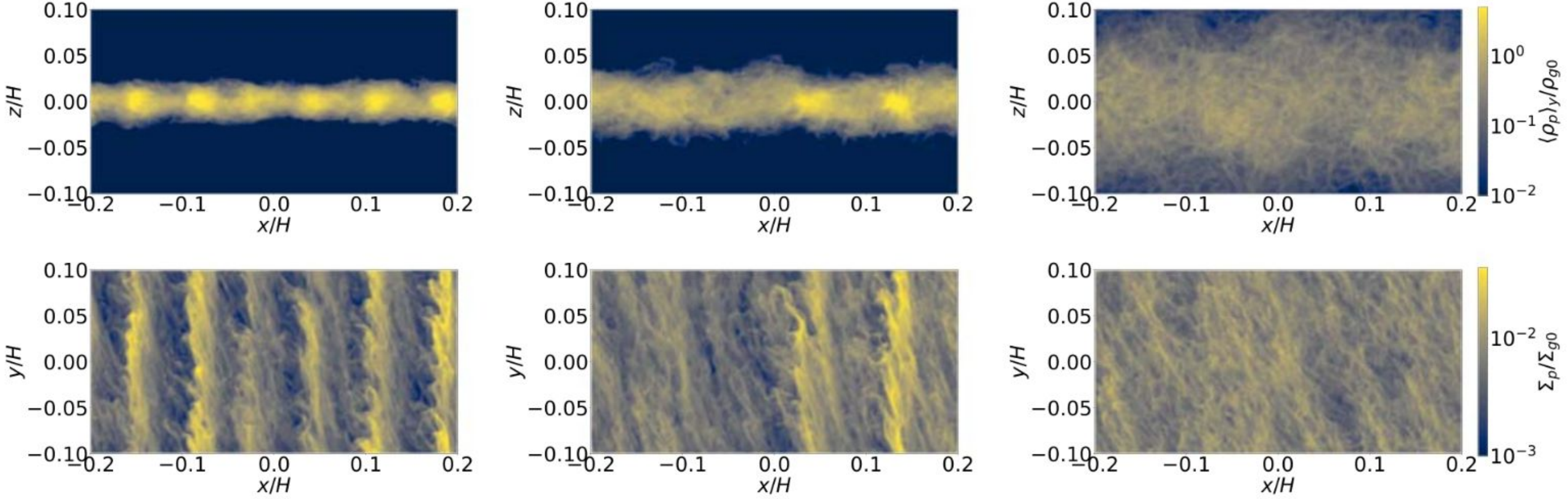
$$\log(Z_{\rm crit}) \approx 0.10(\log {\rm St})^2 + 0.07\log {\rm St} - 2.36$$

final maximum mass:

$$M_{\rm pl} = 5\times 10^{-5}\left(\frac{Z}{0.02}\right)^{1/2}\left(\frac{\gamma}{\pi^{-1}}\right)^{3/2}\left(\frac{h_{\rm g}}{0.05}\right)^{3}\left(\frac{M_\star}{1\,M_\odot}\right) M_\oplus$$

# Streaming instability – the role of turbulence

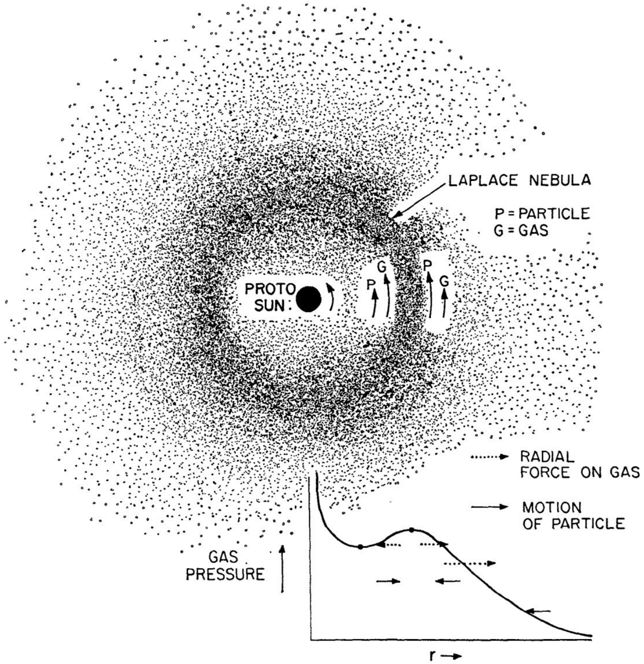
- Qpen question: the SI can typically survive, but turbulence can counteract settling and turbulent diffusion can prevent clumping within filaments
- Numbers from Lim et al. (2024) for τ = 0.01: $Z \sim 0.02$, 0.06, and 0.2 is needed when $\alpha \sim 0, 10^{-4}, 10^{-3}$




Lim et al. (2024)

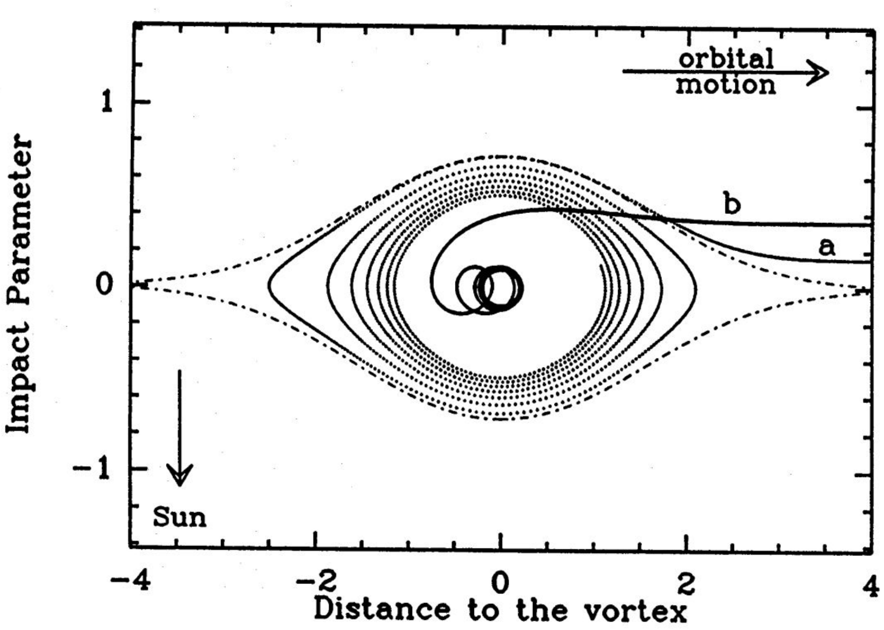
# Streaming instability – elevating the dust-to-gas ratio

- Pressure bumps




Whipple (1972)

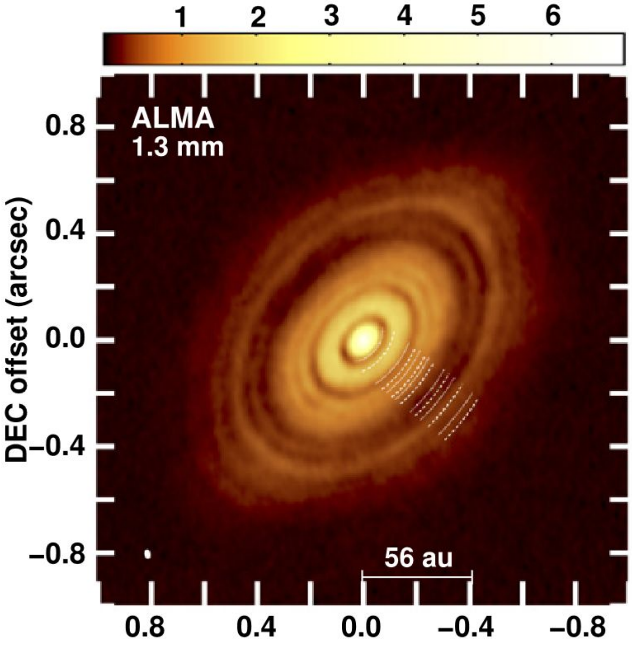
- Vortices




Barge & Sommeria (1995)

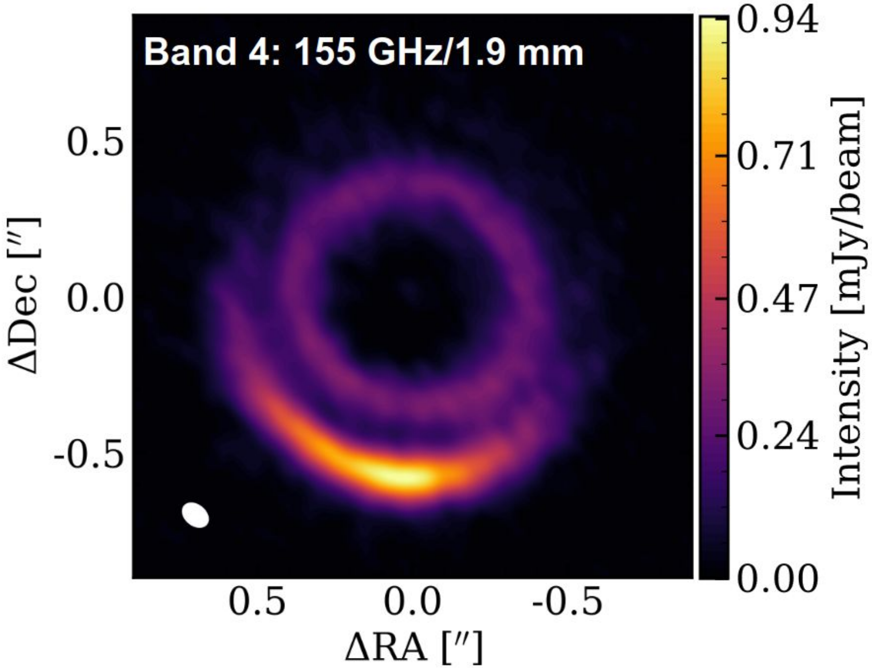
# Streaming instability – elevating the dust-to-gas ratio

- Pressure bumps




Carrasco-Gonzáles et al. (2016) – disk HL Tau

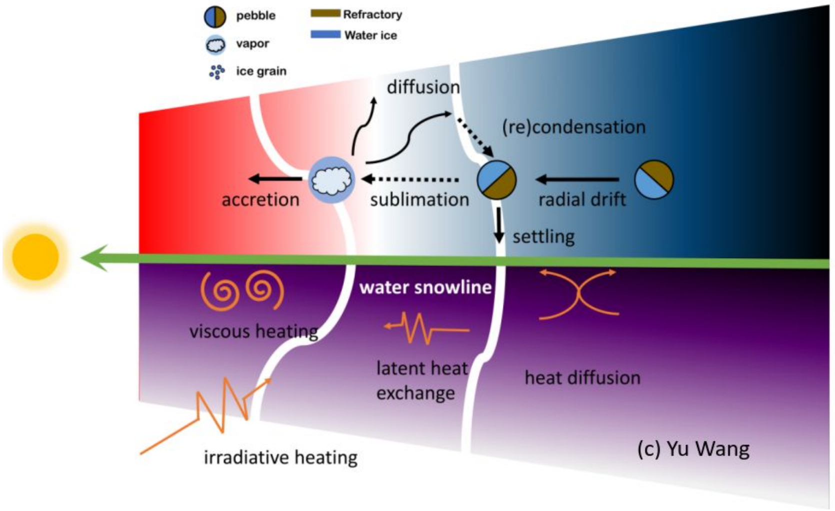
- Vortices

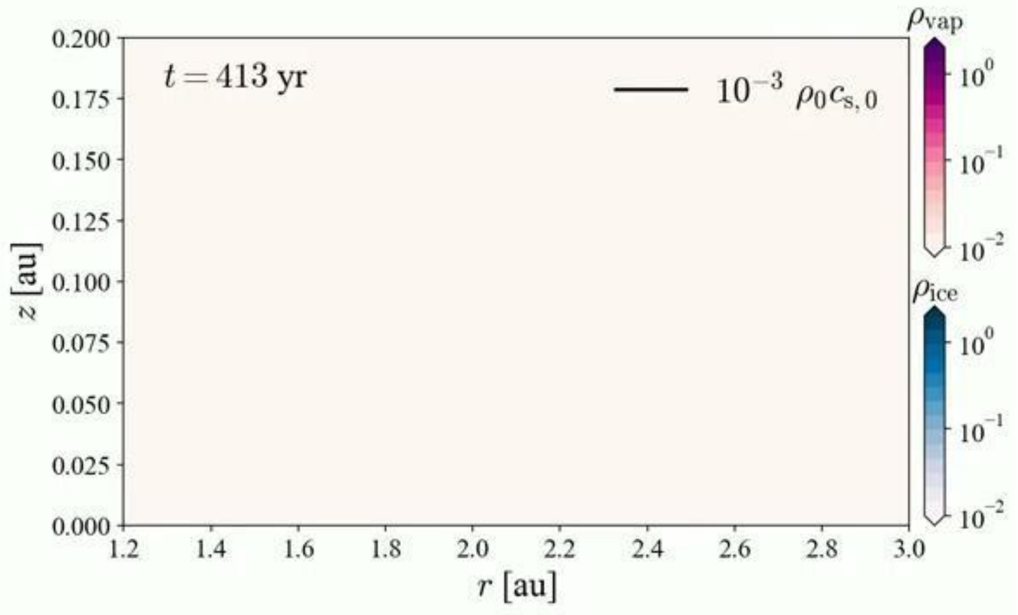



Cazzoletti et al. (2018) – disk HD 135344B

# Streaming instability – elevating the dust-to-gas ratio

- Water ice line: Drazkowska & Alibert (2017), Schoonenberg & Ormel (2017), Wang et al. (2024)

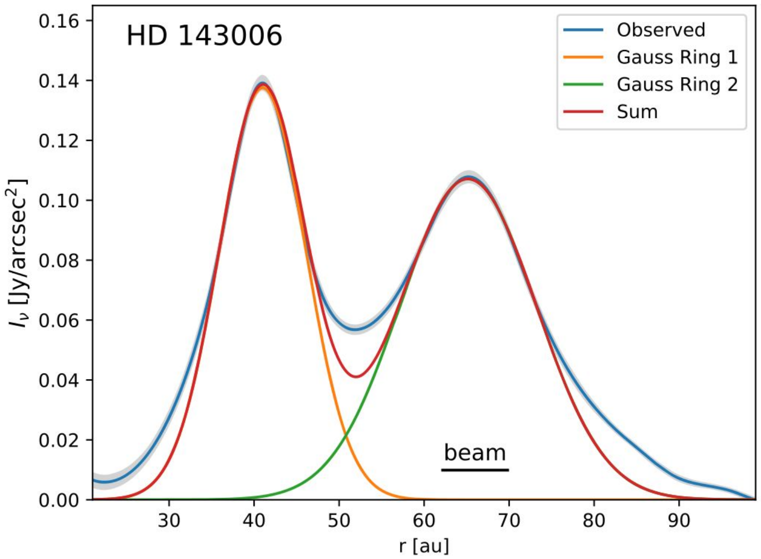



Credit: Yu Wang; Wang et al. (2024)

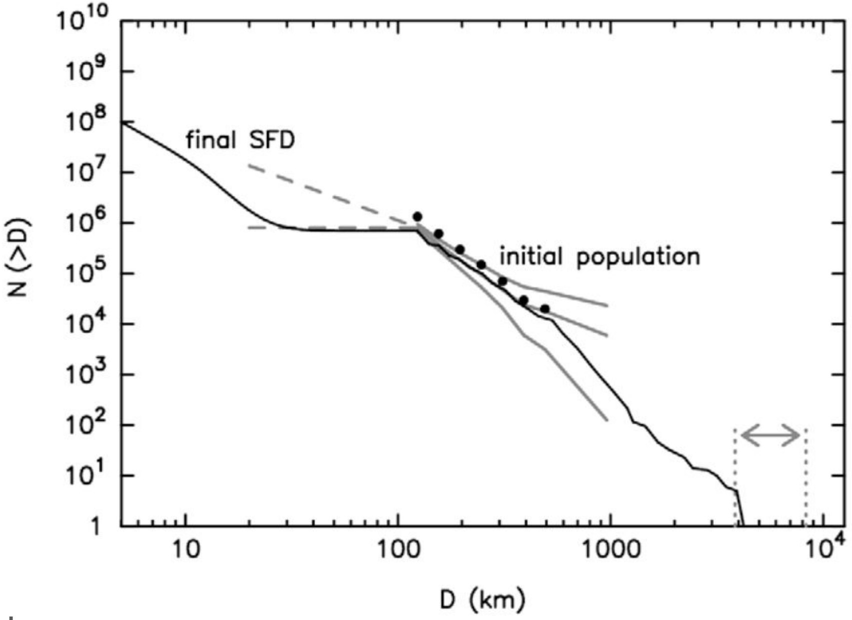
# Streaming instability – evidence

- DSHARP rings have optical depths ≲1 (commonly 0.2–0.5; Dullemond et al. 2018)



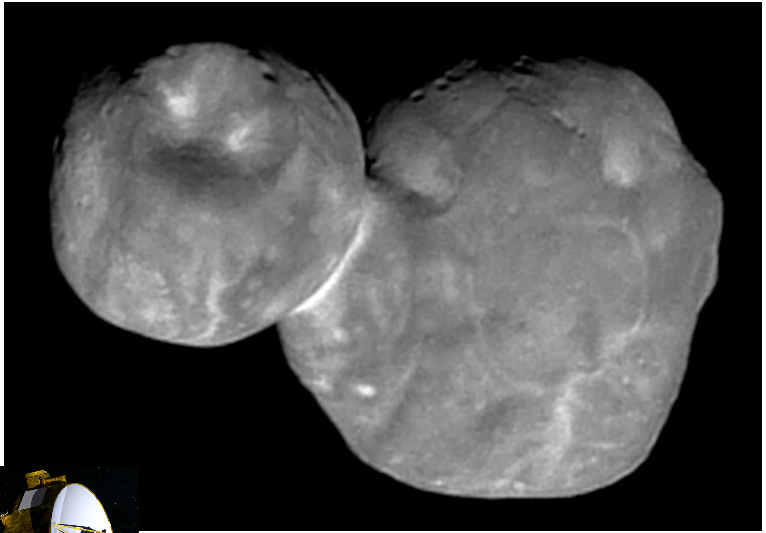

- Stammler et al. (2019): could be the SI converting dust to "invisible" planetesimals and keeping the midplane dust-to-gas ratio at ~unity

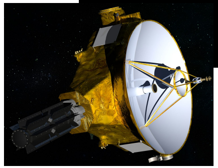
- Asteroids were born big (~100 km, like outcomes from the SI; Morbidelli et al. 2009)



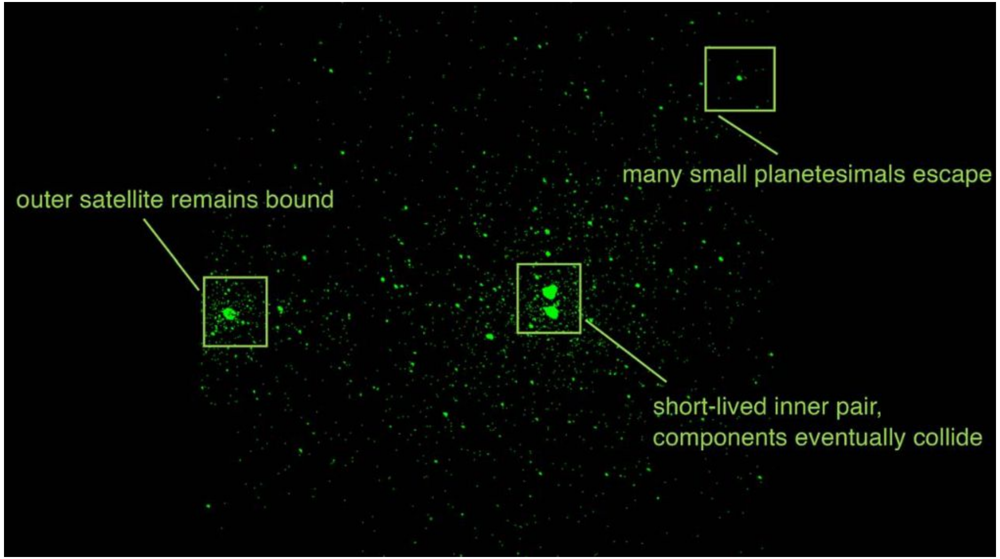
# Streaming instability – evidence

- Arrokoth (Ultima Thule)





- Conditions following the streaming instability (McKinnon et al. 2020):



# Streaming instability – evidence

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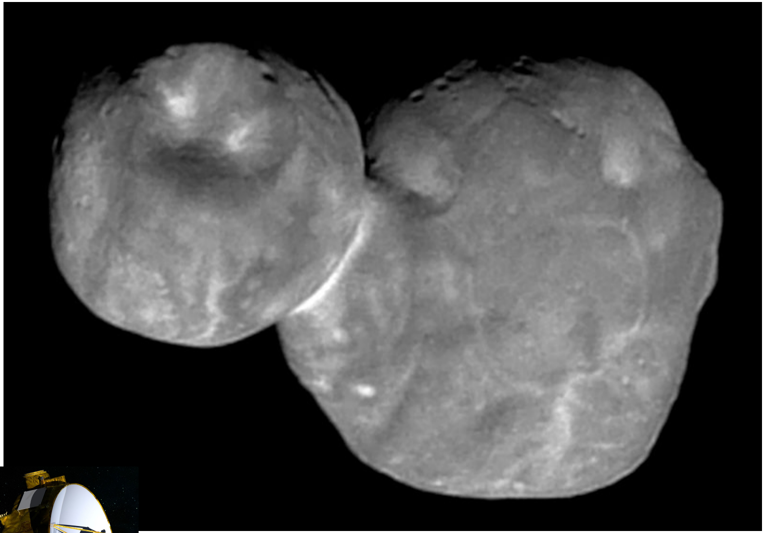
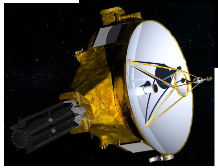
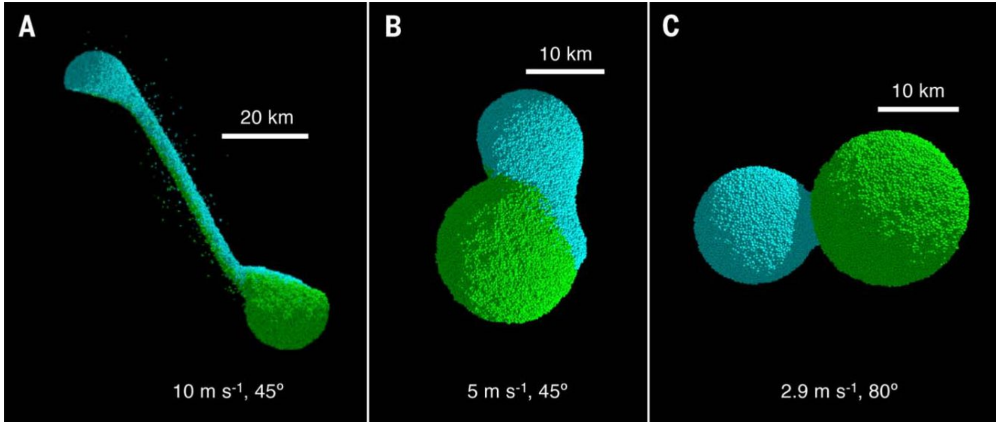


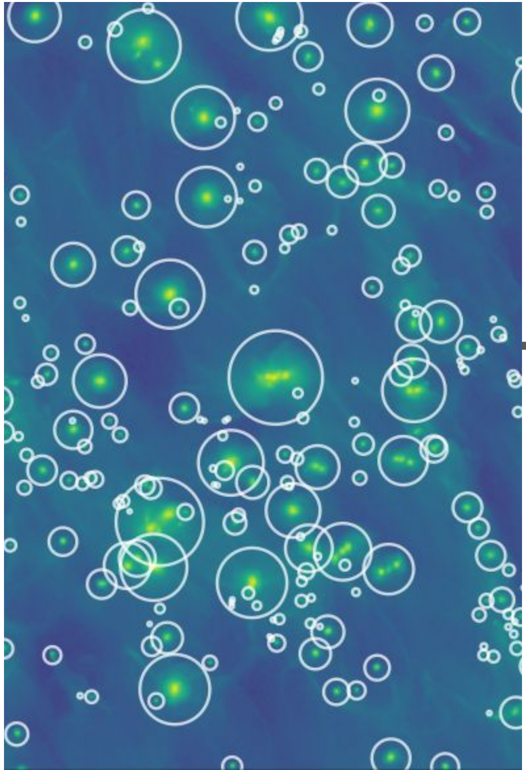


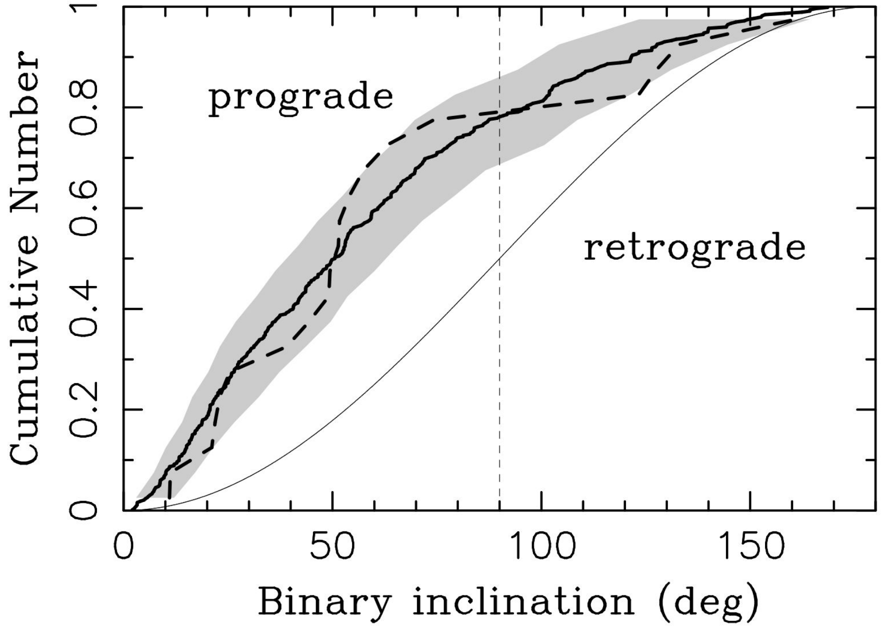
- Gentle collapse of binary configurations possible (McKinnon et al. 2020):



# Streaming instability – evidence

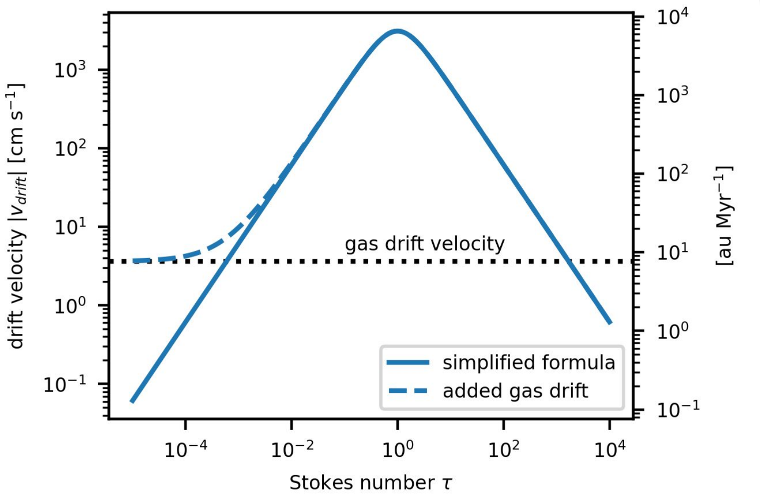
- Kuiper belt binaries: observed (dashed) have 80% prograde orientations (Grundy et al. 2019) which is perfectly reproduced by the SI (solid black; Nesvorný et al. 2019)





# Planetesimal accretion

- Gas drag changes: Epstein → Stokes regime; dimensionless stopping time τ >> 1
- $$F_{\rm drag} = \frac{1}{2} C_D \rho_g v_{\rm rel}^2 A \sim \rho_g v_{\rm rel}^2 R^2$$
  $$a_{\rm drag} \sim \frac{F_{\rm drag}}{m} \sim \frac{\rho_g}{\rho_s} \frac{v_{\rm rel}^2}{R}$$




Increasing particle size →

| **Drag:** | Strong | Intermediate | Weak |
|---|---|---|---|
| **Gas-particle momentum exchange:** | Weak (because $v_{\rm rel}$ kept low) | Ideal ($v_{\rm rel}$ can increase but drag still acts within $t_{\rm orb}$) | $v_{\rm rel}$ can become large but the response to gas is slow |
| **Outcome:** | **Dust** follows gas | **Pebbles** drift | Considerable drift and only for ≲km **planetesimals,** *e*-damping can be important |

# Planetesimal accretion

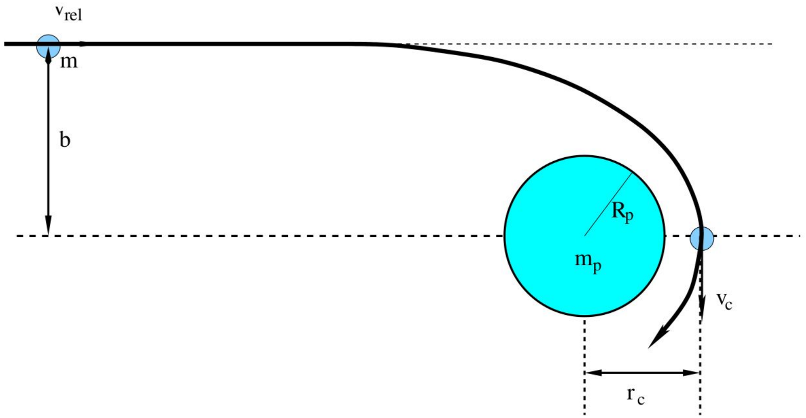
- Planetesimal accretion: growth by pairwise collisions (later also capture of planetesimals by protoplanets)
- Accretion rate = Safronov formula (1969):

$$\frac{dM}{dt} = \pi R^2 \rho_{pla} v_{pla} \left[ 1 + \left( \frac{v_e}{v_{pla}} \right)^2 \right]$$

Gravitational focusing factor: depends on the escape velocity √(2$GM/R$) vs $v_{pla}$

Accreting body

Spatial density and approach velocity of a planetesimal swarm




Kley (2017)

# Planetesimal accretion

- Two regimes of growth: $$\frac{\mathrm{d}M}{\mathrm{d}t} \sim \begin{cases} M^{4/3}, & v_{\mathrm{pla}} \ll v_{\mathrm{esc}} \\ M^{2/3}, & v_{\mathrm{pla}} \lesssim v_{\mathrm{esc}} \end{cases}$$ **<- runaway** **<- oligarchic**

$$t_{\mathrm{grow}} \equiv \frac{M}{(\mathrm{d}M/\mathrm{d}t)} \sim \begin{cases} M^{-1/3} \\ M^{1/3} \end{cases}$$

- Runaway (“rich become richer”): objects which start slightly bigger tend to grow faster and faster, pronouncing the mass difference
- Oligarchic: mass ratios among large bodies converge to unity
- Runaway not sustainable:
  - building larger objects → scattering of smaller objects (near-miss encounters stirr $v_{\mathrm{pla}}$ and planetesimals become dynamically hot towards future encounters)
  - having km-sized planetesimals would help (drag still important), but remember that clumping->collapse (e.g. SI) creates 100-km bodies

# Planetesimal accretion

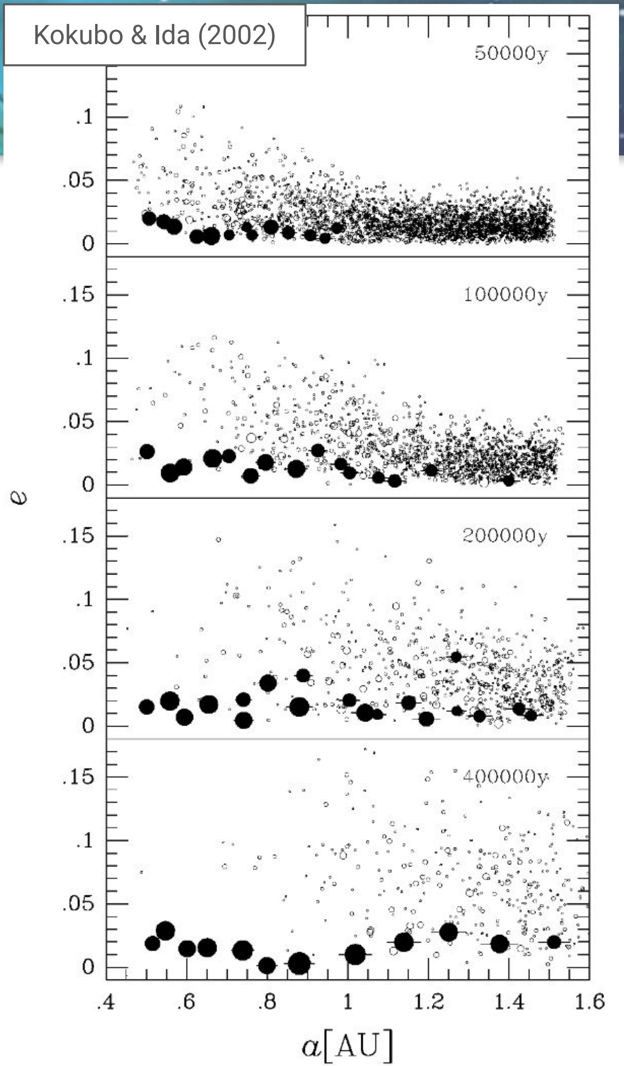
- Typical outcome: bi-modal (similar-sized planets + leftover eccentric planetesimals)
- Spacing between planets: 5–10 mutual Hill radii

$$R_{\mathrm{Hm}} = \frac{a_1 + a_2}{2}\left[\frac{M_1 + M_2}{3M_\star}\right]^{1/3}$$

- Isolation mass:

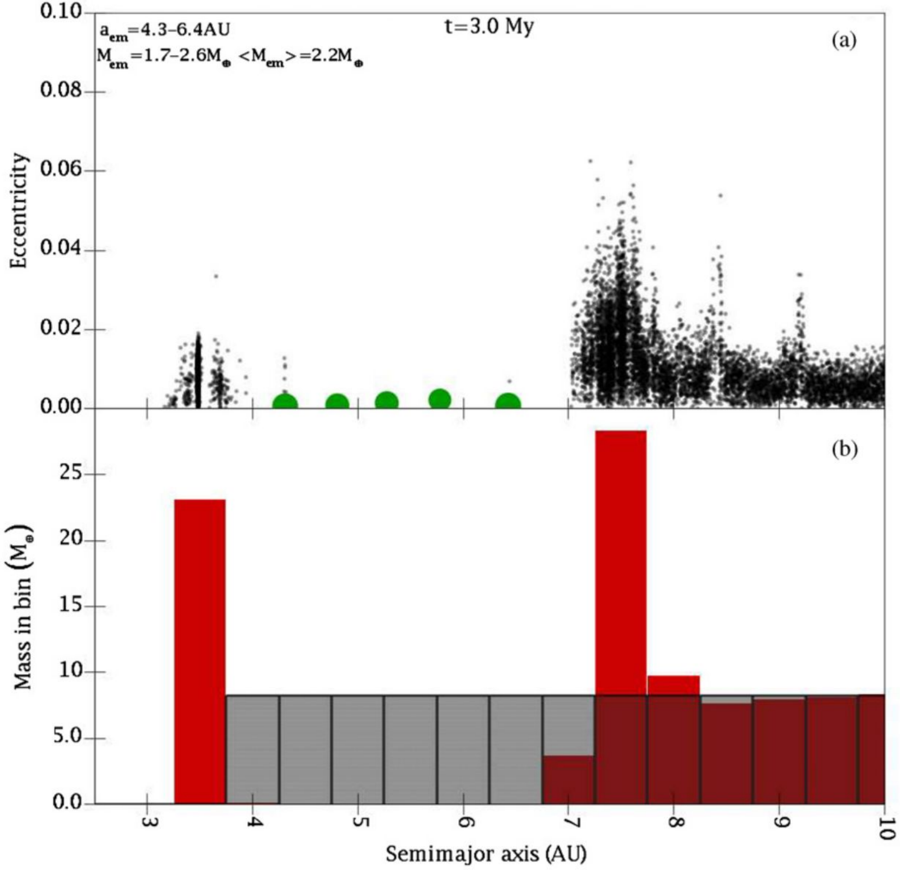
$$M_{\mathrm{iso}} \simeq 2\pi a b \Sigma_{\mathrm{solid}} = 0.16\left(\frac{\tilde{b}}{10}\right)^{3/2}\left(\frac{\Sigma_{\mathrm{solid}}}{10\ \mathrm{g\ cm^{-2}}}\right)^{3/2}\left(\frac{a}{1\ \mathrm{AU}}\right)^{3} \times \left(\frac{M_*}{M_\odot}\right)^{-1/2} M_\oplus\ ,$$

- The isolation mass is rather a hypothetical maximum (consider the time scale and scattering!)



# Does it work in the solar-system context?

- Problem explaining gas giants
- The growth time-scale follows roughly $\sim r^3$
- Scattering dominates over accretion when $\frac{v_{\rm esc}^2}{2v_{\rm K}^2} > 1$
- Once planets empty their neighbourhood, its refilling is problematic to achieve
- See Levison et al. (2010)

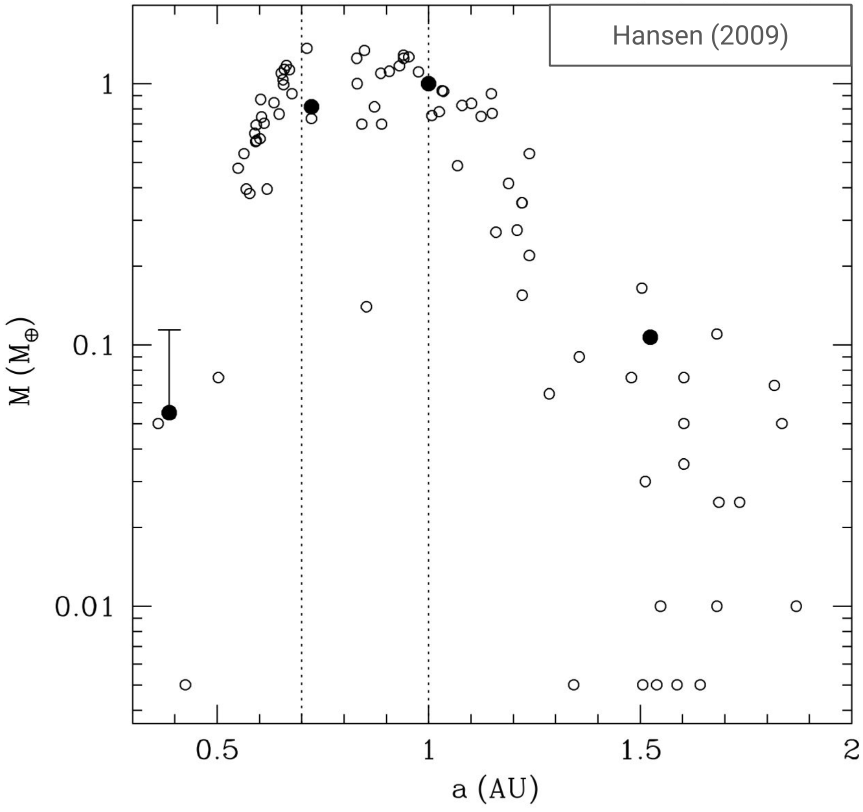



Levison et al. (2010)

# Does it work in the solar-system context?

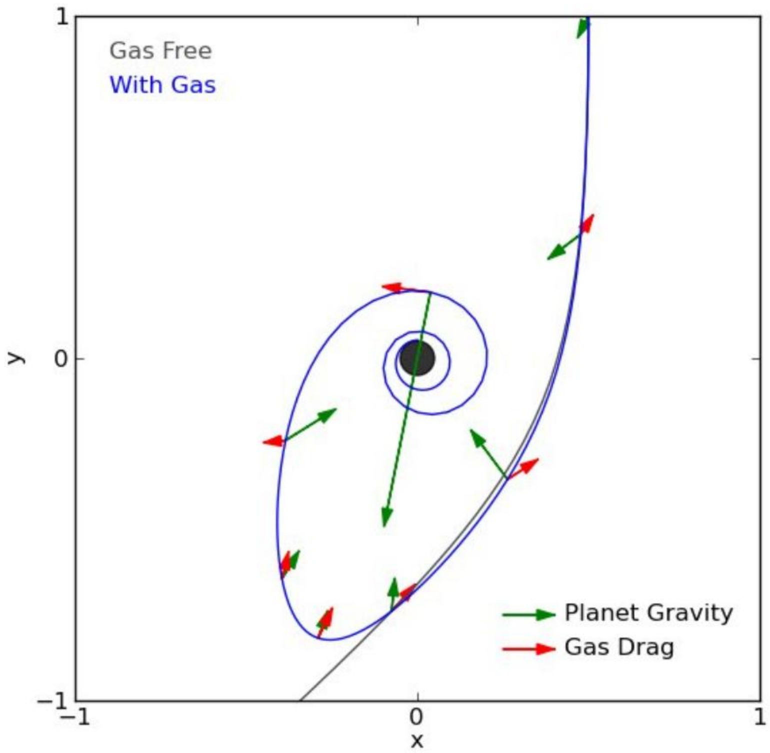
- Okay for the terrestrial region
- Radiometric dating: Earth ~30–100 Myr after CAIs (though Mars earlier; ~2–10 Myr)
- Inner SS dynamically heated (easier to explain *after* the disk phase)
- Giant collisions:
  - Theia hitting Earth and forming Moon
  - Mercury's mantle stripped?

- Growth from a narrow annulus required to avoid overgrowing Mars and Mercury  (so the question is how to create such annulus…)



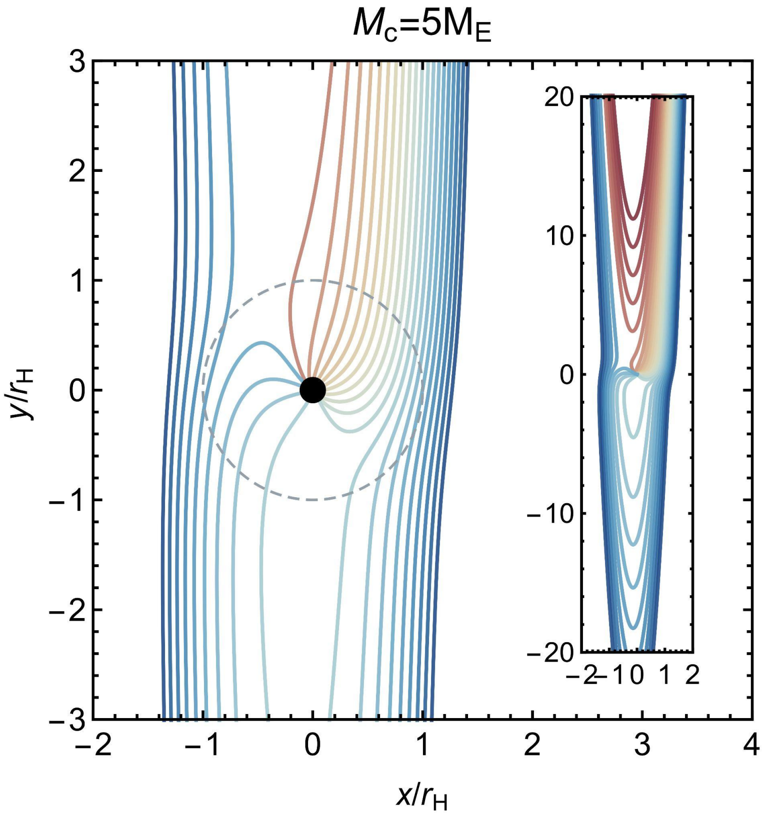
# Pebble accretion

- Starting conditions: planetesimals/embryos embedded in leftover pebbles + gas
- Drag-assisted accretion:
  - Pebble deflected near a bigger body → Misaligned from gas flow → Relative velocity grows, enhancing the drag force → Cross section for pebble capture enhanced significantly
  - Pebble flux through the disk → Orbital neighbourhood of the growing body always replenished

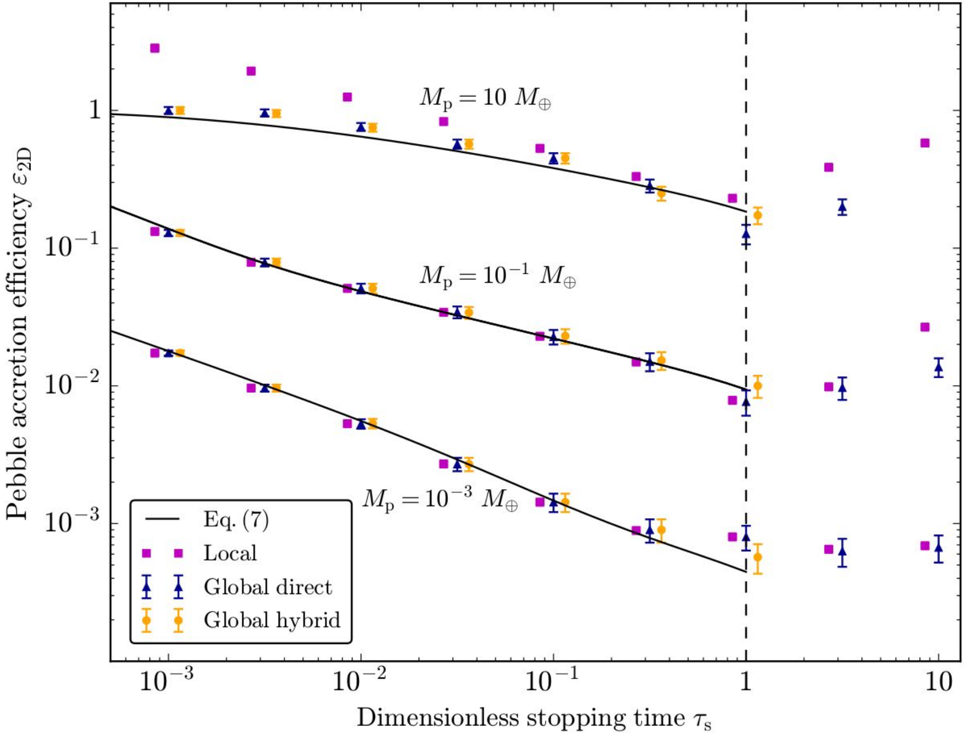



Credit: D. Nesvorný

# Pebble accretion – filtering



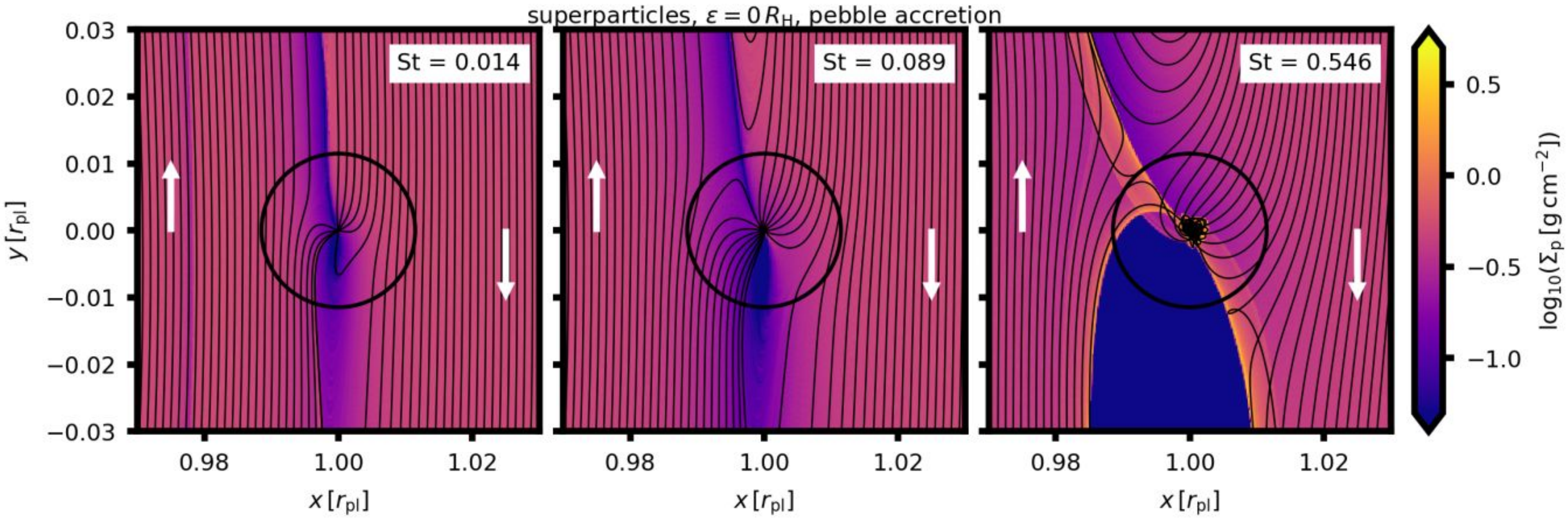

Credit: M. Lambrechts




Liu & Ormel (2018)

# Pebble accretion – accretion radius, surface density

Increasing particle size, more prominent drift (trajectories angled), accretion switches from front-rear to front regime, cross-section grows

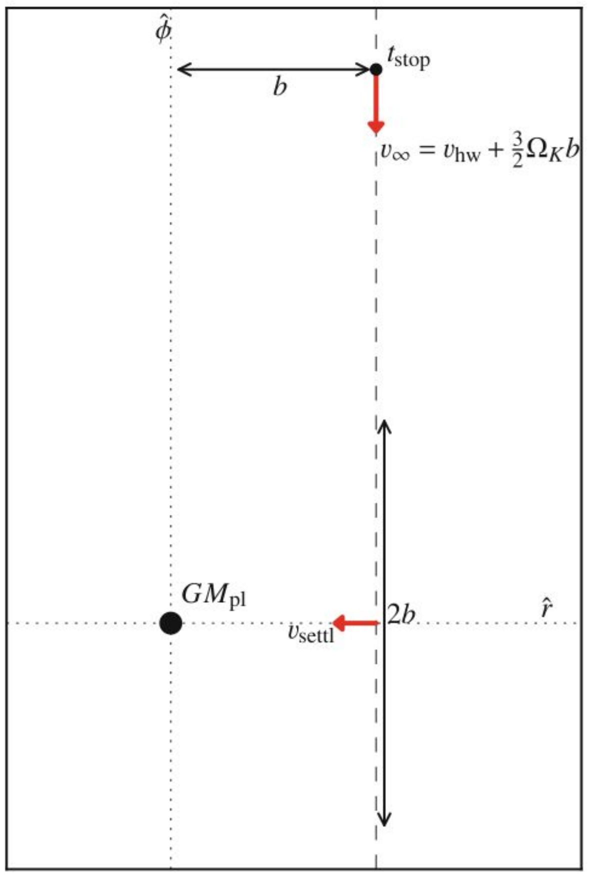



Chrenko et al. (2024)

# Pebble accretion – basic formulae

- To accrete from impact parameter b:
  - **1.** Time to settle onto planet < pebble-planet encounter time
  - **2.** The stopping time < pebble-planet encounter time
- Small particles: fulfill 2. but 1. can be a problem (settling time can be too long if the stopping time is short)
- Large particles: vice versa
- On blackboard: definitions of characteristic timescales and accretion radii

Ormel (2017)



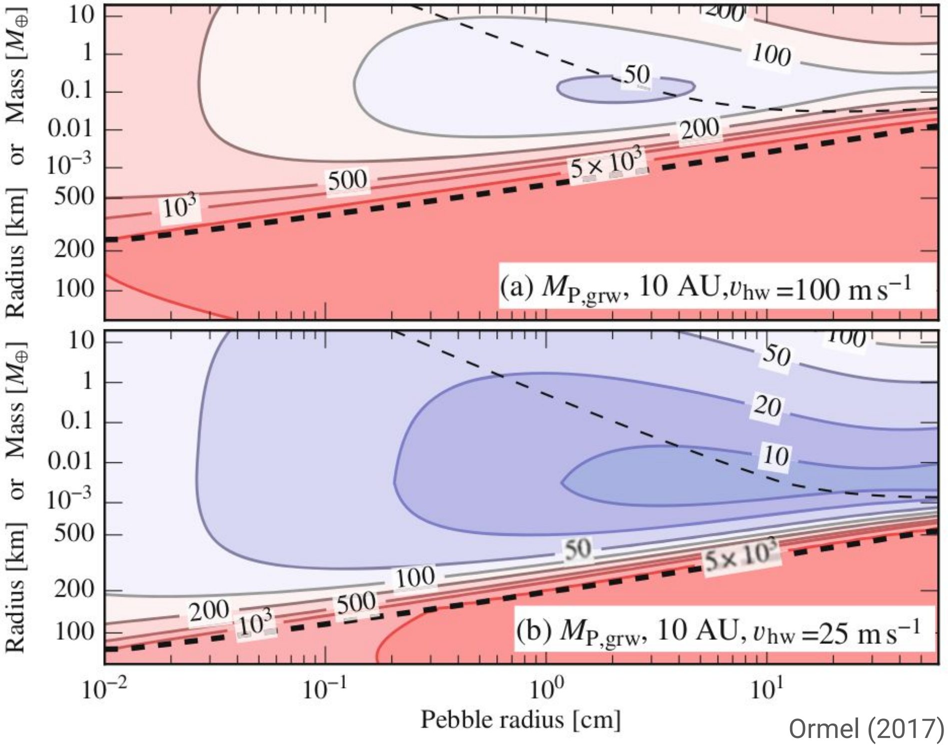
# Pebble accretion – basic formulae

- Regimes deciding the cross section:
  - small embryos & Stokes numbers: headwind dominates → **Bondi regime**
  - larger embryos & Stokes numbers: shear dominates → **Hill regime**
- Regimes deciding the feeding zone:
  - b < scale height of the pebble disk → **3D regime**
  - b > scale height of the pebble disk → **2D regime**

$$
\begin{cases}
H_{\rm peb} = H_{\rm gas}\sqrt{\dfrac{\alpha}{\alpha+\tau}} \\
\dot{M}_{\rm p,2D} = 2bv_\infty\Sigma_{\rm peb} \\
\dot{M}_{\rm p,3D} = \pi b^2 v_\infty \rho_{\rm peb} \\
\Sigma_{\rm peb} = \dfrac{\dot{M}_{\mathcal{F}}}{2\pi r v_{\rm drift}}
\end{cases}
$$

# What pebble mass is needed?

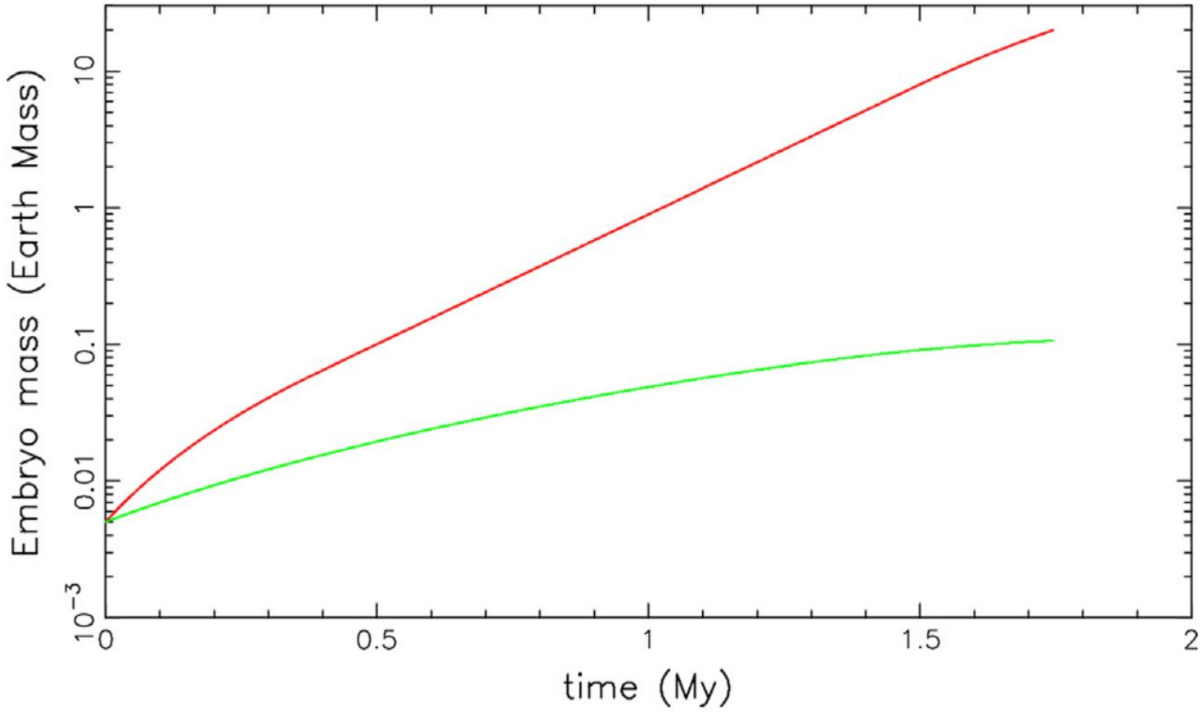
- Necessary total pebble mass (in Earth masses) to grow a body of a given radius/mass




Ormel (2017)

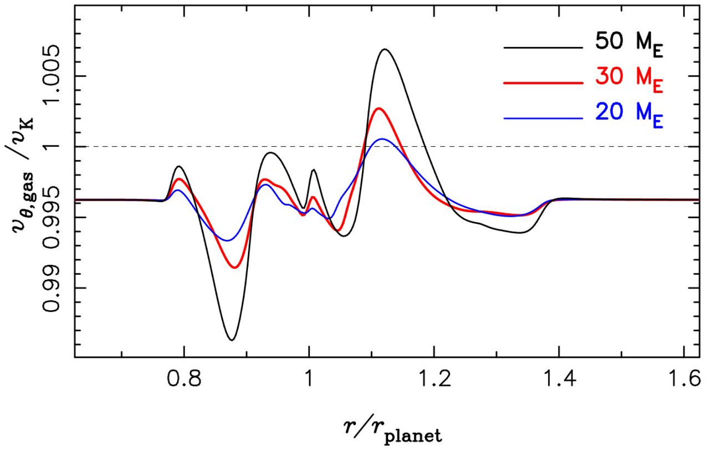
# Pebble accretion vs solar-system dichotomy

- Larger pebbles & ice + rock available beyond iceline → embryos outgrow those in the terrestrial region (Morbidelli 2015):

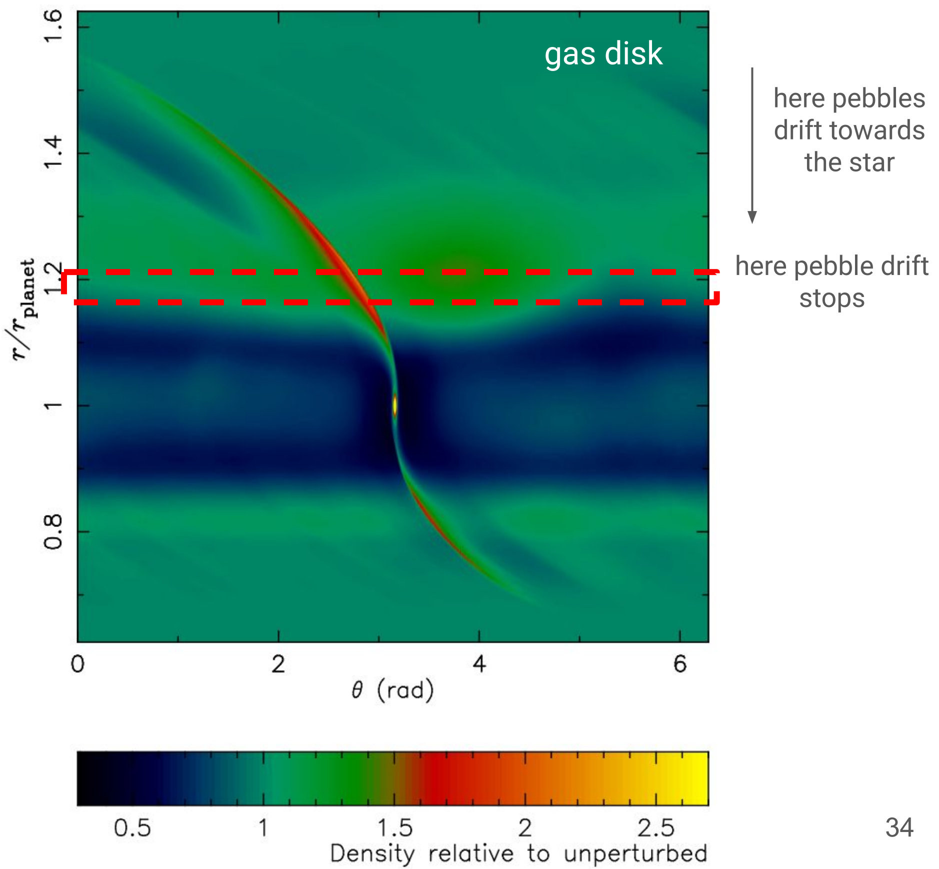


# Pebble isolation mass

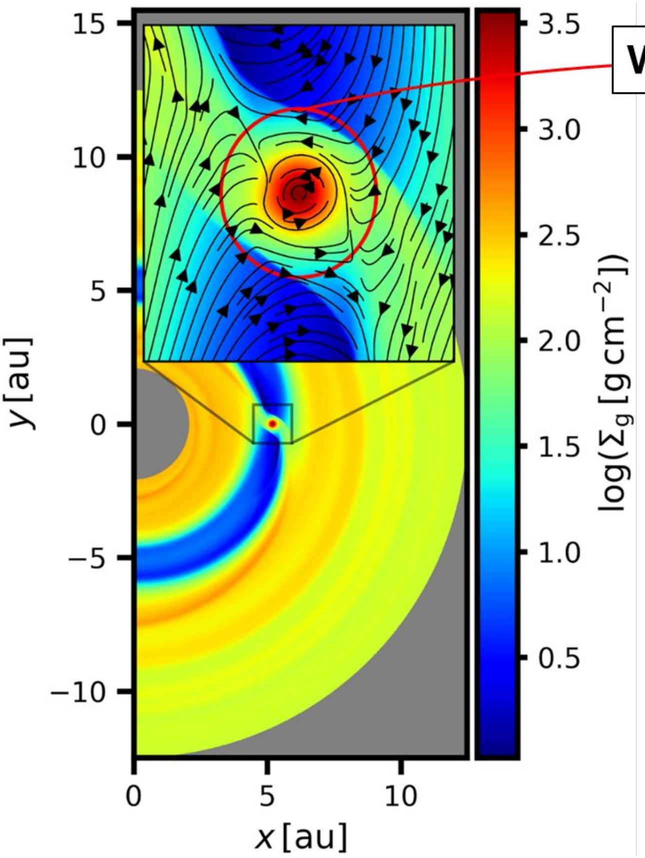
- Pebble accretion is self-limiting: when planet becomes large enough to perturb the gas rotation so that it becomes super-Keplerian (pressure bump is formed), pebble drift is blocked (see Bitsch et al. 2018)

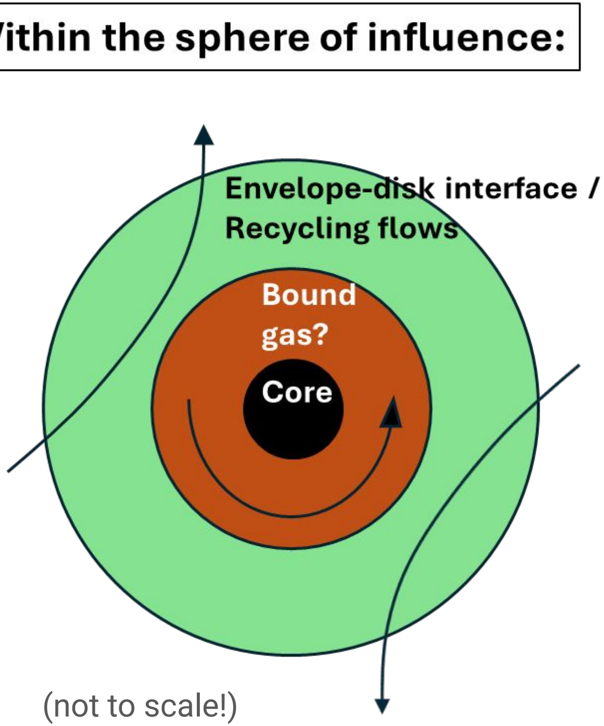



Lambrechts et al. (2014)



# Accreting gas onto cores




First envelopes starts to appear once the Bondi radius

$$R_B \equiv Gm_p/(H\Omega_K)^2$$

exceeds the core radius; fulfilled already at about the Moon mass

Stages (during the disk phase)

- quasi-static Kelvin-Helmholtz cooling/contraction
- runaway accretion
- post-runaway accretion (circumplanetary disk?)

# 1D evolution

- Eqs. similar to stellar interior structure

e.g. Guillot (2005)

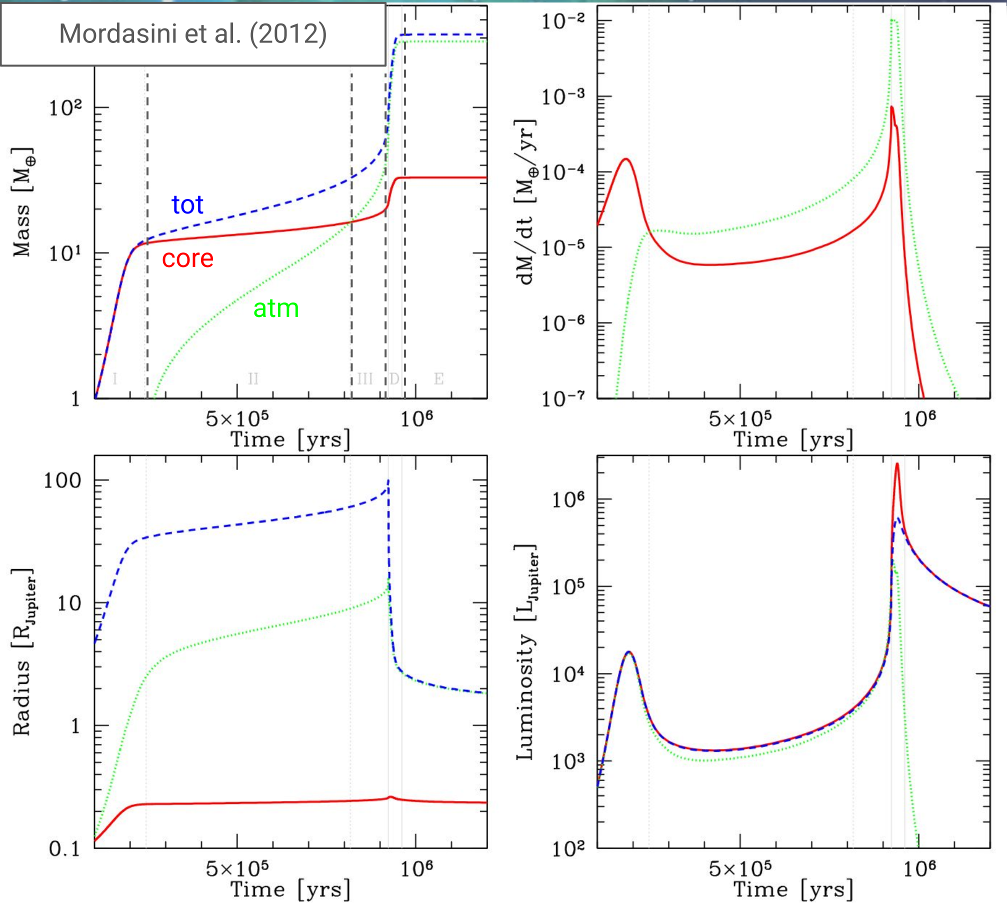
| | |
|---|---|
| hydrostatic balance | $\frac{\partial P}{\partial r} = -\rho g$ |
| energy transport | $\frac{\partial T}{\partial r} = \frac{\partial P}{\partial r}\frac{T}{P}\nabla_T$ |
| mass conserv. | $\frac{\partial m}{\partial r} = 4\pi r^2 \rho$ |
| energy conservation | $\frac{\partial L}{\partial r} = 4\pi r^2 \rho \left(\dot{\epsilon} - T\frac{\partial S}{\partial t}\right)$ |

$$g = Gm/r^2$$

$$\nabla_T \equiv (d\ln T/d\ln P)$$

+ EOS + boundary conditions

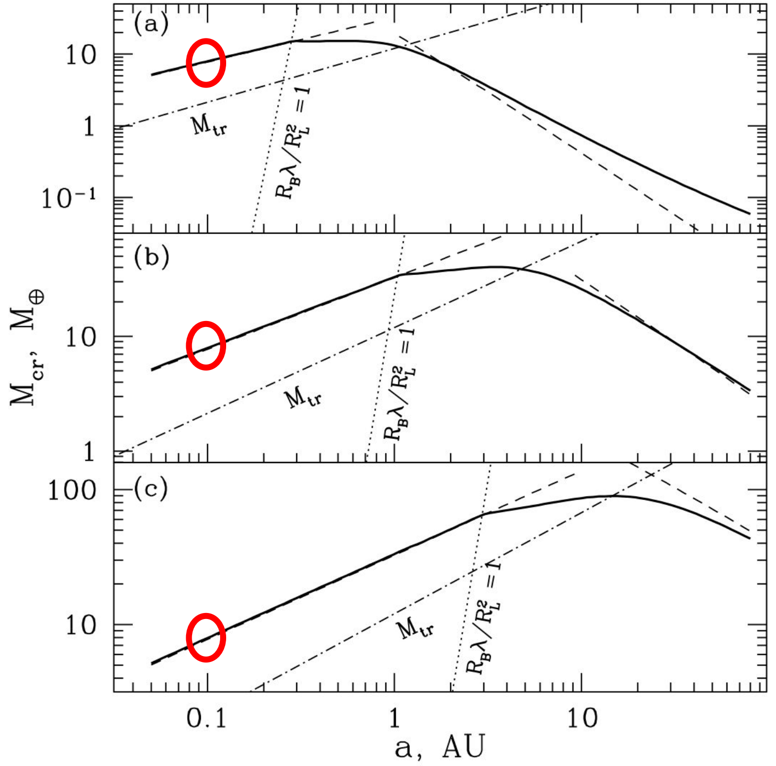
Mordasini et al. (2012)



# Runaway accretion separates giants from sub-giants

- Runaway initiated at (Stevenson 1982, Armitage 2010): $M_{\rm crit} \simeq 20\kappa_{\rm R}^{3/7} M_\oplus$
- Runaway accretion regime (e.g. Choksi et al. 2023):
  - 3D Bondi regime if the planet remains embedded: $\dot{m}_B \sim R_B^2 c_s \Omega_K \rho_{\rm gas}$
  - 2D Hill regime if the Hill radius exceeds the disk scale height: $\dot{m}_{\rm 2d} \sim R_H^2 \Omega_K \Sigma_{\rm gas}$
- The time scale ($m/(dm/dt)$) can be very short (Ormel 2026 gives $10^3$ yr and 50 yr, respectively)

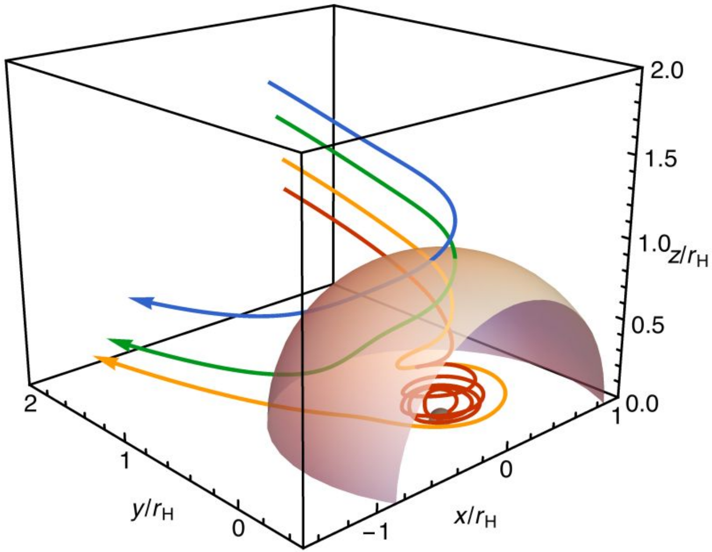
- Runaway onset from Rafikov (2006) → problem for sub-Neptunes ! (also Lee et al. 2014)

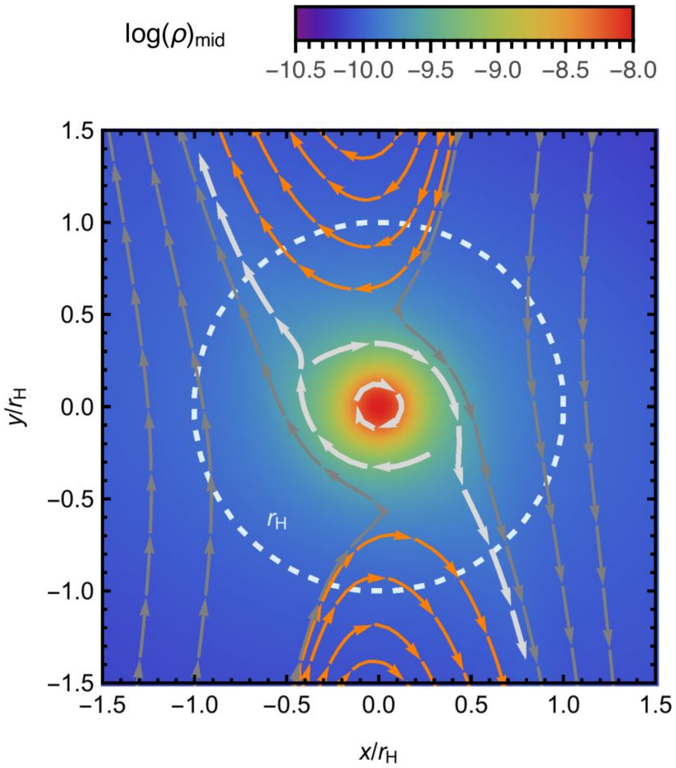


# Quasi-static contraction in 3D

- 3D simulations reveal recycling flows (Ormel et al. 2015): continuous exchange of gas between disk and envelope, often faster than the envelope contraction proceeds
- Recycling flows can extend the phase of quasi-static contraction, possibly explaining the ubiquity of sub-giants

Figs. from Lambrechts & Lega (2017)



# Quasi-static contraction in 3D

- Isothermal limit

$$P = c_\mathrm{s}^2 \rho$$

sound speed prescribed, no energy equation

$\rightarrow$ Envelope typically heavily recycled

- Adiabatic limit

$$P = (\gamma - 1)\epsilon$$
$$\frac{\partial \epsilon}{\partial t} + (\vec{v} \cdot \nabla)\,\epsilon = -\,P\nabla \cdot \vec{v}$$

$\rightarrow$ Envelope typically heavily convective

- Radiative intermed. case

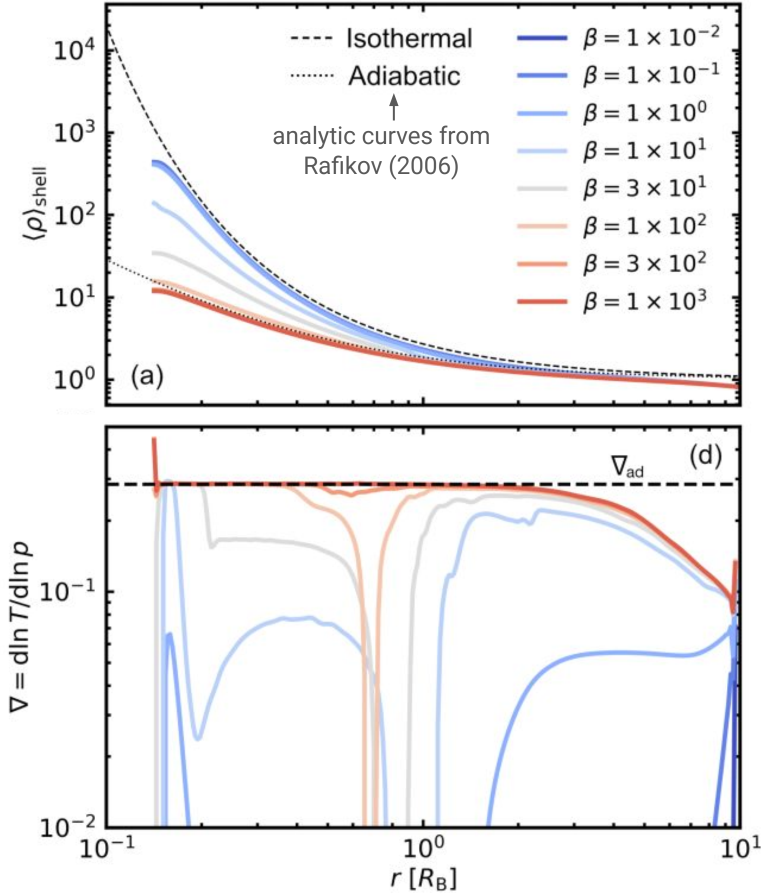
as adiabatic + radiative energy transport

$\rightarrow$ Envelope either:
radiative+recycling
or
convec.+radiative+recycling

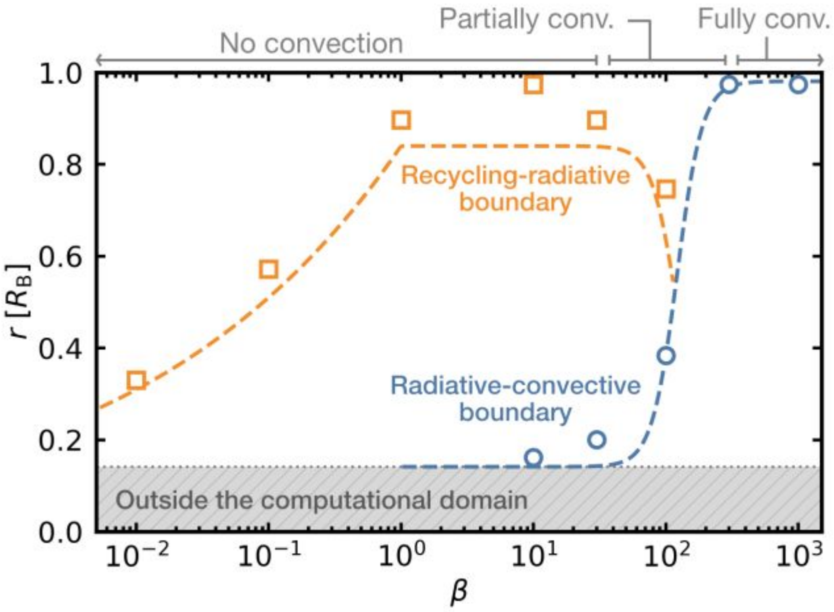
- Kuwahara & Lambrechts (2026): Recent study comparing these cases using the beta-cooling approach:

$$Q_\mathrm{cool} = -\frac{e - e_0}{t_\mathrm{cool}} \qquad \beta \equiv t_\mathrm{cool}\Omega$$

# Quasi-static contraction in 3D

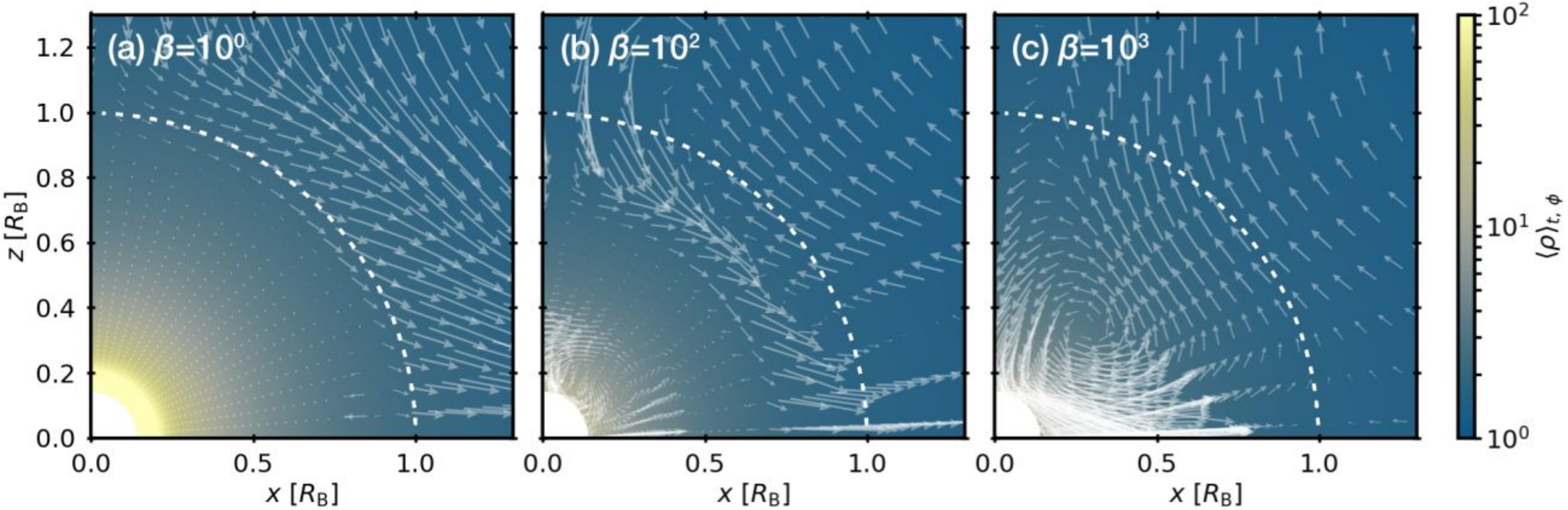



Figs. from Kuwahara & Lambrechts (2026)



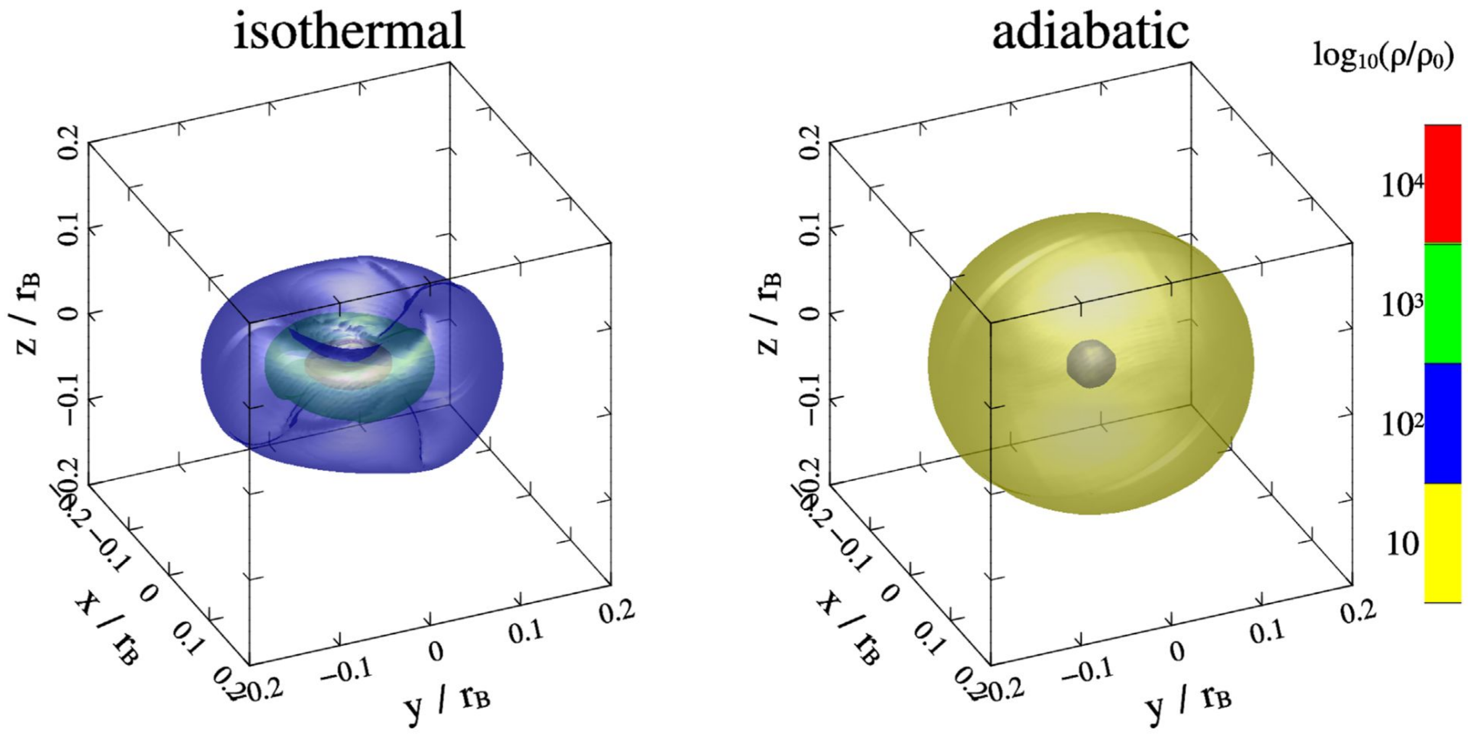
# Quasi-static contraction in 3D

Figs. from Kuwahara & Lambrechts (2026)



# Post-runaway accretion

- Can a circumplanetary disk (CPD) be formed?

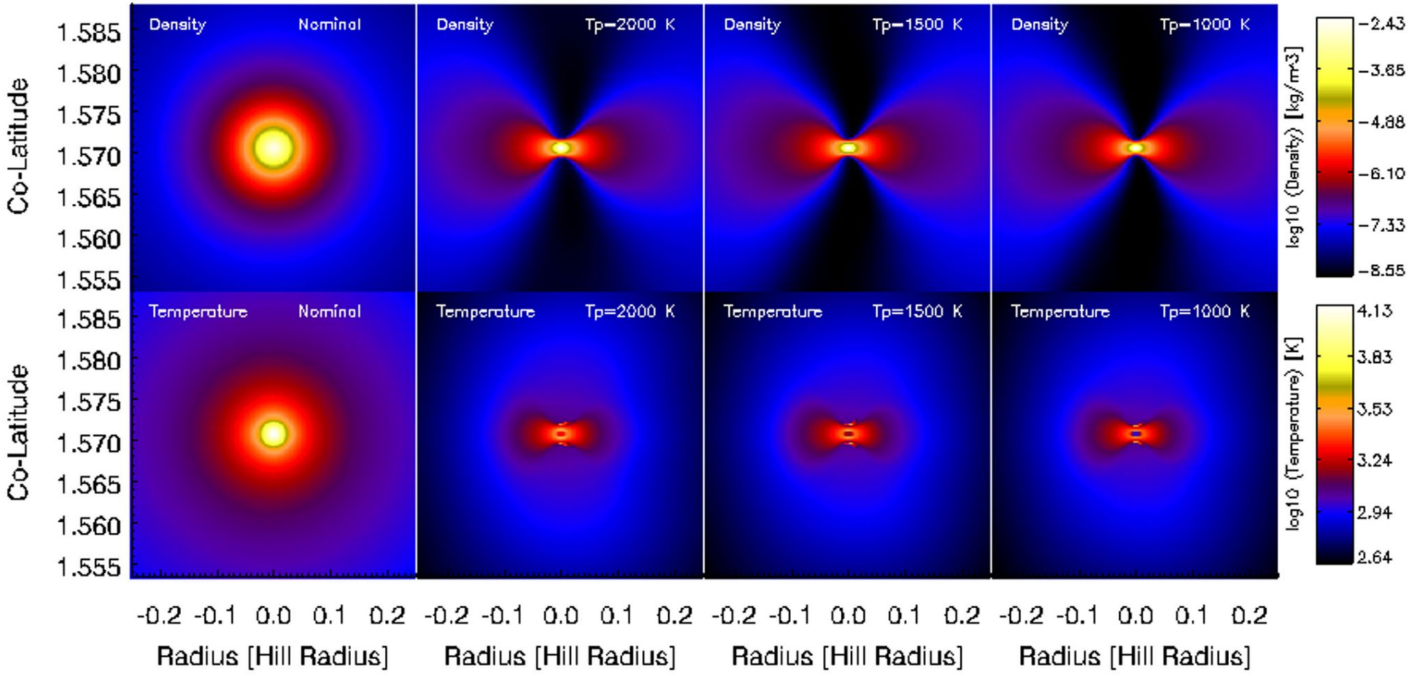



Fung et al. (2019)

# Post-runaway accretion

- Can a circumplanetary disk (CPD) be formed?

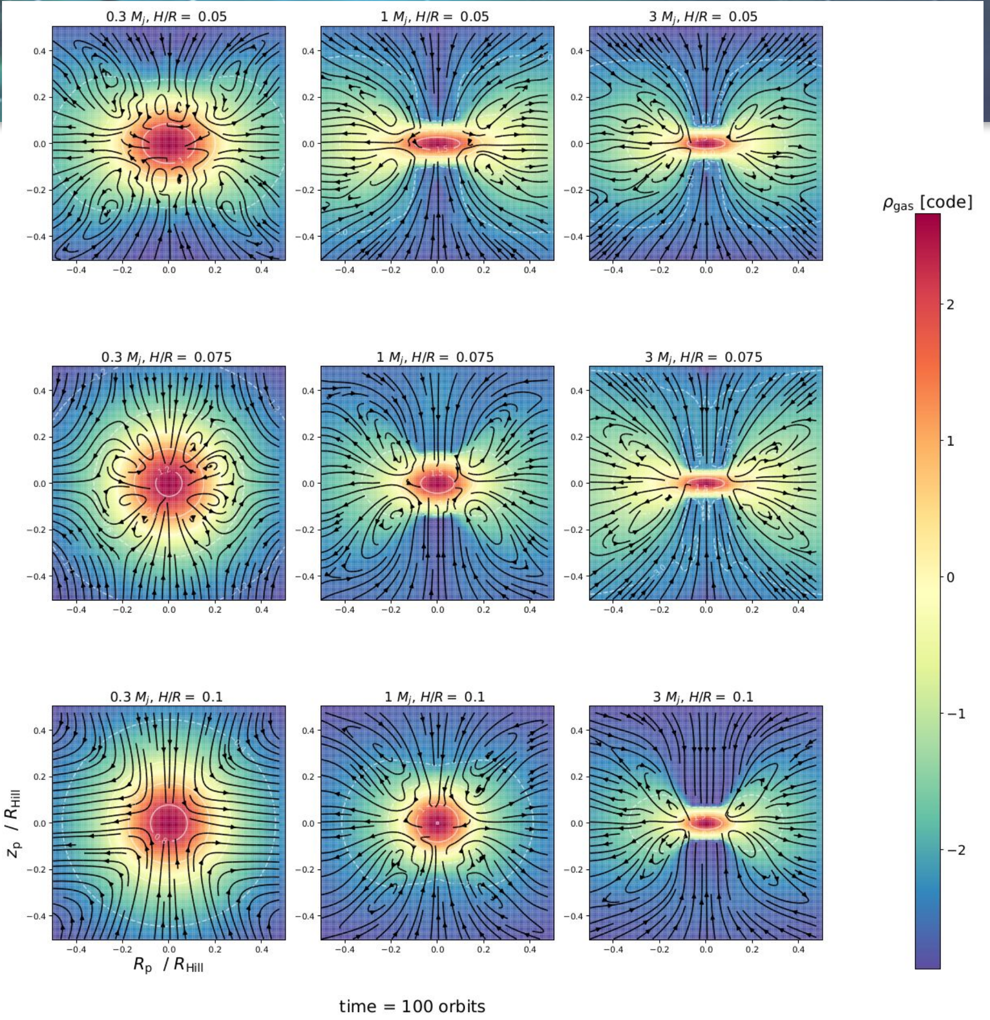
radiative model; artificial cooling added by fixing the temperature at the planet location




Szulagyi et al. (2016)

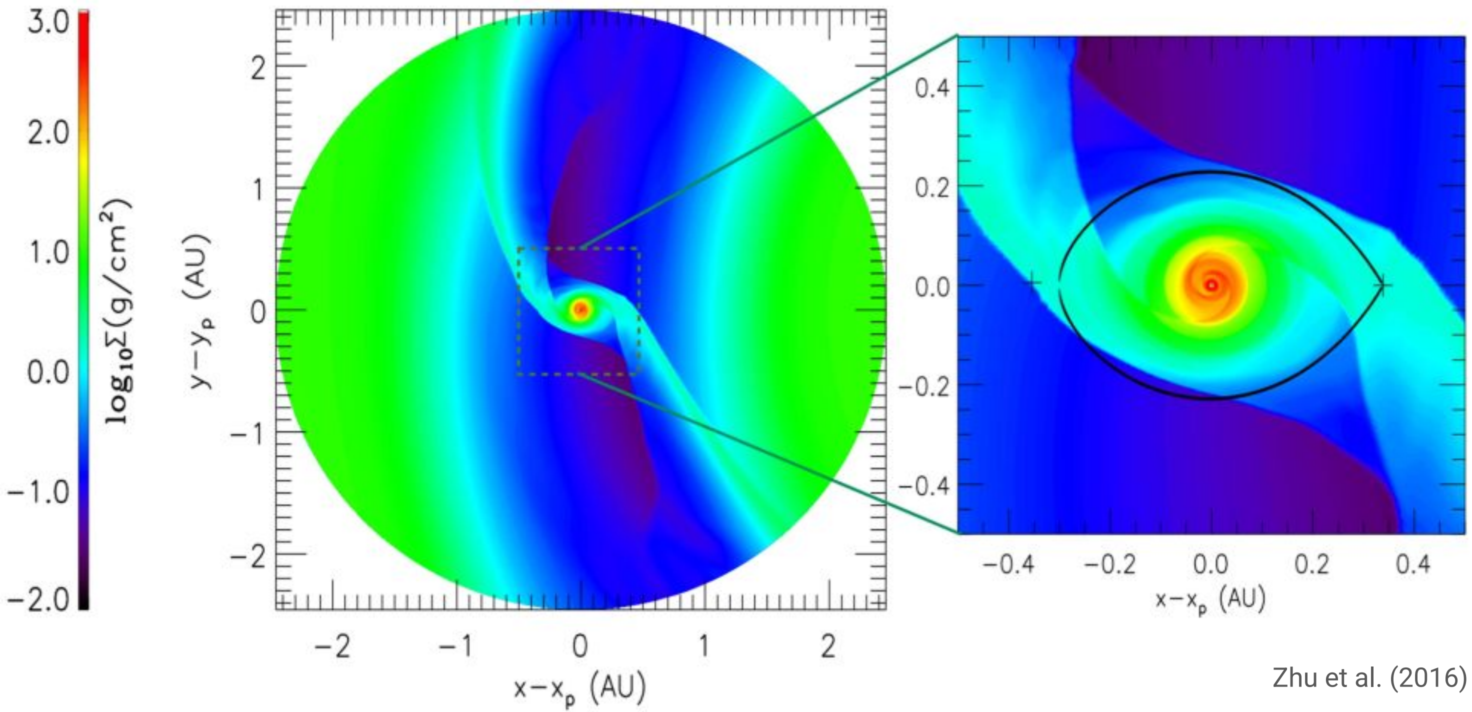
# Post-runaway accretion

- Krapp et al. (2024; radiative simulations):
  - CPD is a rotationally-supported structure, so to form it, enough angular momentum and reduced pressure support is needed
  - if luminosity release → increased pressure support → meridional circulation transporting angular momentum outward: then the outcome is a pressure-supported envelope
  - for a CPD, cooling time has to be <0.1 orbital period
- Sagynbaeva et al. (2024; isothermal simulations): see the **figure** exploring the importance of the local thermal mass



# CPD morphology

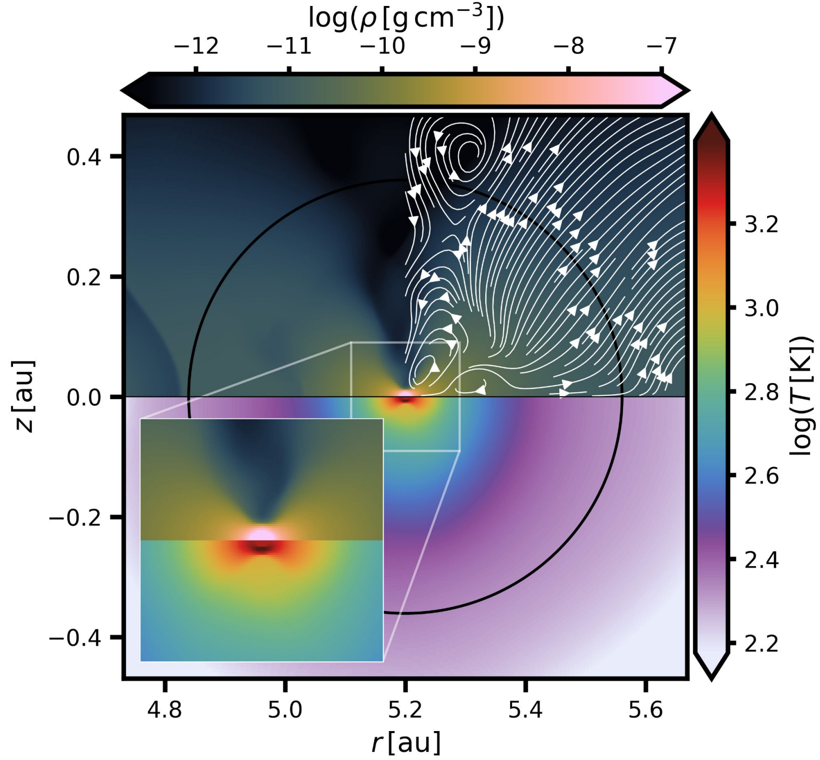
- CPD size: about 30% of the Hill radius, then truncated by the surrounding tidal field




Zhu et al. (2016)

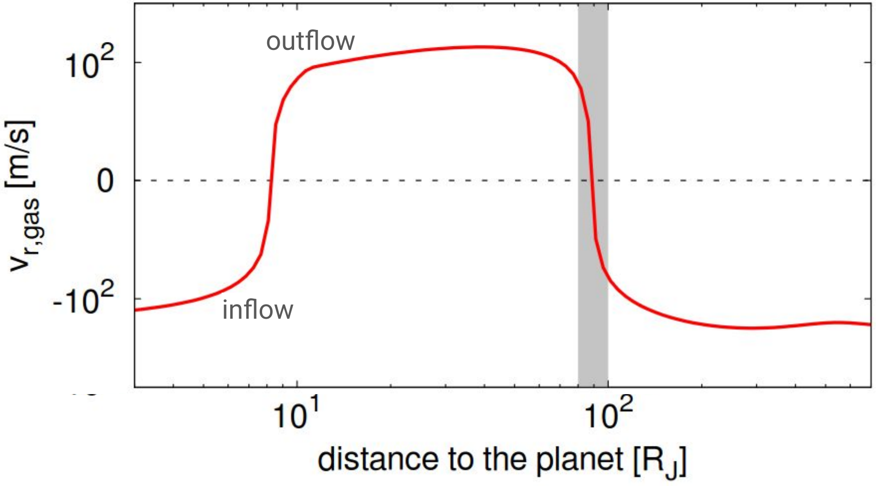
# CPD morphology

- Polar inflow, midplane outflow




my test simulation with Fargo3D

example planetocentric gas velocity in the CPD midplane:




Drazkowska & Szulagyi (2018)