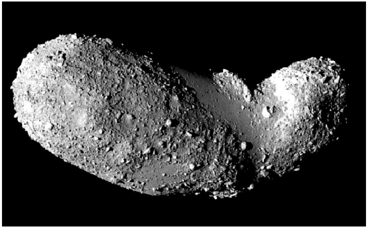
Thermal emission from boulders and their influence on the YORP effect

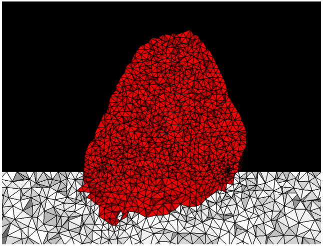
Pavel Ševeček[1], Miroslav Brož[1], David Čapek[2], Josef Ďurech[1]

[1] Institute of Astronomy, Charles University, Prague, Czech Republic

[2] Astronomical Institute of the Academy of Sciences, Ondřejov, Czech Republic

Ševeček et al. (2015), MNRAS, 450, 2104





YORP effect and small-scale topography

- non-gravitational torque acting on asteroids (Rubincam, 2000)
- total torque due to thermal emission:

$$\boldsymbol{T} = \frac{\varepsilon\sigma}{c} \oint_{\partial\Omega} u^4 \, \boldsymbol{h} \times \boldsymbol{r} \, \mathrm{d}\Gamma$$

- existing models — global YORP effect, usually do not include small topographic features (Breiter et al. 2009, Lowry et al. 2014)
- but small-scale topography seems to be important (Statler, 2009)
- transversal heat conduction in surface features
  $\longrightarrow$ non-zero average torque
- Golubov & Krugly (2012), Golubov et al. (2014)
  $\longleftarrow$ idealized boulders

Goals

1. solve the heat diffusion equation inside the boulder:

$$\nabla \cdot (K \nabla u) - \rho C \frac{\partial u}{\partial t} = 0$$

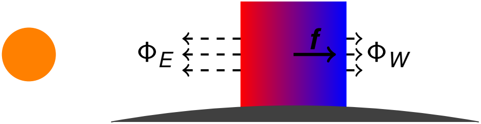
three-dimensional parabolic partial differential equation, non-linear and time-dependent boundary condition:

$$K \boldsymbol{n} \cdot \nabla u + \varepsilon \sigma u^4 = (1 - A)\Phi_{\odot} \cos \vartheta_{\odot} \mu + \mathcal{E}_{\mathrm{th}} + \mathcal{E}_{\mathrm{sc}}$$

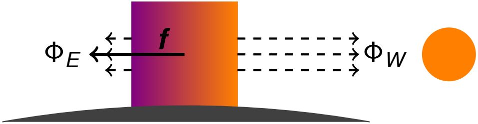
2. derive the dimensionless pressure Π caused by the boulder
3. compute the sum of torques boulders exert on the asteroid
4. compare our result with the models of the global YORP effect

Mean recoil force on symmetric boulders

- east-west asymmetry of emission in boulders of suitable sizes (Golubov & Krugly 2012)
- heat conduction in boulder $\longrightarrow$ higher temperature on western side (essentially similar to the Yarkovsky effect)
- morning

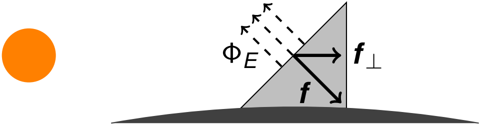


- afternoon

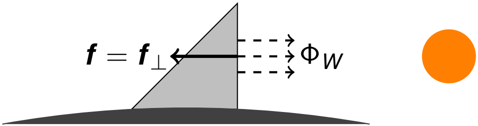


Mean recoil force on asymmetric boulders

- inherently non-zero force due to geometry
- morning

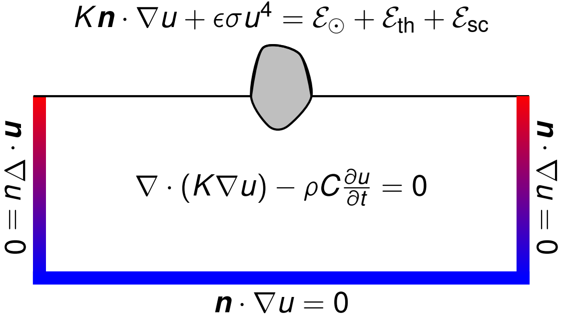


- afternoon



$\longrightarrow$ averaging of the force over different orientations is needed

Boundary conditions



- surface — incoming flux = outgoing flux
- bottom — zero temperature gradient
- sides — only changes of temperature perpendicular to the surface

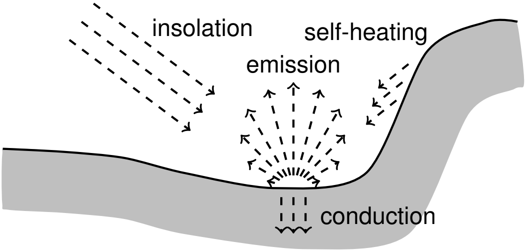
Boundary conditions

- direct insolation $\mathcal{E}_\odot = (1 - A)\Phi_\odot \cos\vartheta_\odot \mu$
- thermal emission

$$\mathcal{E}_{\rm th} = (1 - A) \oint_{\partial\Omega} \varepsilon' \sigma u'^4 \frac{\cos\alpha \cos\alpha'}{\pi \|\boldsymbol{r} - \boldsymbol{r}'\|^2} \nu \, {\rm d}\Gamma'$$

- scattered radiation

$$\mathcal{E}_{\rm th} = (1 - A) \oint_{\partial\Omega} A' \Phi \mu' \cos\vartheta'_\odot \frac{\cos\alpha \cos\alpha'}{\pi \|\boldsymbol{r} - \boldsymbol{r}'\|^2} \nu \, {\rm d}\Gamma'$$



Initial conditions

- linearized analytical solution in a half-space:

$$u_{\text{theory}}(z,t) = u_0 + \sum_{n=1}^{N} \frac{\mathcal{E}_n}{4\varepsilon\sigma u_0^3} \frac{1}{\sqrt{2\Theta^2 + 2\Theta + 1}} e^{-\beta_n z} \cos\left(n\omega t - \beta_n z + \varphi_n\right)$$

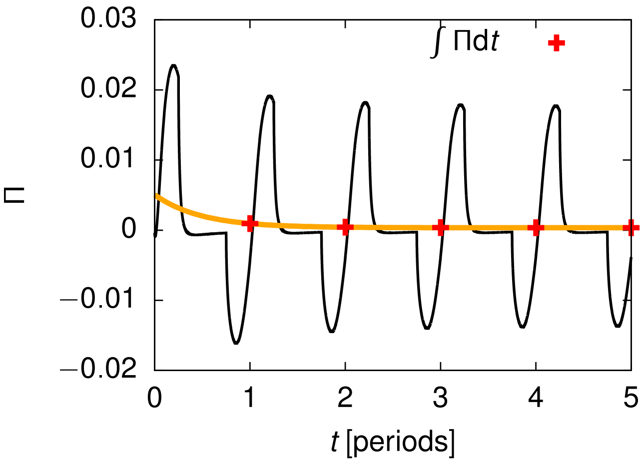
- a linearized solution yields identically zero torque :(nevertheless, we can use it as our initial condition:

$$u(\boldsymbol{r}, 0) = u_{\text{theory}}(z, 0)$$

- we seek for a periodic solution
  $\longrightarrow$ solution should not depend on initial conditions

Initial relaxation of thermal pressure

- vanishing influence of initial conditions



Finite element method (FEM)

- approximation of temperature distribution by a linear combination of basis functions N_i:

$$u(\boldsymbol{r},t) \approx \hat{u}(\boldsymbol{r},t) \equiv \sum_{i=1}^{M} u_i(t) N_i(\boldsymbol{r})$$

- discrete weak formulation, Galerkin method:

$$\int_\Omega \left(\nabla \cdot (K \nabla \hat{u}) - \rho C \frac{\partial \hat{u}}{\partial t} \right) N_i \,\mathrm{d}\Omega = 0$$

- second-order equation, but we can use the divergence theorem:

$$\int_\Omega \nabla \cdot (K \nabla \hat{u}) N_i \,\mathrm{d}\Omega = \oint_{\partial\Omega} K \boldsymbol{n} \cdot \nabla \hat{u} N_i \,\mathrm{d}\Gamma - \int_\Omega K \nabla \hat{u} \cdot \nabla N_i \,\mathrm{d}\Omega$$

$\longrightarrow$ incorporation of boundary conditions

Temporal discretization & linearization

- temporal derivative — implicit Euler scheme

$$\hat{u}(\boldsymbol{r}, t) \to \hat{u}^n(\boldsymbol{r}) \equiv \hat{u}(\boldsymbol{r}, n\Delta t)$$

$\longrightarrow$ algebraic system of non-linear equations

- non-linear term $\hat{u}^4$ — semi-linearization $\hat{u}^4 \approx \hat{u}\hat{u}_0^3$, method of successive substitutions
- convergence issues — relaxation technique
- in a given timestep and iteration, we solve a sparse linear system using conjugate gradient (CG) method

Weak formulation

- weak formulation of the heat diffusion equation:

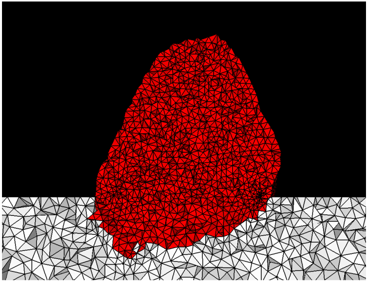
$$\int_{\Omega} K \nabla \hat{u}^n \cdot \nabla N_j \, \mathrm{d}\Omega + \int_{\Omega} \frac{\rho C}{\Delta t} \hat{u}^n N_j \, \mathrm{d}\Omega + \int_{\Gamma_1} \varepsilon \sigma \hat{u}^n \hat{u}_0^3 N_j \, \mathrm{d}\Gamma = \int_{\Omega} \rho C \frac{\hat{u}^{n-1}}{\Delta t} N_j \, \mathrm{d}\Omega + \int_{\Gamma_1} \mathcal{E} N_j \, \mathrm{d}\Gamma$$

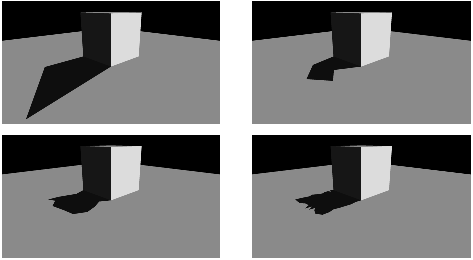
- employing the `FreeFem++` code (Hecht, 2012)

```
problem HDE(u,n,solver=CG)
= int3d(Th) (K/S^2*(dx(u)*dx(n) + dy(u)*dy(n) + dz(u)*dz(n)))
+ int3d(Th) (rho*C*u/dt*n) - int3d(Th) (rho*C*u0/dt*n)
+ int2d(Th, SURFACE) (epsil*sigma/S*u*v^3*n)
- int2d(Th, SURFACE) ((1-A)*Phi/S*n*Shadow()*cos(theta))
- int2d(Th, SURFACE) ( SelfHeating() * n / S)
+ on(BOTTOM, u=U)
+ on(SIDE, u=Utheory());
...
```

Spatial discretization

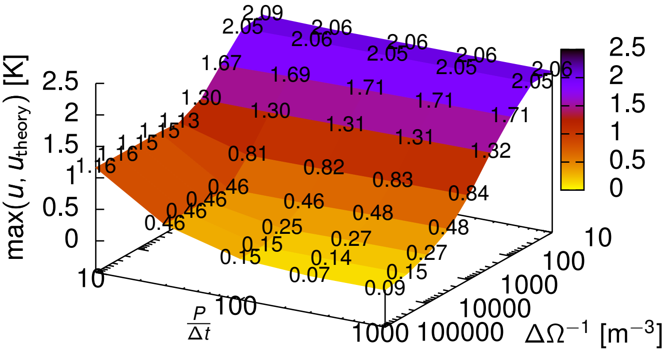
- Delaunay tetrahedralization using `tetgen` code (Si, 2006)
- affects accuracy of FEM and shadows casted by the boulder



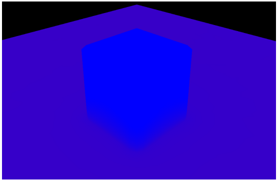


Tests of convergence

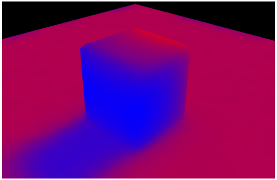
- analytical solution in a half-space usable for checking of numerical solution
- parameters — time step Δt, volume of tetrahedra $\Delta\Omega$



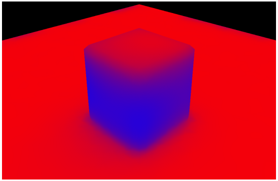
Evolution of the surface temperature (example)



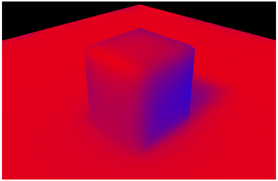
$t = 6\,\mathrm{h}$



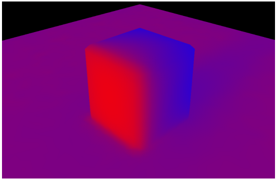
$t = 9\,\mathrm{h}$



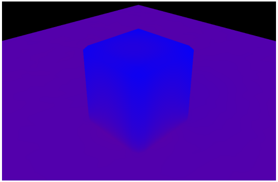
$t = 12\,\mathrm{h}$



$t = 15\,\mathrm{h}$



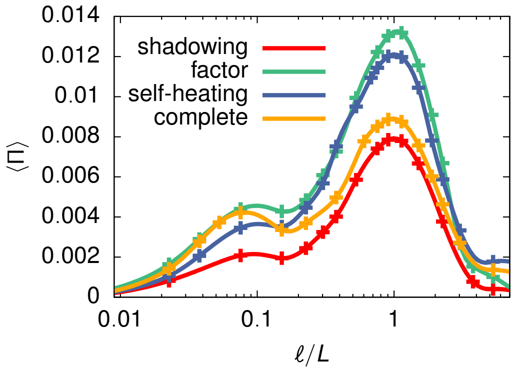
$t = 18\,\mathrm{h}$



$t = 21\,\mathrm{h}$

Four model variants

Shadowing	$\mathcal{E} = (1 - A)\Phi_\odot \cos\vartheta_\odot \mu$	$\boldsymbol{h} = \frac{2}{3}\boldsymbol{n}$
Irradiative factor	$\mathcal{E} = 2(1 - A)\Phi_\odot \cos\vartheta_\odot \mu$	$\boldsymbol{h} = \frac{2}{3}\boldsymbol{n}$
Self-heating	$\mathcal{E} = (1 - A)\Phi_\odot \cos\vartheta_\odot \mu + \mathcal{E}_{th} + \mathcal{E}_{sc}$	$\boldsymbol{h} = \frac{2}{3}\boldsymbol{n}$
'Complete' model	$\mathcal{E} = (1 - A)\Phi_\odot \cos\vartheta_\odot \mu + \mathcal{E}_{th} + \mathcal{E}_{sc}$	$\boldsymbol{h} \neq \frac{2}{3}\boldsymbol{n}$



- self-heating causes an increase of $\langle \Pi \rangle$, close to 'factor'
- 'complete' close to 'shadowing'

Behaviour of the solution at limits

- high-conductivity limit ($K \to \infty$) $\longrightarrow$ isothermal boulder:

$$\boldsymbol{f} = -\frac{2}{3}\frac{\varepsilon\sigma u^4}{c}\oint_{\partial\Omega}\boldsymbol{n}\,\mathrm{d}\Gamma = 0$$

$\longrightarrow$ zero force for all boulders

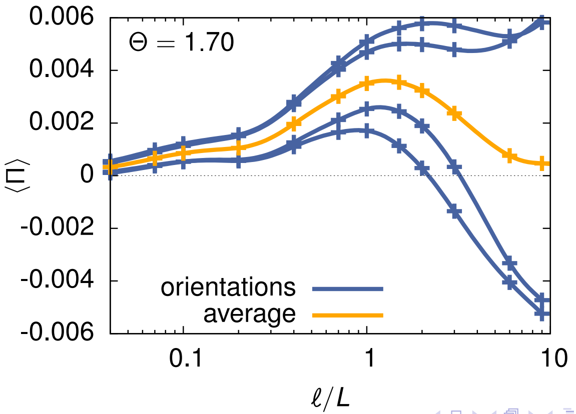
- zero-conductivity limit ($K \to 0$, aka Rubincam approximation):

$$\langle\boldsymbol{f}\rangle = -\frac{2}{3}\frac{\varepsilon\sigma}{c}\oint_{\partial\Omega}\langle\mathcal{E}\rangle\boldsymbol{n}\,\mathrm{d}\Gamma = -\frac{2}{3}\frac{\varepsilon\sigma}{c}\left(\oint_{\Gamma}\langle\mathcal{E}\rangle\boldsymbol{n}\,\mathrm{d}\Gamma + \oint_{\Gamma'}\langle\mathcal{E}\rangle\boldsymbol{n}\,\mathrm{d}\Gamma\right) = 0$$

$\longrightarrow$ zero average force for symmetric boulders

Asymmetric shape

- for asymmetric boulders $\langle \Pi \rangle$ does not approach zero for $K \to 0$
- we average over four orientations of the boulder
  $\longrightarrow$ no systematic error in the limit $K \to 0$



Angular acceleration of a spherical asteroid

- we assume:
  - spherical asteroid of radius $r = 1\,\mathrm{km}$, density $\rho = 2500\,\mathrm{kg/m^3}$, semi-major axis $a = 2.5\,\mathrm{AU}$
  - isotropic distribution of boulders — relative coverage $f = 0.1$
  - mean dimensionless pressure — $\Pi_0 = (2 \pm 1) \times 10^{-3}$
- angular acceleration:

$$\frac{\mathrm{d}\omega}{\mathrm{d}t} \simeq 5.5\,\frac{f\Pi_0}{\rho r^2}\frac{(1-A)\Phi_\odot}{c} \doteq (2 \pm 1) \times 10^{-9}\,\mathrm{rad/d^2}$$

$\longrightarrow$ spin-up timescale $\tau \equiv P/(\mathrm{d}P/\mathrm{d}t) = (32 \pm 16)\,\mathrm{Myr}$

- global YORP effect for Gaussian random spheres (Čapek & Vokrouhlický, 2004)

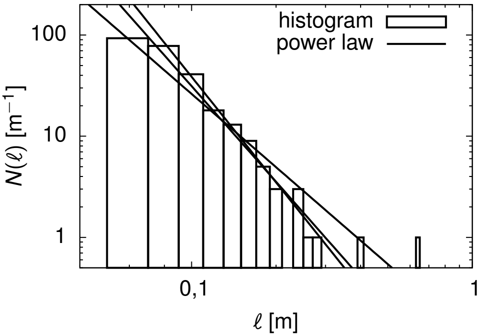
$\tau = 14.3\,\mathrm{Myr}$ i.e. comparable

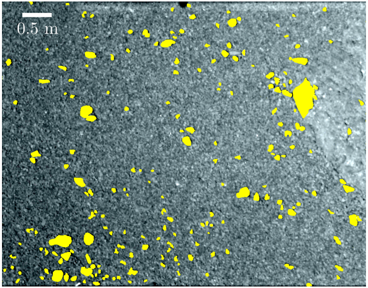
Real boulders on (25143) Itokawa

- Hayabusa mission — detailed images of the surface (Saito et al. 2010)
- size-frequency distribution of boulders

$$N(\ell)\,\mathrm{d}\ell = (3.9 \pm 1.8) \times 10^5 \times [\ell]_{\mathrm{m}}^{-(2.4\pm0.3)}\,\mathrm{d}\ell$$

$\longrightarrow$ boulders cover 19 % to 32 % of the surface





Comparison of the results for Itokawa

- acceleration due to boulders (using derived SFD)

$$\frac{\mathrm{d}\omega}{\mathrm{d}t} = (1.20 \pm 0.11) \times 10^{-7}\,\mathrm{rad/d^2}$$

+ global-shape YORP model prediction:

Breiter et al. (2009) $\quad \frac{\mathrm{d}\omega}{\mathrm{d}t} = -(2.5 \text{ to } 5.5) \times 10^{-7}\,\mathrm{rad/d^2}$

Lowry et al. (2014) $\quad \frac{\mathrm{d}\omega}{\mathrm{d}t} = -(1.80 \pm 1.96) \times 10^{-7}\,\mathrm{rad/d^2}$

= observed acceleration — Lowry et al. (2014)

$$\frac{\mathrm{d}\omega}{\mathrm{d}t} = (0.35 \pm 0.04) \times 10^{-7}\,\mathrm{rad/d^2}$$

Conclusions & future work

- thermal emission from boulders can potentially explain the observed acceleration of (25143) Itokawa
  $\longrightarrow$ alternative explanation to density inhomogeneities (Lowry et al. 2014)
- for accurate quantitative prediction it is necessary to constrain
  - size-frequency distribution of boulders (Tancredi et al. 2015)
  - shapes of boulders (statistics)
- interconnection with global models — global self-heating, global shadowing, mutual shadowing between boulders, ...
- other applications — heat conduction in small meteoroids
  (Brož & Čapek in prep.)