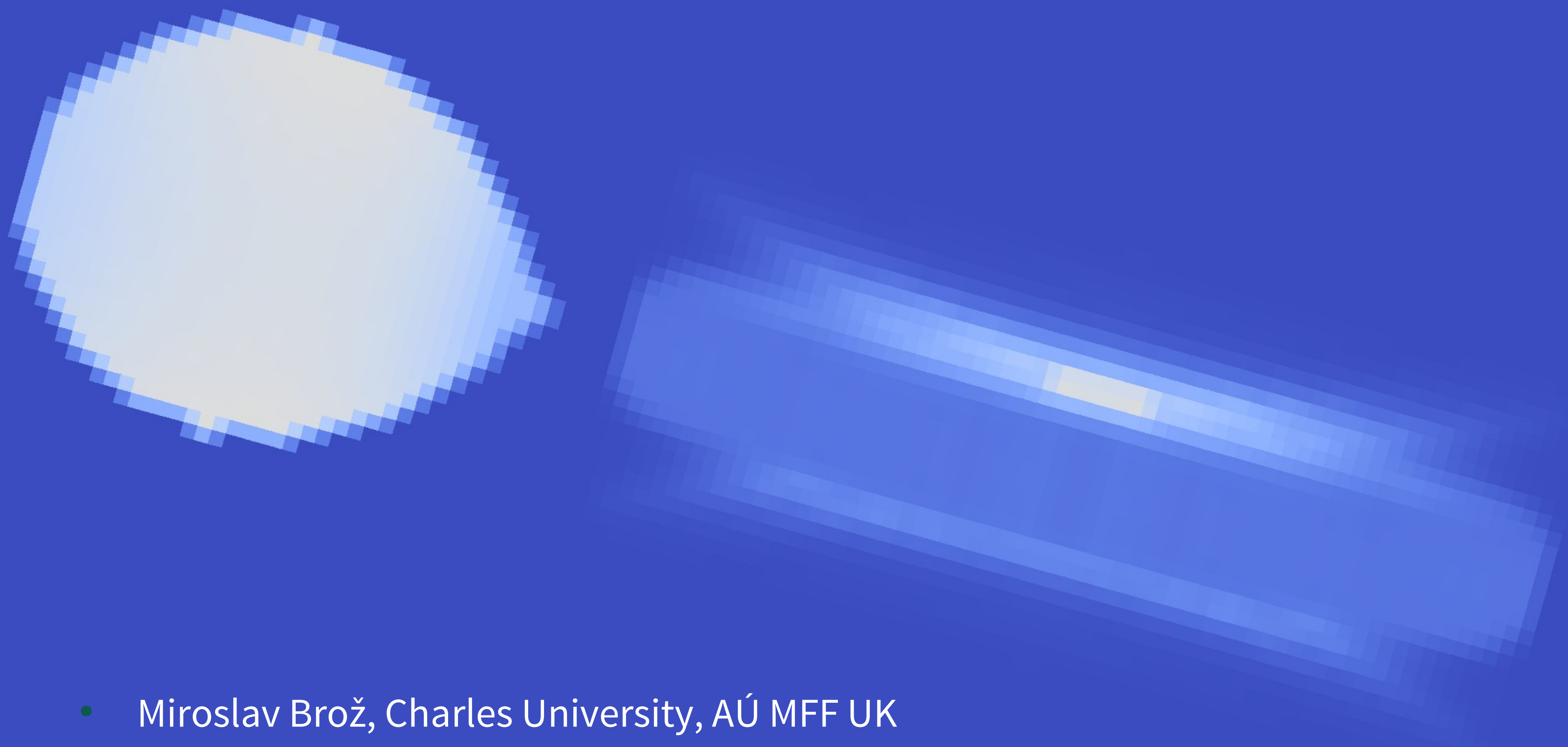
Modelling of complex stellar systems

the role of **multiplicity** && **circumstellar matter**



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- in collaboration with: P. Harmanec, **J. Nemravová**, D. Mourard, J. Budaj, ...

Model (N-body)

← based on SWIFT (Levison & Duncan 1994)

+ numerical integrator (BS)
+ i. to the exact times of observations
+ 4 coordinate systems (b/1/p/J)
+ astrometric positions (photocentric c.)
+ RV (barycentric c.)
+ minima timings (TTVs; 1-centric)
+ eclipse durations
+ interferometric visibility
+ closure phase
+ synthetic spectra (normalized)
+ SED
+ optionally, lightcurves (WD code)
+ joint χ^2, simplex, simulated annealing

Wilson & Devinney (2005, ...)

− ‘good old’ Fortran 77
+ but can call **Pyterpol** (Nemravová etal. 2016) 'on-the-fly' to generate synthetic spectra from Ostar, Bstar, Pollux, Ambre, or Phoenix grids
+ Gnuplot scripts to plot all observations/calculations, datasets, individual χ^2 contributions, χ^2 outliers, zoom-in-zoom-out, ...

dynamical eqs.

$$\ddot{\boldsymbol{r}}_{\mathrm{b}i} = -\sum_{j\neq i}^{N_{\mathrm{bod}}} \frac{Gm_j}{r_{ji}^3}\,\boldsymbol{r}_{ji} + \boldsymbol{f}_{\mathrm{tidal}} + \boldsymbol{f}_{\mathrm{oblat}} + \boldsymbol{f}_{\mathrm{ppn}}; \tag{1}$$

$$\boldsymbol{f}_{\mathrm{tidal}} = -\sum_{j\neq i}^{N_{\mathrm{bod}}} 3k_{\mathrm{L}i}\frac{Gm_j^2}{m_i}\frac{R_i^5}{r_{ij}^8}\boldsymbol{r}_{ji}; \tag{2}$$

$$\boldsymbol{f}_{\mathrm{oblat}} = -\sum_{j\neq i}^{N_{\mathrm{bod}}} \frac{1}{2}k_{\mathrm{L}j}\,\omega_{\mathrm{rot}\,j}^2\frac{R_i^5}{r_{ij}^5}\boldsymbol{r}_{ji}; \tag{3}$$

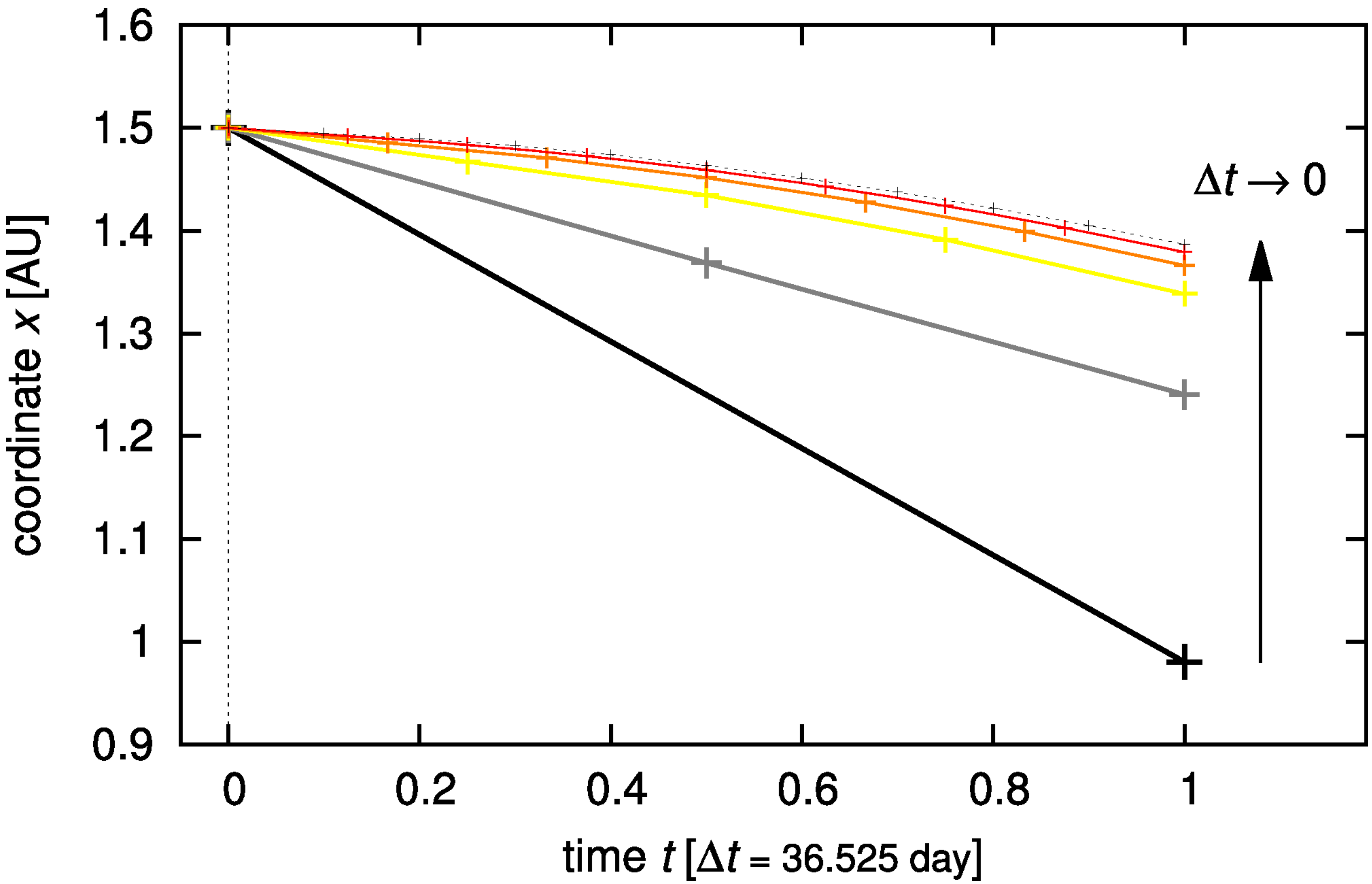
$$\boldsymbol{f}_{\mathrm{ppn}} = -\sum_{j>i}^{N_{\mathrm{bod}}} \frac{G\,(m_j+m_i)}{r_{ij}^2\,c^2}\Bigg\{-2(2-\eta_{ij})\,\dot{r}_{ij}\dot{\boldsymbol{r}}_{ij} + \left[(1+3\eta_{ij})\dot{\boldsymbol{r}}_{ij}\cdot\dot{\boldsymbol{r}}_{ij} - \frac{3}{2}\eta_{ij}\dot{r}_{ij}^{\,2} - 2(2+\eta_{ij})\frac{G\,(m_j+m_i)}{r_{ij}}\right]\frac{\boldsymbol{r}_{ij}}{r_{ij}}\Bigg\}. \tag{4}$$

$$t'_{\mathrm{ecl}} = t_{\mathrm{min}} + \frac{z_{\mathrm{b}1+2} - z_{\mathrm{b}1+2}(t=T_0)}{c} - \frac{z_{\mathrm{h}2}}{c}; \tag{5}$$

$$\epsilon'_{\mathrm{ecl}} = \frac{2}{\bar{v}_{\mathrm{h}2}}\sqrt{(R_1+R_2)^2 - \Delta_{\mathrm{min}}^2}\,; \tag{6}$$

Bulirsch-Stoer integrator

- decreasing time steps Δt → extrapolation by a rational $f(x)$



radiative eqs.

cf. black-body

$$L_j(T_{\mathrm{eff}\,j}, R_j) = 4\pi R_j^2 \times \int_{\lambda-\Delta\lambda/2}^{\lambda+\Delta\lambda/2} F_{\mathrm{syn}}(\lambda, T_{\mathrm{eff}\,j}, \log g_j, v_{\mathrm{rot}\,j}, \mathcal{Z}_j)\,d\lambda; \qquad (7)$$

$$V'(u, v) = \sum_{j=1}^{N_{\mathrm{bod}}} \frac{L_j}{L_{\mathrm{tot}}} \left(\frac{\alpha}{2} + \frac{\beta}{3}\right)^{-1} \times \left[\alpha \frac{J_1(\Theta)}{\Theta} + \beta \sqrt{\frac{\pi}{2}} \frac{J_{3/2}(\Theta)}{\Theta^{3/2}}\right] e^{-2\pi i (u x_{\mathrm{a}j} + v y_{\mathrm{a}j})}; \qquad (8)$$

$$T_3{}' = V'(u_1, v_1) V'(u_2, v_2) V'(-(u_1 + u_2), -(v_1 + v_2)); \qquad (9)$$

$$I'_\lambda = \sum_{j=1}^{N_{\mathrm{bod}}} \frac{L_j}{L_{\mathrm{tot}}} I_{\mathrm{syn}}\left[\lambda\left(1 - \frac{v_{z\mathrm{b}j+\gamma}}{c}\right), T_{\mathrm{eff}\,j}, \log g_j, v_{\mathrm{rot}\,j}, \mathcal{Z}_j\right]; \qquad (13)$$

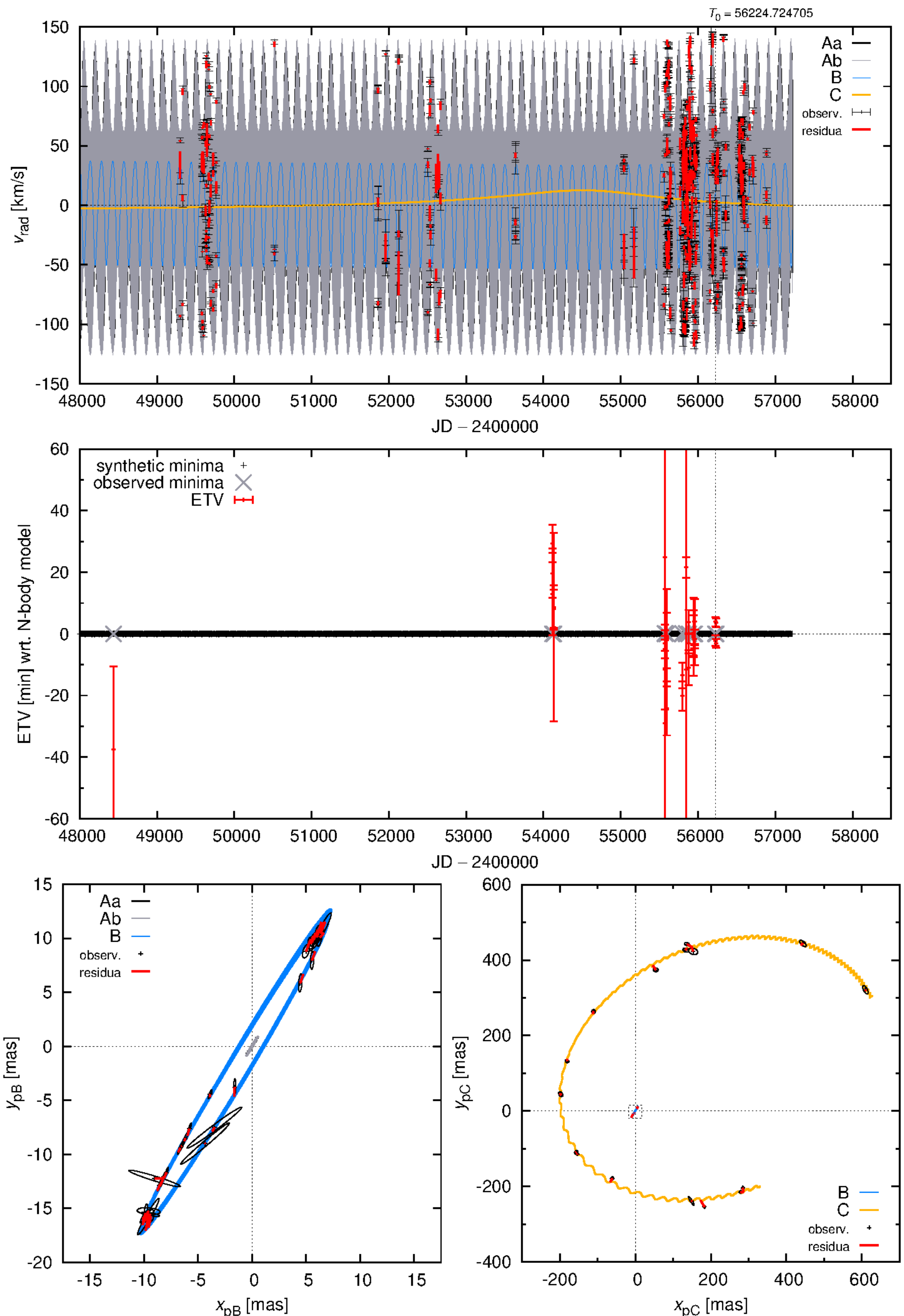
$$F'_V = \sum_{j=1}^{N_{\mathrm{bod}}} \left(\frac{R_j}{d}\right)^2 \int_0^\infty F_{\mathrm{syn}}[\lambda, T_{\mathrm{eff}\,j}, \log g_j, v_{\mathrm{rot}\,j}, \mathcal{Z}_j] f_V(\lambda)\,d\lambda, \qquad (14)$$

```fortran
c geometry_hierarch.f
c Convert elements to barycentric coordinates for geometry like ((1+2)+3)+4.
c Miroslav Broz (miroslav.broz@email.cz), Jun 7th 2016

c      _          \               |
c     / \          |              |
c     1 2          3              4
c     \_/          |              |
c                 /               |

      subroutine geometry_hierarch(nbod, m, elmts, r, v)

      implicit none
      include 'simplex.inc'
      include '../misc/const.inc'
c input
      integer nbod
      real*8 m(NBODMAX)
      real*8 elmts(NBODMAX,6)
c output
      real*8 r(NBODMAX,3), v(NBODMAX,3)
c internal
      integer i, k, ialpha
      real*8 rj(NBODMAX,3), vj(NBODMAX,3)
      real*8 msum, P, a, n, tmp

c convert to radians
      do i = 2, nbod
        do k = 3, 6
          elmts(i,k) = elmts(i,k)*deg
        enddo
      enddo
```

```
c compute Jacobian coordinates
      msum = m(1)
      ialpha = -1
      do i = 2, nbod
        msum = msum + m(i)
        P = elmts(i,1)
        n = 2.d0*pi_/P
        a = (msum / n**2)**(1.d0/3.d0)
        call orbel_el2xv(msum,ialpha,
     :    a,elmts(i,2),elmts(i,3),
     :    elmts(i,4),elmts(i,5),elmts(i,6),
     :    rj(i,1),rj(i,2),rj(i,3),
     :    vj(i,1),vj(i,2),vj(i,3))

c adjust coordinates (the elements were standard stellar-astronomy)
        tmp = rj(i,1)
        rj(i,1) = -rj(i,2)
        rj(i,2) = tmp
        rj(i,3) = -rj(i,3)
        tmp = vj(i,1)
        vj(i,1) = -vj(i,2)
        vj(i,2) = tmp
        vj(i,3) = -vj(i,3)
      enddo

c convert to barycentric frame
      call coord_j2b(nbod,m,
     :  rj(1,1),rj(1,2),rj(1,3),vj(1,1),vj(1,2),vj(1,3),
     :  r(1,1),r(1,2),r(1,3),v(1,1),v(1,2),v(1,3))

      return
      end
```

ξ Tauri

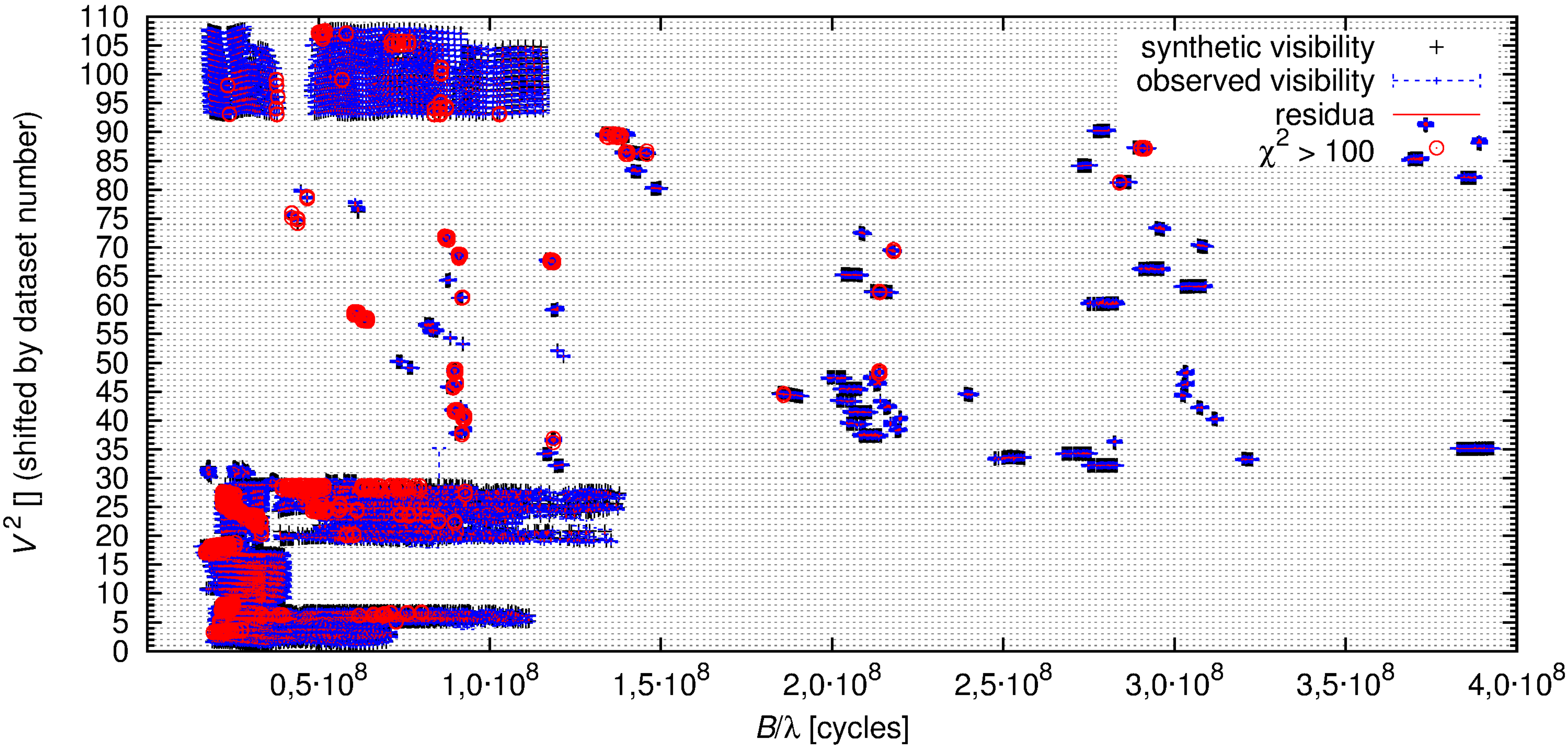
radial velocities
minima timings
astrometry
...



Nemravová et al. (2016),
A&A **594**, 55,
highlight

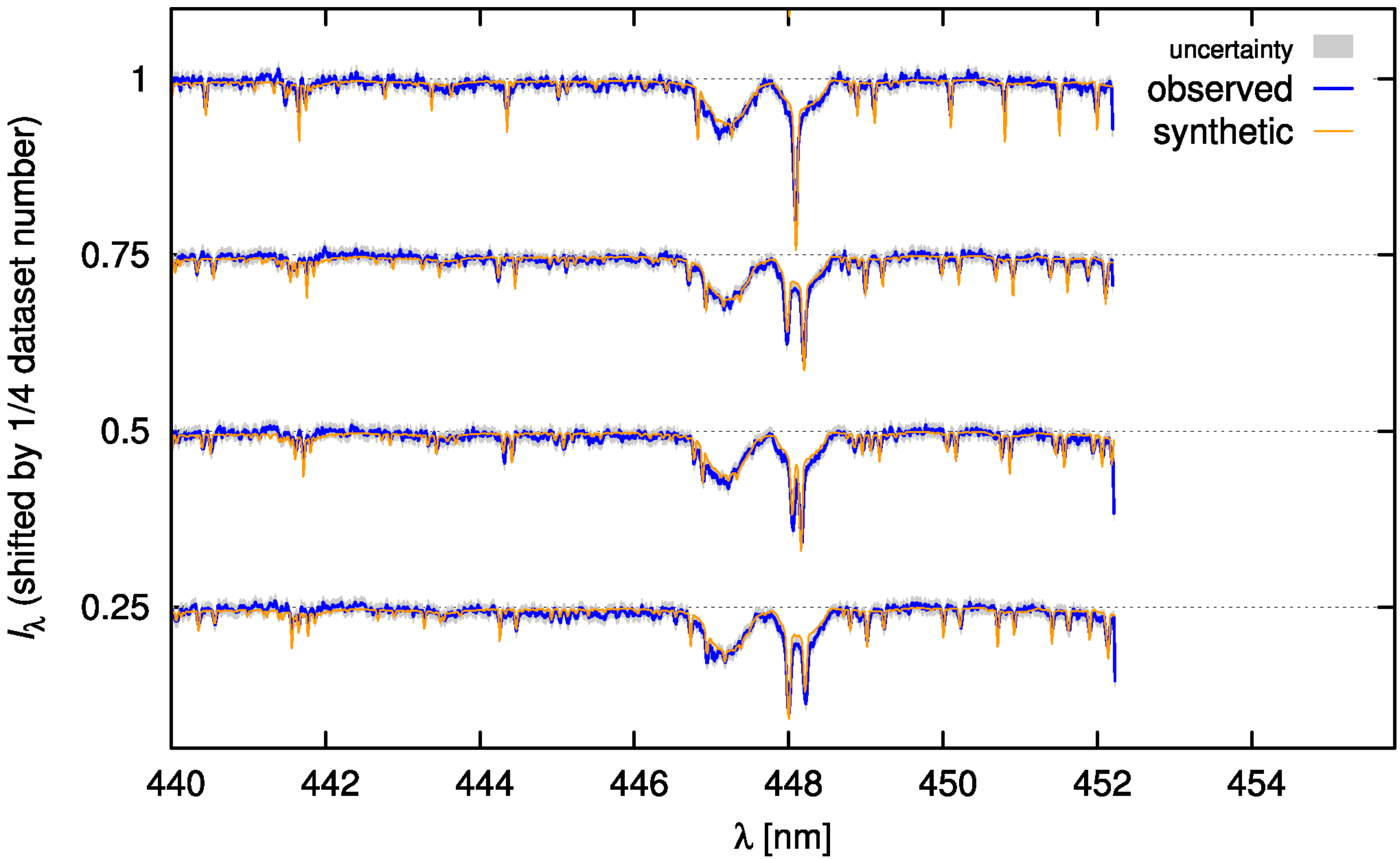
ξ Tauri (VIS)

- NPOI, MARK-III, CHARA, VLTI/AMBER d. ↔ mirror solutions (8)



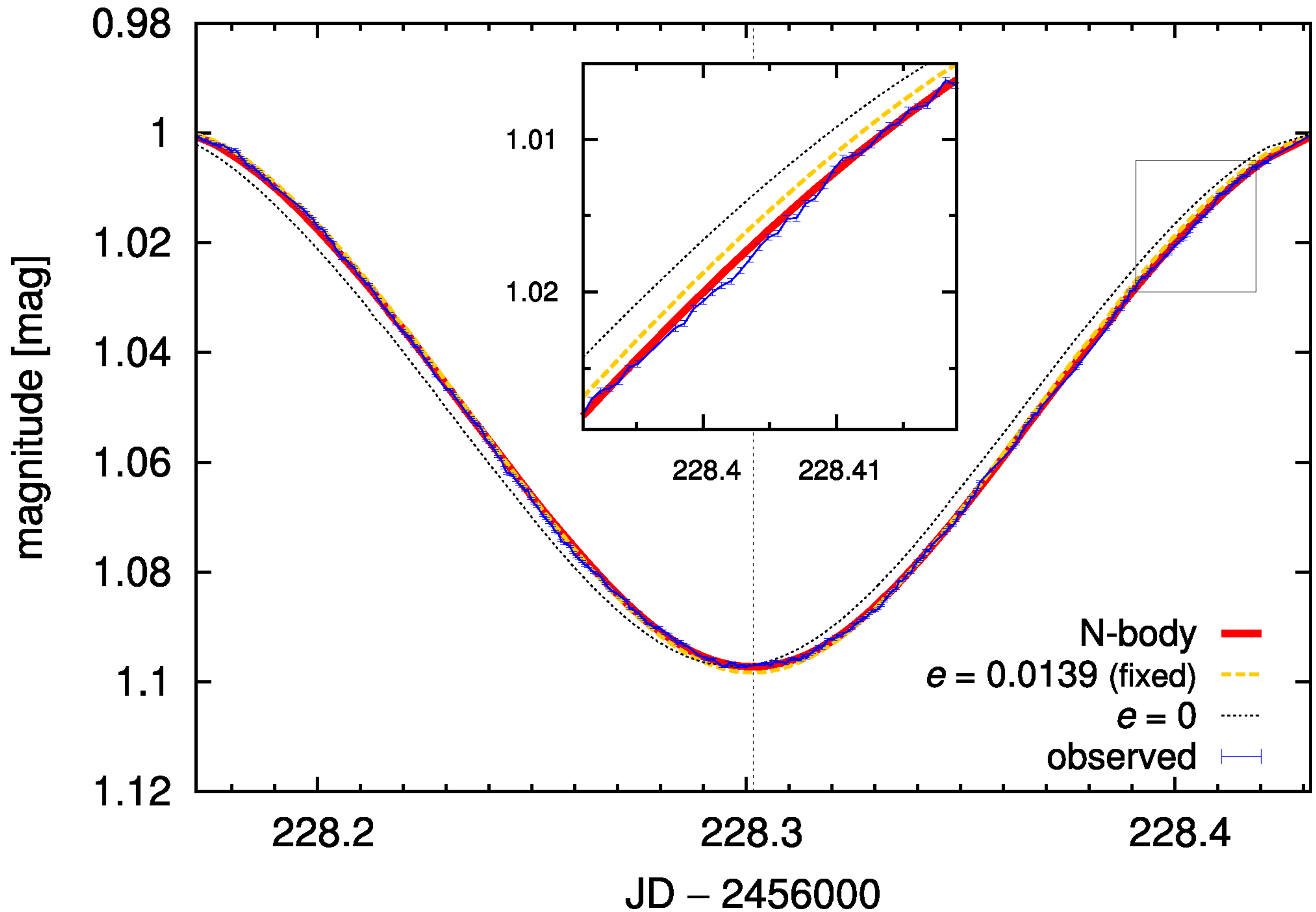
ξ Tauri (SYN)

- c. in direct space, large # of lines; no problem w. blends



ξ Tauri (LC)

- interaction (changing Φ of the 3rd *) on orbital time scale



N-body effects ← see also Fabrycky (2010)

1. ω and Ω precession + tides, oblateness, GR
2. i vs eclipse duration, mutual inclination J
3. e oscillations
4. Kozai cycles
5. variation && evection (4 vs 8)
6. prograde vs retrograde
7. mean-motion/secular/3-body resonances
8. chaotic diffusion
9. long-term stability?
10. close encounters? (unlikely)

⋮

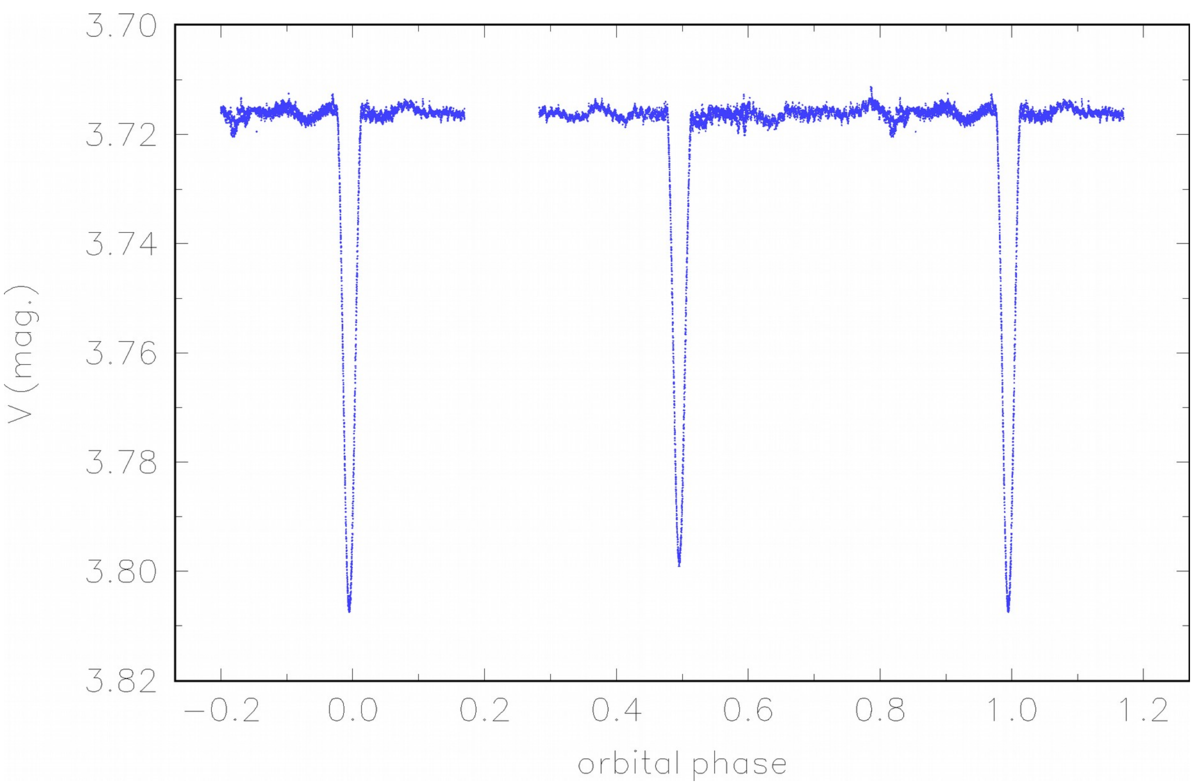
Hidden problems

- discretisation (!)
- three radii (effective, limb-darkened, average) ~ the same
- multiple eclipses, 2+2 eclipsing binaries *not* available
- too cold/hot * outside the grid
- local minima (!)
- convergence (a.w.a. dimensionality) depends on elements
- projection effects ($d < \infty$), differential abberation (Kaplan 2017)
- telluric lines
- strong emission lines, CSM, ...

New developments

new

- MOST observations of ξ Tau, but the opposite evolution of i_1!
- orbital evolution depends on the sign of $\Omega_1 - \Omega_2 \sim \pm 1°$
- planned applications: λ Tau, V907 Sco, HD 91962, ...



Model (Pyshellspec) ← based on Shellspec (Budaj 2011)

+ LTE level populations
+ LTE ionisation levels
+ 1D line-of-sight transfer
− optically-thin (single) scattering ← no 3D, LI or ALI!
− non-isotropic scattering
+ prescribed ρ, T profiles
+ solar abundances
+ Voigt profile (prior to D.)
+ thermal broadening
+ microturbulence
+ natural
+ Stark
+ Van der Waals
+ Doppler shift
+ HI bound-free continuum opacity
+ HI free-free
+ H^- bound-free,
+ H^- free-free

− Thomson scattering on free electrons
− Rayleigh scattering on neutral hydrogen
− Mie absorption on dust
− Mie scattering
− dust thermal emission
− line opacity
+ spherical primary (gainer)
+ Roche secondary (donor)
− black-body approximation (for *)
+ synthetic spectra (for *)
− irradiation
− reflection
+ limb darkening
+ gravity darkening
− heat transport

BC limb darkening

$$I_\nu(0) = \int_0^{\tau_\nu} S_\nu \, \mathrm{e}^{-\tau'_\nu} \, \mathrm{d}\tau'_\nu + I^\star_\nu(\nu_2) f_{\mathrm{LD}} \, \mathrm{e}^{-\tau_\nu}$$

$$S_\nu = \frac{\epsilon_\nu}{\chi_\nu}$$

$$\nu_2 = \nu \left(1 - \frac{v^\star_z}{c} \right)$$

$$\chi_\nu = \kappa_\nu + \sigma_\nu$$

$$\kappa_\nu = \kappa_\nu^{\mathrm{line}} + \kappa_\nu^{\mathrm{odf}} + \kappa_\nu^{\mathrm{HIbf}} + \kappa_\nu^{\mathrm{HIff}} + \kappa_\nu^{\mathrm{H^-bf}} + \kappa_\nu^{\mathrm{H^-ff}}$$

$$\sigma_\nu = \sigma_\nu^{\mathrm{TS}} + \sigma_\nu^{\mathrm{RS}}$$

$$\epsilon_\nu = \epsilon_\nu^{\mathrm{th}} + \epsilon_\nu^{\mathrm{sc}}$$

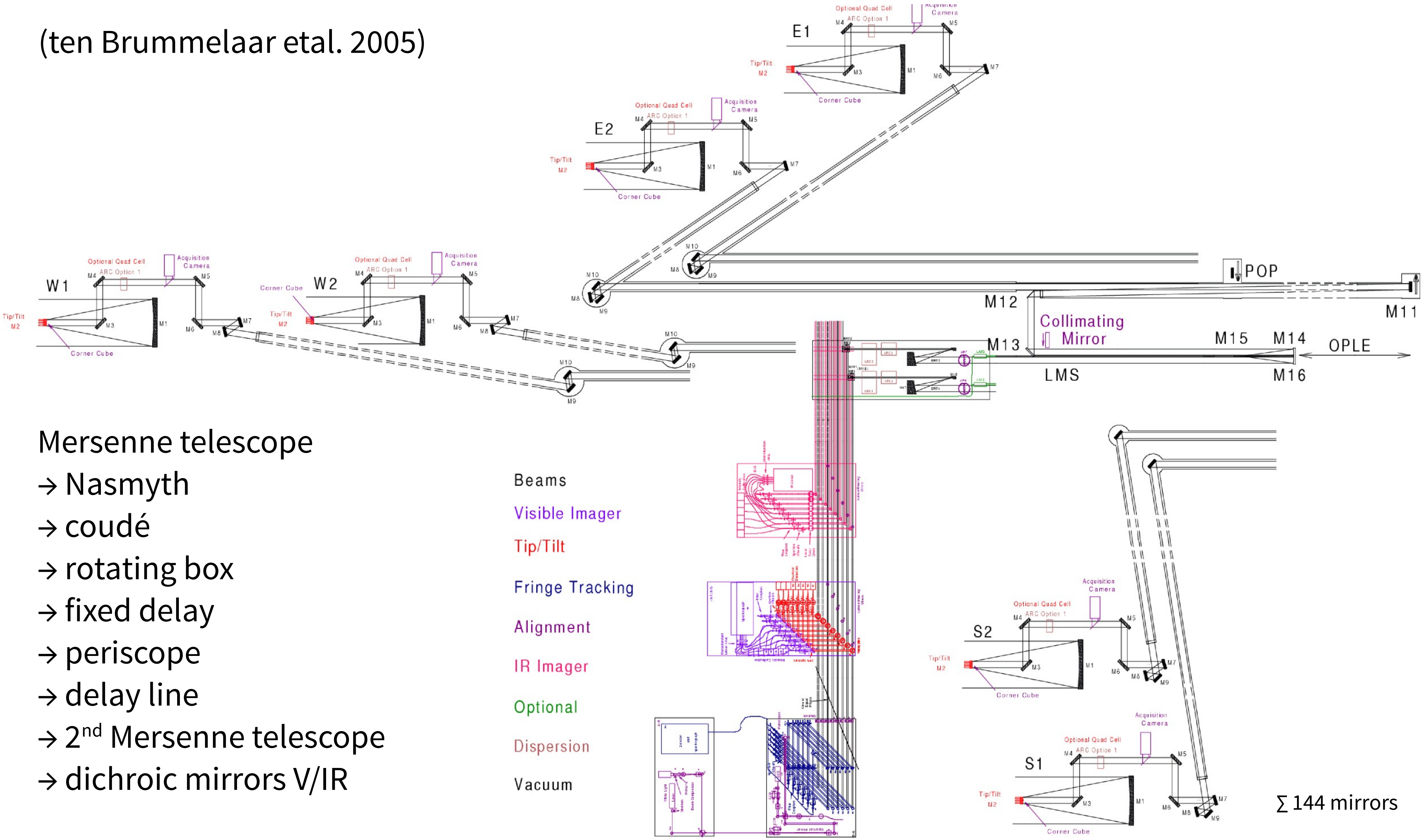
$$\epsilon_\nu^{\mathrm{th}} = B_\nu(T(z))\kappa_\nu$$

$$\epsilon_\nu^{\mathrm{sc}} \doteq \sigma_\nu I^\star_\nu f_{\mathrm{SH}} \frac{\omega}{4\pi}$$

shadow

CHARA Optical scheme

(ten Brummelaar etal. 2005)



Mersenne telescope
→ Nasmyth
→ coudé
→ rotating box
→ fixed delay
→ periscope
→ delay line
→ 2nd Mersenne telescope
→ dichroic mirrors V/IR

∑ 144 mirrors


EXIT
OPLE
Periscopes
PoPs
Metrology
BRT
LDC
BSS

Model (cont.)

- Python interface (JN) — http://sirrah.troja.mff.cuni.cz/~mira/betalyr/
- calculation of interferometric observables (DFT), χ^2
- multiprocessing module (split according to λ; 4-8 cores)
- discretisation $N_x = 160$, $N_y = 60$ (~1 $R_\odot$); variable in z (~τ)

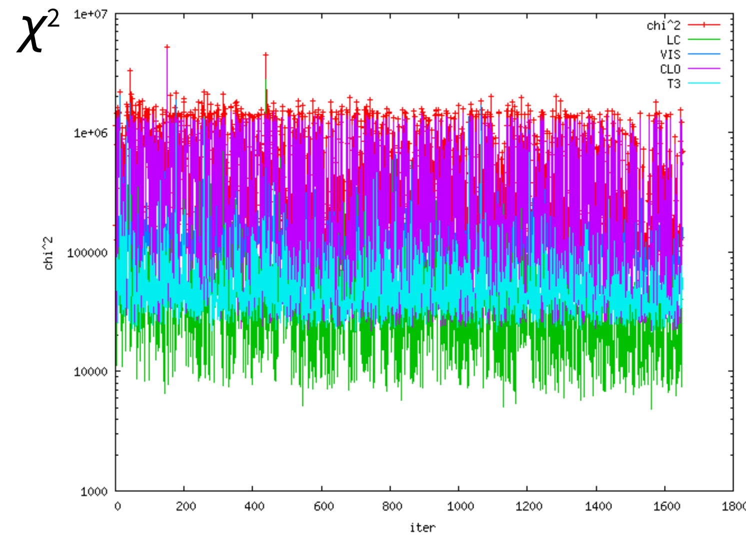
  ↑ trapezoidal rule
- local & global optimisation (simplex, DE, ...)
- 1 iteration: 2392 synthetic images (3 min),
- 1 convergence: >10^3 steps (several days)
- free parameters: H (or θ), $R_{\rm out}$, ρ, T_0 (or T_1), $\alpha_{\rm D}$, $\alpha_{\rm T}$, i, Ω, d, $h_{\rm inv}$, $t_{\rm inv}$, $h_{\rm wind}$, $h_{\rm mul}$
- fixed parameters: P, $\mathrm{JD}_{\rm min}$, $\dot{P}$, $a \sin i$, M_1, $q = M_1/M_2$, e, ω, $f_{\rm ill}$, $R_{\rm g}$, $T_{\rm eff,d}$, $T_{\rm eff,g}$, $x_{\rm bol,d}$, $x_{\rm bol,g}$, $\alpha_{\rm gd,d}$, $\alpha_{\rm gd,g}$, ...

```python
#!/usr/bin/env python

import pyshellspec
...

# constants
ra = (18. + 50. / 60. + 4.79525 / 3600.) * 360./24.
dec = 33. + 21. / 60. + 45.6100 / 3600.
dir_data = '/home/mira/a/betalyr_MYTEST/data_20171126'

def main():

    # data from MIRC
    obs = []
    dir = os.path.join(dir_data, 'mirc2017')
    infile = os.path.join(dir, 'mirc.ascii.lis')
    names = read_if(infile)
    for i in range(0, len(names)):
        print names[i]
        obs.append(pyshellspec.IFData(filename=os.path.join(dir, names[i]),
                                      location='chara', ra=ra, dec=dec,
                                      format='ascii', weight_vis2=0.5, weight_t3amp=0.5))
    ...

    # construct data class
    data = pyshellspec.Data(obs)

    # construct the model
    central = pyshellspec.CentralObject()
    companion = pyshellspec.Companion()
    disk = pyshellspec.Disk()
    nebula = pyshellspec.Nebula()
    orbit = pyshellspec.Orbit()
    objs = [central, companion, disk, nebula, orbit]
    model = pyshellspec.Model(objects=objs)

    # construct the Interface
    itf = pyshellspec.Interface(model=model, data=data, ncpu=4, image_size=180,
                                if_phase_precision=3, lc_phase_precision=2,
                                if_ew_precision=7, lc_ew_precision=9,
                                shellspec_template="template.in", use_offset=True)
    # set grid resolution
    itf.set_parameter('steps', value=1.0)
    itf.set_parameter('stepf', value=1.0)
    itf.set_parameter('stepsz', value=1.0)
    itf.set_parameter('stepfz', value=1.0)
```

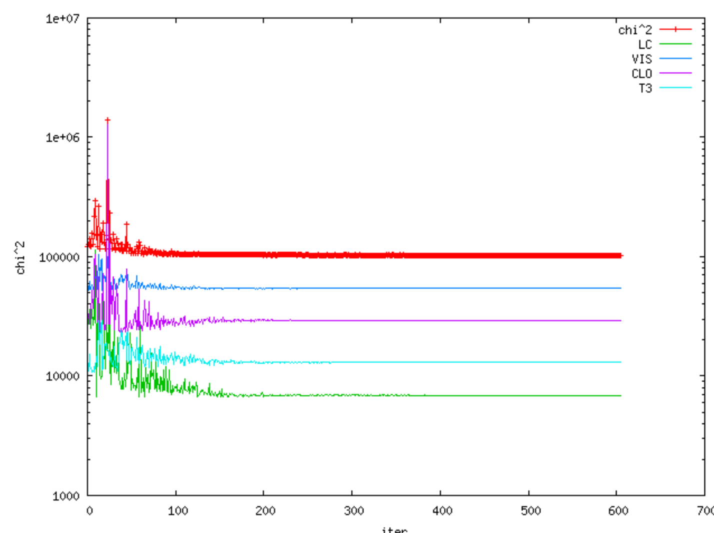
```
    # set shellspec parameters
    itf.set_parameter('inebl', value=1)  # nebula
    itf.set_parameter('itnb', value=3)  # power-law
    itf.set_parameter('ichemc', value=1)
    itf.set_parameter('ielnd', value=1)
    itf.set_parameter('ithom', value=1) # scattering
    itf.set_parameter('irayl', value=1)
    itf.set_parameter('imie', value=0)
    itf.set_parameter('imiepf', value=0)
    itf.set_parameter('ishdnb', value=1)  # shadowing
    itf.set_parameter('iinvnb', value=2)  # inversion of T
    itf.set_parameter('iline', value=1)

    # set fitted parameters
    itf.set_parameter('routnb'  , value=32.01701171 , fitted=True, vmin=26., vmax=35.)
    itf.set_parameter('hinvnb'  , value=7.213479944 , fitted=True, vmin=1.0, vmax=9.0)
    itf.set_parameter('tinvnb'  , value=1.965635409 , fitted=True, vmin=1.0, vmax=9.0)
    itf.set_parameter('hwindnb' , value=4.300571244 , fitted=True, vmin=3.0, vmax=9.0)
    itf.set_parameter('hcnb'    , value=2.641315033 , fitted=True, vmin=1.0, vmax=15.0)
    itf.set_parameter('hshdnb'  , value=3.398141341 , fitted=False, vmin=1.0, vmax=12.0)
    itf.set_parameter('tempnb'  , value=33755.73450 , fitted=True, vmin=23000., vmax=34000.)
    itf.set_parameter('densnb'  , value=4.91712e-06 , fitted=True, vmin=2e-8, vmax=5e-6)
    itf.set_parameter('edennb'  , value=-3.092256032, fitted=True, vmin=-3.5, vmax=-0.8)
    itf.set_parameter('etmpnb'  , value=-1.005127838, fitted=True, vmin=-1.1, vmax=-0.70)
    itf.set_parameter('dinc'    , value=93.64153194 , fitted=True, vmin=91., vmax=97.)
    itf.set_parameter('omega_an', value=254.3871116 , fitted=True, vmin=252., vmax=255.)
    itf.set_parameter('dd'      , value=321.7816453 , fitted=True, vmin=305., vmax=330.)
    itf.set_parameter('aneb'    , value=9.0         , fitted=False, vmin=3.0, vmax=12.0)
    itf.set_parameter('asini'   , value=58.19       , fitted=False, vmin=53., vmax=63.)
    itf.set_parameter('tempcp'  , value=13300       , fitted=False, vmin=12000., vmax=14600.)

    # compute one/fit
    itf.compute_chi2(verbose=True)
#    itf.run_fit(fitter='nlopt_nelder_mead', ftol=1e-6, maxiter=1000)
#    itf.run_fit(fitter='sp_diff_evol', tol=1e-2, maxiter=10000)

    itf.set_model_to_shellspec()
    itf.write_template('final.in')
    itf.write_iterations()
    itf.write_model()

    print "Note: fit.py ended successfully."
    sys.exit(0)

if __name__ == '__main__':
    main()
```

Joint χ^2 metric

$$
\begin{aligned}
\chi^2_{\mathrm{LC}} &= \sum_{i=1}^{N_{\mathrm{P}}} \sum_{j=1}^{N_{\mathrm{M}}} \left(\frac{m_{i,j}^{\mathrm{obs}} - \tilde{m}_{i,j}^{\mathrm{syn}}}{\sigma_{i,j}} \right)^2 , \\
\chi^2_{\mathrm{IF}} &= \chi^2_{\mathrm{IF_{VEGA}}} + \chi^2_{\mathrm{IF_{NPOI}}} + \chi^2_{\mathrm{IF_{MIRC}}} , \\
\chi^2_{\mathrm{IF_{VEGA}}} &= \chi^2_{\mathrm{V^2}} , \\
\chi^2_{\mathrm{IF_{NPOI}}} &= \chi^2_{\mathrm{V^2}} + \chi^2_{\mathrm{CP}} , \\
\chi^2_{\mathrm{IF_{MIRC}}} &= \frac{1}{2} \left(\chi^2_{\mathrm{V^2}} + \chi^2_{\mathrm{T_3}} \right) + \chi^2_{\mathrm{CP}} , \\
\chi^2_{\mathrm{V^2}} &= \sum_{i=1}^{N_{\mathrm{V2}}} \left(\frac{\left|V^{\mathrm{obs}}\right|_i^2 - \left|V^{\mathrm{syn}}\right|_i^2}{\sigma_i} \right)^2 , \\
\chi^2_{\mathrm{T_3}} &= \sum_{i=1}^{N_{\mathrm{T_3}}} \left(\frac{\left|T_3^{\mathrm{obs}}\right|_i - \left|T_3^{\mathrm{syn}}\right|_i}{\sigma_i} \right)^2 , \\
\chi^2_{\mathrm{CP}} &= \sum_{i=1}^{N_{\mathrm{T_3}}} \left(\frac{T_3\phi_i^{\mathrm{obs}} - T_3\phi_i^{\mathrm{syn}}}{\sigma_i} \right)^2 .
\end{aligned}
$$



DE

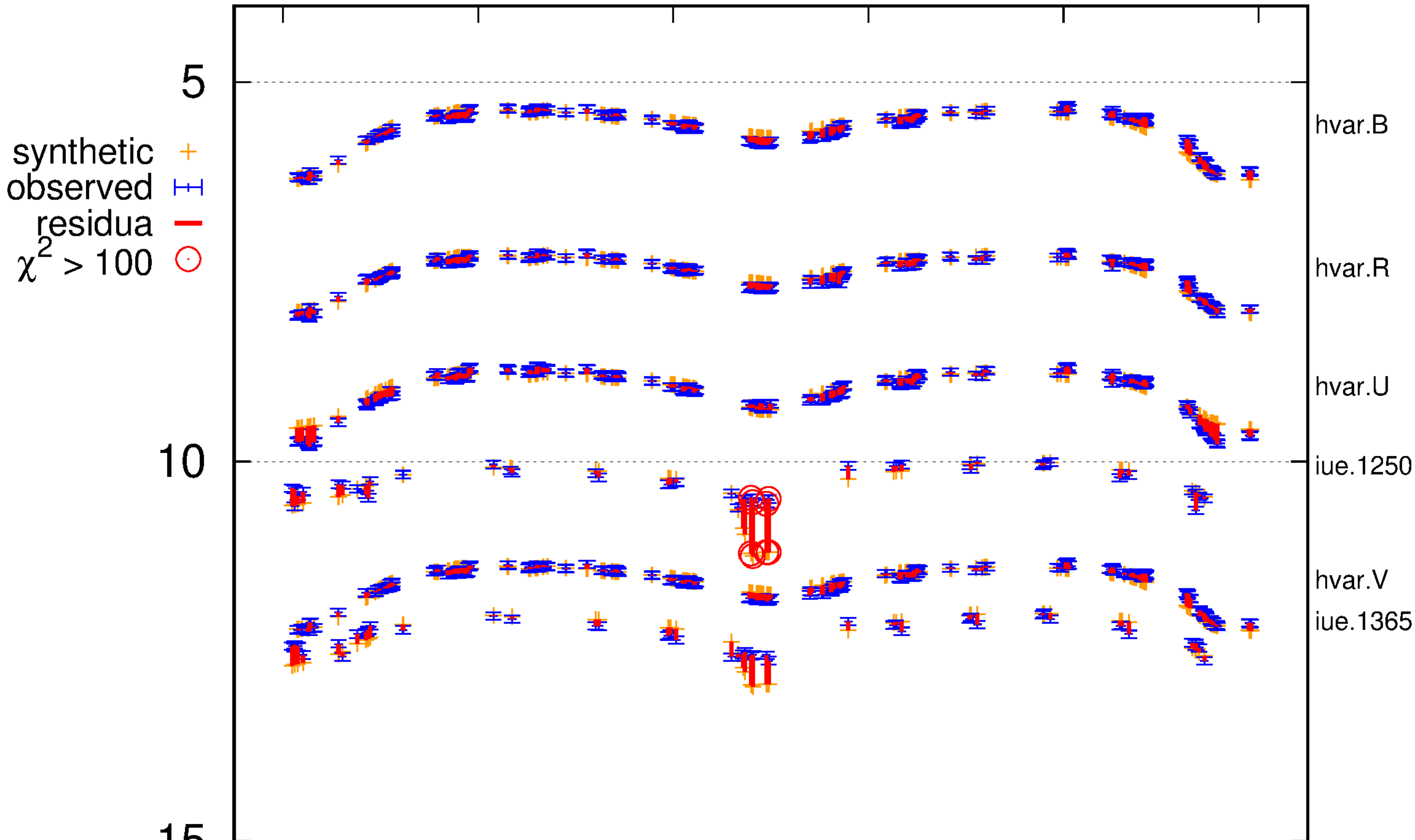


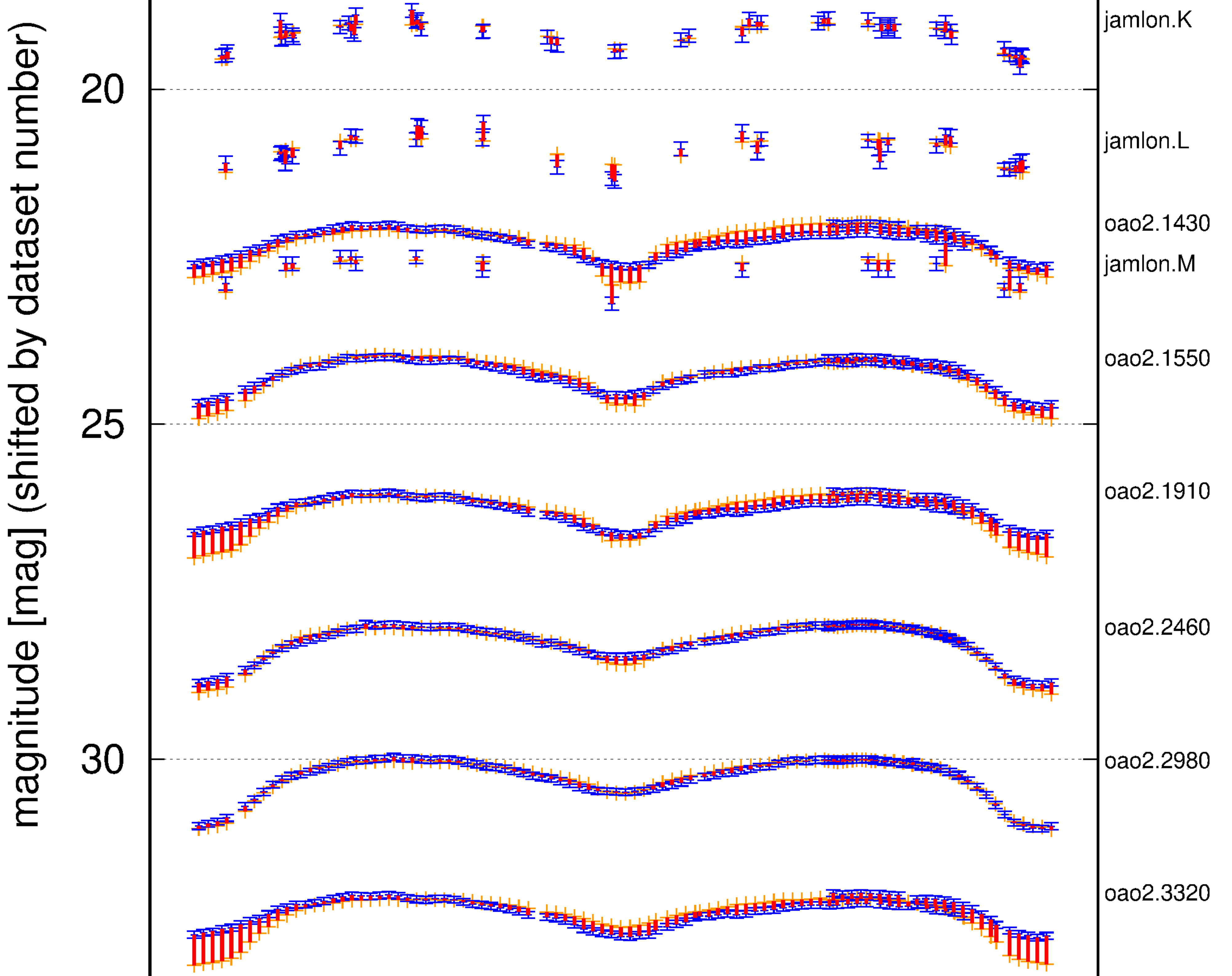
simplex

iteration

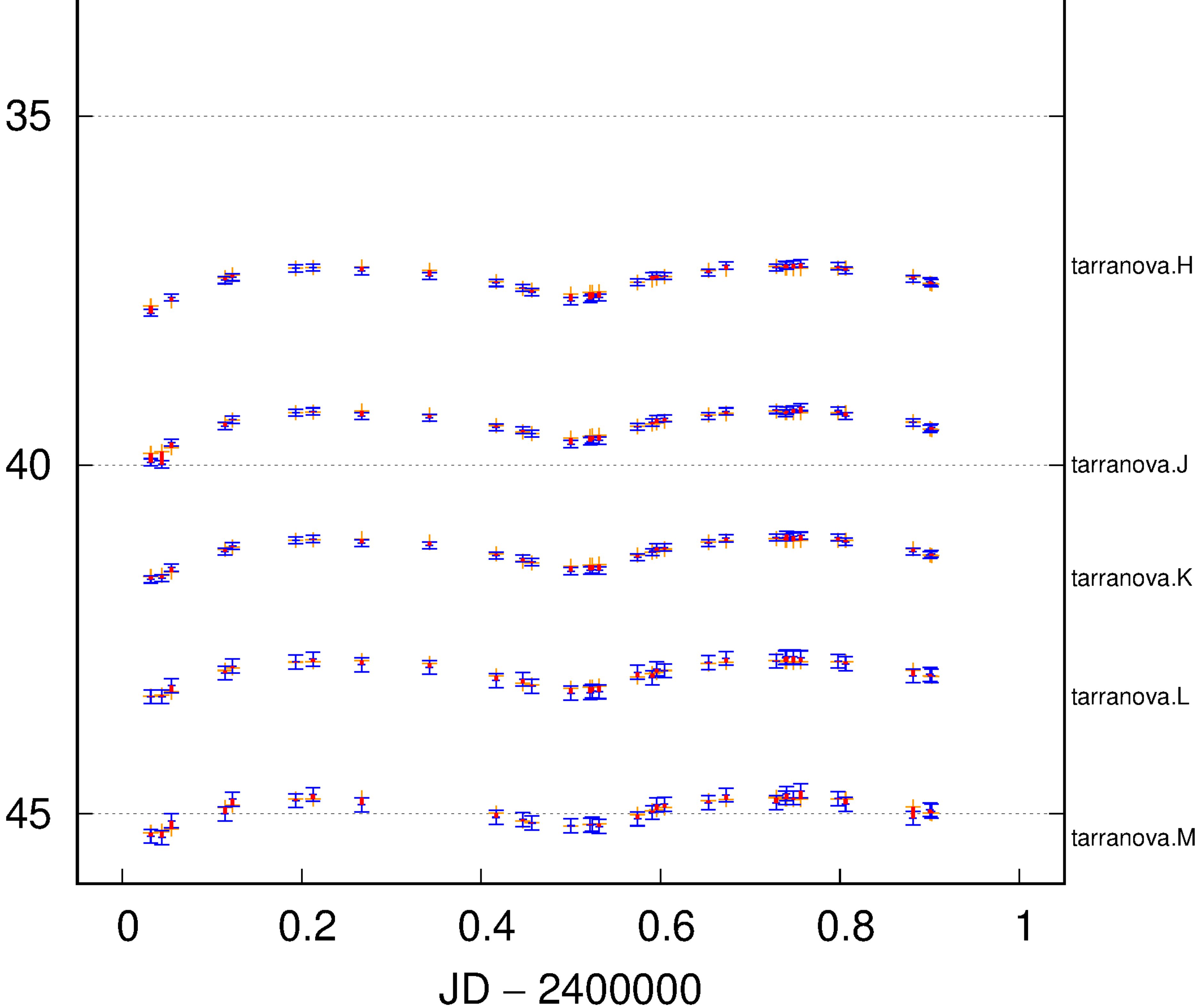
only optically **thick!**

β Lyrae A (LC)



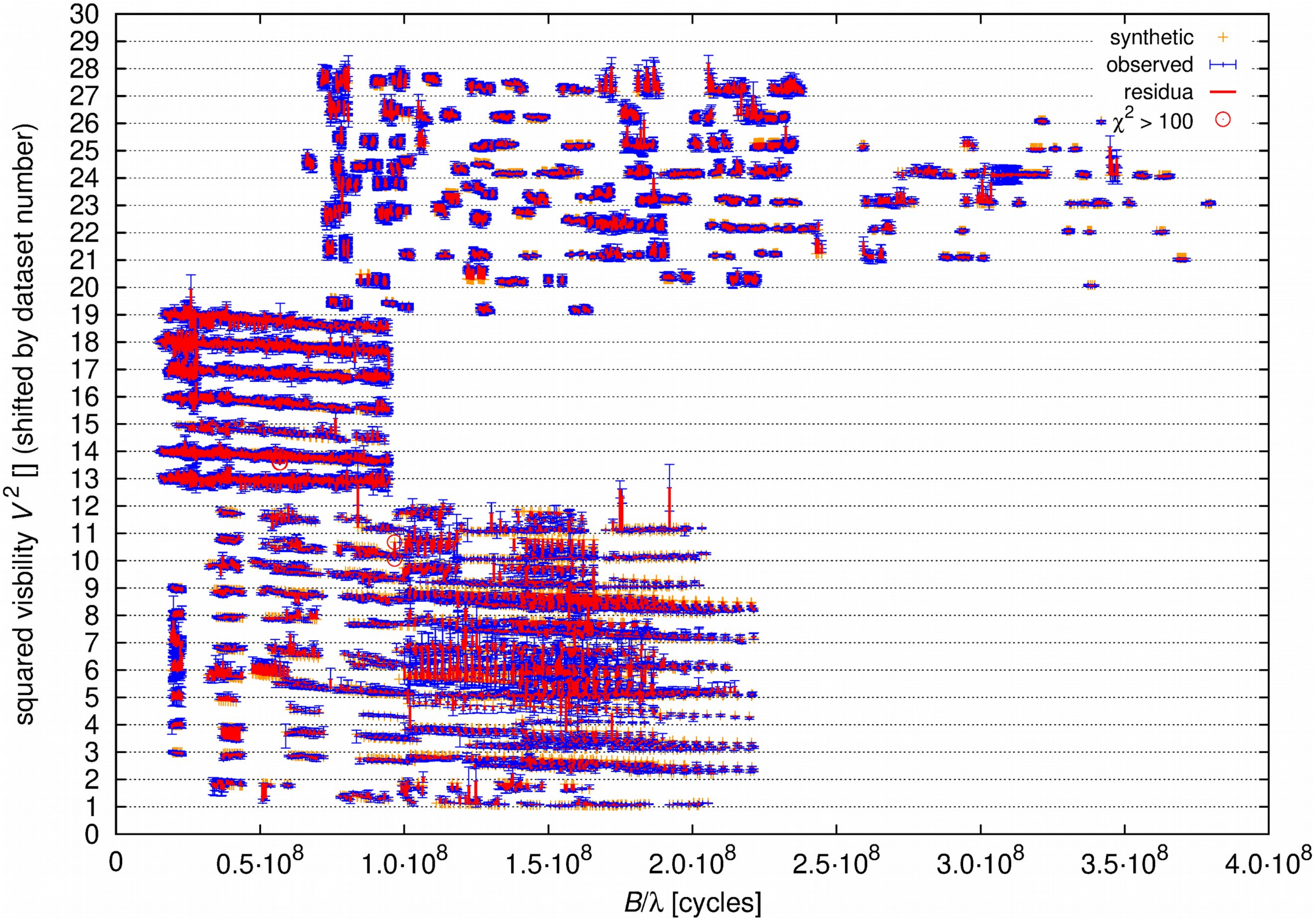


magnitude [mag] (shifted by dataset number)
20
25
30
jamlon.K
jamlon.L
oao2.1430
jamlon.M
oao2.1550
oao2.1910
oao2.2460
oao2.2980
oao2.3320

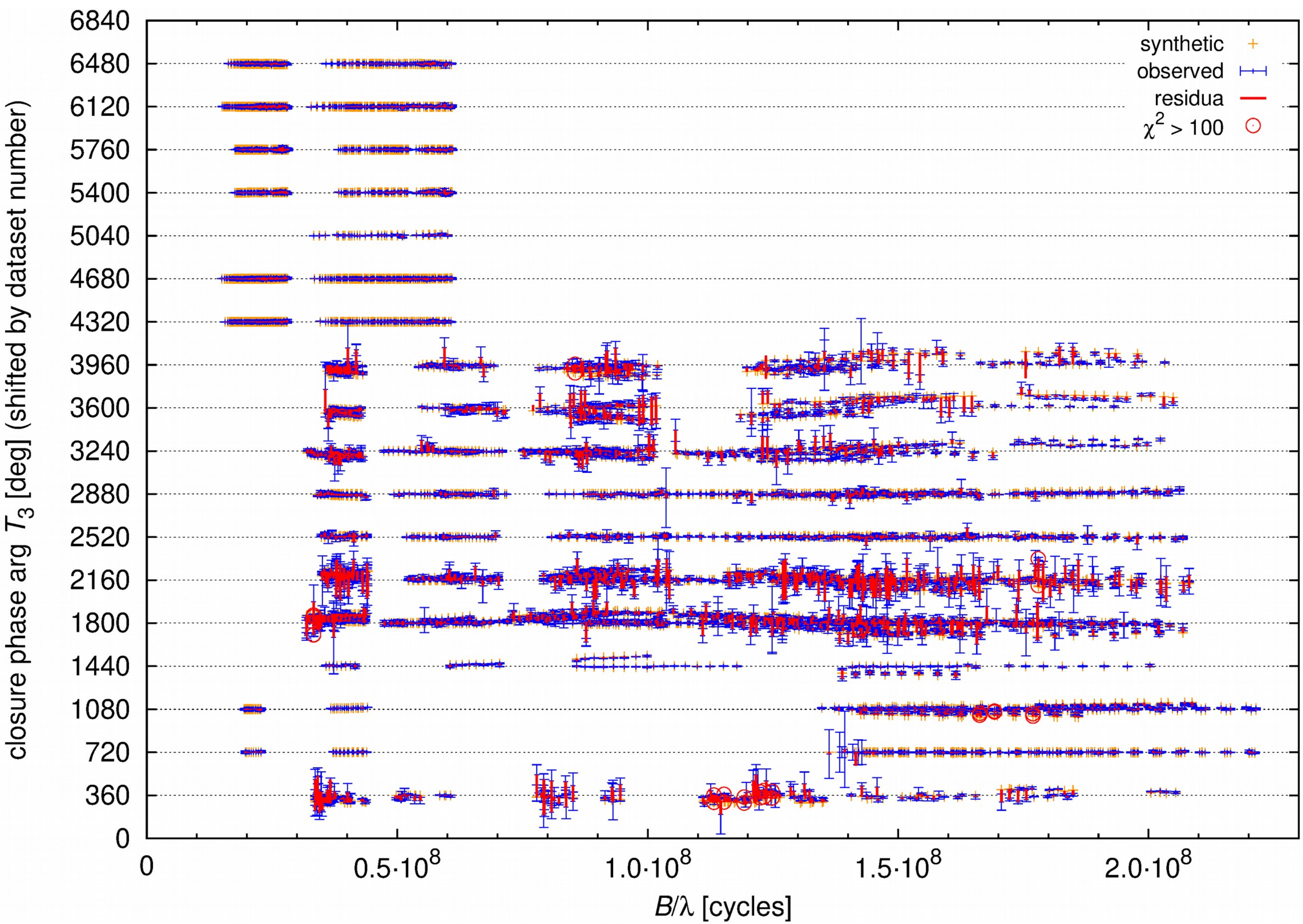


35
40
45
0
0.2
0.4
0.6
0.8
1
JD – 2400000
tarranova.H
tarranova.J
tarranova.K
tarranova.L
tarranova.M

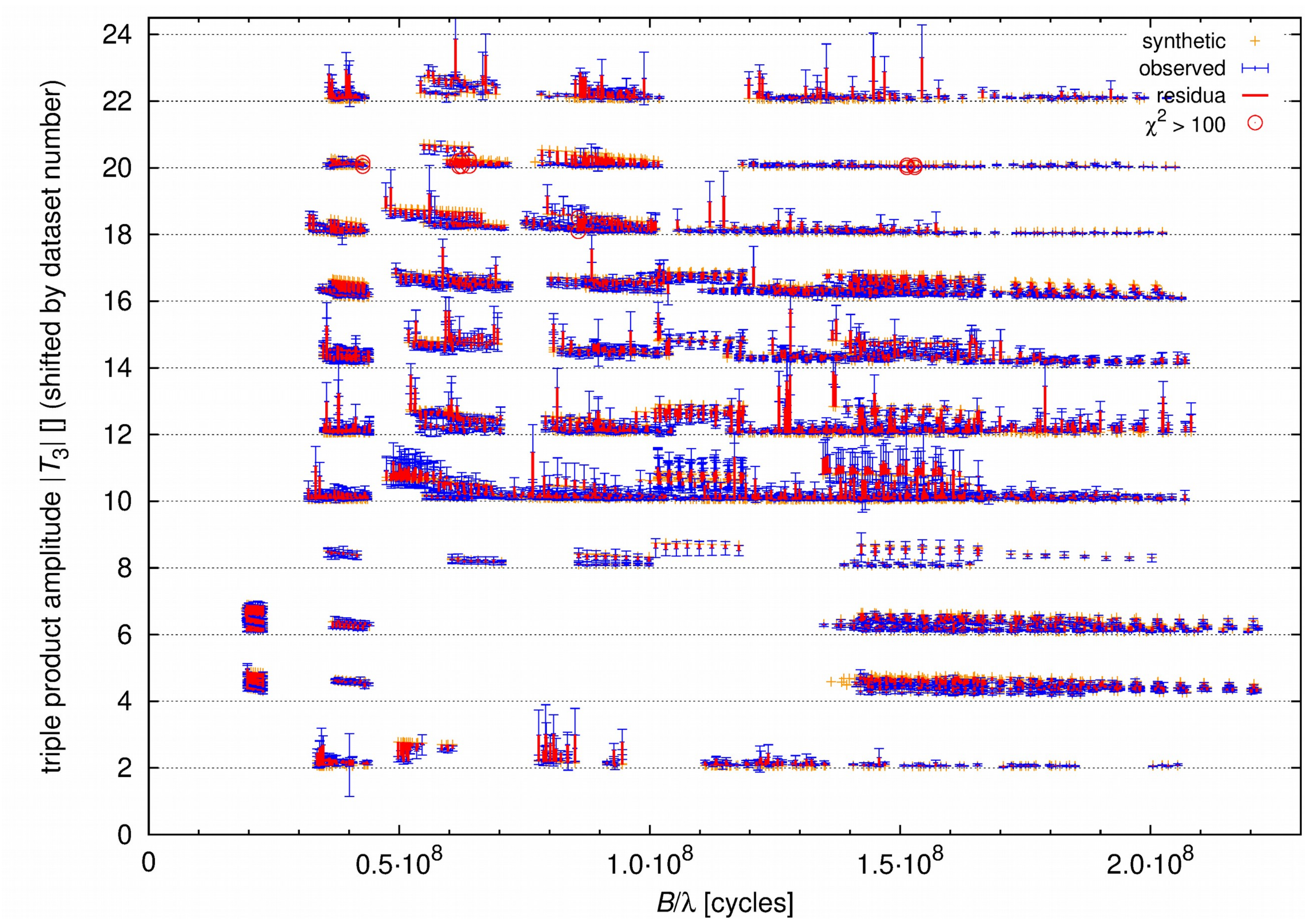
Visibility (VIS)



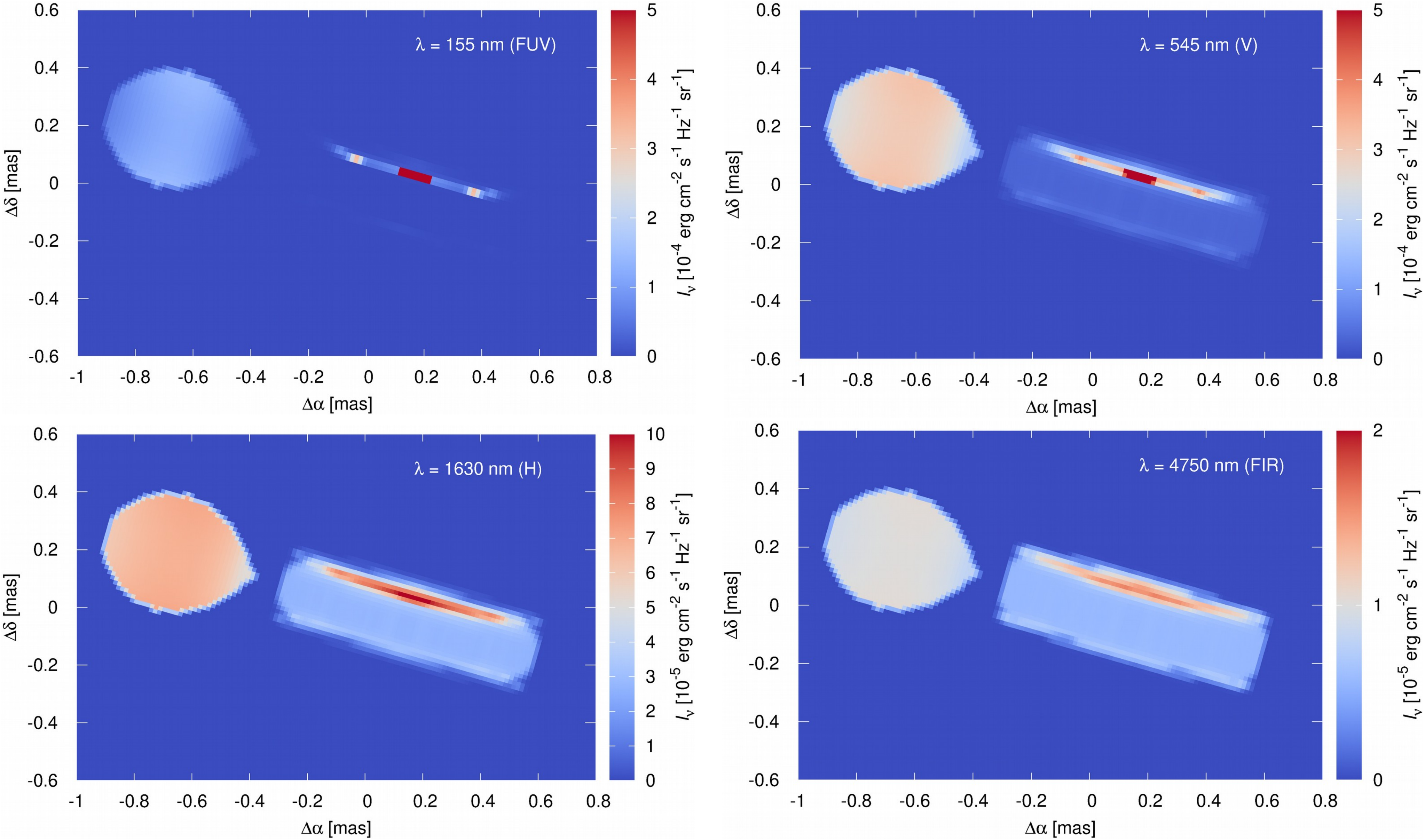
Closure phase (CLO)



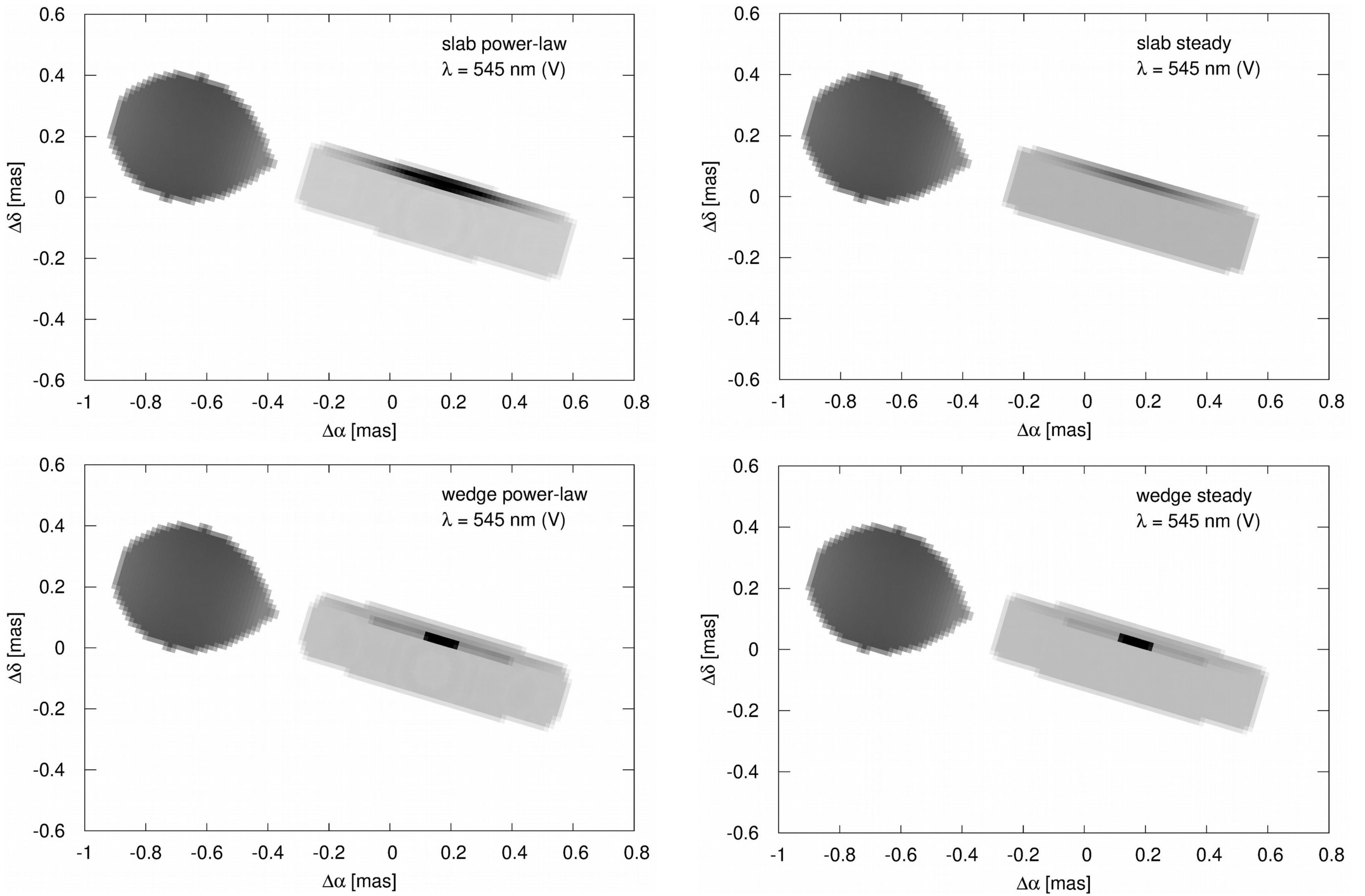
Triple product (T3)



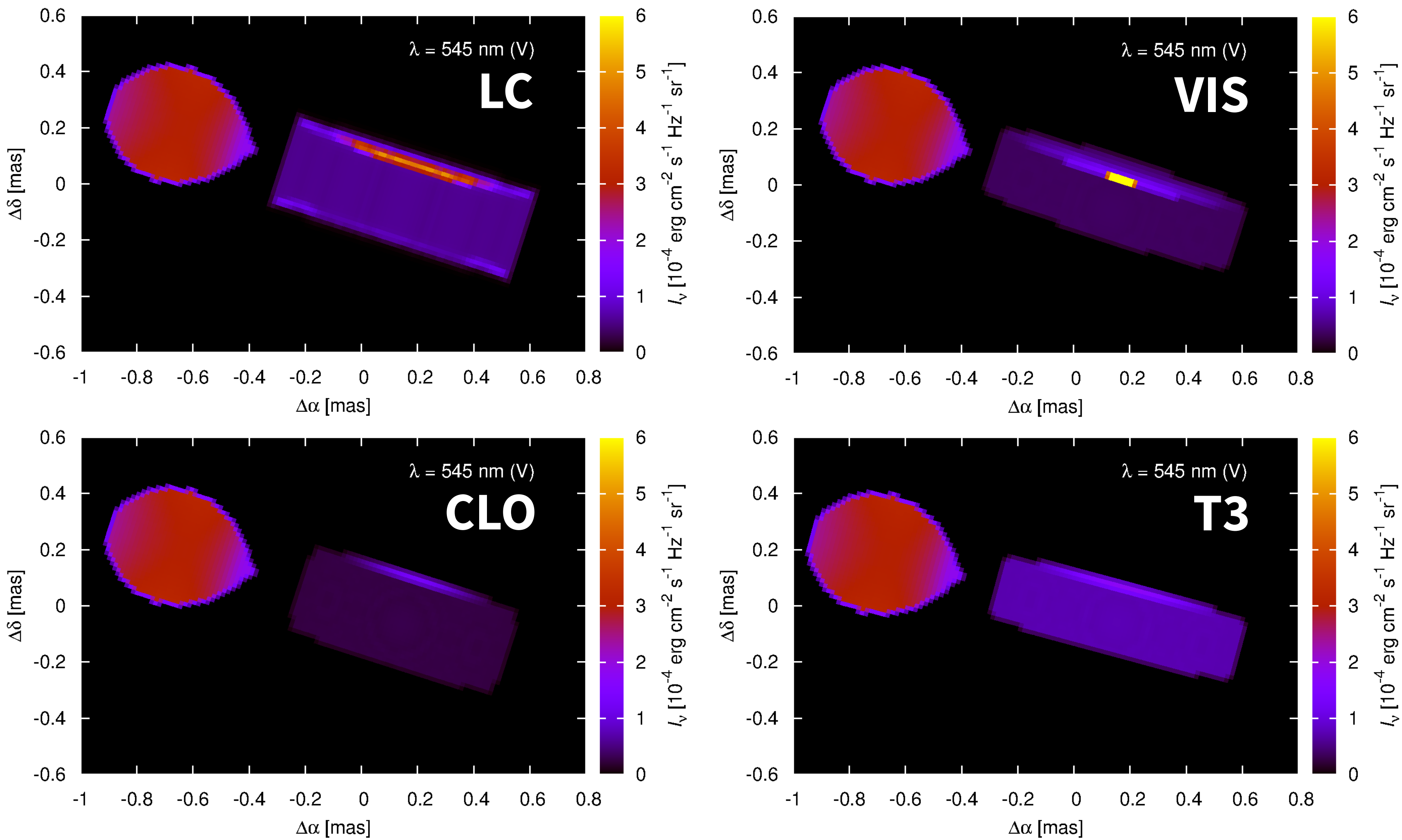
β Lyrae A best-fit model



Alternative shapes

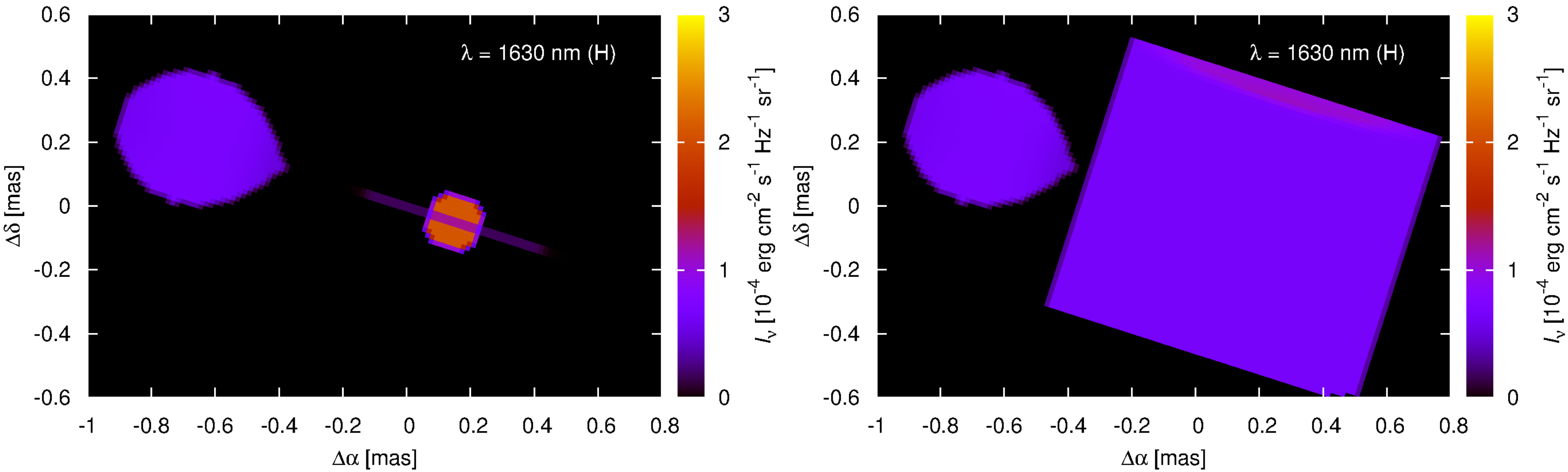


Systematic differences



Parameter space

minimum ⟷ maximum



N: A non-negligible part surveyed, but some p. fixed...

C: Nobody can be sure the model is sufficient! (triviální)

Hidden problems

- missing scattering (shadowing), better m. of atmosphere
- optically thin jets, spot(s), reflection + irradiation
- limited resolution (~ 1 $R_{\odot}$), discretisation errors, RTE artifacts
- line transfer, non-LTE?
- systematics between LC & VIS, CLO, T3
- optically thick vs *very* o. t. ← degenerate problem :(

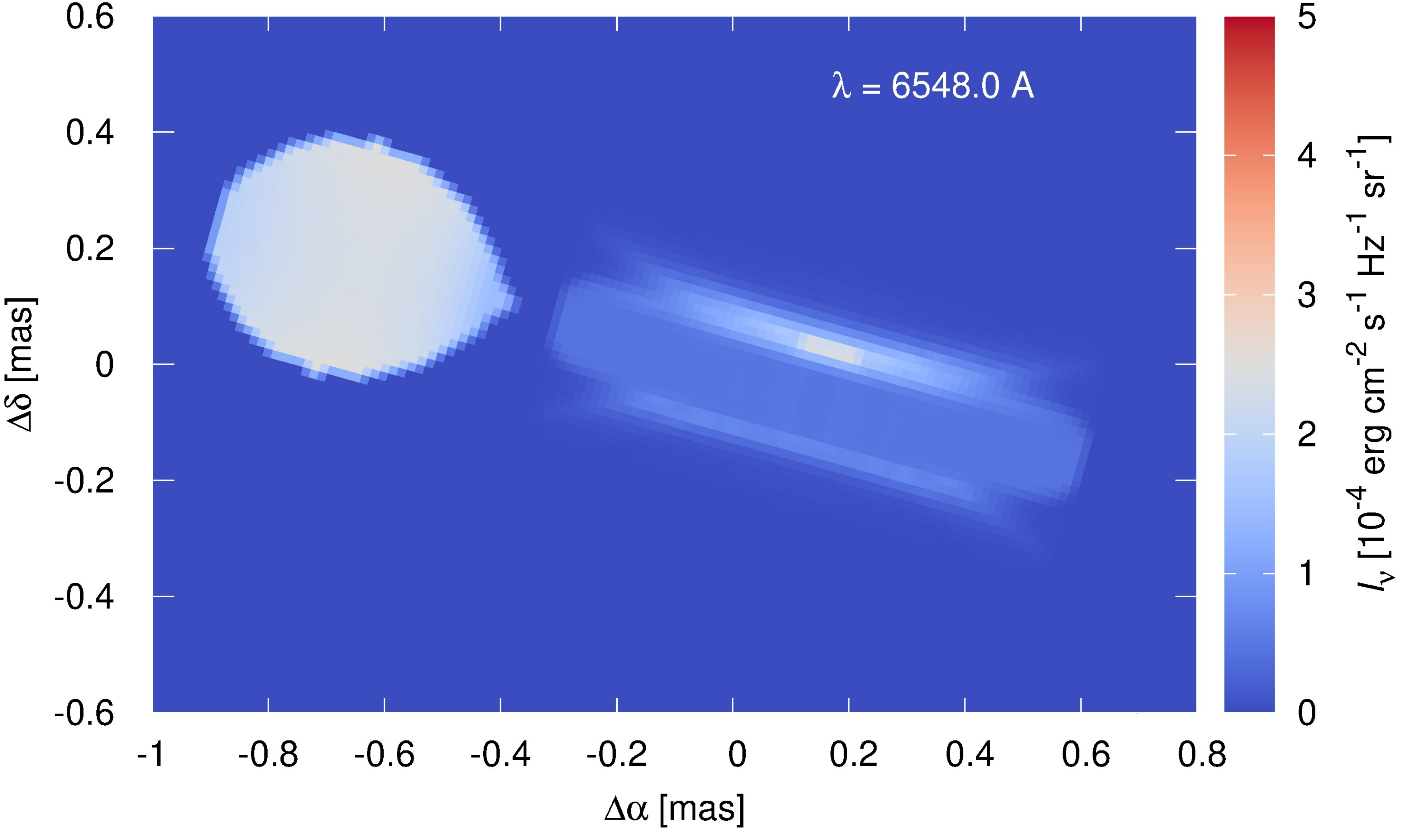
- missing ΔV, Doppler tomography, SED measurements
- kinematics only, missing feedback on HD!
- disk stability, outer edge, precession → dynamical model?

New developments

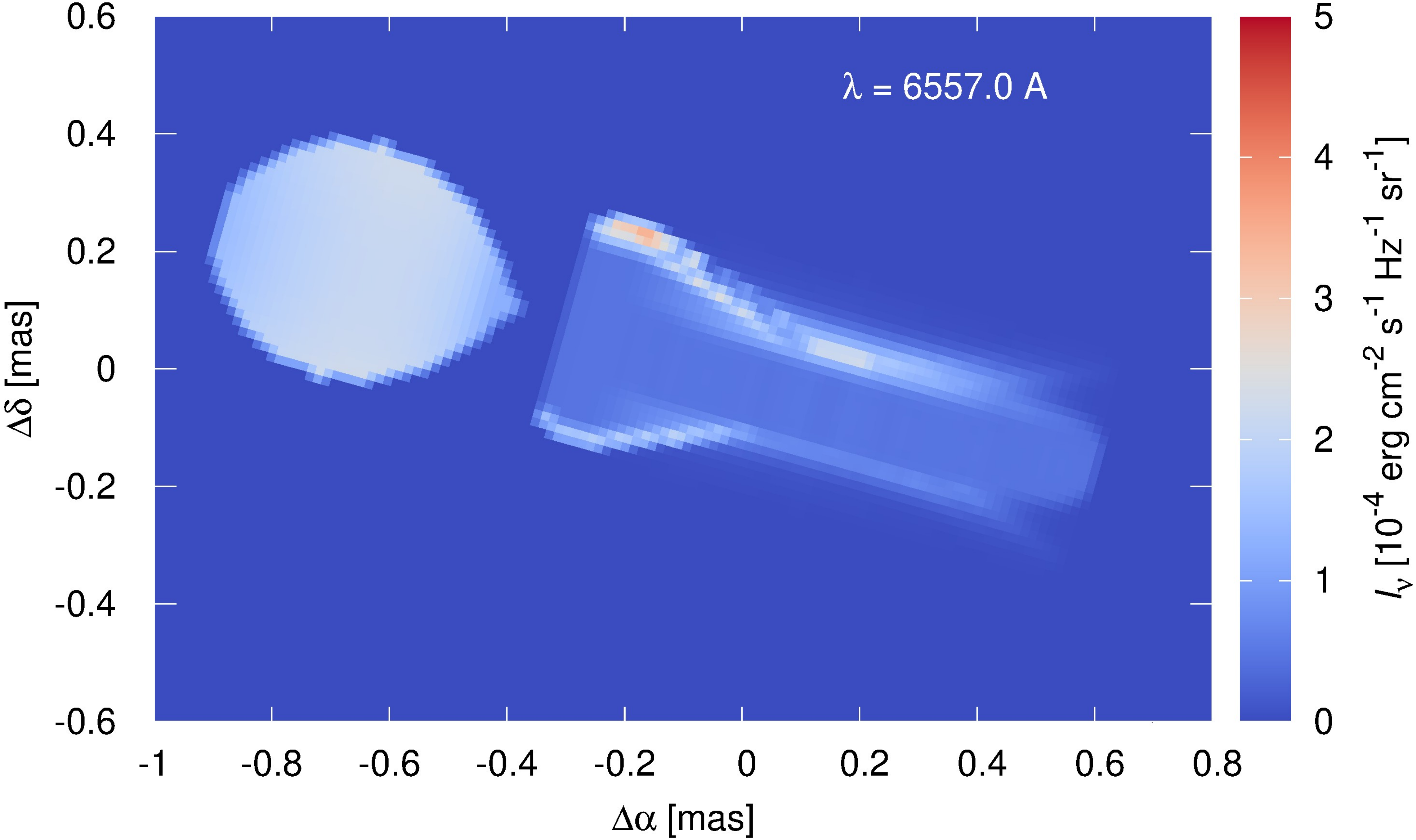
- line transfer (in S.)
- simplified shadowing (dtto)

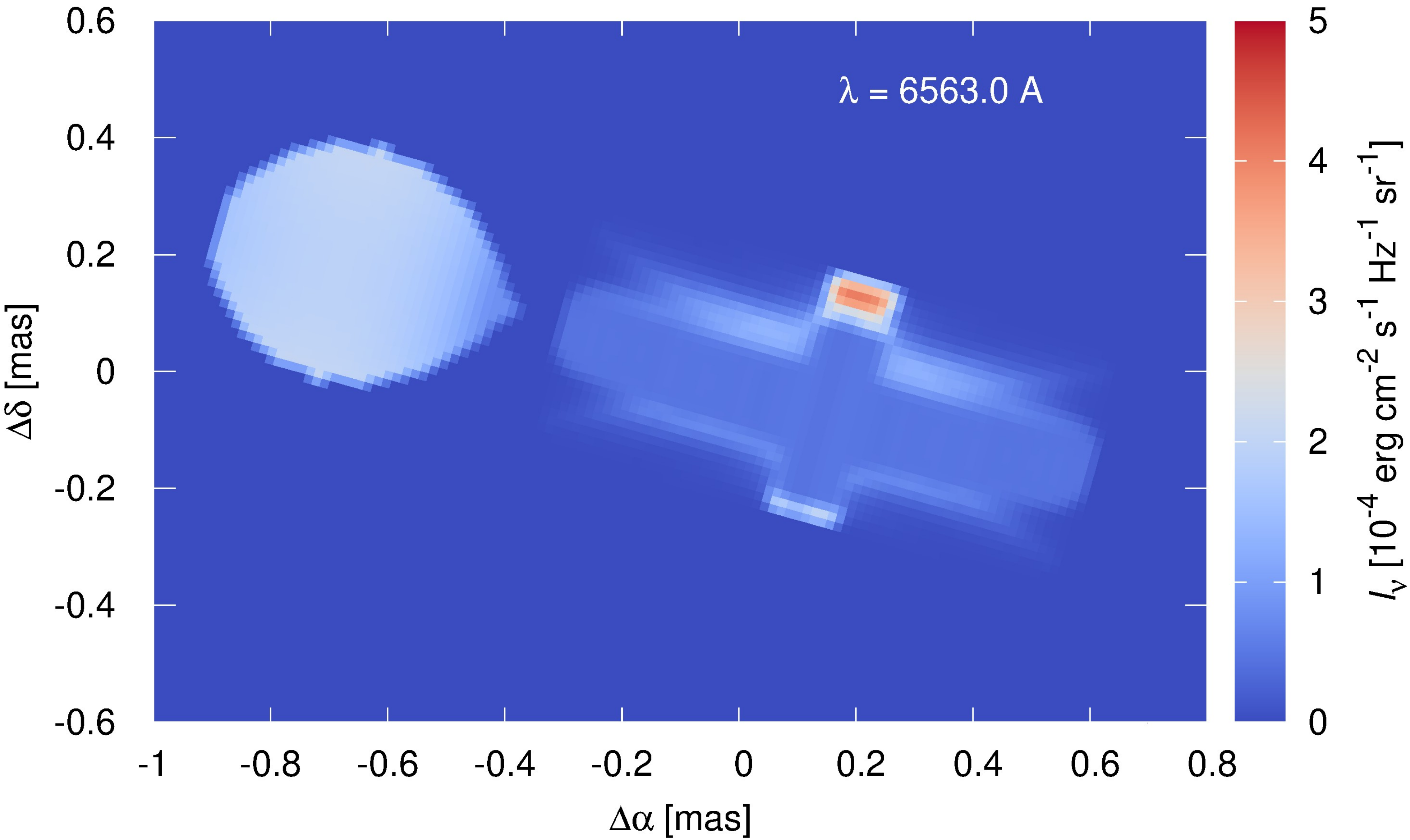
- differential visibility ΔV and phase $\Delta\varphi$ for H_α
- Doppler tomography (SYN)
- spectral-energy distribution (SED)

were implemented in Pyshellspec...

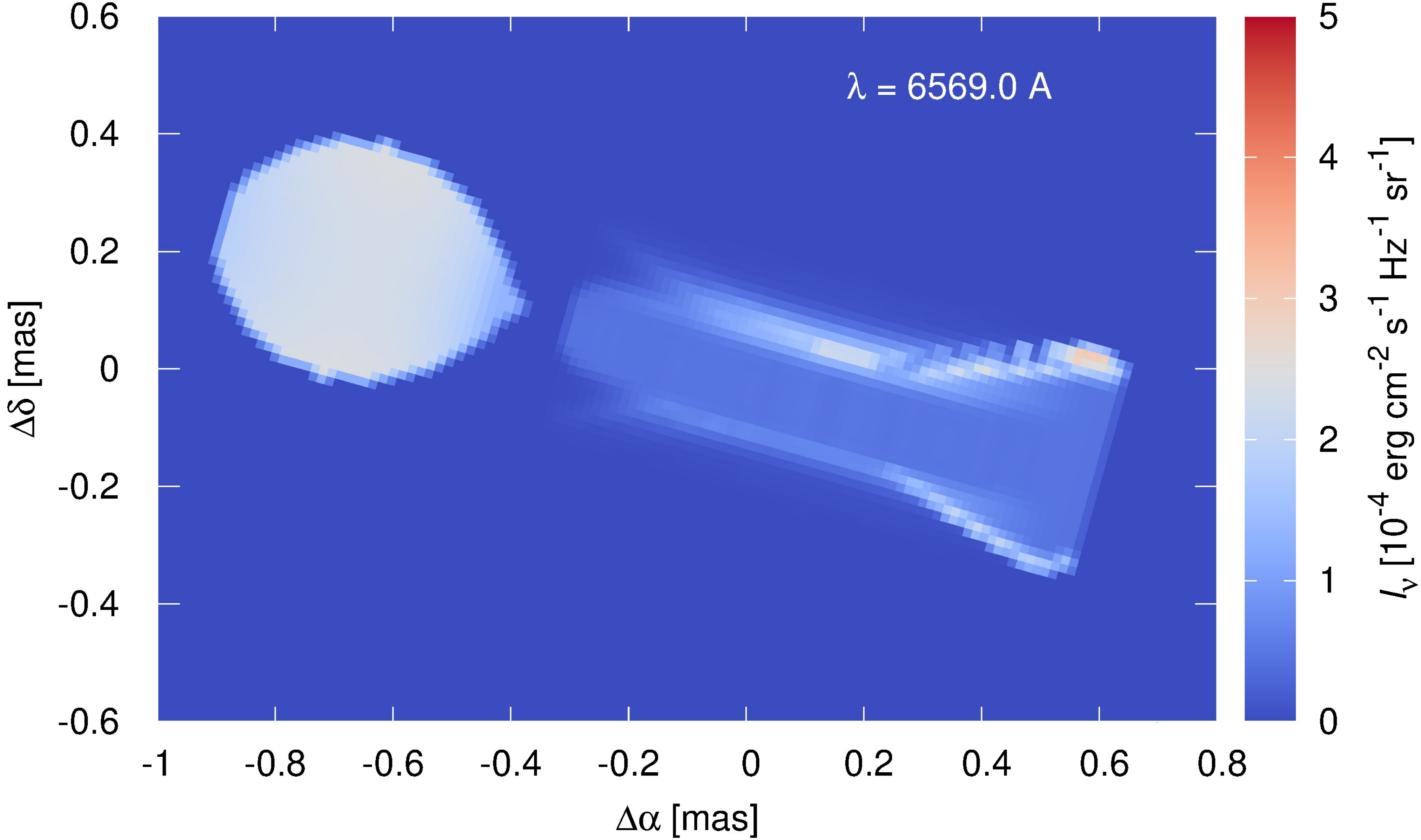


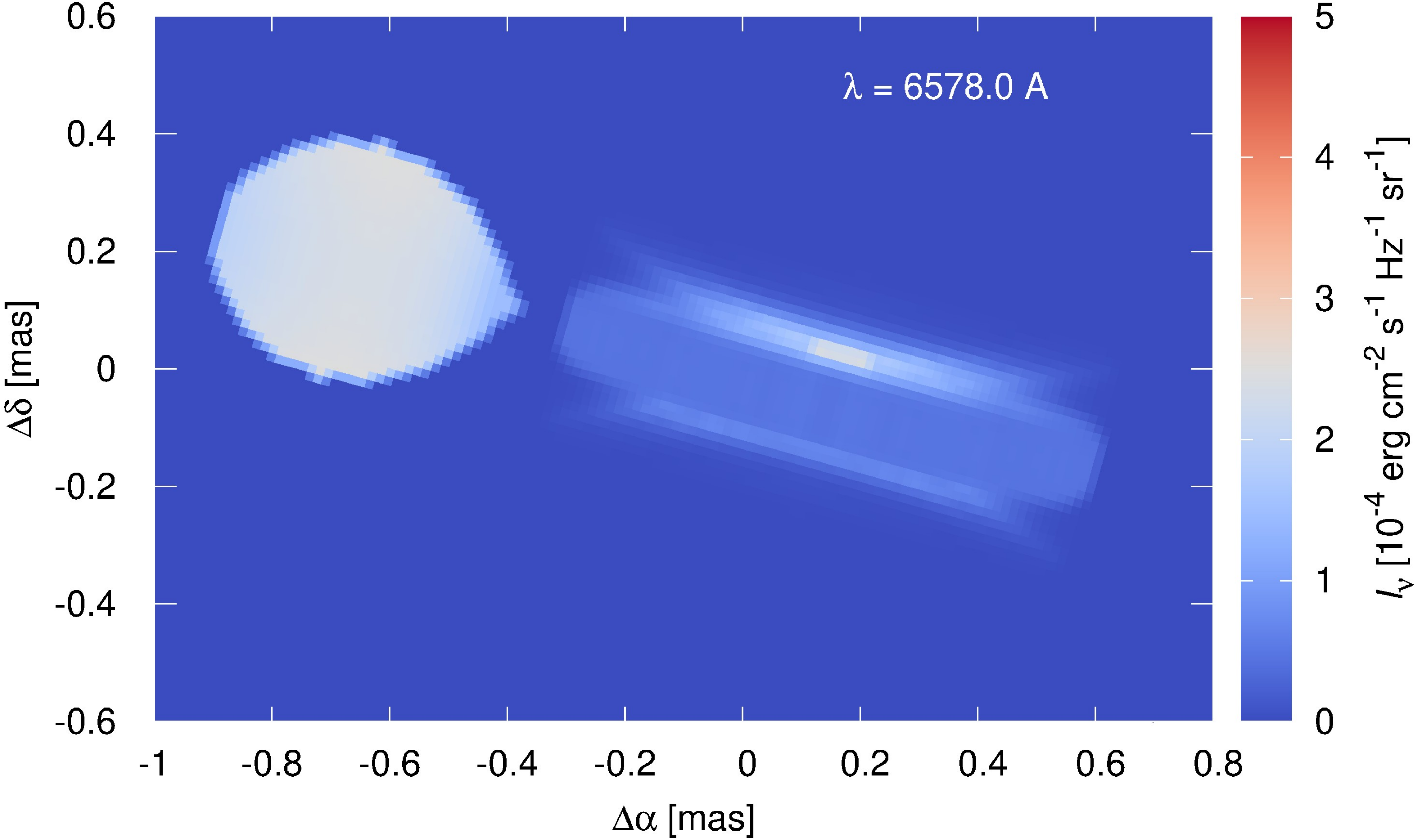
λ = 6548.0 A
Δδ [mas]
Δα [mas]
I_ν [10^{-4} erg cm^{-2} s^{-1} Hz^{-1} sr^{-1}]
0.6
0.4
0.2
0
-0.2
-0.4
-0.6
-1
-0.8
-0.6
-0.4
-0.2
0
0.2
0.4
0.6
0.8
5
4
3
2
1
0


λ = 6557.0 A
0.6
0.4
0.2
0
-0.2
-0.4
-0.6
Δδ [mas]
-1
-0.8
-0.6
-0.4
-0.2
0
0.2
0.4
0.6
0.8
Δα [mas]
5
4
3
2
1
0
Iν [10-4 erg cm-2 s-1 Hz-1 sr-1]



λ = 6563.0 A
0.6
0.4
0.2
0
-0.2
-0.4
-0.6
Δδ [mas]
-1
-0.8
-0.6
-0.4
-0.2
0
0.2
0.4
0.6
0.8
Δα [mas]
5
4
3
2
1
0
I_ν [10^{-4} erg cm^{-2} s^{-1} Hz^{-1} sr^{-1}]

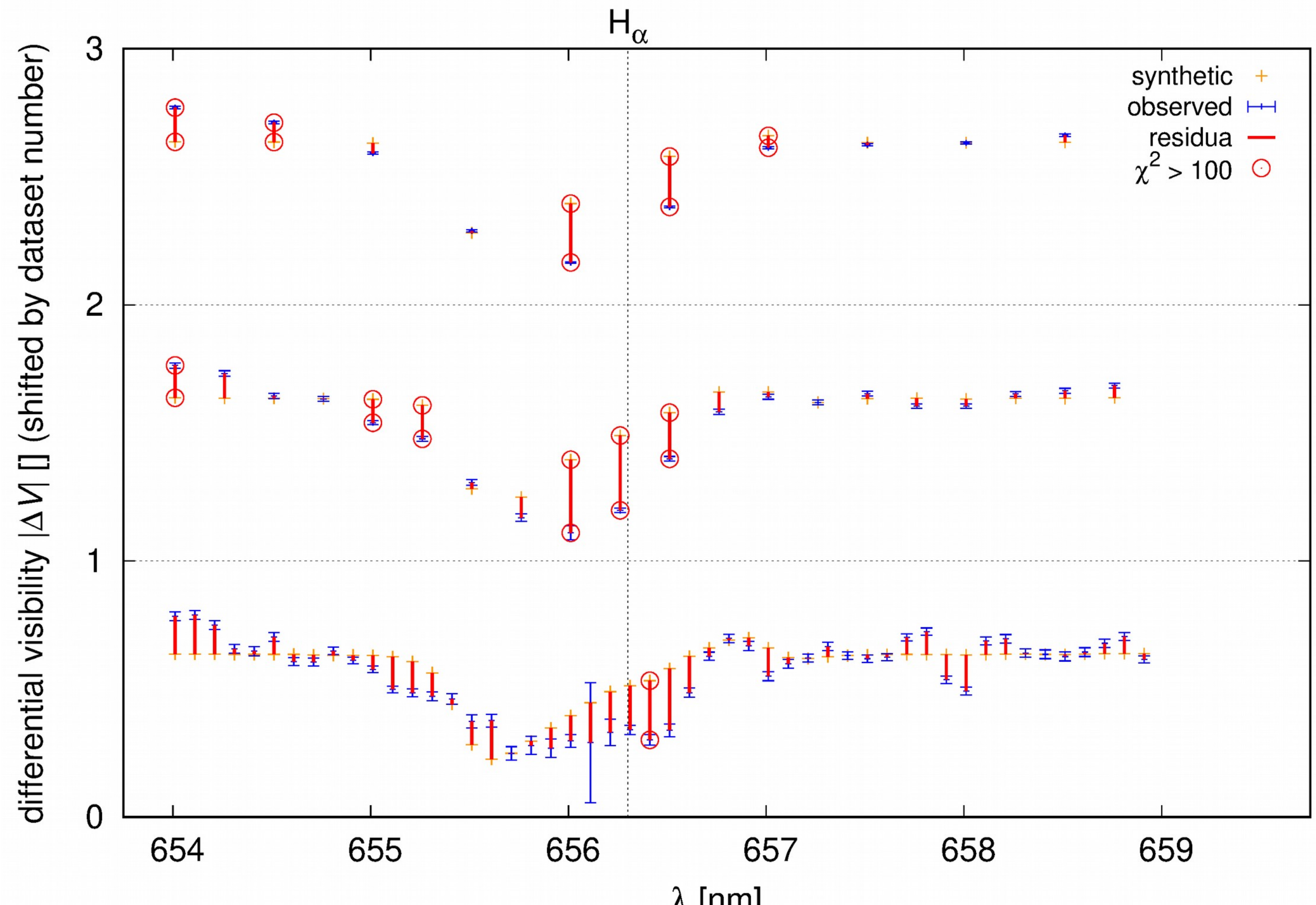

λ = 6569.0 A
Δδ [mas]
Δα [mas]
I_ν [10^{-4} erg cm^{-2} s^{-1} Hz^{-1} sr^{-1}]
0.6
0.4
0.2
0
-0.2
-0.4
-0.6
-1
-0.8
-0.6
-0.4
-0.2
0
0.2
0.4
0.6
0.8
5
4
3
2
1
0


λ = 6578.0 A
0.6
0.4
0.2
0
-0.2
-0.4
-0.6
Δδ [mas]
-1
-0.8
-0.6
-0.4
-0.2
0
0.2
0.4
0.6
0.8
Δα [mas]
5
4
3
2
1
0
I_ν [10^-4 erg cm^-2 s^-1 Hz^-1 sr^-1]

Differential visibility (ΔV)

↓ for Δφ, problems w. phase slips

- no convergence yet, mismatch, but trends probably ok?



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Brož (2017), Astrophys. J. Suppl. **230**, 19

Harmanec et al. (2018), Astron. Astrophys. **609**, 5

Mourard et al. (2018), Astron. Astrophys. **618**, 112

Budaj (2011), Astron. Astrophys. **532**, L12

Brož & Wolf (2017), Astronomická měření, Praha: Matfyzpress

more-sophisticated/alternative approaches:

Prša etal. (2016; PHOEBE2)

Borkovits etal. (2018; Lightcurvefactory)

Horvat etal. (2019)

Wilson & Van Hamme (2015; LCDC2015)

⋮