Charles University, Faculty of Mathematics and Physics
Astronomical Institute

Abstract of the PhD Thesis

Yarkovsky Effect and the Dynamics of the Solar System

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Jarkovského jev a dynamika sluneční soustavy

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1 Introduction

We see an increasing interest in the non-gravitational phenomenon called the *Yarkovsky/YORP effect* during the last 15 years. This is motivated by many observations of small asteroids (and their groups), which properties can be elegantly interpreted with help of the Yarkovsky effect — an electromagnetic recoil force arising due to anisotropic thermal emission from the surface of a celestial body. The implications of this phenomenon on the evolution of small Solar System bodies are the main topic of this thesis.

2 Non-gravitational forces in the Solar System

2.1 A brief history of non-gravitational phenomena

The Yarkovsky effect is called after IVAN OSIPOVICH YARKOVSKY (Figure 1), a person mostly unknown to present-day scientists. His work fell into oblivion, but 'his' effect (described in Yarkovsky (1901)) was rediscovered in 1950's by Öpik and Radzievskii.

Only a very recent publication by Beekman (2006) unveiled some information about his life and work and we present an excerpt in Section 2.1 of the thesis. We also reprint Yarkovsky's 'lost' publication in Section 9.3.

Figure 1: Ivan Osipovich Yarkovsky. From Beekman (2006).

2.2 Non-gravitational forces acting on small bodies

We review recent advances in the studies of non-gravitational forces. We focus on meteoroids and small asteroids in the 10 cm–10 km size range, for which the principal force and torque arise from an anisotropic thermal emission of the absorbed solar radiation energy. Related perturbations of the orbital and rotational motion are called the Yarkovsky and YORP effects. This section is an extended version of the reviews published in Brož *et al.* (2006) and in the Triennial report 2003–2006 of the IAU Commission 7.

The Yarkovsky force is related to the orbital dynamics (Rubincam 1995; Vokrouhlický 1998, 1999). Its diurnal variant (Figure 2), driven by the rotational frequency, dominates for bodies with low thermal conductivity (e.g., with regolith on the surface). It can either increase or decrease semimajor axis a and the change Δa is proportional to the cosine of the obliquity γ. In case of the seasonal variant, the changes of temperature on the surface are mainly driven by the orbital frequency. It is a usual situation for bodies with higher thermal conductivity (regolith-free surface). The semimajor axis a steadily decreases and $\Delta a \propto -\sin^2 \gamma$.

The YORP torque (Rubincam 2000; Vokrouhlický & Čapek 2002) works for non-spherical bodies only. It has an asymptotic behaviour — it pushes the obliquity towards 0 or 180° and the rotation period towards 0 or ∞.

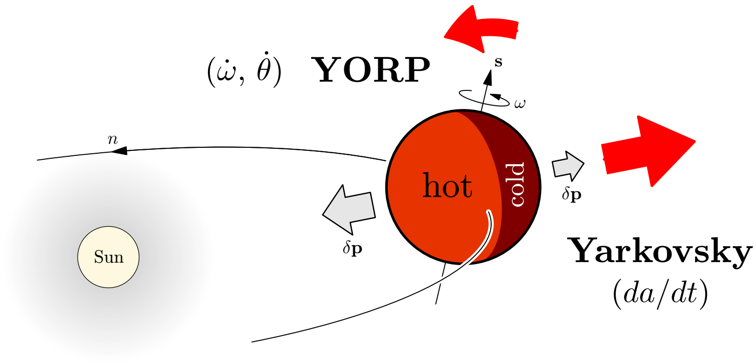


Figure 2: An illustration of the Yarkovsky/YORP effect principle. As an asteroid absorbs the solar radiation, its part facing the Sun becomes hotter than the reverse one. The infrared emission from the surface is then anisotropic, what gives rise to the Yarkovsky force, affecting the orbital motion of the asteroid, and the YORP torque, modifying the spin state. A non-zero transverse component of the force, caused by a thermal lag and the rotational or orbital motion, gives rise to a secular change of the semimajor axis.

The most important applications of the Yarkovsky/YORP models include:

1. precise ephemeris calculations (Chesley *et al.* (2003));
2. explanation of the bi-modal obliquity distribution of the Koronis family members (Vokrouhlický *et al.* (2003));
3. meteorite transport from the Main Belt (e.g., Farinella *et al.* (1998), Vokrouhlický & Farinella (2000));
4. delivery of near-Earth asteroids from the Main Belt (Morbidelli & Vokrouhlický (2003));
5. resupply of unstable resonant populations (Tsiganis *et al.* (2003), Brož *et al.* (2005b));
6. explanation of asteroid families distributions in the proper element space (Bottke *et al.* (2001), Vokrouhlický *et al.* (2006a), Vokrouhlický *et al.* (2006b));
7. measurement of the Yarkovsky semimajor axis drift for the youngest clusters (e.g., Nesvorný & Bottke (2004), Nesvorný *et al.* (2006c)).

2.3 A mathematical formulation of the Yarkovsky/YORP effect

We present, at first, a very simple analytical solution of the 1-dimensional heat diffusion equation, which allows us to quantitatively estimate the Yarkovsky acceleration. This solution, thought being simple and clear, holds basic properties of the Yarkovsky effect, such as its dependence on material, the rotational or orbital frequency; we also discuss the dependence on size and obliquity.

There is a description of the spherically symmetric solution by Vokrouhlický (1998), Vokrouhlický & Farinella (1999) and Vokrouhlický (1999) in the second part, supplemented by notes on its implementation in the `swift_rmvsy` numerical integrator package (i.e., the SWIFT by Levison & Duncan (1994) modified by Brož (1999) to account for the Yarkovsky diurnal and seasonal accelerations). An example, how the semimajor axis drift rates depend on size is depicted in Figure 3.

Finally, we mention 1-D non-linear numerical methods, which Čapek & Vokrouhlický (2004) use to calculate Yarkovsky/YORP effect for realistic shapes of asteroids.

3 Transport of meteoroids ejected from (6) Hebe, (170) Maria or (8) Flora

The Yarkovsky effect and gravitational resonances are the most important transport mechanisms of meteorite precursors, meteoroids or asteroidal fragments in the size-range 1 m to 1 km. We simulate the orbital evolution of putative particles ejected from three parent bodies: (6) Hebe, (170) Maria and (8) Flora. Some results of this study were summarised in the review Bottke *et al.* (2002b). We have verified that meteoroids can be delivered to Mars- and Earth-crossing orbits by the Yarkovsky effect and gravitational resonances on the timescale of the order 10 My. Larger asteroids can escape from the Main Belt due to chaotic diffusion in weaker resonances, which are very common in the inner Main Belt. Transport times to the EC- orbits might be generally consistent with the observed long CRE ages of meteorites.

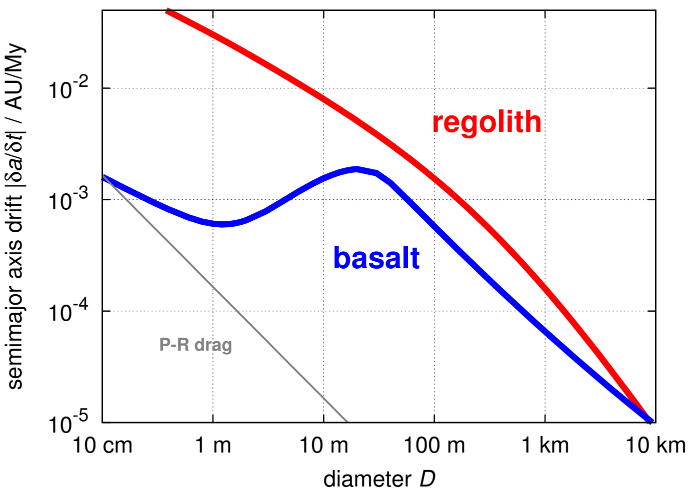


Figure 3: The sum of absolute values $|da/dt|$ of the diurnal and the seasonal semimajor axis drift rates vs. size, calculated for spherical bodies with a moderate value of obliquity $\gamma = 135^\circ$ and consisting of two materials: bare basalt or regolith covered, i.e., with high or low thermal conductivity. The sum plotted here shows clearly an approximate maximum total drift (per 1 My) one can expect. The mean collisional lifetime is roughly 50 My for a 10-m stony meteoroid and 500 My for a 1-km asteroid. Note, that we do not expect very small regolith-covered bodies to exist, thus the drift rates larger than 10^{-2} AU/My are not realistic. The drift rate caused by the Poynting-Robertson drag is also plotted; it prevails for sizes smaller than $\lesssim 10$ cm.

We present examples of various processes involved in the meteoroid transport: i) Yarkovsky drift and captures in low-order mean motion resonances (e.g., J3/1, J8/3) or the powerful ν_6 secular resonance; ii) crossing of resonances (which occurs commonly in the J3/1 case and rarely in the ν_6 case for the fastest drifting meteoroids); iii) interaction with weaker high-order resonances (e.g., J10/3); iv) exterior resonances with Mars (M1/2); v) 3-body resonances (4J−2S−1); vi) capture in high-order secular resonances (z_2); vii) frequent collisional reorientations, which cause a random walk behaviour for low-conductivity bodies and effectively decrease the diurnal Yarkovsky drift. All these processes affect delivery rates to Mars- and Earth-crossing orbits, they can both increase or decrease the rate as compared to a simple Yarkovsky semimajor axis drift towards the nominal position of gravitational resonances; it depends on individual source regions and meteoroid properties. We compute several statistical parameters, e.g., the rate of delivery to Mars-crossing orbits (Figure 4), the probabilities that particles of a given size are captured by a given resonance.

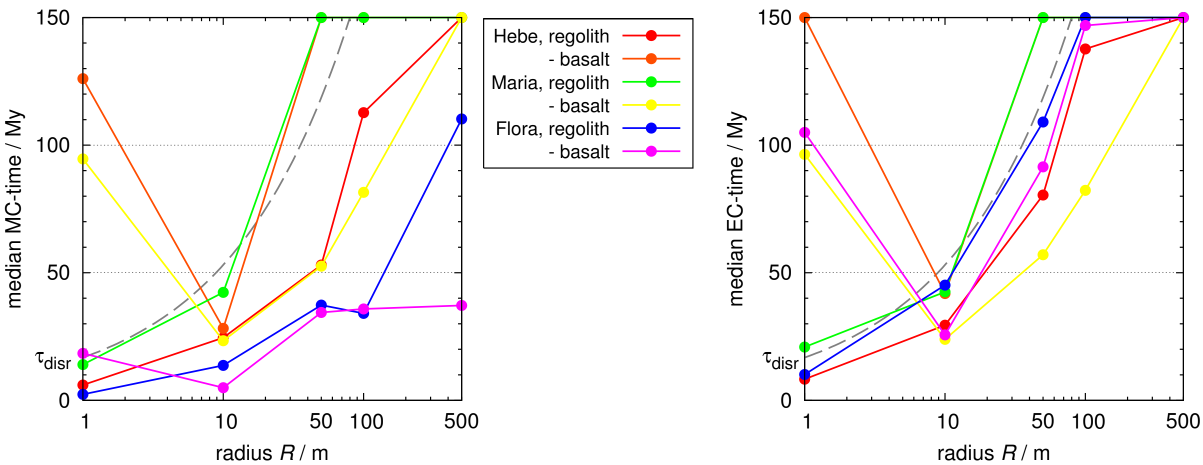


Figure 4: The dependence of the median Mars- and Earth-crossing times on the radius for regolith and basalt meteoroids originating from three parent bodies. Typical values are of the order 10 My, but the meteoroids drifting slowly or located far from the major escape routes have the median equal to 150 My (i.e., the time span of the integration), because less than half of the bodies ever reached MC (EC) orbits. The dashed curves indicate the characteristic disruption timescale $\tau_{\rm disr}(R) = 16.78\,{\rm My}\sqrt{[R]_{\rm m}}$.

Our results are consistent with the results of Bottke *et al.* (2000b) and we extend them in several respects. Compared to their work, we integrated meteoroids not only 1 m in radius, but also larger bodies. The time span of our integration was substantially longer (150 My vs. 50 My), which allowed us to calculate population decay rates for the sizes $R = 1$ m, 10 m, 50 m and 100 m. We use a more precise Mars-crossing criterion and we provide a direct comparison to the analytical decay rates used in the Monte-Carlo model by Vokrouhlický & Farinella (2000), which were calculated from pure Yarkovsky semimajor axis drift rates and fixed positions of major resonances. In the previous work, the initial velocities with respect to the parent bodies were zero, but in our case we included the initial spread. Additionally, we reported interactions with high-order secular resonances and we estimated the crossing probability of the J3/1 resonance.

4 Yarkovsky driven orbit and spin axis of (2953) Vysheslavia

The asteroid (2953) Vysheslavia and several others, located just above the 5/2 mean motion resonance with Jupiter, are known to have unstable orbits. The dynamical lifetime of Vysheslavia is estimated to be of the order of only 10 My. Such a situation poses a problem, since Vysheslavia is a member of the Koronis family, which is likely more than 2 Gy old. Three main hypotheses were developed to solve this apparent contradiction: (i) Vysheslavia might be an outcome of a recent secondary fragmentation event in the family, (ii) Vysheslavia might have been placed on its peculiar orbit by close encounters with nearby massive asteroids, or (iii) the asteroid might have been transported by a slow inward–drift of the semimajor axis due to the Yarkovsky effect.

We present numerical simulations of the orbital evolution, with the Yarkovsky effect included, for Vysheslavia and several neighbouring asteroids and we bring evidence that all these asteroids have been transported to this unstable region from stable regions (with larger semimajor axes) by the Yarkovsky semimajor axis drift (Vokrouhlický *et al.* (2001), Brož & Vokrouhlický (2002); see also Figure 5).

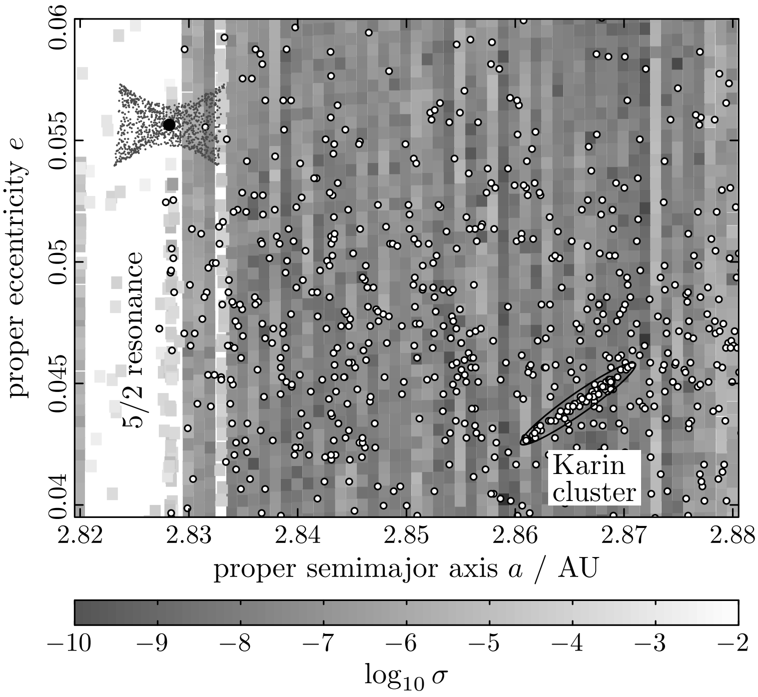


Figure 5: Koronis family members (open circles) projected on the plane of synthetic proper semimajor axis and proper eccentricity; the Karin cluster is indicated in the lower right corner. Assuming an isotropic velocity field, ejecta from a hypothetical secondary collisional disruption near Vysheslavia (full circle) would fall in the region indicated by grey dots. Only one of the observed Koronis asteroids falls in this zone, notably in its stable part exterior to the 5/2 mean-motion resonance with Jupiter. Orbital stability is indicated by grey shading indicating a parameter $\sigma = 1 - n^{(2)}/n^{(1)}$, where $n^{(1)}$ and $n^{(2)}$ are proper mean motions of test particles calculated in the two consecutive time intervals, each spanning 1.2 My (dark zones of small σ value imply stability, white areas of high σ value, such as the 5/2 mean motion resonance with Jupiter, are unstable). The family members were identified using the hierarchical clustering method with the standard metric and the velocity cutoff 60 m/s.

This scenario predicts the asteroids have retrograde orientations of the spin axes (in order to drift towards the J5/2 resonance) and this was confirmed for Vysheslavia by photometric observations during the years 2000–2005 (Vokrouhlický *et al.* (2005e)). We find admissible solutions for ecliptic latitude and longitude of the rotation pole P_3: $\beta_{\rm p} = -64^\circ \pm 10^\circ$ and $\lambda_{\rm p} = 11^\circ \pm 8^\circ$ or P_4: $\beta_{\rm p} = -68^\circ \pm 8^\circ$ and $\lambda_{\rm p} = 192^\circ \pm 8^\circ$. These imply obliquity values $\gamma = 154^\circ \pm 14^\circ$ and $\gamma = 157^\circ \pm 11^\circ$, respectively. The sidereal rotation period is $P_{\rm sid} = (0.2622722 \pm 0.0000018)$ day.

5 Yarkovsky origin of the unstable asteroids in the 2/1 resonance with Jupiter

The 2/1 mean motion resonance with Jupiter, intersecting the main asteroid belt at $\approx$3.27 AU, contains a small population of objects. Numerical investigations (Roig *et al.* (2002)) have classified three groups within this population: asteroids residing on stable orbits (i.e., Zhongguos), marginally stable orbits with dynamical lifetimes on the order 100 My (i.e., Griquas) and unstable orbits. We reexamine the origin, evolution and survivability of objects in the 2/1 population (Brož *et al.* (2005b)).

Using recent asteroid survey data, we have identified one hundred new members since the last search, which increases the resonant population to 153. The most interesting new asteroids are those located in the theoretically-predicted stable island A, which until now had though to be empty (Brož *et al.* (2005a)).

Next, we investigated whether the population of objects residing on the unstable orbits could be resupplied by material from the edges of the 2/1 resonance by the Yarkovsky/YORP effect. Using N-body simulations, we showed that test particles pushed into the 2/1 resonance by the Yarkovsky effect visit the same regions occupied by the unstable asteroids (Figure 6). We also found that our test bodies had dynamical lifetimes consistent with the integrated orbits of the unstable population (Figure 7).

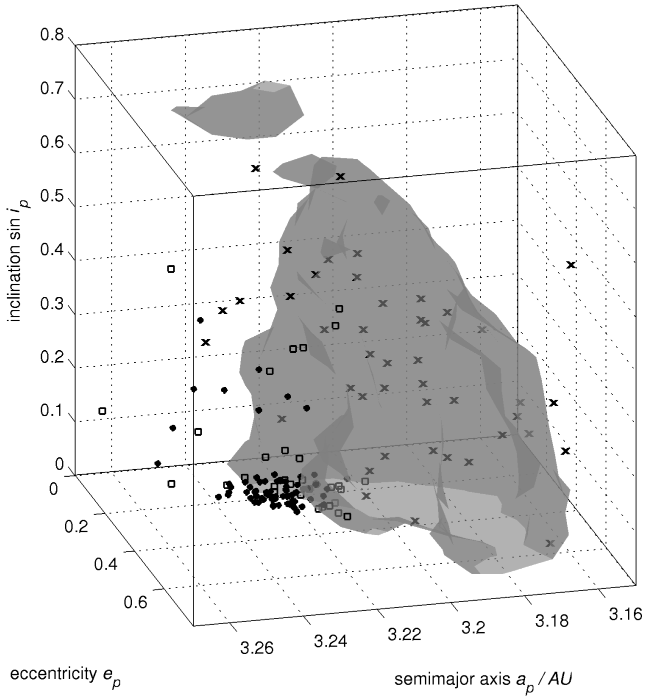


Figure 6: An $n_{\rm TP} = 500$ iso-surface of the number density in the pseudo-proper orbital element space $(a_p, e_p, \sin I_p)$, resulting from our numerical simulation of test particles originating in the neighbouring Main Belt. n_{TP} is given as a weighted mean of the contributions by the three source populations: the background population (contributing by 84.5 %), the Themis family (14.2 %) and the Hygiea family (1.3 %). $n_{\rm TP}$ reaches its maximum value of $\simeq 3 \times 10^4$ particles per cell per 1 Gy inside this zone. Symbols denote positions of the observed populations inside the 2/1 resonance: (i) Zhongguos (filled circles), (ii) Griquas (squares), and (iii) the unstable asteroids (crosses). The 3-D surface is plotted as semi-transparent and one can distinguish the objects, which are in front of, inside or behind the surface, because they are gradually more and more gray/hidden. We see the test particles visit mainly the space, where the unstable asteroids are located; on the other hand, they avoid positions of long-lived Zhongguos and Griquas. An illustrative animation with several coloured and partially transparent iso-surfaces can be found on *Yarko-site*.

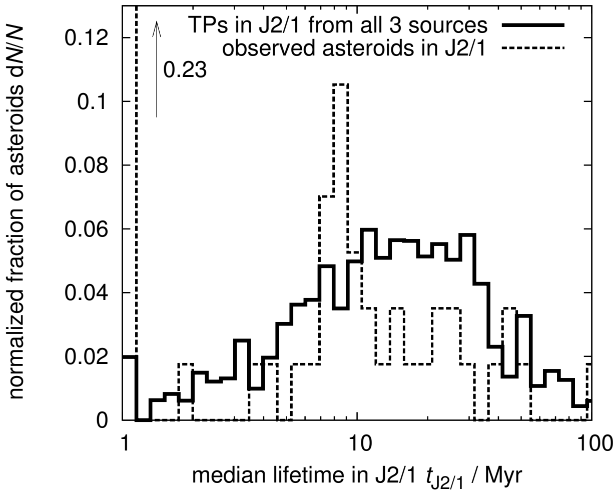


Figure 7: Distribution of the residence time inside the J2/1 for: (i) test bodies which were delivered into the J2/1 by the Yarkovsky effect (bold solid curve) and (ii) observed members of the unstable J2/1 population currently residing in the resonance (dashed curve). In the case (i), the residence time records the time interval from entry into the J2/1 till escape out of the J2/1, while the case (ii) records the time interval from the present day to escape. The number of bodies dN in each logarithmic bin has been normalised by their total number N.

Using a semi-analytical Monte-Carlo model, we computed the steady-state size distribution of magnitude $H < 14$ asteroids on unstable orbits within the resonance. Our results provide a good match with the available observational data. The J2/1 objects with extremely short dynamical lifetimes ($\leq 2\,$My) are probably Jupiter family comets or near-Earth asteroids. We were not able to unveil the origin of the long-lived Zhongguos and Griquas.

6 Yarkovsky footprints in the Eos family

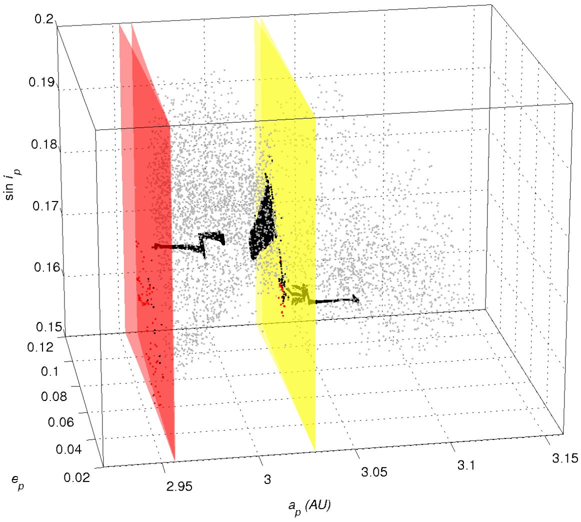
The Eos asteroid family is the third most populous, after Themis and Koronis, and one of the largest non-random groups of asteroids in the main belt. It has been known and studied for decades, but its structure and history still presented difficulties to understand. We first revise the Eos family identification as a statistical cluster in the space of proper elements. Using the most to-date catalogue of proper elements we determine a nominal Eos family, defined by us using the hierarchical-clustering method with the cut-off velocity of 55 m/s, contains some 4400 members. This unforeseen increase in known Eos asteroids allows us to perform a much more detailed study than was possible so far (Vokrouhlický *et al.* (2006a)).

We show, in particular, that most of the previously thought peculiar features are explained within the following model: (i) collisional disruption of the parent body leads to formation of a compact family in the proper element space (with characteristic escape velocities of the observed asteroids of tens of metres per second, compatible with hydrocode simulations), and (ii) as time goes, the family dynamically evolves due to a combination of the thermal effects and planetary perturbations. This model allows us to explain sharp termination of the family at the J7/3 mean motion resonance with Jupiter, uneven distribution of family members about the J9/4 mean motion resonance with Jupiter, semimajor axis distribution of large vs. small members in the family and anomalous residence of Eos members inside the high-order secular resonance z_1 (Figure 8). Our dynamical method (Vokrouhlický *et al.* (2006b)) also allows us to estimate Eos family age to $1.3^{+0.15}_{-0.2}$ Gy.

Several formal members of the Eos family are in conflict with our model and these are suspected interlopers. We use spectroscopic observations, whose results are also reported here, and results of 5-color wide-band Sloan Digital Sky Survey photometry to prove some of them are indeed spectrally incompatible with the family.

7 The Agnia family embedded inside the z_1 secular resonance

The Agnia asteroid family, a cluster of asteroids located near semimajor axis $a = 2.79$ AU, has experienced significant dynamical evolution over its lifetime. The family, which was likely created by the



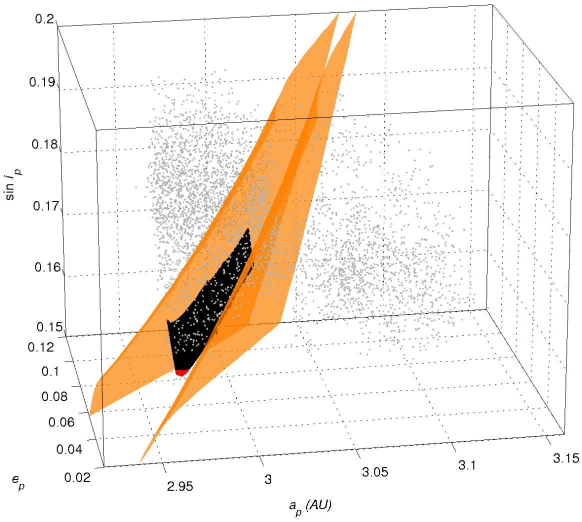


Figure 8: The Eos family in the 3-dimensional space of proper elements a_p, e_p and $\sin I_p$. The three resonances, J7/3 and J9/4 (left) and z_1 (right) are plotted together with examples of bodies drifting due to the Yarkovsky effect from the center of the family and interacting with these resonances. An illustrative animation can be downloaded from *Yarko-site*.

breakup of a diameter $D \sim 50$ km parent body, is almost entirely contained within the high-order secular resonance z_1. This means that unlike other families, Agnia's full extent in proper eccentricity and inclination is a byproduct of the large-amplitude resonant oscillations produced by this resonance. Using numerical integration methods, we found that the spread in orbital angles observed among Agnia family members (Figure 9) would have taken at least 40 My to create; this sets a lower limit on the family's age (Vokrouhlický *et al.* (2006c)).

To determine the upper bound on Agnia's age, we used a Monte Carlo model to track how the small members in the family evolve in semimajor axis by Yarkovsky thermal forces. Our results indicate the family is no more than 140 My old, with a best-fit age of 100^{+30}_{-20} My. Using two independent methods, we also determined that the $D \sim 5$ km fragments were ejected from the family-forming event at a velocity near 15 m/s. This velocity is consistent with results from numerical hydrocode simulations of asteroid impacts and observations of other similarly-sized asteroid families. Finally, we found that 57% of known Agnia fragments are prograde rotators. The reason for this limited asymmetry is unknown, though we suspect it is a fluke produced by the stochastic nature of asteroid disruption events.

8 Conclusions

We studied the influence of non-gravitational forces on the long-term evolution of meteoroids and small asteroids. The major results presented in this thesis can be summarized as follows:

1. We reviewed the applications of the Yarkovsky/YORP effect and its mathematical formulation.

2. We described major aspects of the orbital evolution of meteoroids and small asteroid fragments under the influence of gravitational resonances and the Yarkovsky effect. We computed several statistics (e.g., the crossing probability of the J3/1 resonance) which may be used in future to improve Monte-Carlo models of meteoroid transport.

3. The short-lived orbits of (2953) Vysheslavia and several other asteroids located just above 5/2 mean motion resonance with Jupiter can be explained as a consequence of the Yarkovsky effect, which pushes the asteroids from stable regions into the current unstable positions. We supported this conclusion by photometric observations of (2953) Vysheslavia — they reveal the retrograde spin-axis orientation, which is in concert with negative Yarkovsky semimajor axis drift.

4. The unstable asteroids inside 2/1 mean motion resonance with Jupiter have been most probably transported from the neighbouring Main Asteroid Belt; the Yarkovsky/YORP effect is efficient enough to keep this transient population in steady-state. Unfortunately, we were not able to unveil the origin of the long-lived Zhongguos and Griquas yet.

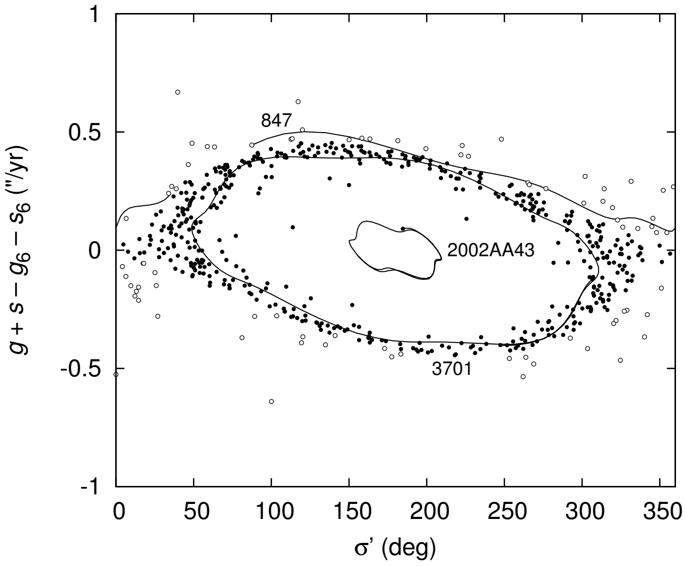


Figure 9: Agnia family members (symbols) projected onto the plane of critical angle σ of the z_1 resonance vs. the associated frequency $\dot{\sigma} = g + s - g_6 - s_6$. Bulk of the family asteroids reside inside this resonance and close to its separatrix. Filled symbols for librating orbits, open symbols for circulating orbits. The solid lines show evolution paths as determined from 10 My numerical integration for three characteristic cases: (i) asteroid (3701) Purkyne is the most typical evolutionary track in the family (large-amplitude libration), (ii) asteroid 2002 AA43 is a rare case of small-amplitude libration in the resonance, and (iii) asteroid (847) Agnia is located right outside the resonance, but very near to its separatrix. Libration period for (3701) Purkyne is about 8 My, circulation of (847) Agnia, very close to the separatrix, is even slower. Dispersion of the family members along all values of the resonance angle σ shows the age of the family to be at least several times the characteristic libration period.

5. We studied the Eos asteroid family, namely the three dynamical processes, how the Yarkovsky drifting orbits interact with resonances: "bracketing" by strong mean motion resonances, "crossing" of weaker resonances, and "trapping" in secular resonances. The Yarkovsky effect seems to be essential to understand the current observed shape of the family in the space of proper orbital elements. We were also able to determine the family age to be 1.3 Gy.

6. The Agnia family is located almost entirely inside the z_1 secular resonance, a lucky coincidence, which allowed us to disclose the family age (surely more than 40 My and less then 140 My) and the magnitude of the mutual velocities (15 m/s for 5 km fragments) gained due to the original disruption event.

7. We implemented computation of proper and resonant orbital elements into the commonly used SWIFT integration package.

9 Appendices

9.1 A catalogue of synthetic proper elements

In order to study fine details of orbital dynamics and small structures in the (a, e, I) elements space, we have to apply a suitable digital averaging to the osculating elements produced by numerical integrators. We use the following approach for regular (non-resonant) orbits: i) we sample osculating orbital elements; ii) we apply a Kaiser windows filter (Quinn *et al.* (1991)) to remove fast-period oscillations; iii) we apply frequency modified Fourier transform (Šidlichovský & Nesvorný (1997)) to obtain proper frequencies, amplitudes and phases; iv) we drop the terms with planetary frequencies and the remaining largest amplitudes are the required proper elements.

We implemented the filtering outlined above in the framework of the SWIFT integrator. As a particular test of our algorithms, we prepared a catalogue of proper elements (`prop_fmft.dat`; presented at *Yarko-site*). The system allows an automated calculation of proper elements for all numbered asteroids, listed the Bowell's AstOrb catalogue of osculating elements, and a comparison with the already existing AstDyS proper elements catalogue by Milani and Knežević.

9.2 The SWIFT-MVS2 integrator, a faster variant of the MVS

The numerical integrator package SWIFT (Levison & Duncan (1994)) is a well known and commonly used tool for solar system studies. We have modified the original code in the following manner: i) We have incorporated Yarkovsky and Poynting-Robertson dissipative accelerations into the integrators and checked their results against analytic predictions (see Section 2.3). ii) We have implemented on-line digital filters based on the Kaiser windows (see Section 9.1). iii) We have implemented the 2^{nd} order symplectic integration scheme (MVS2) by Laskar & Robutel (2001), which seems to be at least 2 times faster than MVS, while keeping the same relative energy error (Figure 10). However, the algorithm is not "regularised", i.e., no close encounters are allowed. iv) We have parallelised integrators (namely the calculation of the TP accelerations) according to the OpenMP standard, which allows to run SWIFT on multiprocessor machines. We present results of various tests involving the new integrators. The `swift_rmvsy` package is available on our *Yarko-site*.

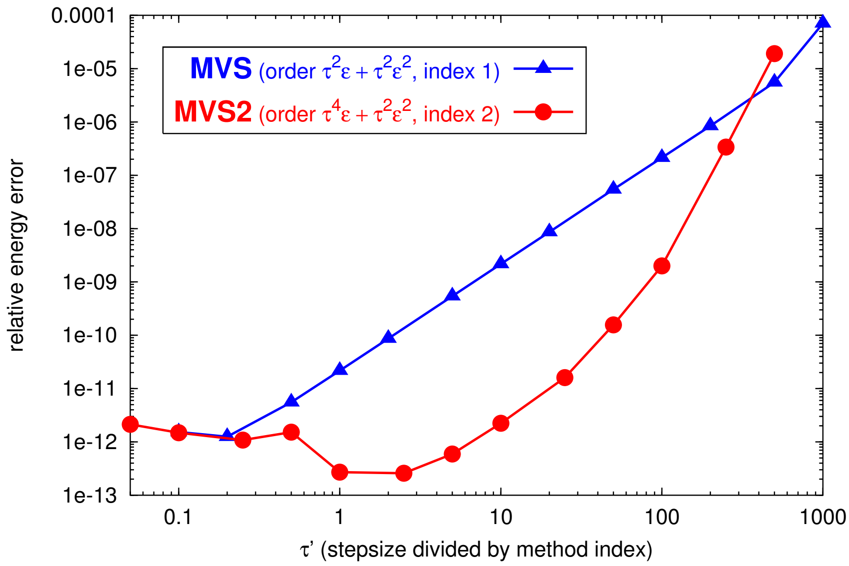


Figure 10: The relative timestep τ' vs. the relative energy error, computed for the Sun-Jupiter-Saturn system evolving for 0.1 My. τ' is the timestep $\mathrm{d}t$ divided by the method index, i.e., 1 for the classical leap-frog MVS and 2 for the new MVS2. The relative energy error is computed as the standard deviation σ_E of the total energy $E(t)$, in course of the integration, divided by the mean total energy $\langle E \rangle$.

9.3 A reprint of Yarkovsky's 'lost' pamphlet

The reprint of Yarkovsky's 'lost' pamphlet quoted by Öpik (1951) is included in the thesis, which can be downloaded from the *Yarko-site*. All credit goes to George Beekman (e-mail: `gbeek@xs4all.nl`), who rediscovered the publication in the library of the Sternberg Astronomical Institute in Moscow in 2002.

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