Hektor — peculiar family among Jovian Trojans

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ABSTRACT

In this work we analyze Jovian Trojans in the space of proper elements of Jovian Trojans and we identify clusters of possible collisional orgin by two independent methods: HCM and *randombox*. Compared to our previous work (Brož & Rozehnal 2011), we updated our database of suitable resonant elements and thus we can study a twice larger sample than before. Apart from the Eurybates and Ennomos families, we also found four clusters consisting of small asteroids – namely families around asteroids (20961) Arkesilaos, (624) Hektor and (9799) 1996 RJ in L_4 cloud and (247341) in L_5 cloud.

Using the WISE and AKARI albedos and diameters (Grav et al. 2011, 2012 and Usui et al. 2011), we constructed size-frequency distributions of Trojans in both the leading/trailing clouds. As the families fulfill our stringent criteria (i.e. a high statistical significance, an albedo homogeneity, a steeper SFD than that of background), we tried to determine their parent-body sizes. Then we simulate their subsequent collisional evolution using the SPH simulations (Durda et al. 2007) and the Boulder code (Morbidelli et al. 2009), and also dynamical evolution (using the SWIFT code, Levison and Duncan, 1994). Within the framework of our evolution model, we determine the age of the Hektor family to be between XXX.

Since (624) Hektor is a close binary with a sattelite (Marchis et al. 2014), i.e. an exceptional object, we want to address its association with the family and we show the whole system could be indeed created during one event (i.e. cratering) XXX.

Key words: celestial mechanics – minor planets, asteroids – methods: *N*-body simulations.

1 INTRODUCTION

Jovian Trojans are two numerous populations of minor bodies in 1:1 MMR with Jupiter, librating around L_4 and L_5 points. There are in general two theories about their origin: i) theory in the frame of accretion model (e.g. Goldreich 2004) and ii) theory in the frame of the Nice model (Gomes et al. 2005, Tsiganis et al. 2005, Morbidelli et al. 2005, Morbidelli et al. 2010). Since the librating regions are very stable and the probability of capture of small bodies from other source regions (e.g. Main belt, Centaurs, Jupiter family comets) is very low (cite XXX), Jupiter Trojans represent primordial population of small bodies orbiting in the Solar system in the early stages of its evolution.

As we have shown in Brož & Rozehnal 2011, no family could not survive even late phases of slow migration of Jupiter, which is assumed by the Nice model (Morbidelli et al. 2010). Hence, we can determine whether the observed propperties of families among Jovian Trojans are consistent with the Nice model, i.e. if ≈ 3.8 Gyr of collisional and dynamical evolution is able to produce the Trojan population we observe today.

2 NEW OBSERVATIONAL DATA

Earlier works about the Jovian Trojans (e.g. Roig et al. 2008, ... XXX) were faced with a lack of observational data, what could lead in some cases to the incorrect conclusions. For example, as we have shown in Brož & Rozehnal 2011, total number of observable families amnog Jovian Trojans is significantly lower than was previously thought – this conlusion we made on the basis of analysing about twice larger sample than was studied in Roig et al. 2008. The difference lay in the fact that due to the lack of data, there occured empty areas in the space of proper elements, leading to erroneous inclusion of bodies to the collisional families. As new obser-

vational data filled this empty areas, mistakenly identified families "disappeared".

Hence, we need to evaluate our conclusions on even larger sample of data, what could also allow us to reveal possibly yet–unknown structures in the space of proper elements and unweil possible links between orbital and physical propperties (e.g. albedos, diameters) of Jovian Trojans.

2.1 Resonant elements

Using the integration with the SWIFT integrator (Levison & Duncan, 1994), we computed proper resonant elements (i.e. the libration amplidude Δ , eccentricity e, inclination I) of 3907 Trojans in L_4 cloud and 1945 Trojans in L_5 cloud. For this purpose we used osculating elements listed in the AstOrb catalogue (cite xxx), released in July 2014. A detailed description of the resonant elements computing can be found in Brož & Rozehnal 2011. Images of Trojans in the space of proper elements (a_p, I_p) and (e_p, I_p) are on figure 1, together with their sizes and albedos, see later.

2.2 WISE and AKARI albedos and diameters

To construct Size-frequency distributions of the whole Trojan clouds and compact groups, we used WISE albedos and diameters derived by Grav et al. 2012, which we also compared to AKARI albedos and diameters as reported in Usui et al. 2011.

- treatment aboud WISE/AKARI coincidence (just 1-sigma level!)???

3 PHYSICAL CHARACTERISATION OF TROJAN POPULATIONS

3.0.1 Albedo/size dependence

- goal: to show, if there is albedo/size depnendence or note - purpose: construction of reliable SFD

While some work suggests that the albedo of Trojans depends on their size (e.g. XXX), most agree that it is not actually a physical effect (e.g. XXX).

There are in general two possibilities: i) there are missing small ($D \lesssim 5$ km) bodies with low albedo in the WISE sample, or ii) there is a systematic error in determination of albedo of these small object, resulting in fact that albedo of these bodies seems to be higher than it actually is. It is crucial to determine which case is true, because it could have far-reaching consequences: If there is actually present the group of bodies with very high albedo compared to all Trojans, we can split Trojans to sub-populations with respect to their albedo and look up for possible connections between albedo and dynamical properties. On the other side, if the high albedo of small objects is just an artifact, it will affect the shape of the size-frequency distribution, and will conseqently change the collisional model.

3.0.2 Albedo distribution

3.1 Size-frequency distributions

The WISE data provide us very useful source of information about diameters we need to construct SFD of whole Trojan population in L_4 and L_5 cloud. However, the Trojans sample measured by the WISE satellite is not complete and, what is more important, is not debiased. XXX Consequences???

In our previous work, we have constructed SFD of the whole Trojan population assuming constant albedo of Trojans, which we set to be equal to the median albedo of Trojans that was measured then. Since the number of measured albedo was then very low, this was the only one reasonable way to create SFD. However, when we construct SFD using the new WISE data, we obtain completely different picture, as one can see on figure 2.

As we have about 1609 albedos measured by WISE (about one half of these albedos are measured during cryophase, the second half is measured in post-cryo-phase) and resonant elements for more than 5800 Trojans, we constructed SFD by random assingning the albedo from the WISE sample to those Trojans, whose albedo is still unmeasured. To avoid bias, we tried to compare different SFDs constructed with different "WISE distribution" random generator and we realized, that overall shape of SFD does not change noticeably, what means the slope of SFD γ varies in the range of ± 0.1 for different generators xxx najit poznamky s presnymi cisly!!!.

SFDs for L_4 and L_5 clouds look different in the size range from 60 km to 100 km. This part of SFD is not influenced by Eurybates family, the largest family among Trojans, because all its members have diameter D < 50km, so this part of SFD is not connected to any known collisional event. While the slope of L_4 SFD is ($\gamma = -2.0 \pm 0.1$) in this range, the slope of L_5 SFD is ($\gamma = -1.1 \pm 0.1$) in the same range. However, this could natural phenomenon as in the case of L_5 there is smaller number of bodies in this size range. As mentioned by (Grav et al., 2012), they estimate the ratio of $N_{\text{leading}}/N_{\text{trailing}}$ to be 1.4 ± 0.2 , lower than the 1.6 ± 0.1 value derived by Szabó et al. (2007).

But this may also suggest a different scenario of collisional evolution of each cloud, or, if we consider this part of SFD as primordial (see chapter 6), partially different source regions for each cloud.

- goal: Do L4 and L5 originate from a common source population?

4 METHODS OF DETECTION

4.1 Randombox method

Besides the commonly-used Hierarchical clusterring method, we used a "Randombox method", based on the Monte Carlo simulations. This method allows us to compute the statistical significance of the concentrations of bodies in the space of proper elements (a, e, sin I). We can also use an analytical formula:

$$p = \frac{\sum C(n,k)V'(n_{box} - 1, n - k)}{V'(n_{box}, n)},$$
(1)

We plot the results on the picture 3 for both the L4 (left) and L5 (right) clouds. Probabilities p that clusters of bodies in the space of proper elements (green dots) are random, are marked by boxes of different colours, ranging from the dark blue (i.e. low significance) to yellow (high significance, see the scale next to the pictures). Using this method, we evaluated all families identified by common Hierarchical



Hektor — peculiar family among Jovian Trojans 3

Figure 1. The resonant elements $(a_p \equiv \bar{a} + \Delta, I)$ and (e, I) for L_4 and L_5 Trojans. The circles indicate relative diameters of bodies, as determined by WISE (Grav et al. 2011), or, when unmeasured by WISE, computed from absolute magnitude H and geometric albedo p_V , which we assumed to be $p_V = 0.08$ for both the L_4 and L_5 Trojans (WISE median value is $p_V = 0.082$ for L_4 and $p_V = 0.077$ for L_5 Trojans).



Figure 2. Size-frequency distributions for both L4 and L_5 Trojans, created using the albedos measured by WISE sattelite (Grav et al. 2012). Since WISE data cover just about 18% of L_4 and 29% of L_5 Trojans known today, we assigned albedo randomly chosen from the WISE sample to those Trojans, which albedo was not determined by WISE. Together with overall distribution of L_4 and L_5 clouds, we also present SFDs of families discussed in the main text. Together with SFDs we show our fits of each SFD in the range 12 – 30 km by the power law $N_{(>D)} \propto CD^{\gamma}$. As we can see, whole clouds are near the collisional equilibrium ($\gamma \simeq -2.5$, (Dohnanyi 1969)), while most families have slope γ much steeper.

designation	$v_{cutoff}[m/s]$	$N_{\rm memb}$	$p_v(WISE)$	tax. type	diameter [km]	$D_{PB_{min}}$	D_{Durda}	LF/PB	$v_{esc}[m/s]$	age [G
(624) Hektor	110	90	0.087 ± 0.016	D	164 ± 7	171	216	0.0005	73	0 to 3
(3548) Eurybates	60	310	0.060 ± 0.016	C/P	59.4 ± 1.5	100	155	0.03	46	1.0 to 3
(9799) 1996RJ	140	17	0.082 ± 0.014	_	58.3 ± 0.9	61	88	0.006	26	-
(20961) Arkesilaos	55	35	n/a	-	24 ± 5	37	87	0.01	16	-
(17492) Hippasos	100	104	0.064 ± 0.012	_	55.2 ± 0.9	67 - 154	95 - 168	0.06	29 - 66	1 to 2
(247341) 2001UV209	120	30	0.088 ± 0.023	-	16.3 ± 1.1	32	80	0.005	14	-

Clustering Method (Zappala et al., 1990), what makes our decision wheather the cluster is real family or not much more objective.

4.2 Hierarchical clustering method

We used HCM to detect "suspect clusters", which may be originated by collisional disruption. For those clusters, we constructed dependece of the number of members of the cluster $N_{\rm memb}$ on the cutoff velocity $v_{\rm cutoff}$, because this is another clue to collisional origin of the family. As we have mentioned in (Brož & Rozehnal, 2011), the number of members of the real collisional family rises first slowly with rising $v_{\rm cutoff}$, in contrast with random clusters, which are merging wery quickly with the background. For all families listed in the table 1 we was convinced that they fullfill this criterion.

5 PROPERTIES OF STATISTICALLY SIGNIFICANT GROUPS

5.1 Eurybates

As we have already shown in (Brož & Rozehnal, 2011), the family associated with the asteroid (3548) Eurybates is the largest collisional family, and it is the only one family among Trojans, which originated by catastrophic disruption (this means that the ratio of mass of the largest fragment to the mass of the parent body $M_{\rm LR}/M_{\rm PB} < 0.5$) of the parent body with the size $D_{\rm PB} > 100$ km. Using albedos derived by (Grav et al., 2012) from the WISE measurements, we recalculated overall slope of SFD of this family. We determine this value to be $\gamma = -3.46 \pm autobusxxx$, significantly steeper than previous calucation of $\gamma = -2.5 \pm 0.1$, derived assuming constant albedo of all members of the family. With the new data we derived the new value of diameter of the parent body, which is still above the border of 100 km: the minimum value given by simple sum of the sizes of the family members is $D_{\rm PB_{min}} \simeq 100$ km, while the approximation of the SFD by power law gives the value $D_{\rm PB} \simeq 140$ km. Finally, by fitting the synthetic SFD from SPH simulations (Durda et al., 2007) we obtained value $D_{\rm PB_{SPH}} \simeq 155 \rm km$.

There is one interesting fact concerning the taxonomical classification of the Eutybates family, which is widely classified as C-type (e.g. XXX). But when we look at Trojan asteroids larger than 50 km, we found that number of C type asteroids is about order magnitude less than number of D-type asteroids (Grav et al., 2012). As we observe just one family created by catastrophic disruption of parent body larger than 100 km among Trojans, why it belongs to the C-type? Beside random coincidence, there are two alternatives to explain that fact: One possibility is that there are more C-type asteroids among bodies with diameter larger than D > 50km. However, it seems unlikely - according to (Grav et al., 2012), there is 85% D-type and 15% C/P-type asteroids among objects with diameters larger than 60 km. The second possibility is that catastrophic disruptions of D-type parent bodies do not lead to the origin of asteroid families. So far, this was supported by observational fact, that we did not observe D-type families, but as we show later, it is not entirely true. But, if this would be true, there should be about order magnitude more catastrophic disruptions of bodies larger than 50 km. But this is not consistent with results of our simulations of collisional evolution, as we will see later. For this reasons we consider orgin of C-type family among the bodies dominated by D-type asteroids to be accidental.

5.2 (624) Hektor — first D-type family?

Since (624) Hektor is a close binary with a sattelite (Marchis et al. 2014), i.e. an exceptional object, we want to address its association with the family. The cluster around the largest Trojan asteroid appears in the space of proper elements as relatively compact group, which is limited particullary in proper inclinations in the range $I_{\rm p} \in \langle 18.13^{\circ}; 19.77^{\circ} \rangle$ and with pseudo-proper axes lying in the interval $a_{\rm p} \in \langle 5.234 \, {\rm au}; 5.336 \, {\rm au} \rangle$. Number of members of this group is slowly increasing with increasing cutoff velocity up to $v_{\rm cutoff} \simeq 110 \, {\rm ms}^{-1}$, above which it is quickly joining with the background. With our Randombox method, we estimated probability that the family is just a random cluster to be $P_{\rm random} \simeq 2 \cdot 10^{-3}$.

The nominal diameter of asteroid (624) Hektor derived from its albedo measured by WISE is 164 km (Grav et al., 2012) but we found that albedo measured by AKARI $p_V = 0.034 \pm 0.001$ (Usui et al., 2011) totally differs from that measured by WISE $p_V = 0.087 \pm 0.016$ and these values do not match even within the error limits¹, we do not determine Hektor's diameter from its albedo, but from fits of Marchis et al. (2014), which is $D = 250 \pm 26$ km for all possible geometries (convex, bilobe and binary). For other bodies in family we use a nominal value $p_V = 0.072$, which is median of WISE measurements.

 $^{^1}$ This may be caused by the bilobed shape of the asteroid (Marchis et al., 2014), which albedo was measured in different phases.







Figure 3. Statistically significant groups in L4 ans L5 Trojans, determined by "Randombox method".

5.2.1 Simulation of long-term dynamical evolution

To get an upper limit of the Hektor family age, we simulated a long-term evolution of the synthetic families created in different geometries. Our model included four giant planets on current orbits, integrated by the symplectic integrator SWIFT (Levison and Duncan 1994) modified according to Robutel and Lascar (2001), with the timestep of 91 days.

We created the synthetic family by assigning random velocities to 234 bodies (i.e. 3 times more than the number of the observed Hektor family members) assuming the model with isotropic velocity field with typical velocity of 70 ms^{-1} corresponding to escape velocity from parent body (Farinella et al., 1993).

Then we simulated long-term dynamical evolution of 7 synthetic families, created by disruption in different geometries. To create a synthetic family in the same place in the space of proper elements occupied by the observed Hektor family, we integrated orbit of asteroid (624) Hektor with osculating elements from AstOrb catalogue (xxx cit) untill we got appropriate values of true anomaly f and arument of pericentre ω . We tried values of f ranging from 0° to 180° with step 30° and ω satisfying the condition $f + \omega = 60°$, i.e. we fixed angle distance from the node to ensure a comparably large perturbations.

Initial shapes of synthetic families just after the disruption compared to the observed Hektor family are on the picture 6. To make quantitative comparison of the distribution in the space of proper elements, we used two-dimensional Kolmogorov-Smirnov test to compute K-S distance of the synthetic family to the observed one with timestep of 1 Myr of evolution. The result for different initial geometries is shown on the picture 5.

Our two best fits corresponding to the lowest K–S distance are on the picture 7. Left picture shows evolution of the synthetic family originated in $(f = 0^{\circ}, \omega = 60^{\circ})$ in the space of proper elements (a, e) after 364 Myr from disruption.

However, as we can see from the image of the whole Trojan L4 population, Hektor is near the border of stability of librating region. On picture 6 we can se, that there are almost no asteroids on the right side $(a_p > 5.32 \text{ au})$ of the graph of real family, but we can see them on the graph of our syntetic family.

On the other side, when we look on the picture describing our secon best fit (picture 7, right) in the space (a, e)(initial geometry $f = 150^{\circ}$, $\omega = 270^{\circ}$) in the snapshot of age approx 3000 Myr, we can see that there is much less bodies outside the long-term stable librating region.

Hence, it is probable, that the right geometry in which disruption occured, is $(f = 150^{\circ}, \omega = 270^{\circ})$ and the age lies between 1 – 4 Gyr rather then 250 – 500 Myr for geometry $(f = 0^{\circ}, \omega = 60^{\circ})$.

Texty z posteru:

XXXFor f = 180 deg the shape of the syntetic family can be compatible with the observed one, even at t = 0, which is the lower limit of the family age.XXX

XXXThe structures in the space of proper elements may dissapear only after approximately 1 Gyr. We think the sructure will persist up to 4 Gyr, so we can exclude this initial geometry (f = 0 deg).

5.2.2 SFD fitting by SPH

We tried to estimate the parent body size of Hektor family and other families by the method described in Durda et al., (2007). To this point, we calculated a "pseudo-chi-square" for the whole set of size-frequency distributions as given by the SPH simulations results (see figure 4. We will use these as initial conditions for simulations of collisonal evolution. The parent body size for the Hektor family is about ($260 \pm$ 20) km.

5.3 1996RJ - extremely compact

In our previous work, we mentioned small cluster associated with the asteroid (9799) 1996 RJ, which consisted of just 9 bodies. With the contemporary sample of resonant elements we can confirm, that this cluster is even better visible. It is composed of 18 bodies situated near the edge of the librating zone on high inclinations in the range $I_p \in \langle 31.38^\circ; 32.27^\circ \rangle$



Figure 5. Distribution of proper elements (a, e) (left) and (a, i) (right) of 7 synthetic families created in different geometries compared to proper elements of the observed Hektor family by Kolmogorov-Smirnov test over 4 Gyr of dynamical evolution.



Figure 6. Initial conditions of simulation of long-term evolution of synthetic family (red) compared to the observed Hektor family (blue) in space of proper elements (a, e). Each figure shows different disruption geometry with different values of true anomaly f and argument of pericentre ω .

and $a_{\rm p} \in \langle 5.225 \, {\rm au}; 5.238 \, {\rm au} \rangle$. As it is detached from other bodies in the space of proper elements, it is also extremely compact with respect to cutoff velocity – it is detached from the background even at $v_{\rm cutoff} = 160 \, {\rm ms}^{-1}$. $\overline{p_{V}} = XXX \pm XXX$, they are relatively bright. XXX May it suggest recent origin???

xxxSimulovat konvergenci pericenter nebo uzlu???

Unfortunately, we have albedos measured by WISE for just 4 members of this family. Beside next "dramatic" difference between AKARI ($p_V = 0.037 \pm 0.004$) and WISE ($p_V = 0.082 \pm 0.014$), WISE albedos are not much dispersed (they range from $p_V = 0.079 \pm 0.019$ to $p_V = 0.109 \pm 0.029$) and, compared to the mean albedo of Trojan population

5.4 Arkesilaos

5.5 Ennomos (Hippasos)

In our previous work we reported discovery of possible family associated with the asteroid (4709) Ennomos. With new



7

Figure 7. Our best fits of the evolution of synthetic family (red) in the space of proper elements (a, e) in comparison with the observed Hektor family (blue). Left picture shows evolution of synthetic family which originated by disruption at $f = 0^{\circ}$ and $\omega = 60^{\circ}$ after 364 Myr of evolution, right picture shows evolution of synthetic family which originated by disruption at $f = 150^{\circ}$ and $\omega = 270^{\circ}$ after 3100 Myr of evolution.



Figure 4. Our best fit of Hektor family SFD by SFD from SPH simulations results (Durda et al. 2007). Our fits with similar χ^2 suggest the impact velocity $v_{imp} = (4\pm 1) \mathrm{kms}^{-1}$. XXX SFD shape is more dependent on impact geometry than on impact velocity (pictures in Durda et al. 2007) XXX

data, we can still confirm, that there is significant cluster near this body, but when we take into account our " $N_{\rm memb}/v_{\rm cutoff}$ " criterion described above, it turns out that the family is rather associated with asteroid (17492) Hippassos. It is relatively numerous group composed of ~ 100 bodies, situated near the stable librating zone at high inclinations ranging from $I_p \in \langle 26.86^{\circ}; 30.97^{\circ} \rangle$ and pseudoproper axes in the range $a_{\rm p} \in \langle 5.225 \, {\rm au}; 5.338 \, {\rm au} \rangle$.

Table 2. Comparison of dispersion in I_p of all families among Trojans

family	$I_{p_{\min}}[deg]$	$I_{p_{\max}}[deg]$	$\Delta I_p[deg]$
 (624) Hektor, L4 (3548) Eurybates, L4 (9799) 1996RJ, L4 (20961) Arkesilaos, L4 	$18.13 \\ 6.90 \\ 31.38 \\ 8.52$	19.77 7.85 32.27 9.20	$1.64 \\ 0.95 \\ 0.89 \\ 0.68$
(17492) Hippasos, L5 (247341) 2001UV209, L5	$26.86 \\ 24.02$	$30.97 \\ 26.56$	$\begin{array}{c} 4.11 \\ 2.54 \end{array}$

5.6 (247341)

We discovered "new" family around asteroid (247341), which is the second and last observable family in our sample of resonant elements for L_5 cloud. Similar to Ennomos/Hippassos family, it is located near the border of stable librating zone on high inclinations $I_p \in \langle 24.02^\circ; 26.56^\circ \rangle$ and $a_p \in \langle 5.218 \text{ au}; 5.320 \text{ au} \rangle$. This family has very steep slope of SFD, with $\gamma = -8.59$

!!!Check the dispersion in I_p for L4 and L5 families. Is it interesting???

6 COLLISIONAL MODEL OF TROJAN POPULATION

We simulated collisional evolution of Trojans with the Boulder code (ref). Collisional model show only little evolution above D > 50km over last 3.85 Gyr (i.e. post-LHB phase). The expected and observable number of catastrophic disruptions ($M_{\rm LR}/M_{\rm PB} < 0.5$) with $D_{\rm PB} > 100$ km is only 0.67 (an average over 100 simulations; we require $N_{\rm fragments}$ with diameter (D > 10km > 10), which roughly matches the observations (i.e. the Eurybates family). The number of ob-

8 J. Rozehnal and M. Brož

servable cratering events $(M_{\rm LR}/M_{\rm PB} > 0.5)$ is even lower: although there is more likely to occur the cratering event rather than catastrophic disruption, to observed remnants of cratering is much more complicated to observe.

7 SPH SIMULATION OF DISRUPTION OF THE HEKTOR FAMILY

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Figure 8. Left: A simulation of the collisional evolution of L4 Trojans with the Boulder code (Morbidelli et al., 2009). The evolution of bodies larger than D > 50km is very slow, hence we can consider this part of the SFD as primordial. Right: The dependence of the total number of catastrophic disruptions (average over 100 simulations) on the target diameter DPB, and a subset of the families, which should be detected in contemporary observational data.



Figure 9. Comparison of SPH simulation of disruption of single body (basalt) with diameter 260 km by impactor of diameter 48 km (silicate ice) (*left*) and disruption of bilobe basalt target with diameters of 198 km each by impactor of diameter 46 km (silicate ice) (*right*). Time elapsed t = 45s in both cases.