



~~Protoplanetary and circumstellar disks~~

~~Turbulent accretion disks~~

~~Cold and hot disks~~

~~Optically thick disks~~

~~Disks: dynamics & observations~~

~~Radiation transfer in whatever disks~~

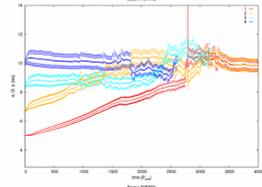
Disks ← why?

Miroslav Brož, in collaboration w. Ondřej Chrenko, David Nesvorný, Jana Nemravová,
Denis Mourard, Petr Harmanec, ...

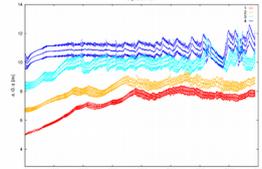
Evolution of 3-Earth-mass protoplanets towards eventual gap opening

Miroslav Brož¹, Ondřej Chrenko¹, David Nesvorný² - ¹Charles University in Prague, V Holešovičkách 2, 18000 Prague, Czech Republic; ²SWRI - email: mira@sirrah.troja.mff.cuni.cz

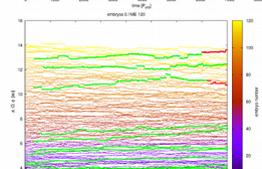
Abstract: Several-Earth-mass protoplanets interact with the gaseous and pebble disk in a complex way (see Chrenko et al. 2017, or Eklund & Masset 2017). The **hot-trail effect** arises as a consequence of accretion heating, it raises planetary eccentricities, and may prevent resonant captures of migrating planets. Here we study the dependence of this effect on parameters such as the surface density, viscosity, or the number of protoplanets. After mergers, planets are massive enough to accrete massive gas envelopes, open gaps, and eventually Type-I migration changes to Type-II. We are also using hydrocode results and radiation transfer code to compute how disk would appear in ALMA observations and whether this may constrain the properties of embedded planets.



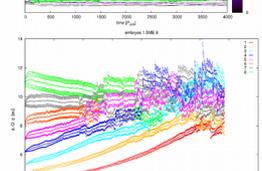
Case III nominal — As presented in Chrenko et al. (2017). Starting with 4 embryos, $3 M_E$, initial spacing $10 R_{Hill}$, pebble flux $2 \times 10^5 M_E$ per yr, approx. MMSN, with 0.5 \mathcal{Z} slope, kinematic viscosity $\nu = 10^9$ [c.u.], proto-Sun, resolution 1024×1536 , damping BC's, artificial inclination damping (Tanaka & Ward 2004), no Hill cut. Results: hot-trail effect, high eccentricities (cf. talk by O. Chrenko), 0-torque at approx. 9 au, no low-order mean-motion resonances (MMR), because embryos were too close, capture difficult anyway (because $e > 0$), two successful mergers $13.8 M_E$ and $4.3 M_E$, but co-orbitals, their long-term evolution? We performed 6 additional simulations, always with a single modified parameter...



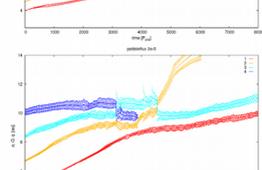
Sigma 3MMSN — initial surface density \mathcal{Z} 3x larger; 0-torque radius further out at 11 au, e often smaller, slower evolution (even though timespan 2x longer), embryos do NOT interact so strongly, rather stay next to each other, because damping is too large? sometimes inward migration of inner embryos @ larger e , possible interference of (massive) co-orbital regions? 10+ attempts of the outer 4th embryo to enter the co-orbital region of the 3rd one, only temporary co-orbitals.



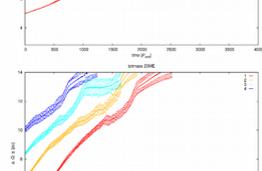
embryos 0.1ME_120 — 120 low-mass $0.1 M_E$ embryos, spacing 2 mutual R_{Hill} , disk up to 16 au, resolution still low (3 pixels per Hill sphere), at least 2048×3072 would be needed, convergence tests show that Δt is overestimated, very slow computation anyway (120 disk = planet interactions), it was run on Pleiades, caveat: collisional radii increased only during merger events, overlapping weak spiral arms, slow evolution dominated by encounters, e up to 0.06, 10+ quick mergers $0.2 M_E$, pebble accretion up to $0.45 M_E$, but strong filtering for inner embryos, $0.2 M_E$ mergers are either inside (short periods) or outside, the "winner" is outside (no filtering), longer simulation needed?



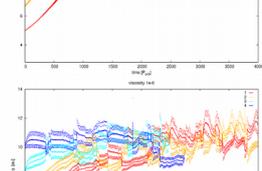
embryos 1.5ME_8 — 8 embryos with $1.5 M_E$, clear convergence to 0-torque, slower evolution, a number of encounters, more opportunities to merge, especially when an additional embryo arrives and starts to interact, 2 mergers 13.2 and $6.5 M_E$ as of yet, more outer embryos should be added and an extended disk (20 au) should be used?



pebbleflux 2e-5 — $10 \times$ lower pebble flux $2 \times 10^5 M_E$, i.e. $0.25 M_E$ per $4000 P_{orb}$ (more realistic?), lower eccentricity excitation (!), consequently smooth evolution, all embryos initially drift outwards, 0-torque at about 11 au, 1 yellow merger with $6 M_E$ quickly drifts outwards (!), only temporarily decelerated by the red embryo, runaway migration mode as in Pierens & Raymond (2016)? planet [X?] is it a rule for low pebble fluxes? Possible clearing of the outer disk? More outer embryos should be probably added...



totmass 20ME — initial masses $5 M_E$, all embryos quickly drift outwards (!), even though w/o heating the 0-torque should be at 7 au; lower e , practically NO interactions, because real 0-torque is further out, unwanted interactions with the disk edge; larger disk & more embryos should be used...



viscosity 1e-6 — low-viscosity disk; same e , BUT faster migration $d(a)/dt$, i.e. like ν in the denominator (!!), surroundings more easily affected by the embryo, many encounters, only temporary co-orbitals, 2 mergers $8 M_E$ as of yet, an onset of gap opening even without gas-accretion term? many attempts to form a co-orbital pair, BUT failed co-orbital formation? (cf. Figs. above)

$$\frac{\partial \Sigma}{\partial t} + \mathbf{v} \cdot \nabla \Sigma = -\Sigma \nabla \cdot \mathbf{v} - \frac{\partial \Sigma}{\partial t}_{acc}$$

$$\frac{\partial \mathbf{v}}{\partial t} + \mathbf{v} \cdot \nabla \mathbf{v} = -\frac{1}{\Sigma} \nabla P + \frac{1}{\Sigma} \nabla \cdot \mathbf{T} - \frac{\partial \Sigma}{\partial t}_{acc} \frac{\mathbf{u} - \mathbf{v}}{\tau}$$

$$\frac{\partial E}{\partial t} + \mathbf{v} \cdot \nabla E = -E \nabla \cdot \mathbf{v} - P \nabla \cdot \mathbf{v} + Q_{visc} + \frac{2\sigma T_{eff}^4}{\tau_{eff}} + 2H \nabla \cdot \frac{16\sigma \lambda_{lim} T^3 \nabla T}{\rho_0 \tau} + \frac{GM}{RS_{coll}}$$

$$\frac{\partial \Sigma_p}{\partial t} + \mathbf{u} \cdot \nabla \Sigma_p = -\Sigma_p \nabla \cdot \mathbf{u} - \left(\frac{\partial \Sigma_p}{\partial t} \right)_{acc}$$

$$\frac{\partial \mathbf{u}}{\partial t} + \mathbf{u} \cdot \nabla \mathbf{u} = \int \rho_p \nabla \phi dz - \frac{\Omega_K}{\tau} (\mathbf{u} - \mathbf{v})$$

$$P = \Sigma \frac{RT}{\mu} = (\gamma - 1)E$$

System of equations; changes w.r.t. Chrenko et al. (2017): gas accretion (Kley's prescription for 3D orbits), corresponding gas-accretion heating, fragmentation-limited pebbles, improved SOR convergence.

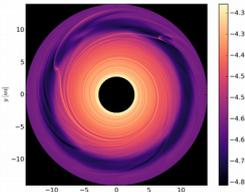


Figure: Evolved gas disk @ time $t = 4000 P_{orb}$ @ 5.2 au, hot-trail effect visible, failed co-orbital, viscosity 1e-6.

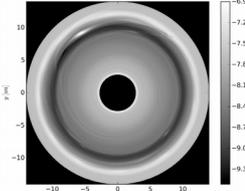


Figure: Pebble disk, corresponding 1:1 to the gas disk.

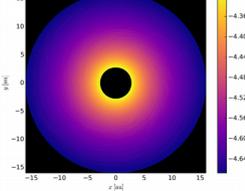


Figure: Gas disk with initially 120 embryos $0.1 M_E$, and many weak overlapping spiral arms, $t = 4000 P_{orb}$.

Conclusions:

Clearly, the dependence on parameters is complex! Apart from the very origin of gas-giant cores, there are a number of possible applications here: studies of different parts of the disk, origin of Uranus & Neptune, dynamics of other (compact) planetary systems. However, we have to face several serious problems: (i) find IC's suitable for the inner disk edge; (ii) resolve different pebble isolation in 2D vs 3D; (iii) gas accretion in 2D is not self-consistent, produces too much heating. It's possible that the deposition is below opaque atmosphere. A parameterisation of 3D in- and outflows (Lambrechts & Lega 2017) would be needed for this purpose.

Rw. the code is available @ <http://sirrah.troja.mff.cuni.cz/~chrenko/>

References:
 Bell & Lin (1994) *AstJ*, **768**, 35
 Binzel et al. (2012) *ACA*, **539**, A148
 Chrenko et al. (2017) *ALMA*, in press
 Criga & Bitsch (2017) *ICarus*, **285**, 145
 Dullemond et al. (2012) *ASL*, **1202**, 015
 Eklund & Masset (2017) *MNRAS*, **469**, 206
 Lambrechts & Lega (2017) *ALMA*, in press
 Pierens & Raymond (2016) *MNRAS*, **462**, 4130
 Tanaka & Ward (2004) *AJ*, **602**, 388

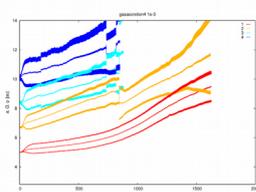


Figure: Simulation with gas accretion (incl. heating). Key parameter $f_{acc} = 10^3$, $\Delta M \approx f_{acc} \Sigma_p \Delta t [\rho] dz$.

Observability: The disk is optically thick in the vertical direction $\tau = \kappa_H H = 10^2 \gg 1$, with Bell & Lin (1994) integral opacities. It is thus necessary to properly model the disk atmosphere. In the midplane, the mean-free path λ of gas molecules is small enough to assure a sufficient thermal contact and equilibrium between the gas and dust. This is no more true far from the midplane and one has to use 3D, non-equilibrium model, and monochromatic opacities (cf. eqs.). While surface-area distribution of solids is dominated by sub-micron dust, the mass distribution is dominated by pebbles (as in Binzel et al. 2012); in principle we can use $\Sigma_p M_E < H$, $\kappa_p \ll \kappa_g$, but it could hardly produce observable effects.

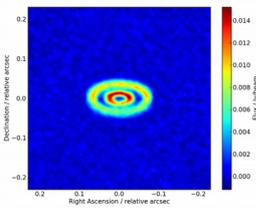
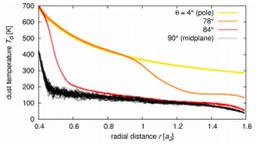
We tried to use Radmc-3D code (Dullemond et al. 2012), assuming LTE, dust absorption, isotropic scattering, central star, possibly also embryos heated by pebble accretion, and viscous heating (i.e. an extended source). Synthetic image for 10^5 photons was processed by ALMA OST, assuming high $\nu = 900$ GHz, 7.5 GHz bandwidth, 3-hour observation, 1 visit starting at 78°, PWV 0.475 mm, and extended (full) configuration with baselines up to $B/A = 3.6 \times 10^3$ cycles.

Problems: (i) only fully-opened gaps seem observable; (ii) only escape-probability, no A -iterations, ALI or Ng-acceleration \rightarrow a slow convergence with extended source inside optically-thick disk interior!

$$\Sigma_d = \Sigma_p \int_{-H}^H \frac{1}{\Omega} \frac{d\Omega}{dV} dV$$

$$\rho_d = \frac{\Sigma_p}{\sqrt{2H}} \exp\left(-\frac{z^2}{2H^2}\right)$$

$$\frac{dL}{dt} = \kappa_{peb}^{abs} \rho_d E_{\star}(T) + \frac{e_{peb}^{em}}{4\pi} \int I_{\nu} d\Omega - (\kappa_{peb}^{abs} + \kappa_{peb}^{em}) \rho_d L$$



Q: How to compute $a(t)$? of planets

- mutual gravity (N-body)
- tides, spin-orbit coupling, relativistic precession, ...
- disk gravity, spiral arms, Lindblad torque (HD) ←
- corotation region, c. torque →
- cold finger (RHD) →→
- hot trail (in e) ←
- ⋮

$$\frac{\partial \Sigma}{\partial t} + \overbrace{\mathbf{v} \cdot \nabla \Sigma}^{\text{convection}} = - \overbrace{\Sigma \nabla \cdot \mathbf{v}}^{\text{expansion}} - \overbrace{\left(\frac{\partial \Sigma}{\partial t} \right)_{\text{acc}}}^{\text{gas accretion}}$$

$$\frac{\partial \mathbf{v}}{\partial t} + \mathbf{v} \cdot \nabla \mathbf{v} = - \frac{1}{\Sigma} \nabla P + \overbrace{\frac{1}{\Sigma} \nabla \cdot \mathbb{T}}^{\text{bulk viscosity}} - \overbrace{\frac{\int \rho \nabla \phi dz}{\Sigma}}^{\text{gravity}} + \overbrace{\frac{\Sigma_p \Omega_K}{\Sigma \tau} (\mathbf{u} - \mathbf{v})}^{\text{back-reaction}}$$

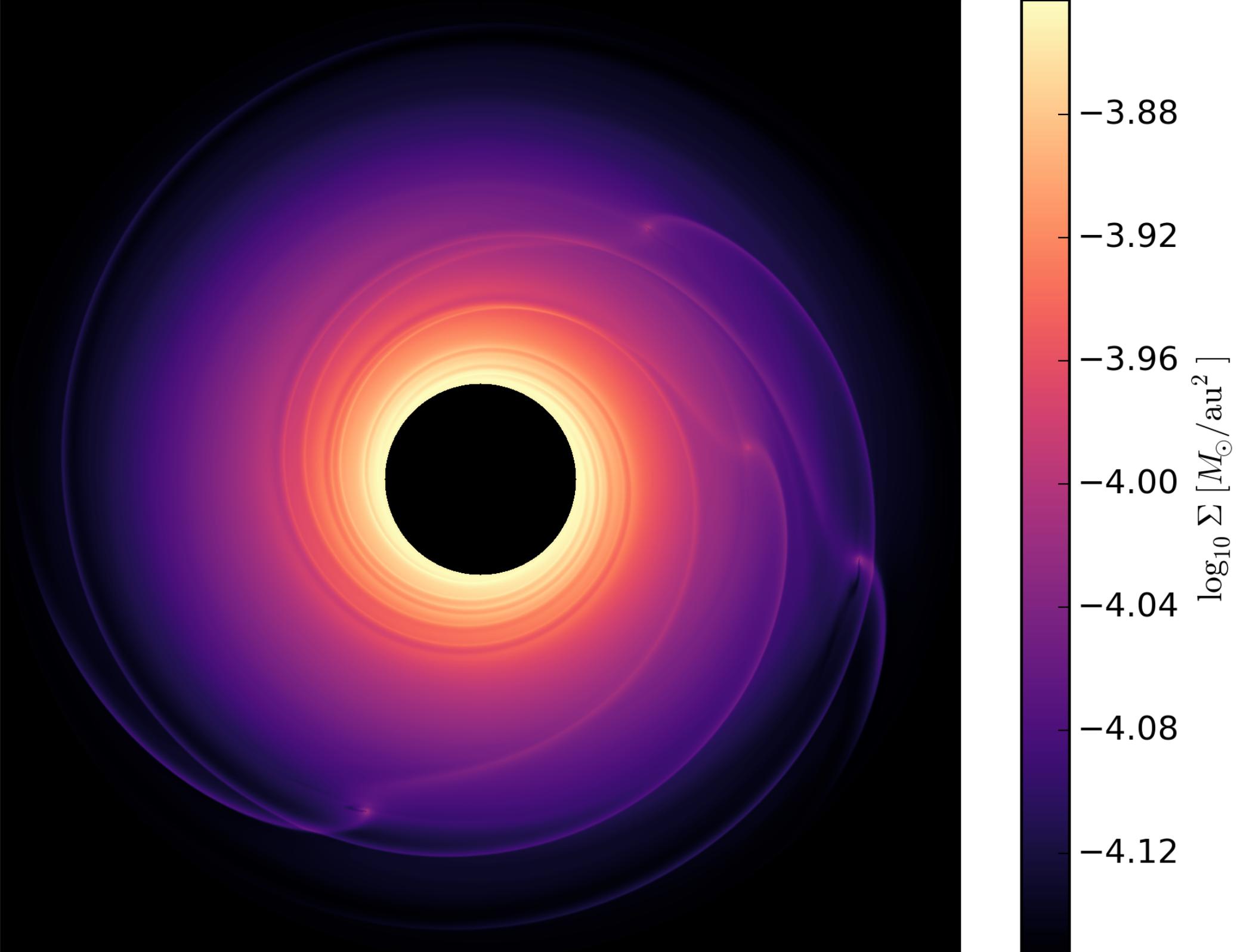
$$\frac{\partial E}{\partial t} + \mathbf{v} \cdot \nabla E = - E \nabla \cdot \mathbf{v} - P \nabla \cdot \mathbf{v} + \overbrace{Q_{\text{visc}}}^{\text{dissip.}} + \overbrace{\frac{2\sigma T_{\text{irr}}^4}{\tau_{\text{eff}}}}^{\text{irradiation}} - \overbrace{\frac{2\sigma T^4}{\tau_{\text{eff}}}}^{\text{vertical cooling}} + \overbrace{2H \nabla \cdot \frac{16\sigma \lambda_{\text{lim}} T^3 \nabla T}{\rho_0 \kappa}}^{\text{radiative diffusion}} + \overbrace{\frac{GM\dot{M}}{RS_{\text{cell}}}}^{\text{accretion}}$$

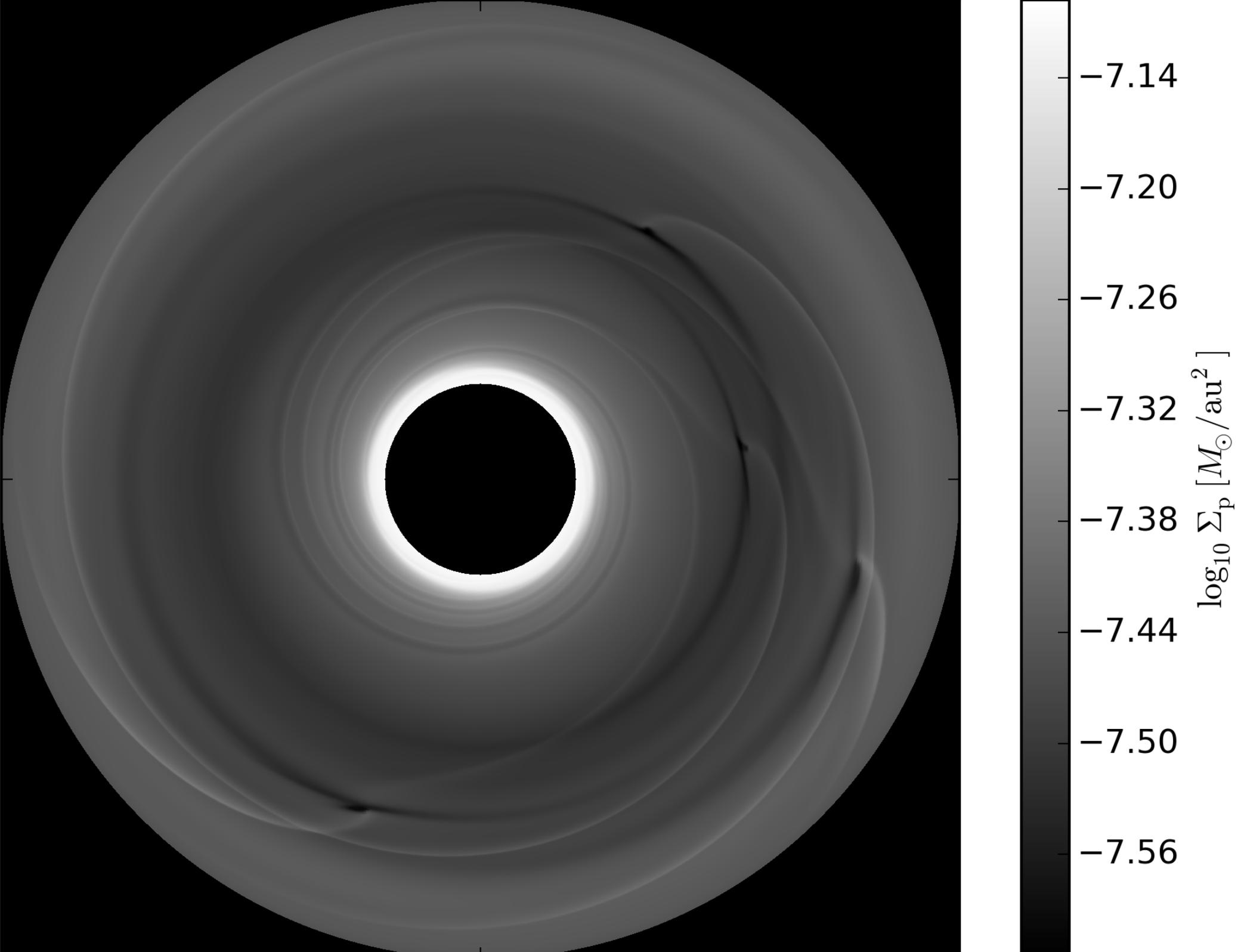
$$\frac{\partial \Sigma_p}{\partial t} + \mathbf{u} \cdot \nabla \Sigma_p = - \Sigma_p \nabla \cdot \mathbf{u} - \overbrace{\left(\frac{\partial \Sigma_p}{\partial t} \right)_{\text{acc}}}^{\text{pebble accretion}}$$

$$\frac{\partial \mathbf{u}}{\partial t} + \mathbf{u} \cdot \nabla \mathbf{u} = - \frac{\int \rho_p \nabla \phi dz}{\Sigma_p} - \overbrace{\frac{\Omega_K}{\tau} (\mathbf{u} - \mathbf{v})}^{\text{aerodynamic drag}}$$

$$P = \Sigma \frac{RT}{\mu} = (\gamma - 1)E$$

RHD
FMT
BC
IC?





Stellar-solar context

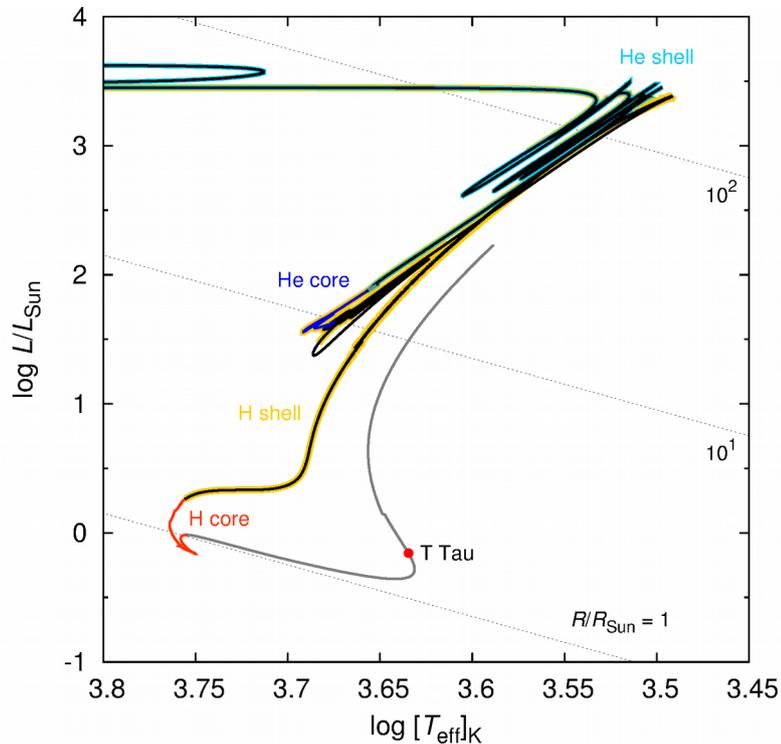
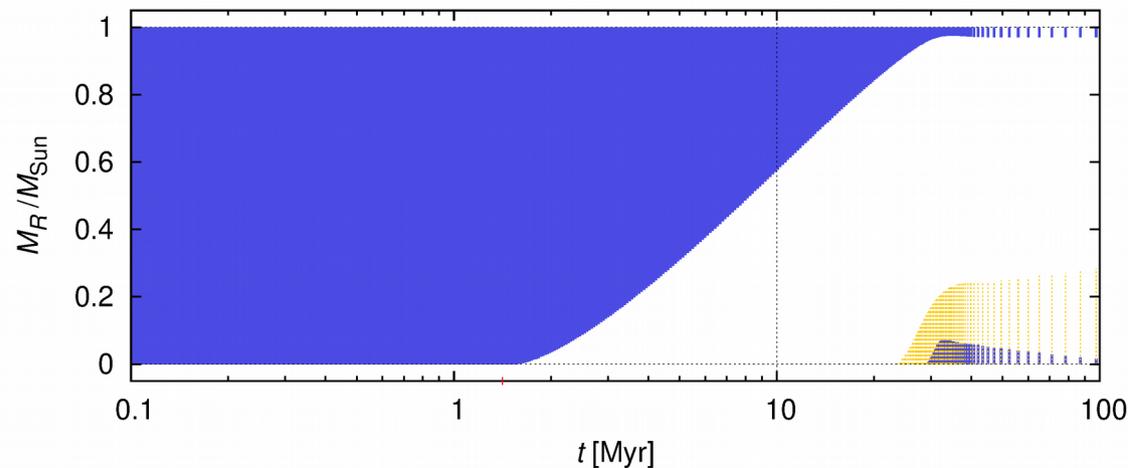


Fig. 1. The Hertzsprung–Russel diagram for a star (Sun) with the initial mass $M = 1 M_{\odot}$, helium abundance $Y = 0.274$, metallicity $Z = 0.0195$, mixing length parameter $\alpha = 2.1$, Reimers RGB wind with the scaling factor $\eta_R = 0.6$, Blocker AGB wind with $\eta_B = 0.1$. The element diffusion was also accounted for. Colours indicate four major burning phases: H core, H shell, He core and He shell burning. The red point denoted “T Tau” corresponds to the pre-main sequence object we use in our protoplanetary disk simulations. The evolution was calculated by Mesastar code, rev. 8845; Paxton et al. (2015).

- T Tauri (hydrostatic) phase
- receding convective zone (tachocline) & fast rotation (~ 2 d) \rightarrow magnetic dynamo
- corotation cavity in the disk
- d. turbulence: VSI, SBI, MRI $\rightarrow \nu$



Viscosity context

$\nu = \text{const.}$ or $\nu = \alpha c_s H$

- vertical shearing instability **VSI** (a.k.a. Kelvin–Helmholtz in z direction; Nelson et al. 2013)
- subcritical baroclinic instability **SBI** (essentially, Rayleigh–Taylor with heat diffusion; Klahr & Bodenheimer 2003)
- magneto–rotational instability **MRI** (Balbus & Hawley 1973, Turner et al. 2014)
- spiral wave instability **SWI** (resonant coupling between spiral arms induced by an embedded planet and inertial-gravity waves; Bae et al. 2016)
- strong stellar wind (Günther 2013, Turner et al. 2014)
- X-wind at the disk edge (Shu et al. 1994)
- magneto-centrifugal wind and loading of ions (Anderson et al. 2005, Stute et al. 2014, Bai et al. 2016)

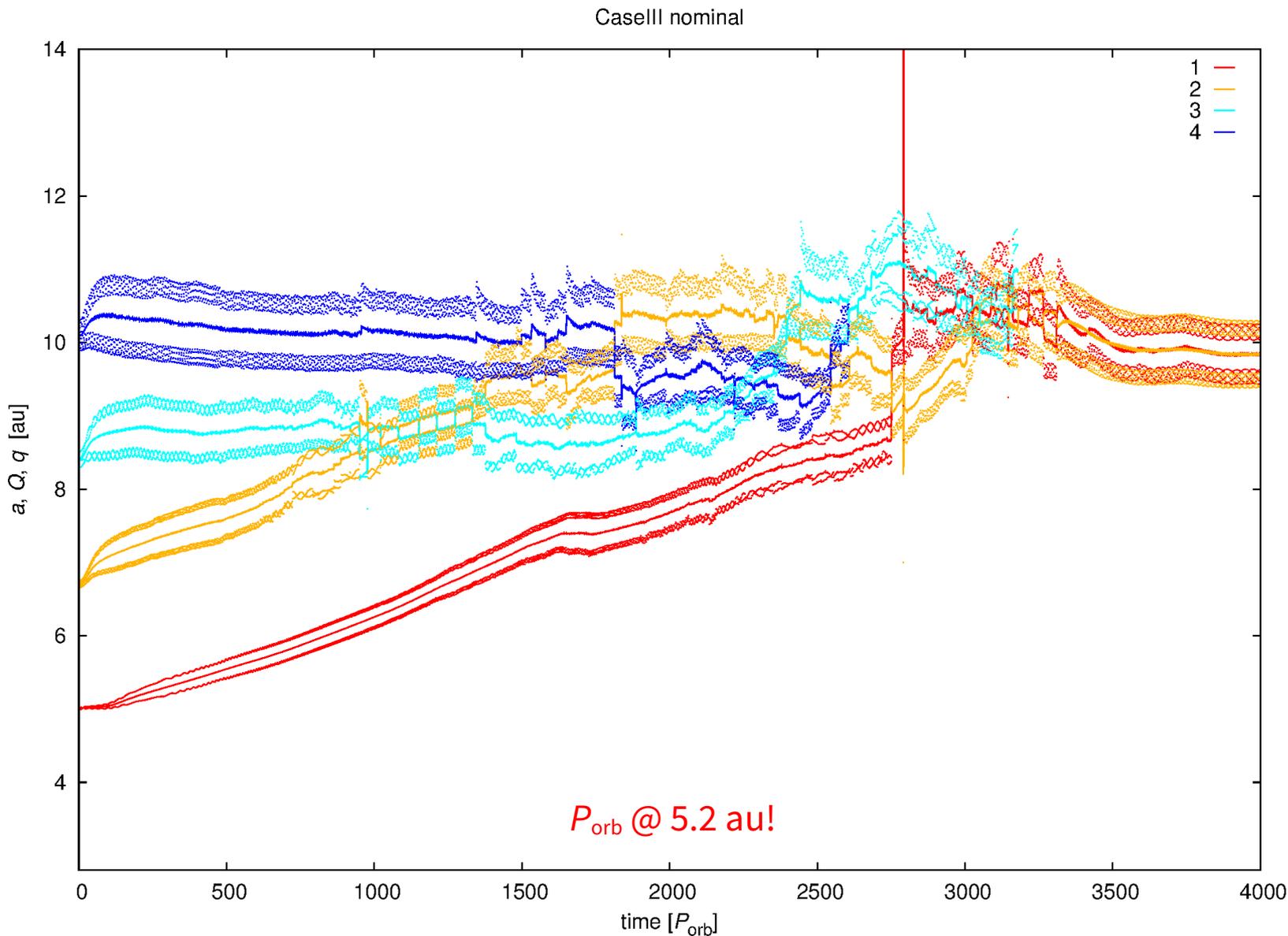
$\nu = 0$

Model ingredients (Fargo-Thorin)

- based on Fargo (Masset 2000) ← extended by OC
- N-body interactions u. Rebound (Rein & Spiegel 2015)
- inclination damping (Tanaka & Ward 2004)
- integral LTE opacities (Bell & Lin 1994)
- pebble accretion: Bondi regime, Hill r.
- Type-I migration → Type-II (gas accretion, gap opening)
- FVM discretisation $N_r = 1024$, $N_\phi = 1563$; implicit SOR (RTE), CFL
- MPI, OpenMP
- free parameters: $\Sigma(r)$, M_{em} , $\#_{\text{em}}$, \dot{M}_F , ν (or α), but not $h = H/r$
- fixed parameters: M_\odot , T_\odot , R_\odot , α_p , γ , μ , c_K , A , Sc , ρ_b , f_{acc}

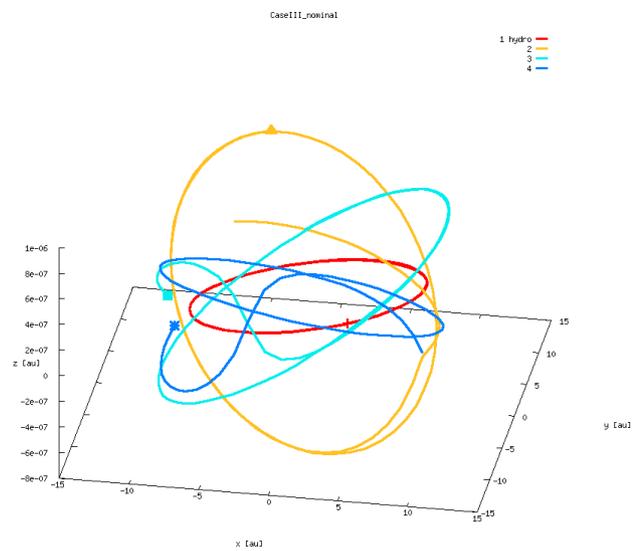
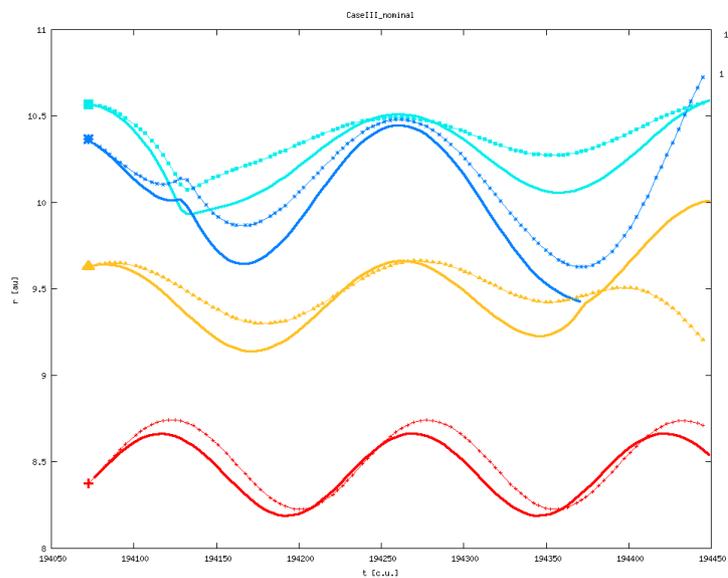
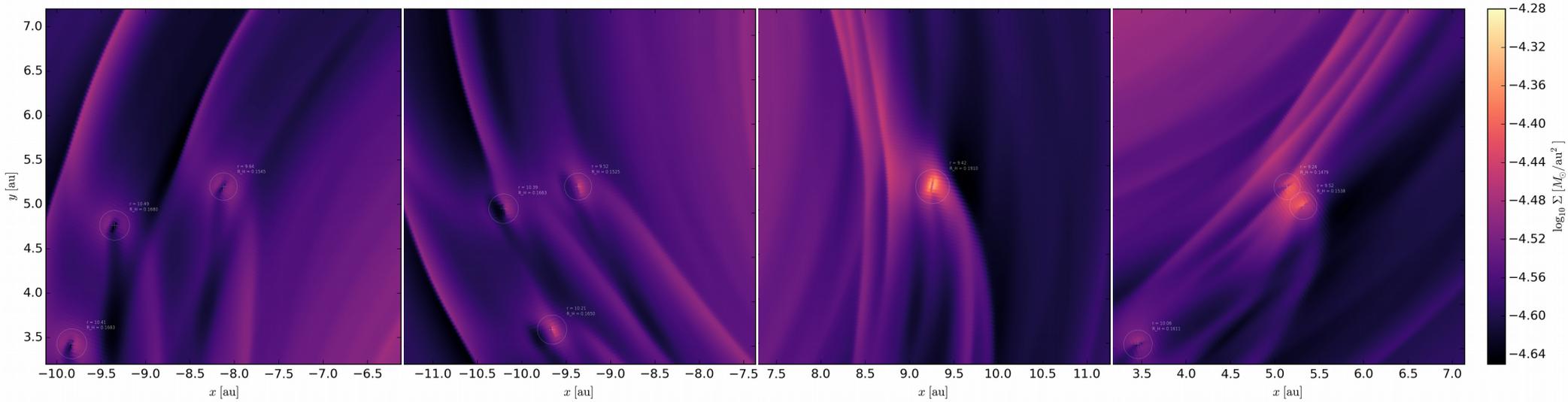
Case III nominal

As presented in Chrenko et al. (2017).

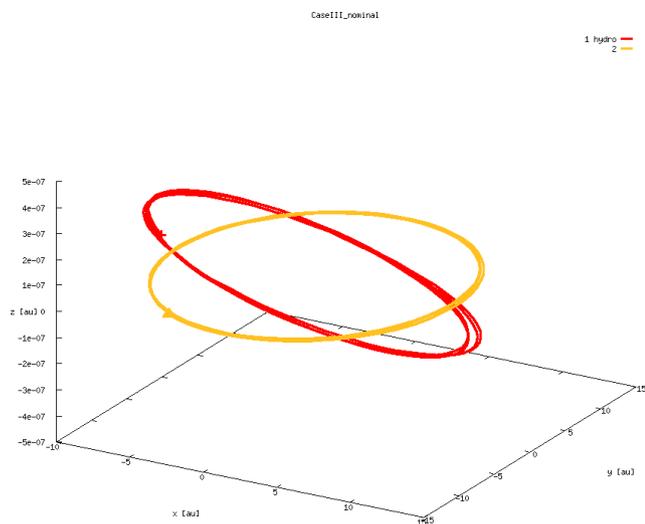
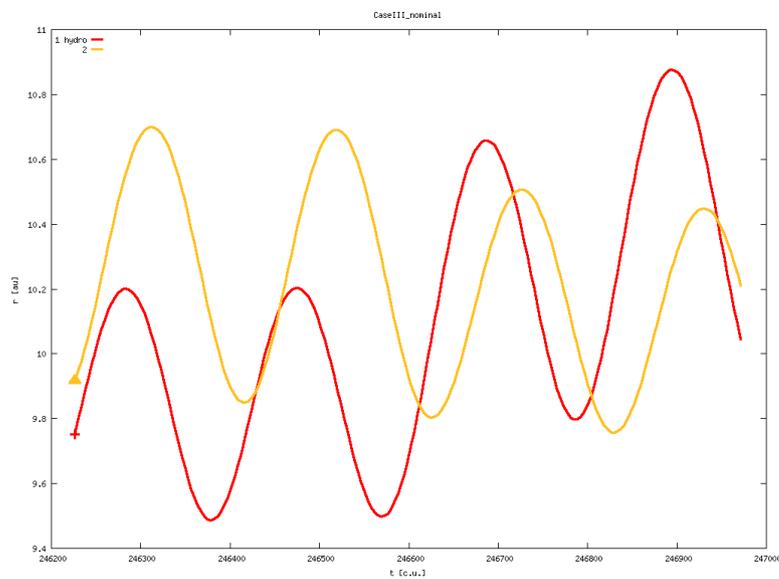
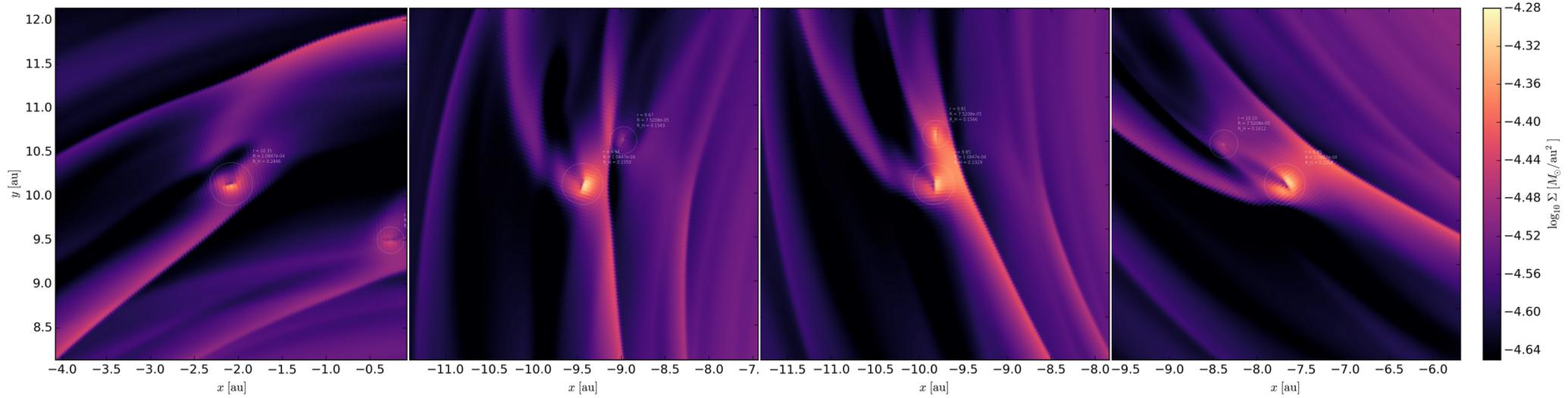


Starting with 4 embryos, 3 ME, initial spacing 10 RHill, pebble flux 2×10^{-4} ME per yr, approx. MMSN, with 0.5 $\Sigma(r)$ slope, kinematic viscosity $\nu = 1e-5$ [c.u.], proto-Sun, resolution 1024×1536 , damping BC's, artificial inclination damping (Tanaka & Ward 2004), no Hill cut. Results: hot-trail effect, high eccentricities (cf. talk by O. Chrenko), 0-torque at approx. 9 au, no low-order mean-motion resonances (MMR), because embryos were too close, capture difficult anyway (because $e > 0$), two successful mergers 13.8 ME and 4.3 ME, but co-orbitals, their long-term evolution? 3-body interactions are needed for successful mergers! a scattering event occurs prior to every merger (2 out of 2); see details below...

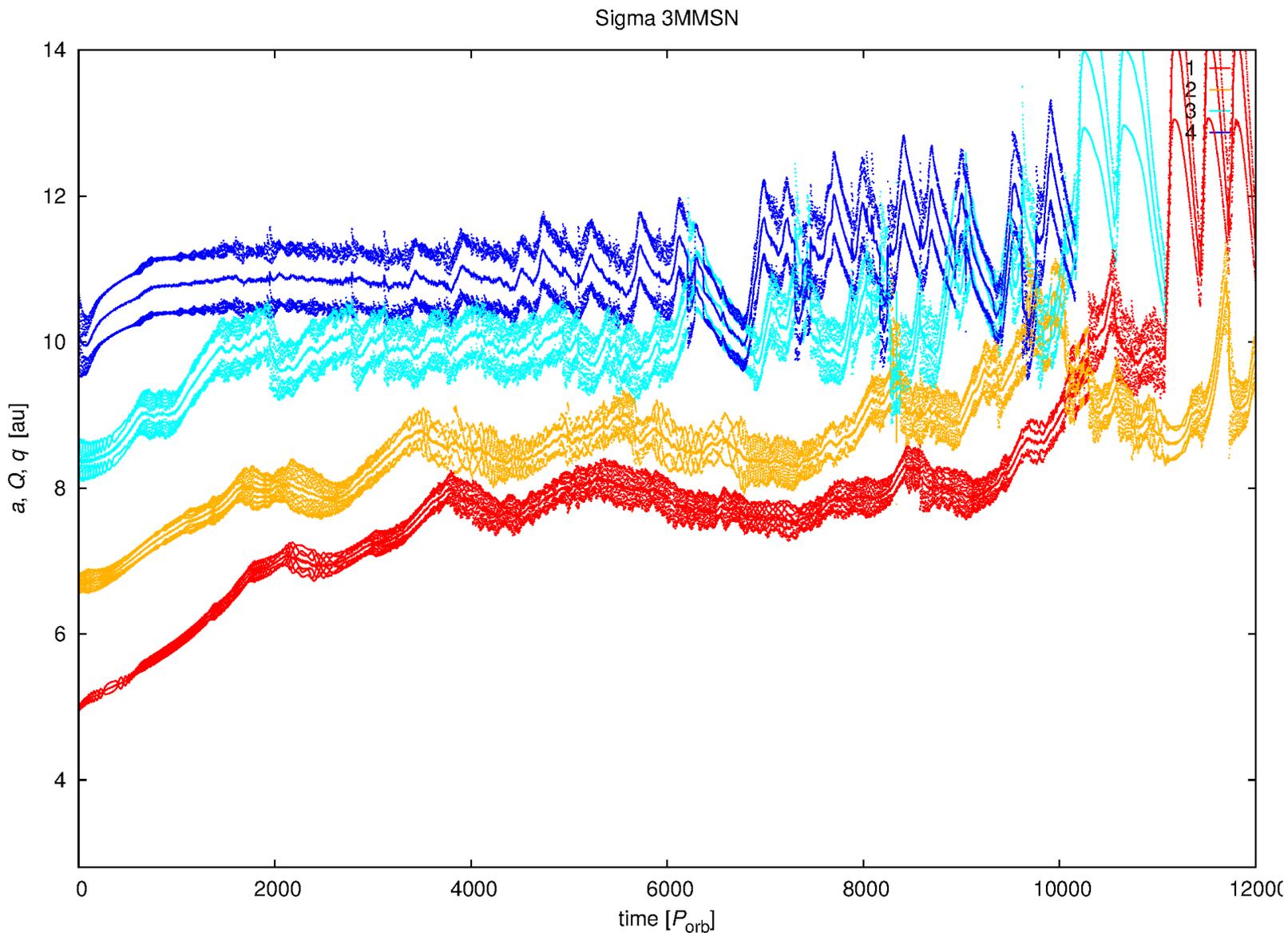
Case III nominal: Merger



Case III nominal: Coorbital

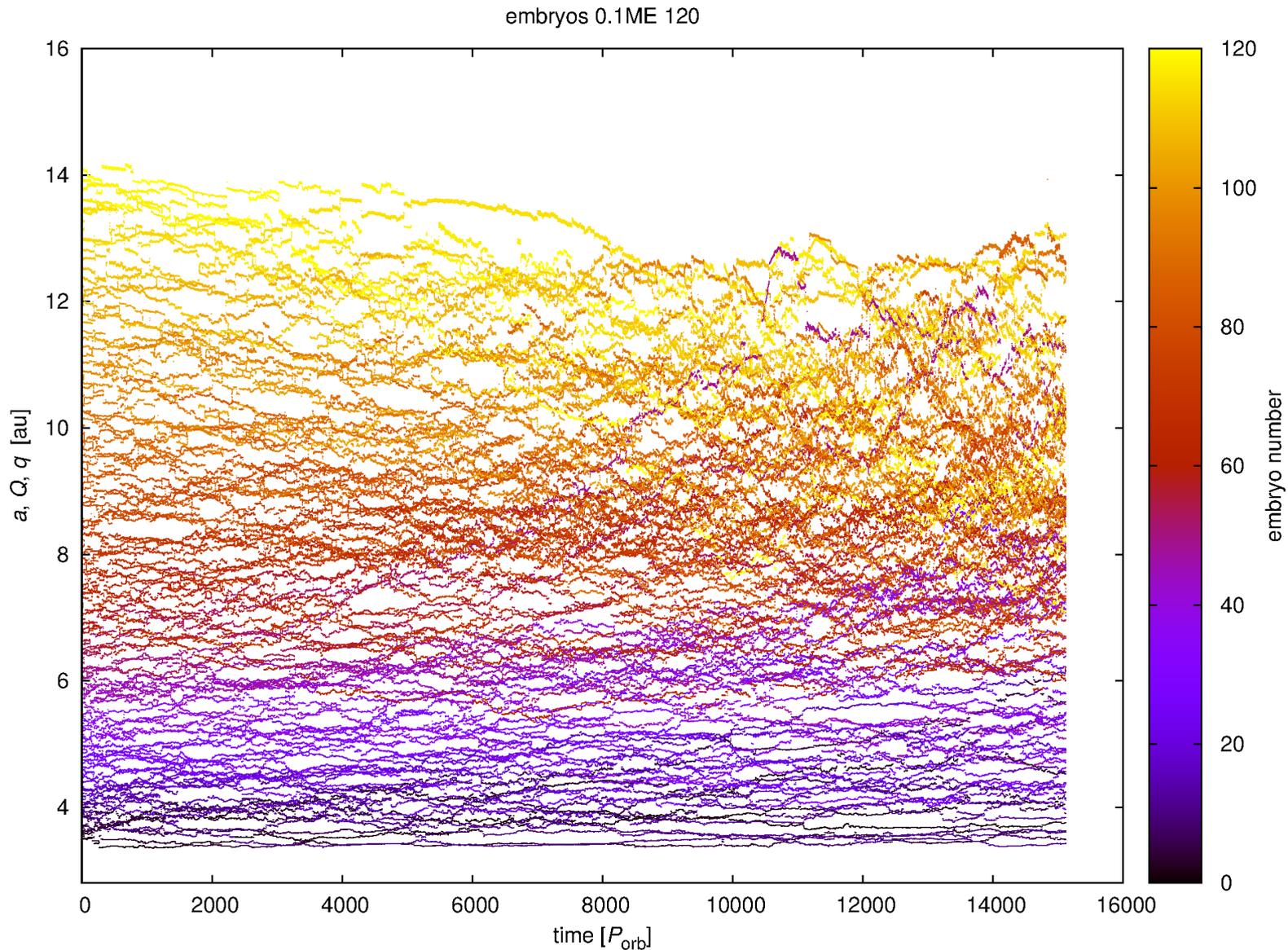


Sigma 3MMSN

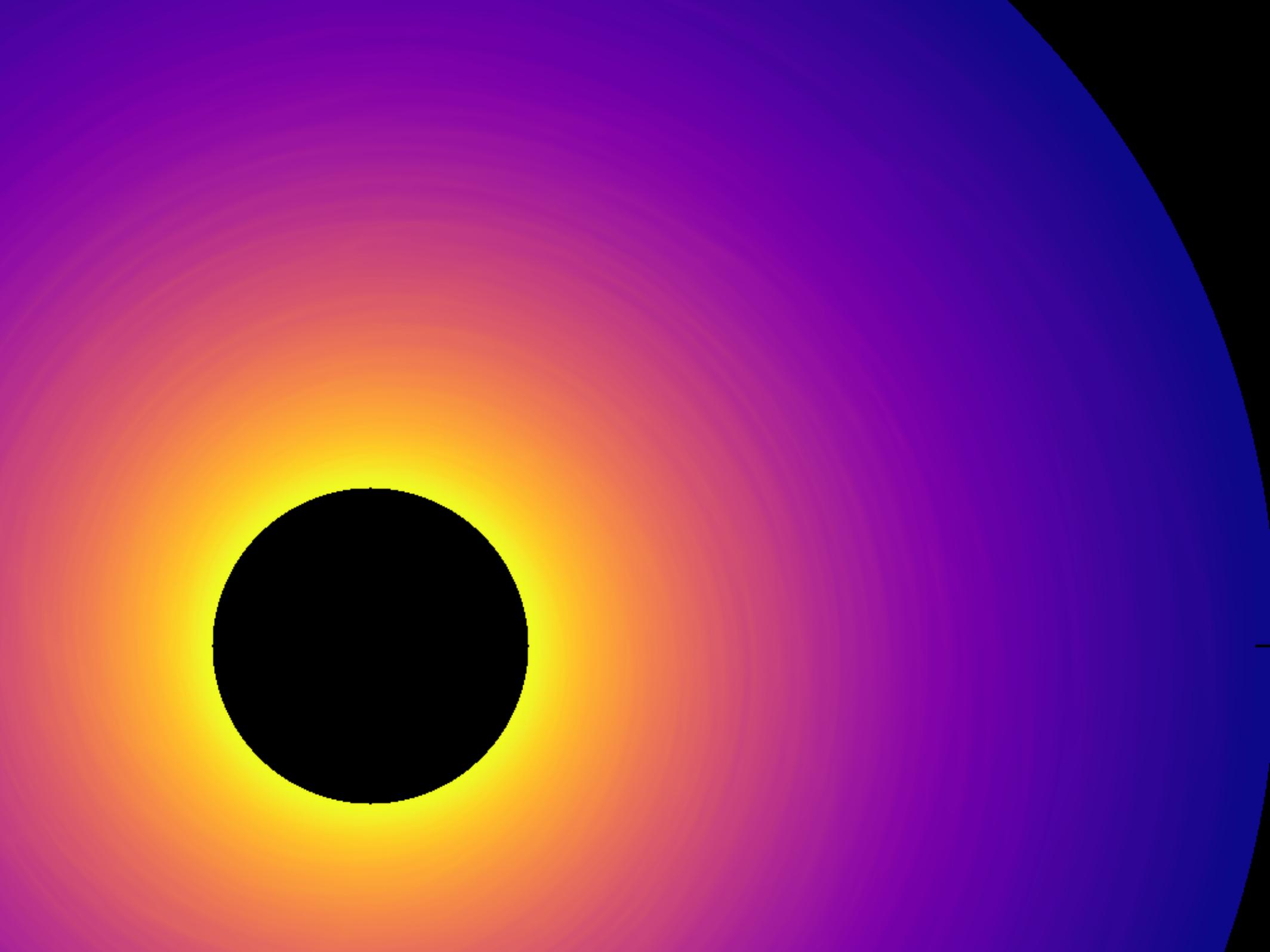


private notes: initial surface density 3x larger; 0-torque radius further out at 11 au, e often smaller, slower evolution (even though timespan 2x longer), embryos do NOT interact so strongly, rather stay next to each other, because damping is too large? sometimes inward migration of inner embryos @ larger e , possible interference of (massive) co-orbital regions? 10+ attempts of the outer 4th embryo to enter the co-orbital region of the 3rd one, only temporary coorbitals; the last part affected by interactions with the disk edge and damping zone, which kills the outer spiral arm.

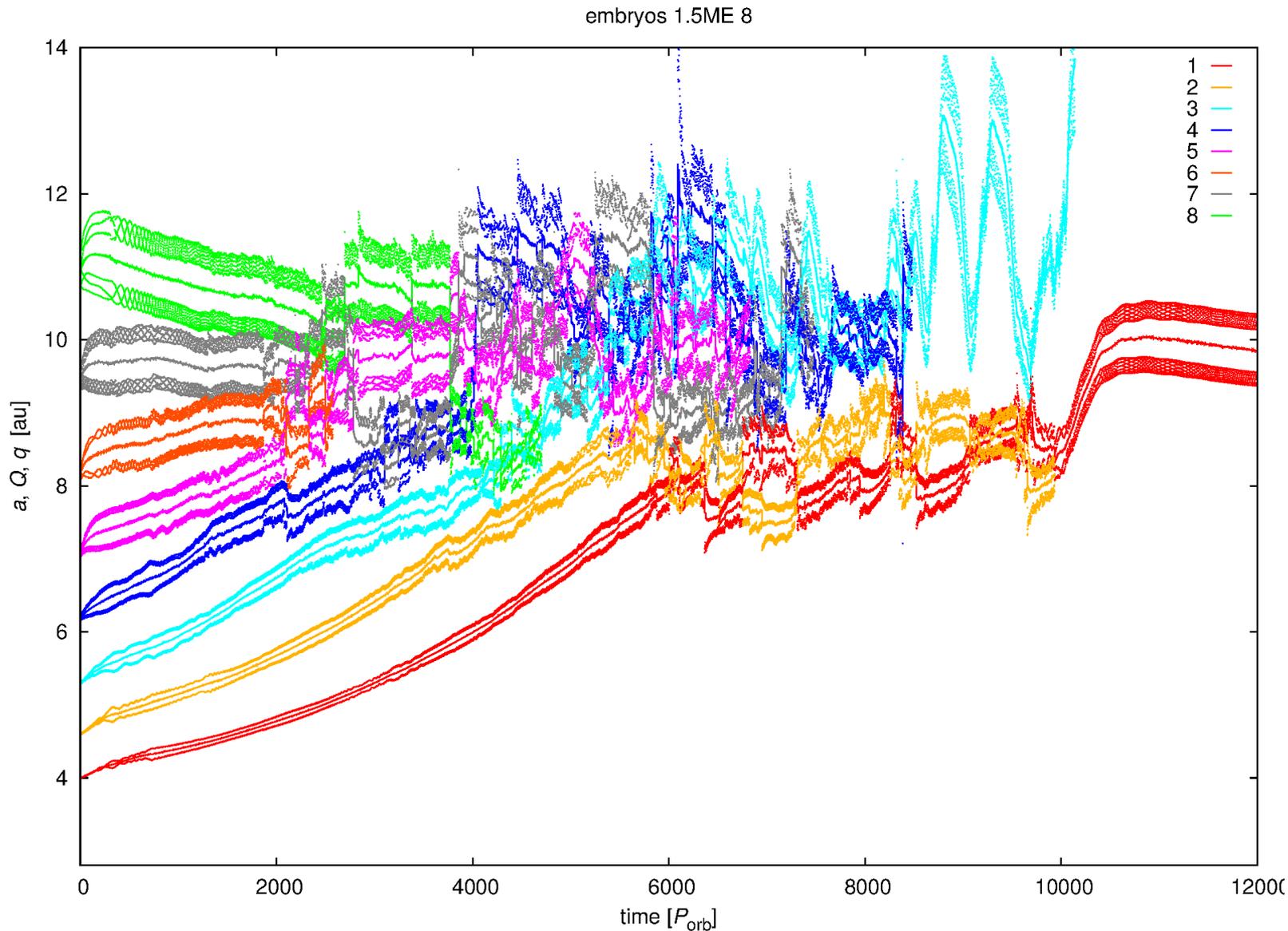
Embryos $0.1 M_E$ 120



120 low-mass $0.1 M_E$ embryos, spacing 2 mutual R Hill, disk up to 16 au, resolution still low (3 pixels per Hill sphere); overlapping weak spiral arms, slow evolution dominated by encounters, e up to 0.06, 10+ quick mergers $0.2 M_E$, pebble accretion up to $1.4 M_E$, but strong filtering for inner embryos, $0.2 M_E$ mergers are either inside (short periods) or outside, the "winner" is outside (no filtering) - see the plot wrt. mass, or enhanced massive bodies, several mergers or pebble-accreted bodies up to $3 M_E$, we are apparently at the beginning of Caselli_nominal, but the bodies concentrate in the outer disk :-|, e excitation by hot trail alone is 0.02 only, but this is NOT final value, nevertheless, it serves as an initial 'kick'!

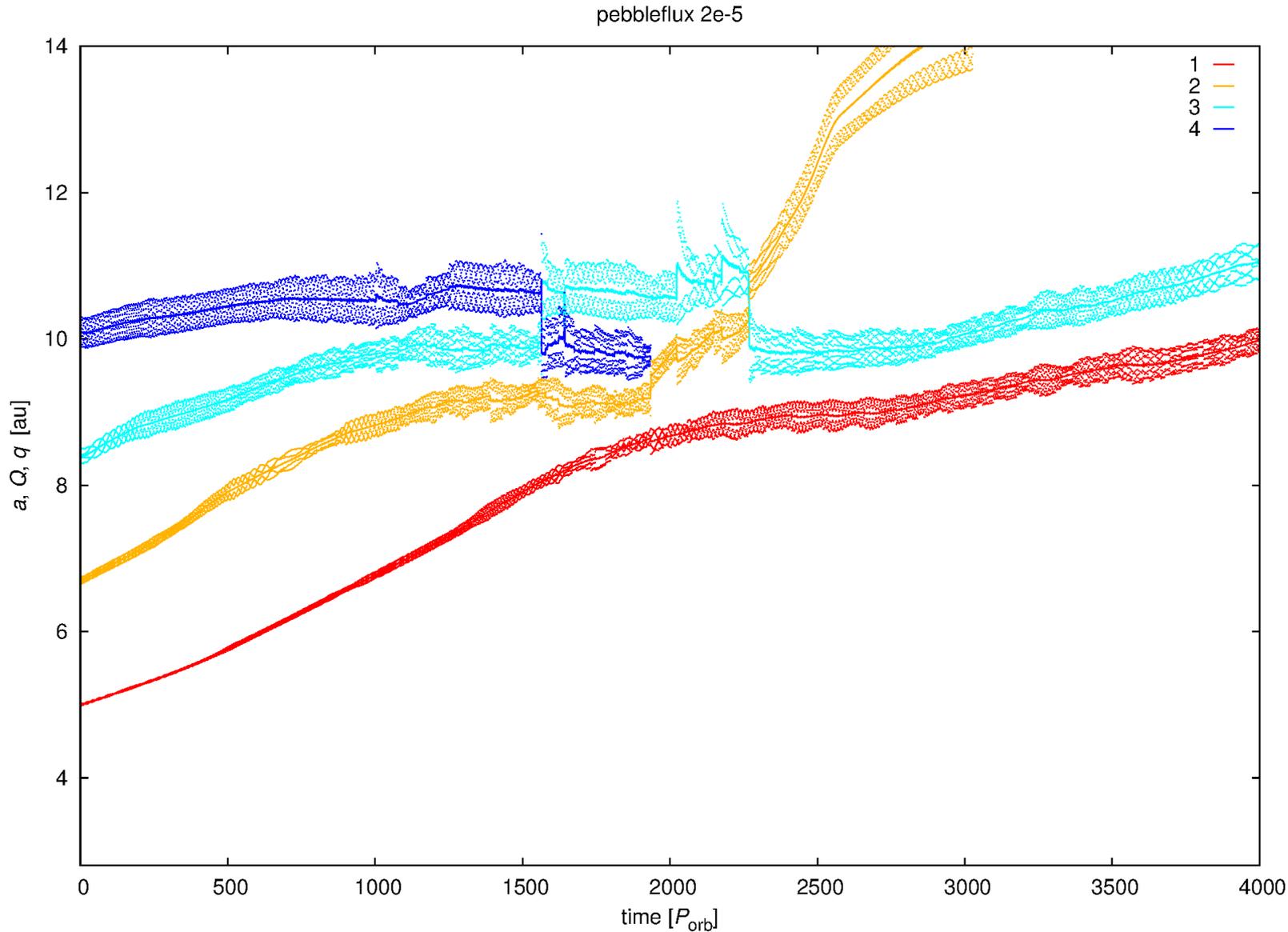


Embryos $1.5M_E$ 8



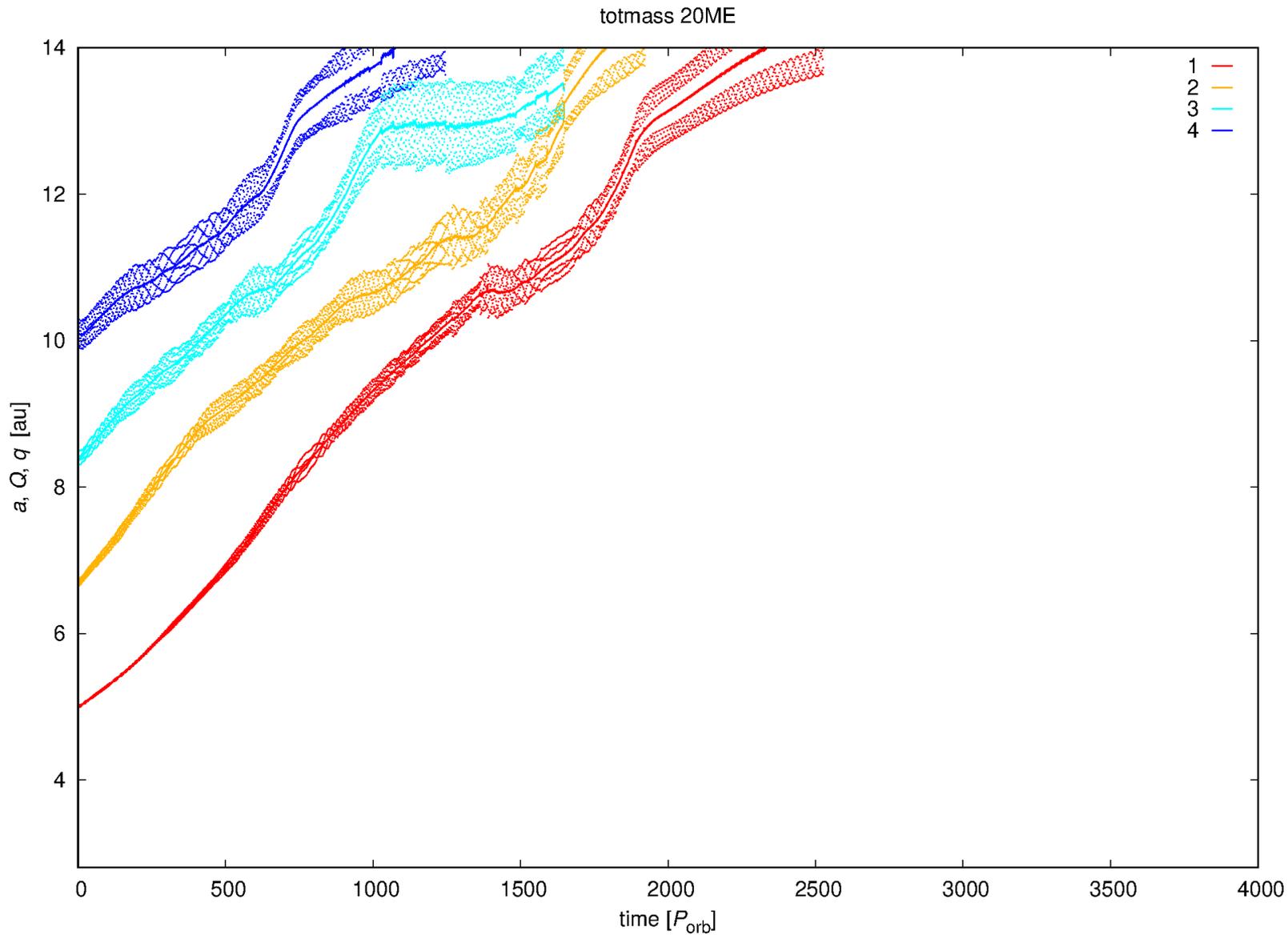
8 embryos with $1.5 M_E$, clear convergence to 0-torque, slower evolution, a number of encounters, more opportunities to merge, especially when an additional embryo arrives and starts to interact, several mergers, the final mass of the escaping embryo is $25 M_E$, the heating may be actually lower (pebble isolation), and there is NO gas accretion in this model; btw. it's strange that 5-6 M_E embryos (cf. below) migrate out of the disk without problems, similar final position of the remaining embryo, more outer embryos should be added and an extended disk (20 au) should be used? is the long duration of the high pebble flux ok?

Pebbleflux 2e-5



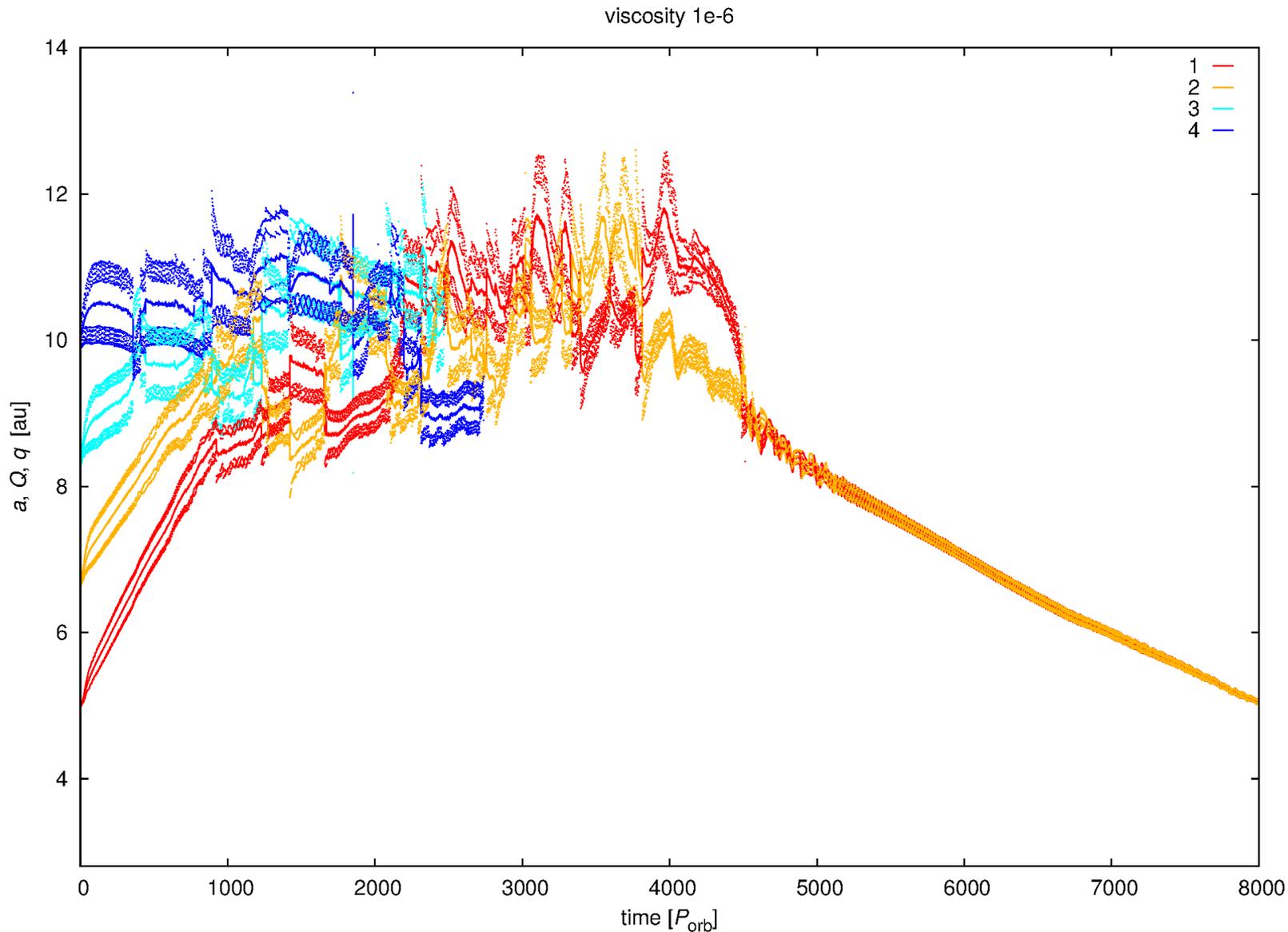
10× lower pebble flux 2×10^{-5} ME, i.e. 0.25 ME per 4000 P_{orb} (more realistic?), lower eccentricity excitation (!), consequently smooth evolution, all embryos initially drift outwards, 0-torque at about 11 au? 1 yellow merger with 6 ME quickly drifts outwards (!), only temporarily decelerated by the 3rd embryo, runaway migration mode as in Pierens & Raymond (2016)? planet IX? :) Is it a rule for low pebble fluxes? Possible clearing of the outer disk? Why the 2 remaining embryos migrate outward? (initially the convergence zone is at 10 au) More outer embryos should be probably added...

Totmass $20M_E$



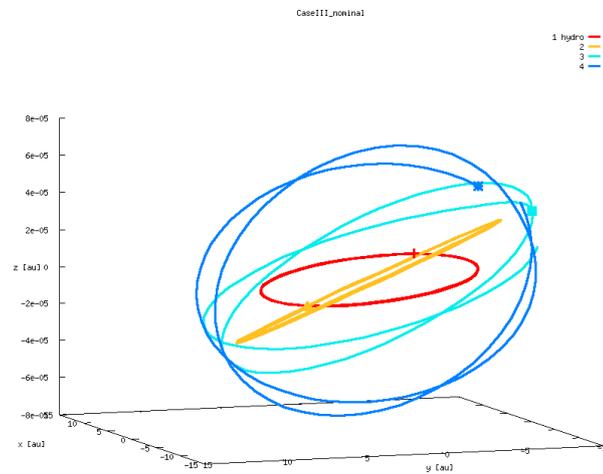
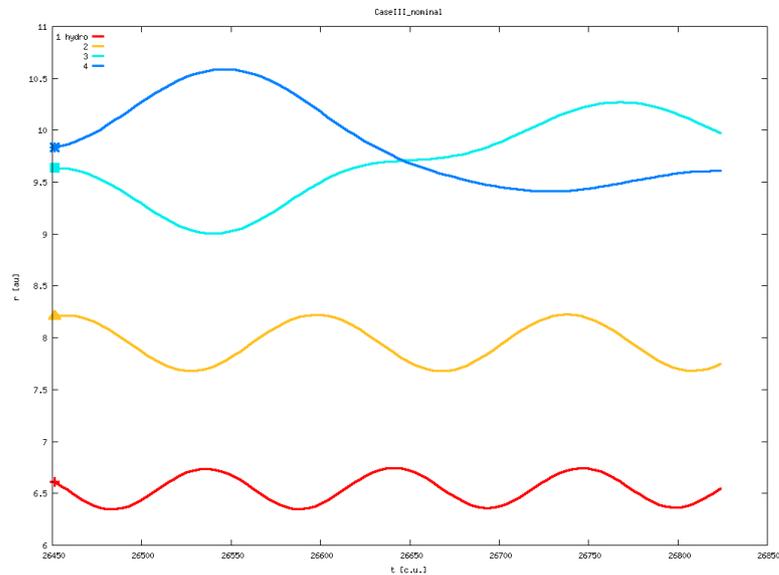
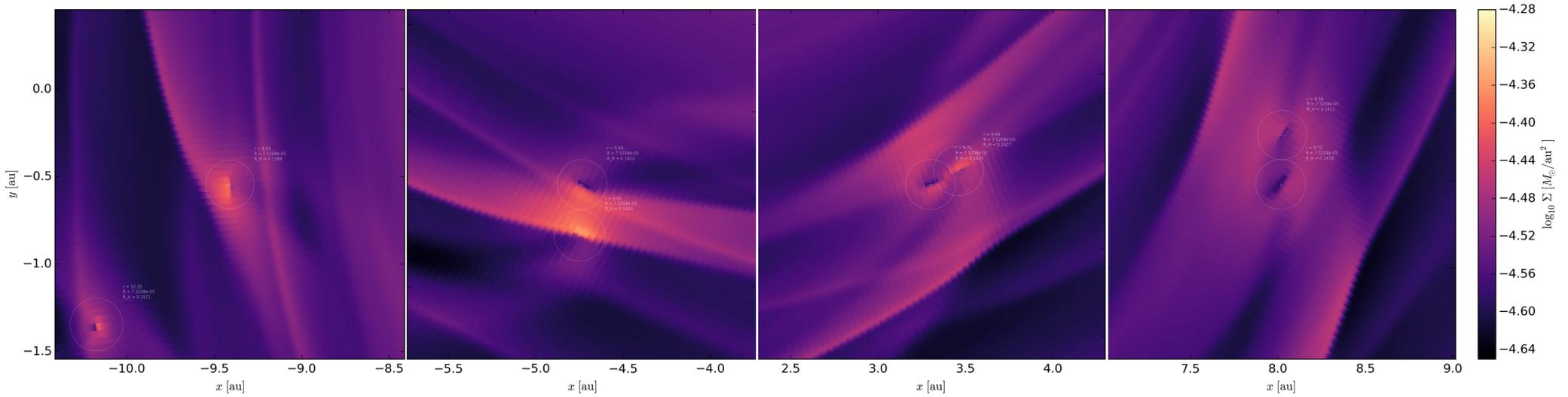
initial masses $5 M_E$; all embryos quickly drift outwards (!), even though wo. heating the 0-torque should be at 7 au; lower e, practically NO interactions, because real 0-torque is further out, unwanted interactions with the disk edge; larger disk & more embryos should be used...

Viscosity 1e-6



low-viscosity disk; same e , BUT faster migration da/dt , i.e. like ν in the denominator (!), surroundings more easily affected by the embryo, many encounters, only temporary co-orbitals, 2 mergers 8 ME as of yet, an onset of gap opening even without gas-accretion term? many attempts to form a co-orbital pair, finally a co-orbital is formed and further stabilised, a change of regime around $t = 4500 P_{\text{orb}}$ is due to developed pebble isolation (see below)

Viscosity 1e-6: Exchange



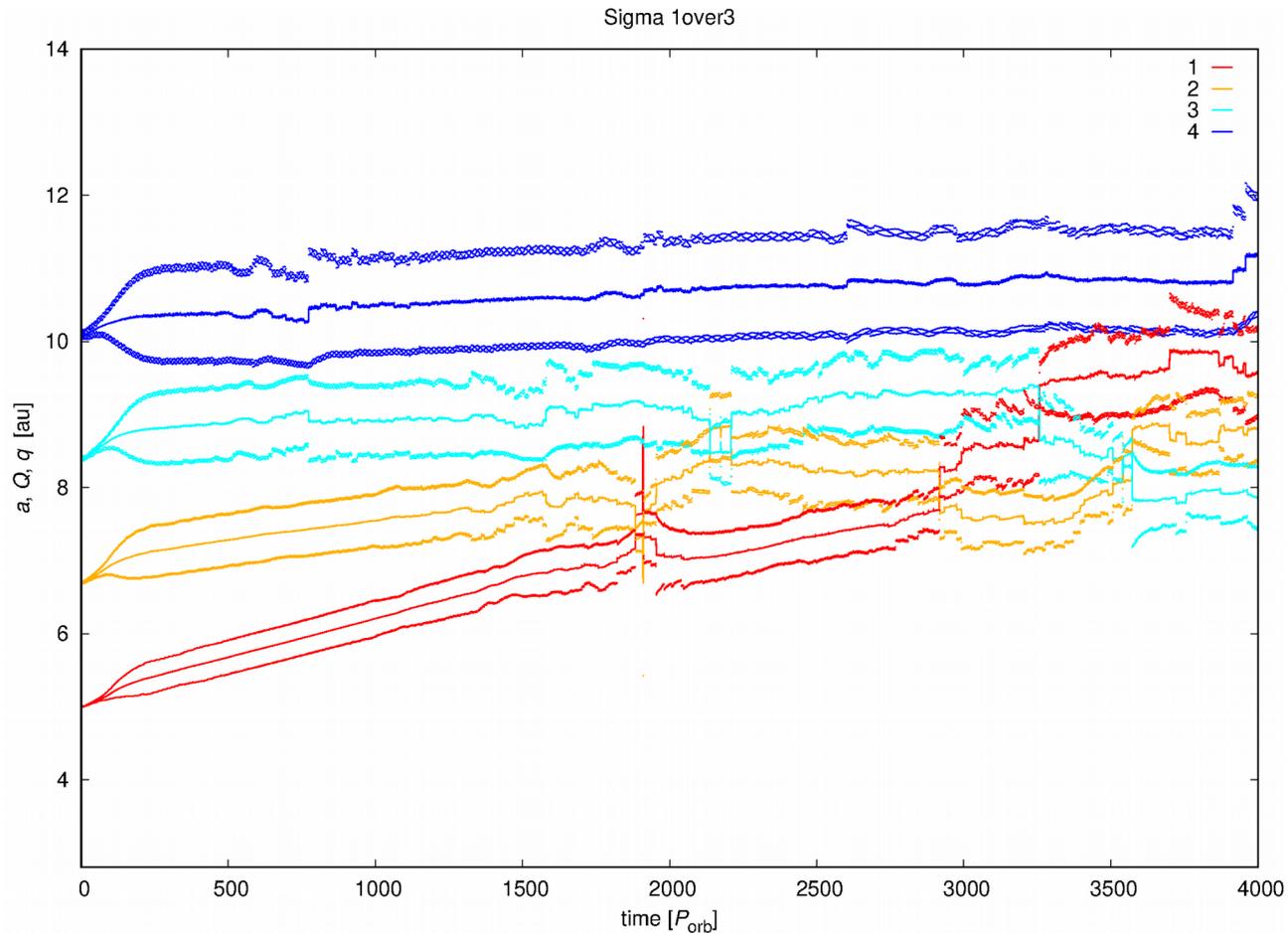
The Bet :: Sigma I over 3

Q: Will the hot trail effect be smaller or larger for $\Sigma = \frac{1}{3}\Sigma_{\text{MMSN}}$?

$\Sigma \rightarrow 0$: no gas, no perturbation, $e = 0$

$\Sigma \rightarrow \infty$: large thermal capacity, no ΔT , no expansion, $e = 0$

Sigma 1 over 3



A: The hot-trail effect is *larger* (and seems more stable)!

Conclusion: Evolution does depend on parameters! (surprisingly)

↑ migration map difficult

Hidden problems

- **2D → 3D**
- hot trail in I , pebble isolation (Eklund & Masset 2017, Bitsch et al. 2018)
- LTE, no \star ← falling snowflakes
- mean-field, no outliers
- inner edge setup (stability)
- unresolved turbulence ← ν isn't free
- resolution for small masses! ← discretisation errors, N^4 scaling
- opacity in FUV, V ? ← sideways irradiation, windows
- inflow vs outflow, advective atmospheres (Lambrechts & Lega 2017)
- GPUS (Benítez-Llambay & Masset 2016)

Q: How to resolve disks (IC)?

- photometry (unresolved)
- interferometry, spectro-interferometry, w. supersynthesis
- absolute SED, calibrated flux F_λ [$\text{erg s cm}^{-2} \text{cm}^{-1}$]
- differential interferometry (in line profiles)
- spectroscopy, a.k.a. Doppler tomography

- IFT difficult, if not impossible ← audio vs video, distorted image
- synthetic image && DFT

Model (Pyshellspec)

← based on Shellspec
(Budaj 2011)

- + LTE level populations
- + LTE ionisation levels
- + 1D line-of-sight transfer
- optically-thin (single) scattering ← no 3D, LI or ALI!
- non-isotropic scattering
- + prescribed ρ , T profiles
- + solar abundances
- + Voigt profile (prior to D.)
- + thermal broadening
- + microturbulence
- + natural
- + Stark
- + Van der Waals
- + Doppler shift
- + HI bound-free continuum opacity
- + HI free-free
- + H free-free
- Thomson scattering on free electrons
- Rayleigh scattering on neutral hydrogen
- Mie absorption on dust
- Mie scattering
- dust thermal emission
- line opacity
- + spherical primary (gainer)
- + Roche secondary (donor)
- black-body approximation (for *)
- + synthetic spectra (for *)
- irradiation
- reflection
- + limb darkening
- + gravity darkening
- heat transport

$$I_\nu(0) = \int_0^{\tau_\nu} S_\nu e^{-\tau'_\nu} d\tau'_\nu + I_\nu^*(\nu_2) f_{\text{LD}} e^{-\tau_\nu}$$

$$S_\nu = \frac{\epsilon_\nu}{\chi_\nu}$$

$$\nu_2 = \nu \left(1 - \frac{v_z^*}{c} \right)$$

$$\chi_\nu = \kappa_\nu + \sigma_\nu$$

$$\kappa_\nu = \kappa_\nu^{\text{line}} + \kappa_\nu^{\text{odf}} + \kappa_\nu^{\text{HIbf}} + \kappa_\nu^{\text{HIff}} + \kappa_\nu^{\text{H}^- \text{bf}} + \kappa_\nu^{\text{H}^- \text{ff}}$$

$$\sigma_\nu = \sigma_\nu^{\text{TS}} + \sigma_\nu^{\text{RS}}$$

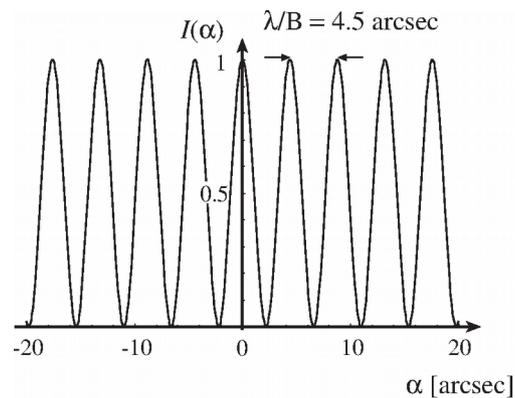
$$\epsilon_\nu = \epsilon_\nu^{\text{th}} + \epsilon_\nu^{\text{sc}}$$

$$\epsilon_\nu^{\text{th}} = B_\nu(T(z)) \kappa_\nu$$

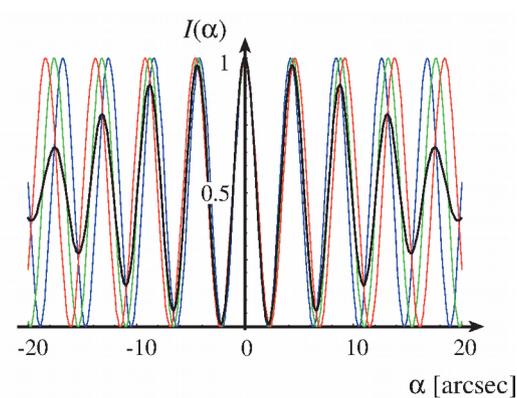
$$\epsilon_\nu^{\text{sc}} \doteq \sigma_\nu I_\nu^* f_{\text{SH}} \frac{\omega}{4\pi}$$

Young experiment

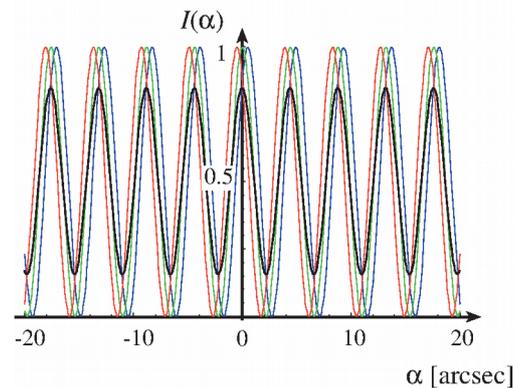
- monochromatic vs polychromatic vs extended
- experiment: waves and shadows



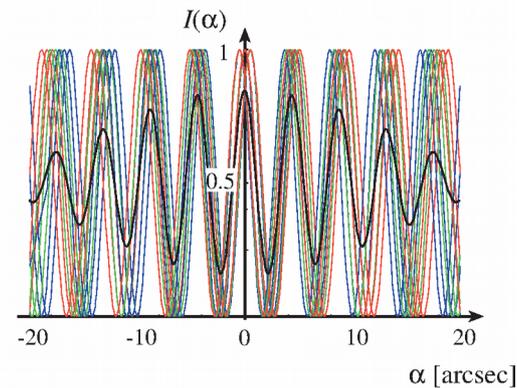
a) point source, monochromatic



b) point source, K-band

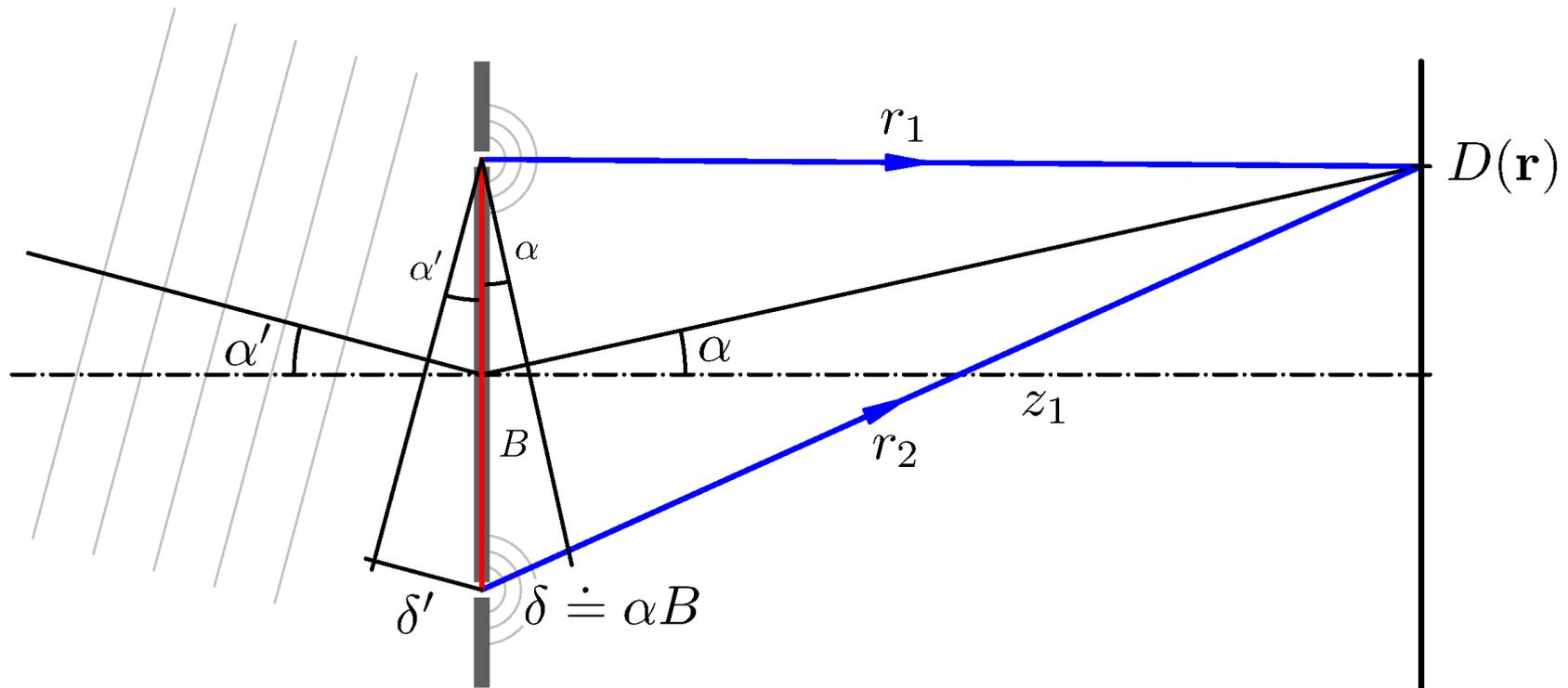


c) extended source, monochromatic



d) extended source, K-band

Young experiment (cont.)



Obrázek 6.51: Uspořádání Youngova experimentu, kde B označuje vzájemnou vzdálenost štěrbin (základnu), z_1 vzdálenost stínítka od překážky, r_1 , r_2 vzdálenost studovaného místa na stínítku od štěrbin, α odpovídající odchylka od osy překážky, α' úhel dopadu vlny na překážku, δ , δ' dráhové rozdíly vznikající za a před překážkou.

Visibility

Brož & Wolf (2017)

Viditelnost. V Youngově experimentu dopadá na zmiňovanou překážku rovinná monochromatická elektromagnetická vlna. Namísto jednotlivých složek polí \mathbf{E} , \mathbf{B} budeme používat bezrozměrný vzruch D (angl. disturbance) v komplexní notaci:

$$D(\mathbf{r}, t) = D_0 e^{-i(\omega t - \mathbf{k} \cdot \mathbf{r})} = D(\mathbf{r}) e^{-i\omega t}, \quad (6.151)$$

kde $\omega = \frac{2\pi c}{\lambda}$ označuje úhlovou frekvenci a \mathbf{k} vlnový vektor, $k = \frac{2\pi}{\lambda}$. Podle Huygensova principu spočteme vzruch na stínítku jako součet dvou kulových vln, šířících se z otvorů (obr. 6.51):

$$\begin{aligned} D(\mathbf{r}) &= \frac{D_0}{r_1} e^{ikr_1} + \frac{D_0}{r_2} e^{ikr_2} \doteq \frac{D_0}{z_1} (e^{ikr_1} + e^{ikr_2}) = \\ &= \frac{D_0}{z_1} e^{i\frac{1}{2}k(r_1+r_2)} 2 \cos\left[\frac{1}{2}k(r_1 - r_2)\right]. \end{aligned} \quad (6.152)$$

Nepozorujeme ovšem přímo D , nýbrž tok daný Poyntingovým vektorem $\mathbf{S} = \mathbf{E} \times \mathbf{H}$, který lze pro naše účely středovat, normalizovat a považovat za bezrozměrnou intenzitu:

$$I(\mathbf{r}) \equiv \langle DD^* \rangle = |D(\mathbf{r})|^2. \quad (6.154)$$

Visibility (cont.)

V místě odchýleném od osy překážky o úhel α je pak:

$$\begin{aligned} I(\alpha) &= |D(\alpha)|^2 = \left(\frac{D_0}{z_1}\right)^2 4 \cos^2\left[\frac{1}{2}k(r_1 - r_2)\right] = \left(\frac{D_0}{z_1}\right)^2 2\{1 + \cos[k(r_1 - r_2)]\} = \\ &\doteq I_0\{1 + \cos[k\alpha B]\}, \end{aligned} \quad (6.155)$$

kde B označuje vzájemnou vzdálenost otvorů. Pokud navíc vlna sama dopadá na překážku pod úhlem α' :

$$I(\alpha, \alpha') = I_0\{1 + \cos[k(\alpha + \alpha')B]\}. \quad (6.156)$$

Jako jednoslovný popis jevu se zavádí *viditelnost*, neboli kontrast interferenčních proužků:

$$\mathcal{V} \equiv \frac{I_{\max} - I_{\min}}{I_{\max} + I_{\min}}. \quad (6.157)$$

Protože zde $I_{\min} = 0$, $I_{\max} = 1$, je $\mathcal{V} = 1$. Pro rozlehlý zdroj nebo polychromatické záření bývá ovšem viditelnost menší, protože přicházející vlny nejsou prostorově respektive časově koherentní (viz obr. 6.52).

Van Cittert & Zernike theorem

Teorém van Citterta a Zernikeho. Budeme-li přes rozlehlý zdroj (úhly α') integrovat:

$$I(\alpha) = \int I(\alpha, \alpha') d\alpha' = \overbrace{\int I(\alpha') d\alpha'}^{= I_0} + \overbrace{\int I(\alpha') \cos[k(\alpha + \alpha')B] d\alpha'}^{= \Re[e^{ik\alpha B} \int I(\alpha') e^{ik\alpha' B} d\alpha']}, \quad (6.158)$$

uzříme, že se vlastně jedná o reálnou část Fourierovy transformace rozložení intenzity zdroje $I(\alpha')$, násobené jakýmsi faktorem. Obecněji zapsáno:

$$I(\vec{\alpha}) = I_0 \left\{ 1 + \Re \left[\mu(\vec{B}) e^{-ik\vec{\alpha} \cdot \vec{B}} \right] \right\}, \quad (6.159)$$

kde komplexní *funkce viditelnosti*:

$$\mu(\vec{B}) \equiv \frac{\int I(\vec{\alpha}') e^{-ik\vec{\alpha}' \cdot \vec{B}} d\alpha'}{I_0}, \quad (6.160)$$

tj. tvrzení teorému van Citterta a Zernikeho. Absolutní hodnota $|\mu(\vec{B})|$ evidentně určuje viditelnost \mathcal{V} (tj. kontrast), kdežto příslušná fáze $\phi(\vec{B})$ polohu „prostředního“ bílého proužku

Closure phase

← asymmetry of the source!

Vzruch je na každém z dalekohledů pozměněn (Haniff 2006):

$$\tilde{D} = GD = |G| e^{i\Phi} D, \quad (6.169)$$

kde $|G|$ označuje zisk dalekohledu, zohledňující mj. odrazivost zrcadla, Φ fázový posun, ovlivněný seeingem, teplotní roztažností atd. Funkce viditelnosti je přitom $\mu \propto D_1 D_2^*$, čili skutečně měřená („rozvlněná“) funkce viditelnosti:

$$\tilde{\mu} = G_1 G_2^* \mu = |G_1| |G_2| e^{i(\Phi_1 - \Phi_2)} \mu. \quad (6.170)$$

Amplituda je evidentně zmenšena, fáze kamsi posunuta. Definujeme-li však *trojný součin* (zvaný též bispektrum):

$$T_3 \equiv \mu_{12} \mu_{23} \mu_{31}, \quad (6.171)$$

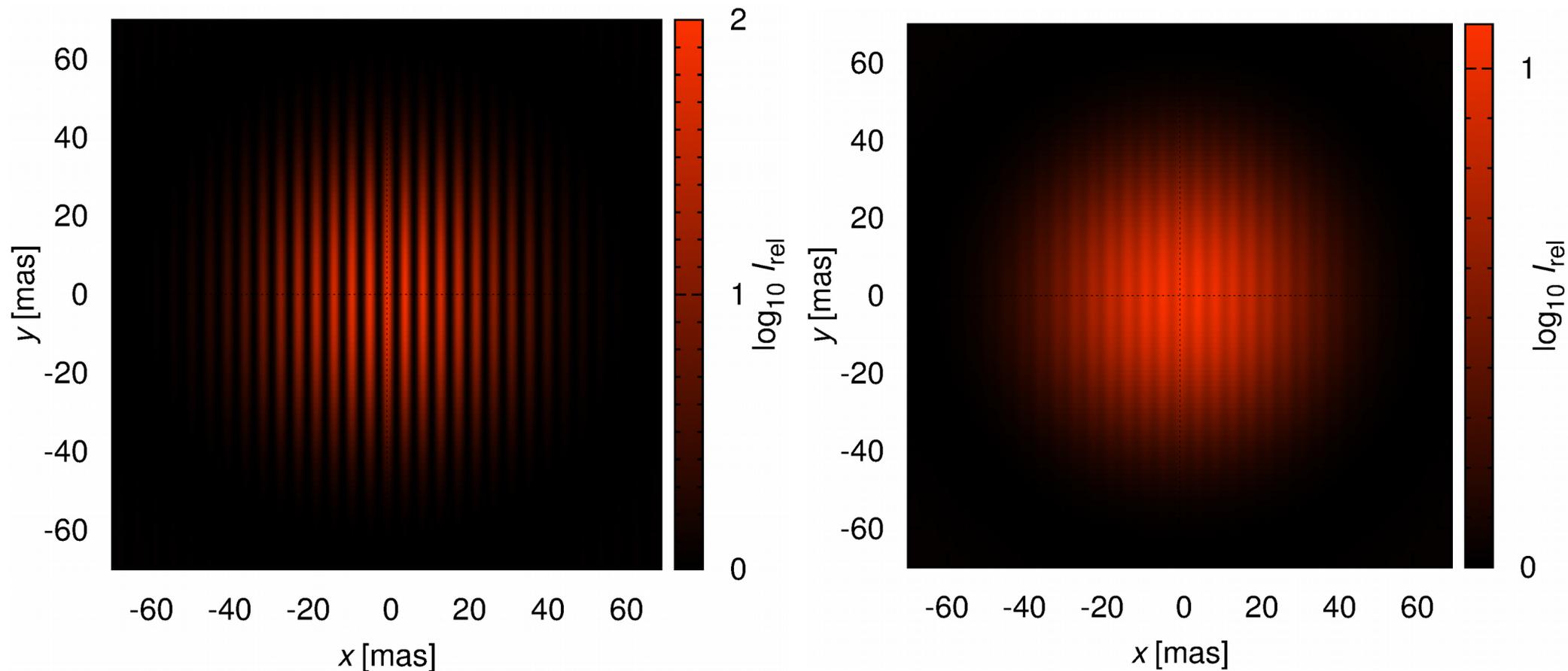
zjistíme úžasnou věc:

$$\begin{aligned} \tilde{T}_3 &= \tilde{\mu}_{12} \tilde{\mu}_{23} \tilde{\mu}_{31} = \\ &= |G_1| |G_2| e^{i(\Phi_1 - \Phi_2)} \mu_{12} |G_2| |G_3| e^{i(\Phi_2 - \Phi_3)} \mu_{23} |G_3| |G_1| e^{i(\Phi_3 - \Phi_1)} \mu_{31} = \\ &= |G_1|^2 |G_2|^2 |G_3|^2 \mu_{12} \mu_{23} \mu_{31}. \end{aligned} \quad (6.172)$$

Amplituda je sice zmenšena, ale fáze nikam *neposunuta*! Uzavírací fáze je pak $\arg T_3$.

Fringes (for VLTI)

- $D = 8$ m, $B = 100$ m, observations of a disk $\theta = 1$ or 5 mas, but without seeing, bandwidth $\Delta\lambda_{\text{eff}}$, and other incoherence

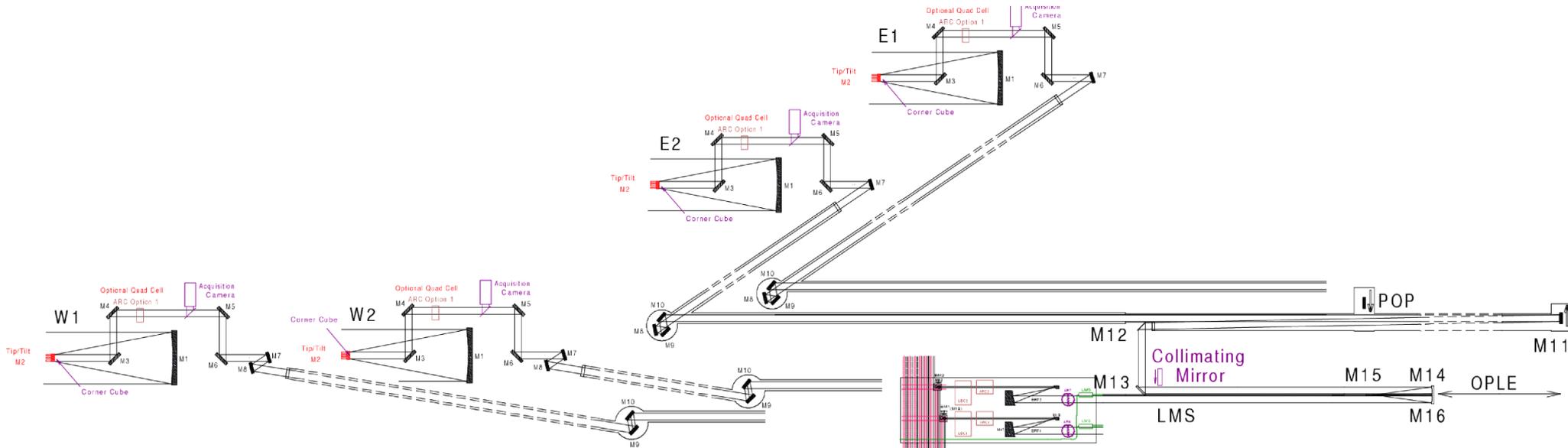


CHARA

- Mt. Wilson observatory, 6 telescopes, Y configuration, 10 baselines up to 331 m, $B/\lambda = 331 \text{ m}/550 \text{ nm} = 6 \cdot 10^8 \text{ c. per b.}$
- Mersenne afocal system, primary diameter 1 m (ten Brummelaar et al. 2005)



Optical scheme



Mersennův dalekohled

- Nasmyth
- coudé
- otáčecí krabice
- fixní zpoždění
- periskop
- zpoždovací dráha
- 2. Mersennův dalekohled
- dichroická zrcadla V/IR

Beams

Visible Imager

Tip/Tilt

Fringe Tracking

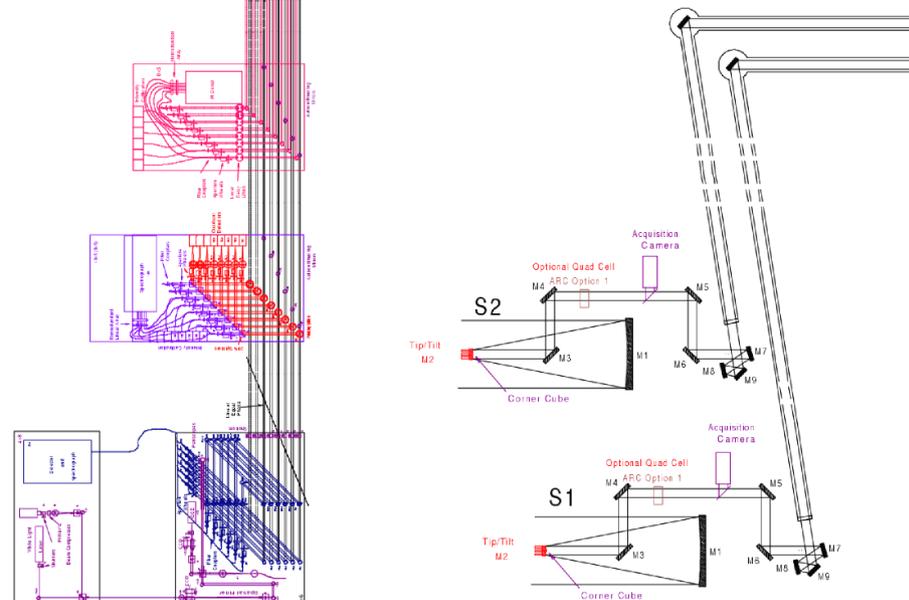
Alignment

IR Imager

Optional

Dispersion

Vacuum



Σ 144 zrcadel



EXIT

Periscopes

OPLE

PoPs

Metrology

BRT

LDC

BSS

Model (cont.)

- Python interface by JN — <http://sirrah.troja.mff.cuni.cz/~mira/betalyr/>
- computation of interferometric observables (DFT), joint χ^2
- multiprocessing module (split on wavelengths; 4-8 cores)
- discretisation $N_x = 160$, $N_y = 60$ ($\sim 1 R_\odot$); variable in z ($\sim \tau$)
- local & global optimisation (DE, simplex, ...) ↑ trapezoidal rule
- 1 iteration: 2392 synthetic images per iteration (3 min),
1 convergence: $>10^3$ steps (several days)
- free parameters: H (or θ), R_{out} , ρ , T_0 (or T_1), α_D , α_T , i , Ω , d , h_{inv} ,
 t_{inv} , h_{wind} , h_{mul}
- fixed parameters: P , JD_{min} , \dot{P} , $a \sin i$, M_1 , $q = M_1/M_2$, e , ω , f_{fill} , R_g ,
 $T_{\text{eff,d}}$, $T_{\text{eff,g}}$, $X_{\text{bol,d}}$, $X_{\text{bol,g}}$, $\alpha_{\text{gd,d}}$, $\alpha_{\text{gd,g}}$, ...

Joint χ^2 metric

$$\chi_{\text{LC}}^2 = \sum_{i=1}^{N_P} \sum_{j=1}^{N_M} \left(\frac{m_{i,j}^{\text{obs}} - \tilde{m}_{i,j}^{\text{syn}}}{\sigma_{i,j}} \right)^2,$$

$$\chi_{\text{IF}}^2 = \chi_{\text{IF}_{\text{VEGA}}}^2 + \chi_{\text{IF}_{\text{NPOI}}}^2 + \chi_{\text{IF}_{\text{MIRC}}}^2,$$

$$\chi_{\text{IF}_{\text{VEGA}}}^2 = \chi_{V^2}^2,$$

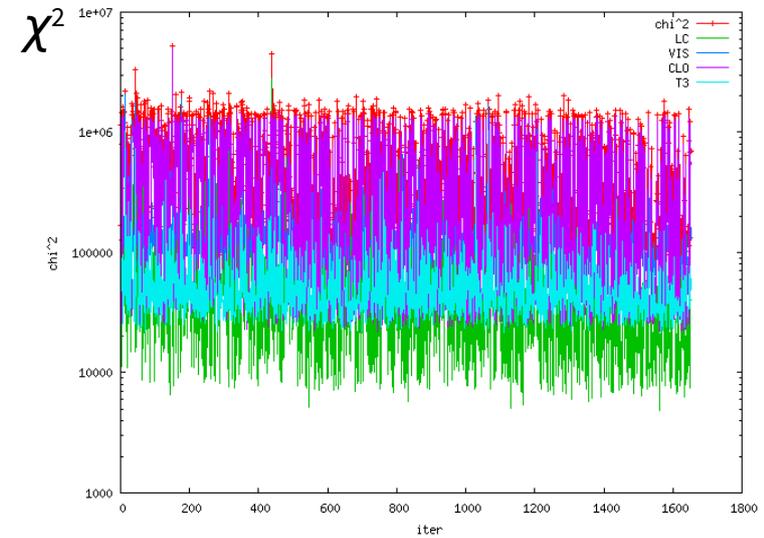
$$\chi_{\text{IF}_{\text{NPOI}}}^2 = \chi_{V^2}^2 + \chi_{\text{CP}}^2,$$

$$\chi_{\text{IF}_{\text{MIRC}}}^2 = \frac{1}{2} (\chi_{V^2}^2 + \chi_{T_3}^2) + \chi_{\text{CP}}^2,$$

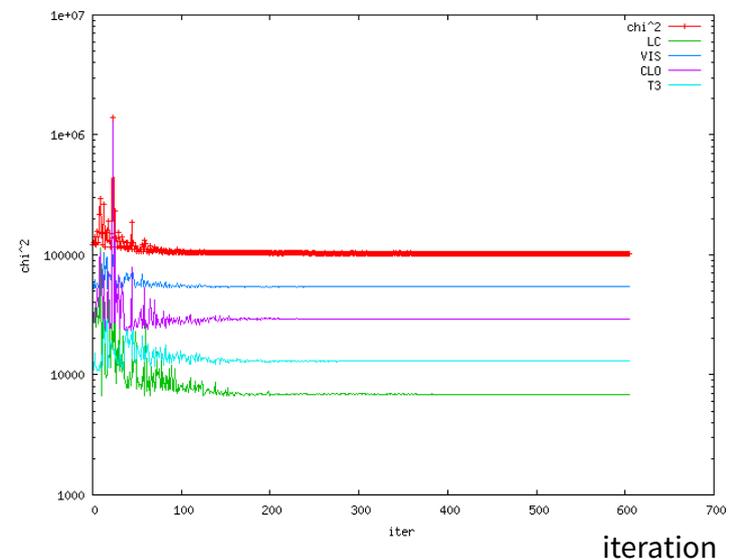
$$\chi_{V^2}^2 = \sum_{i=1}^{N_{V^2}} \left(\frac{|V_{\text{obs}}|_i^2 - |V_{\text{syn}}|_i^2}{\sigma_i} \right)^2,$$

$$\chi_{T_3}^2 = \sum_{i=1}^{N_{T_3}} \left(\frac{|T_3^{\text{obs}}|_i - |T_3^{\text{syn}}|_i}{\sigma_i} \right)^2,$$

$$\chi_{\text{CP}}^2 = \sum_{i=1}^{N_{T_3}} \left(\frac{T_3 \phi_i^{\text{obs}} - T_3 \phi_i^{\text{syn}}}{\sigma_i} \right)^2.$$

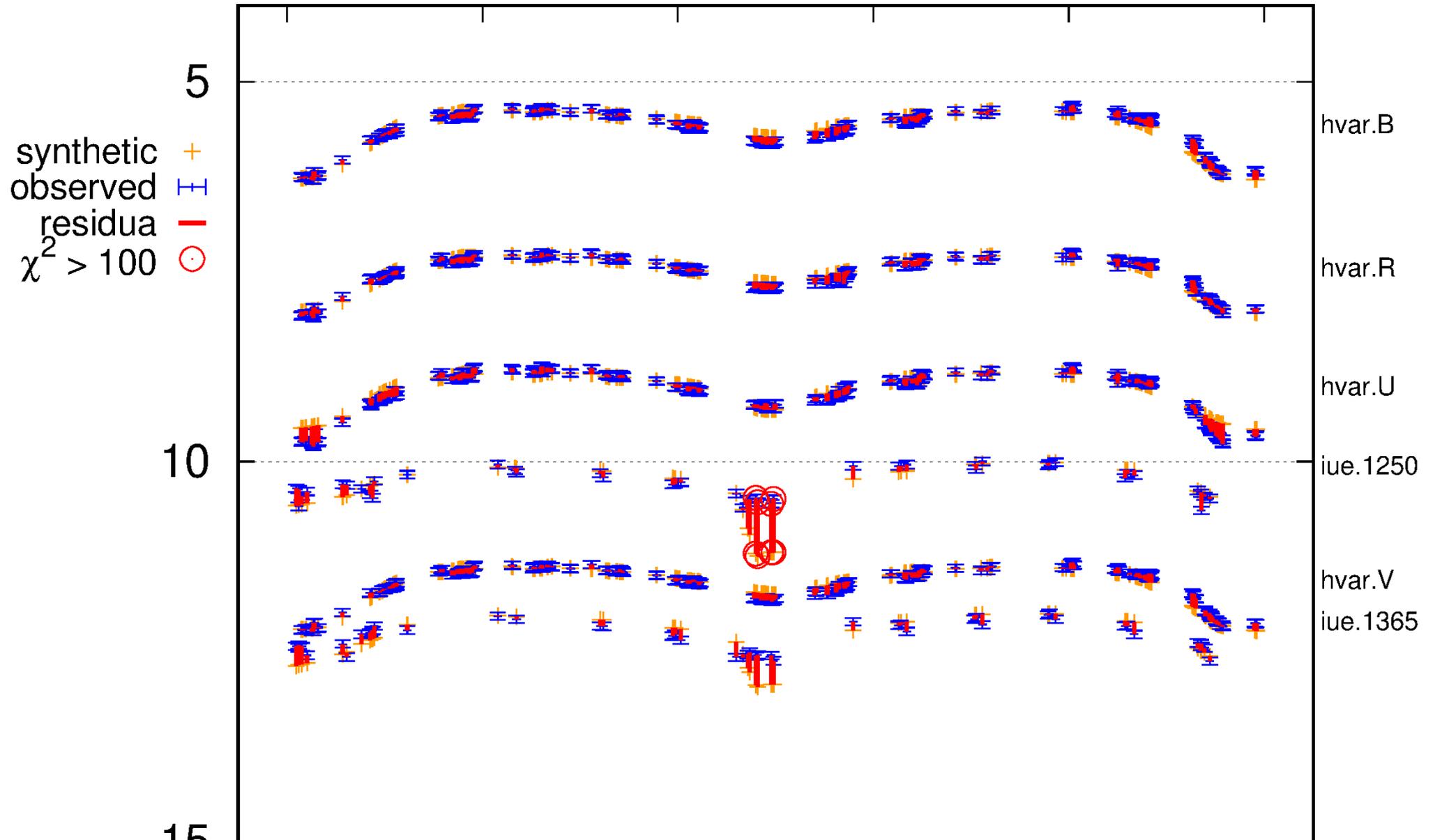


DE



simplex

Lightcurves (LC)



magnitude [mag] (shifted by dataset number)

20

jamlon.K

jamlon.L

oao2.1430

jamlon.M

25

oao2.1550

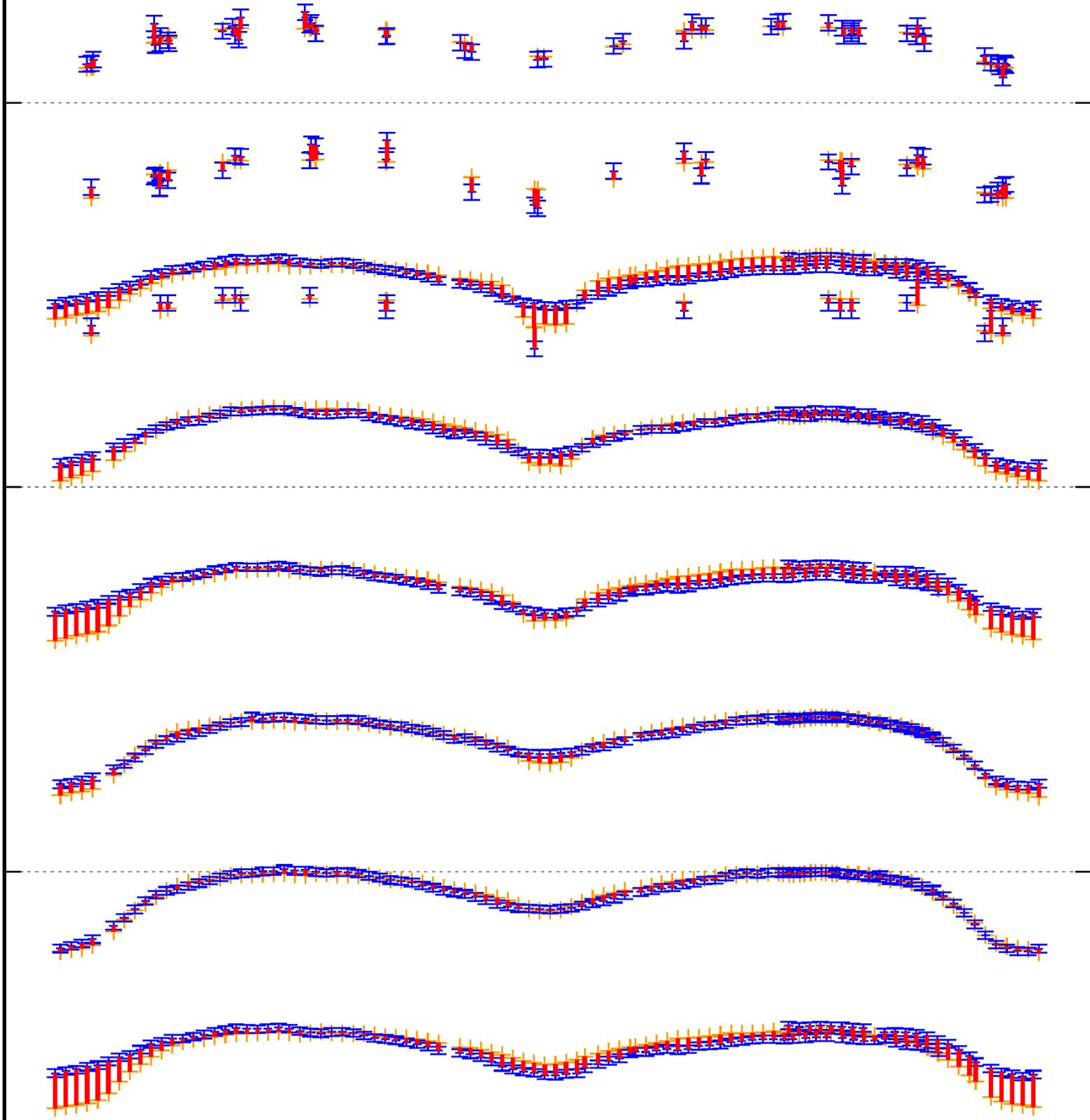
oao2.1910

oao2.2460

30

oao2.2980

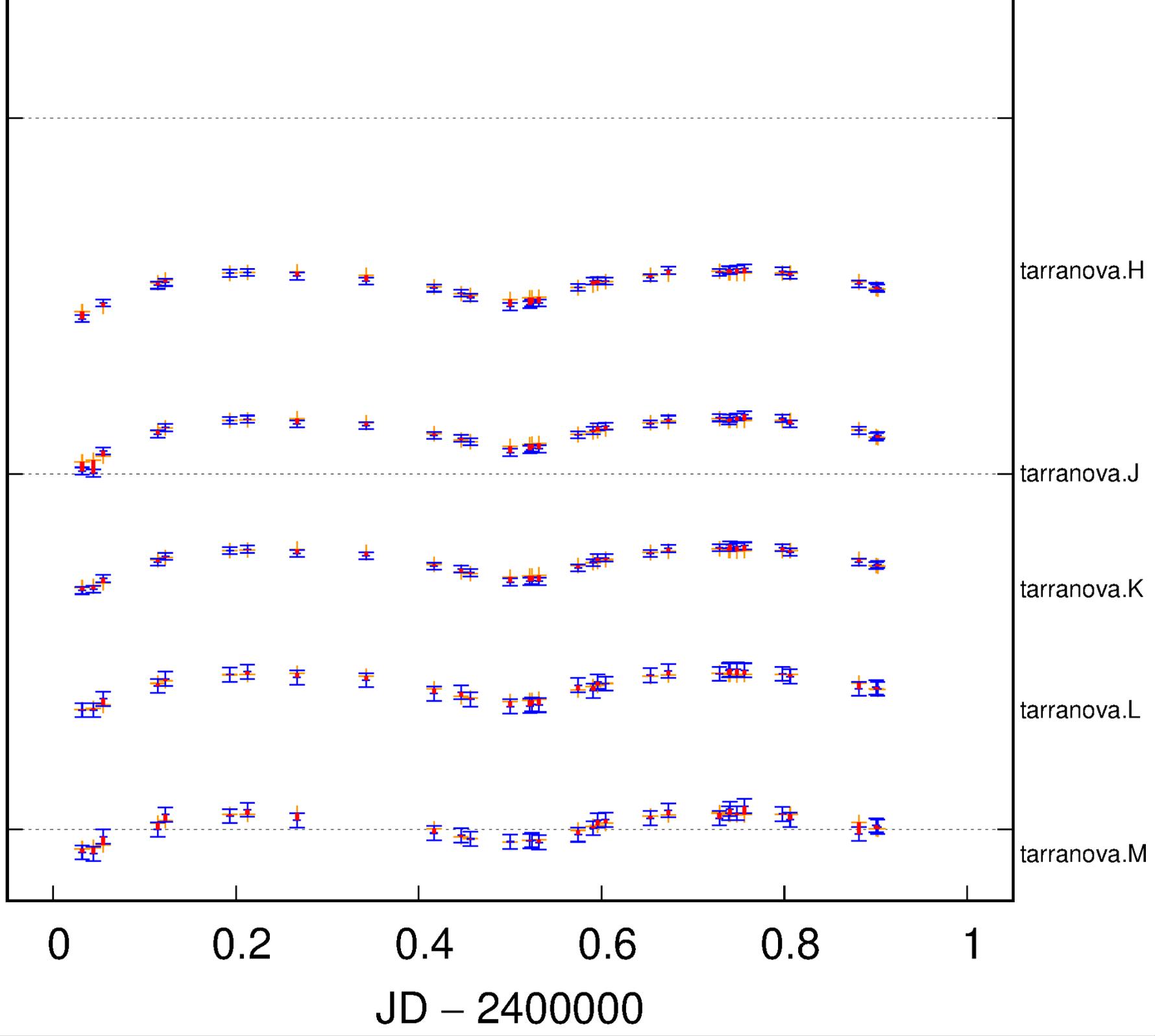
oao2.3320



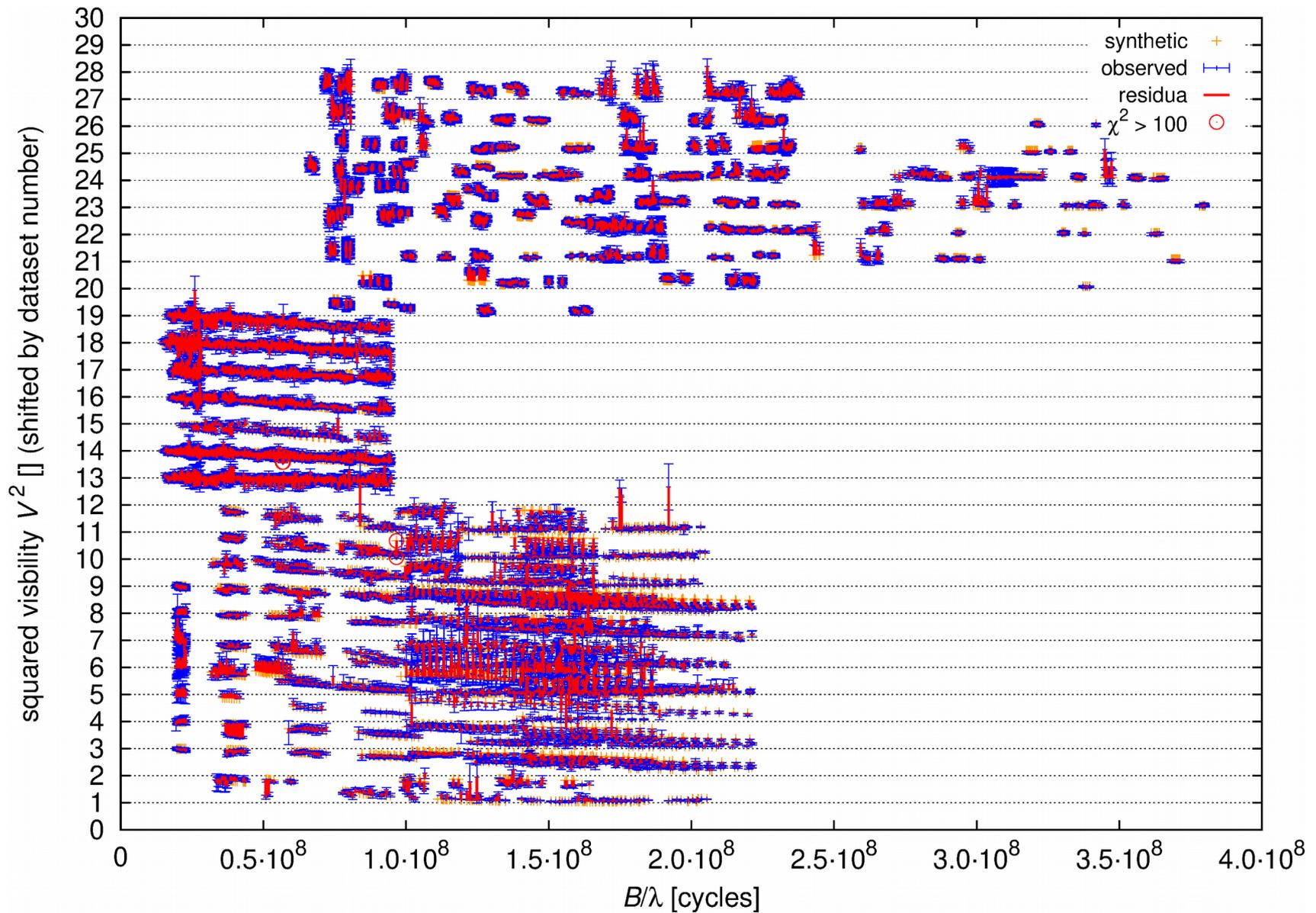
35

40

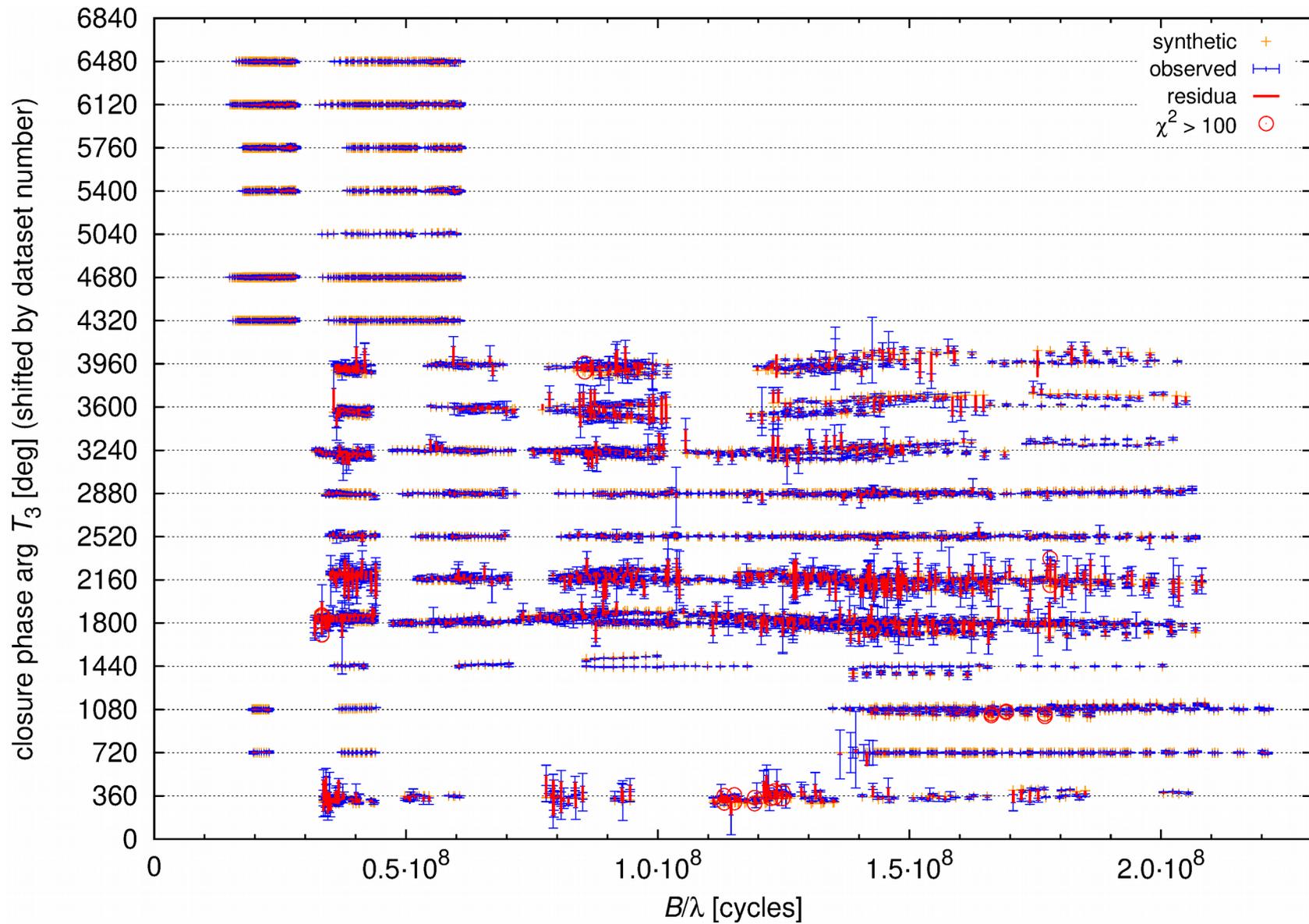
45



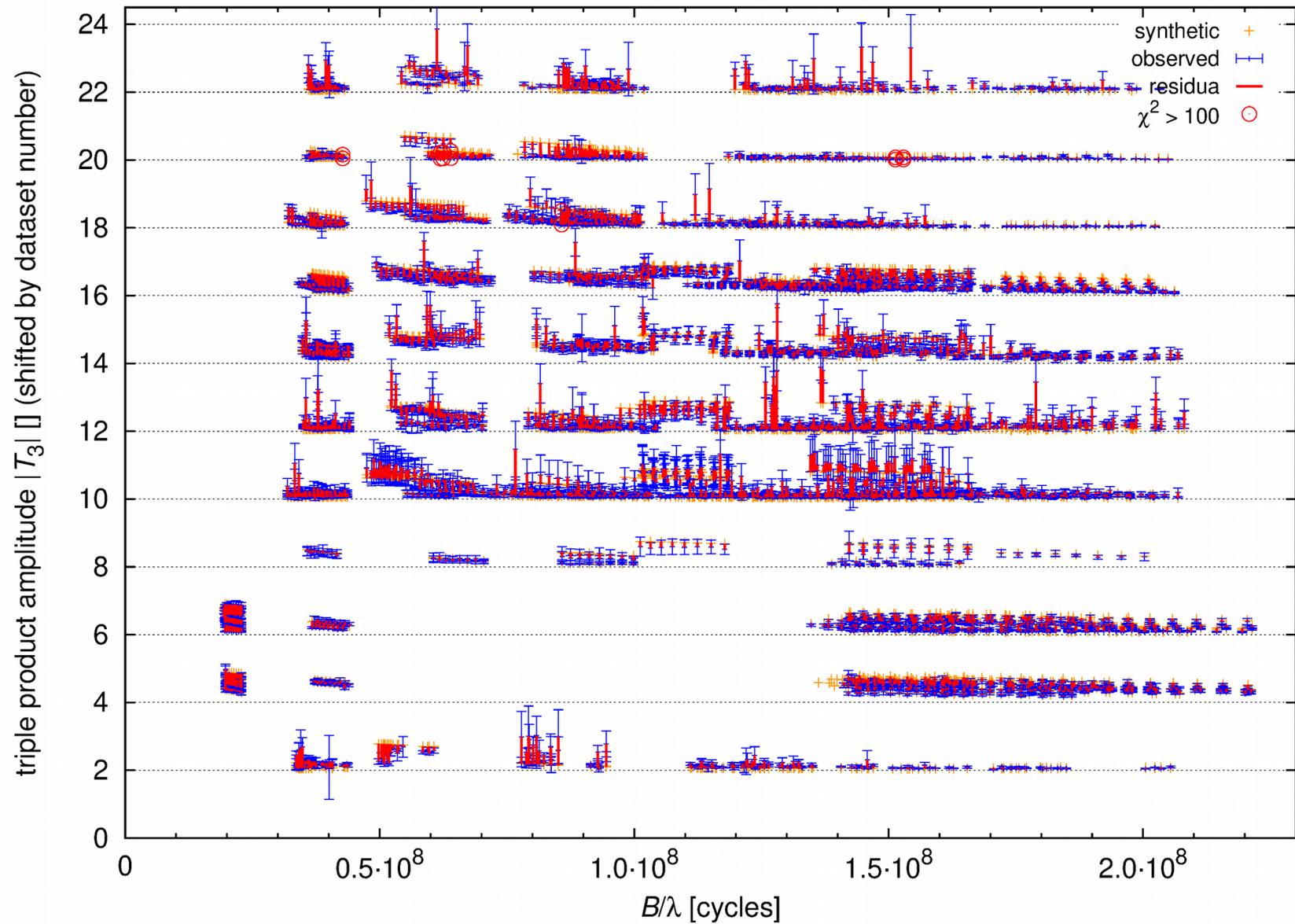
Visibility (VIS)



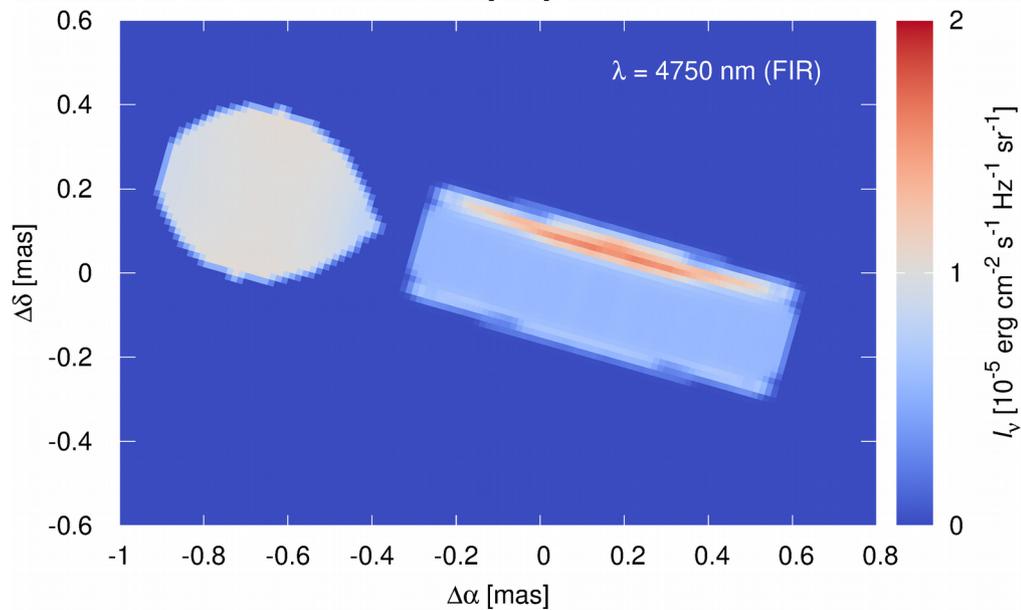
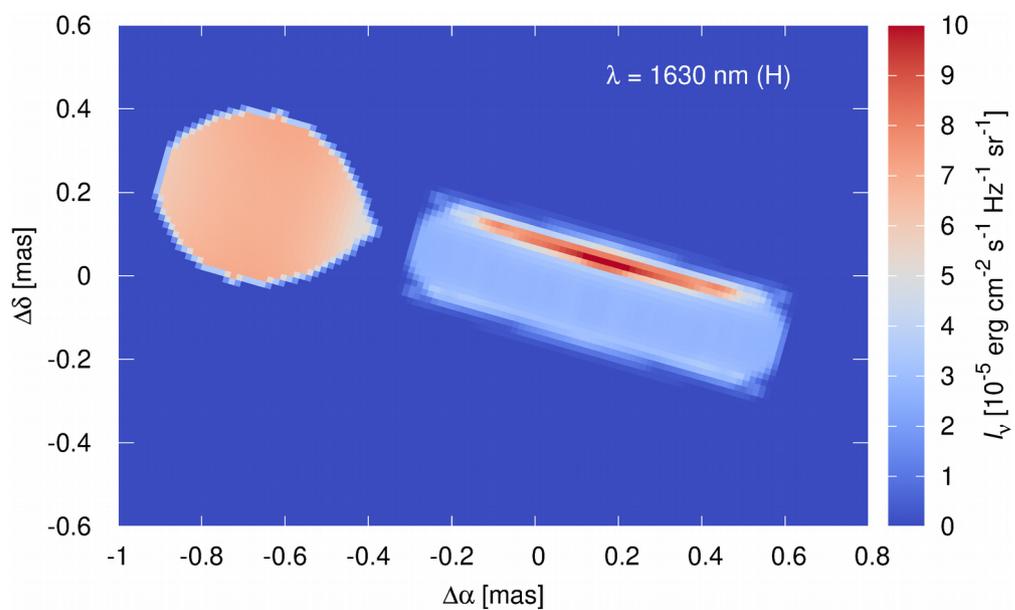
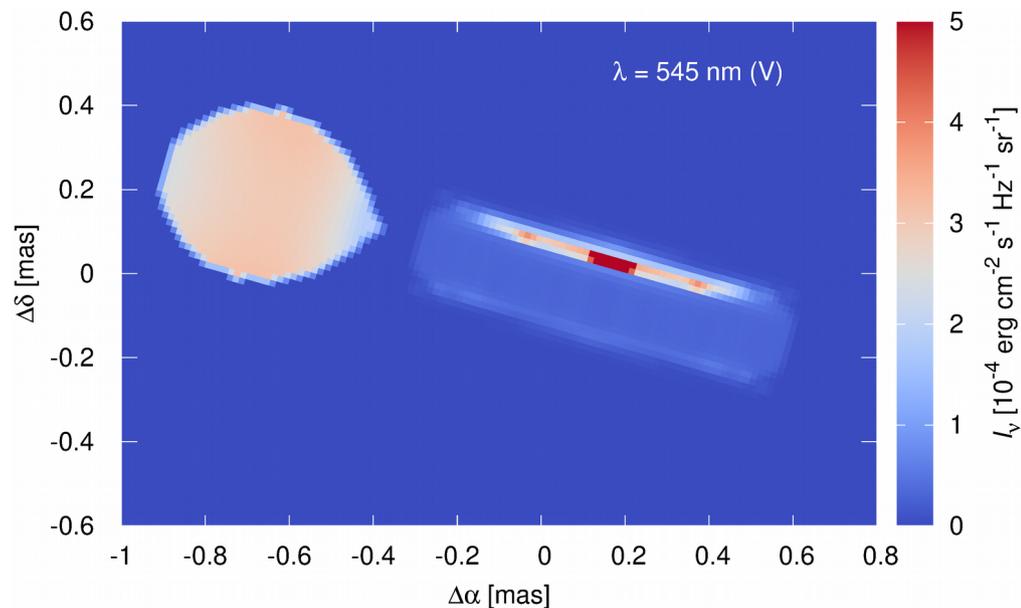
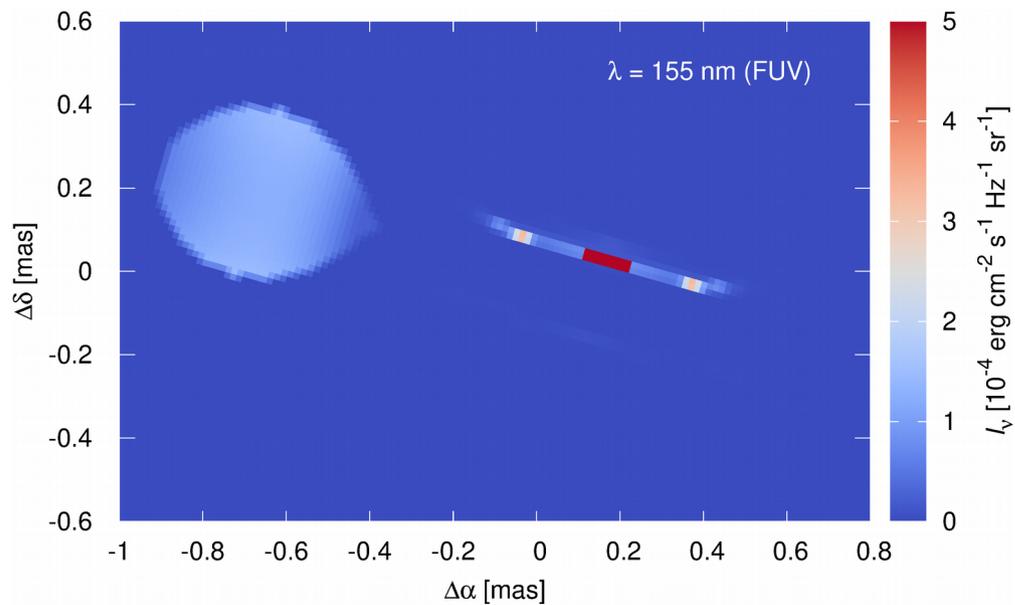
Closure phase (CLO)



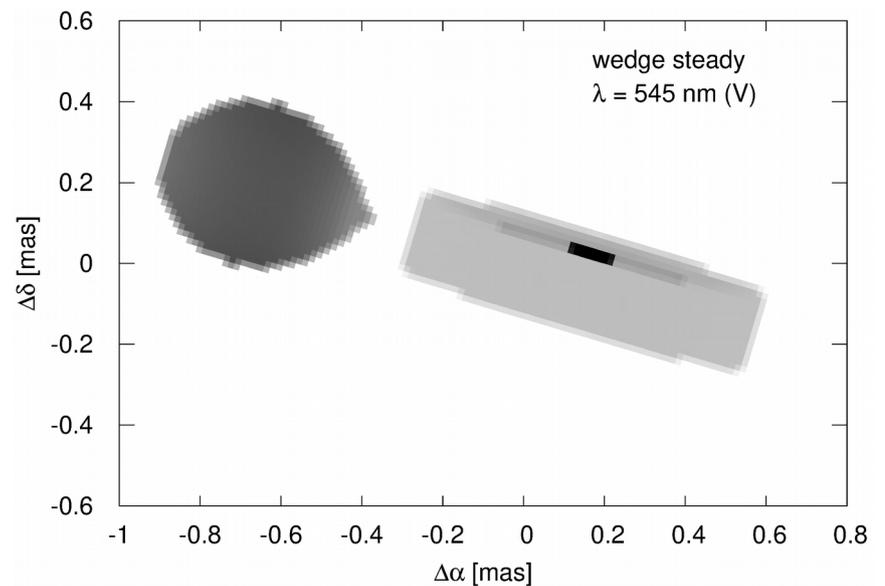
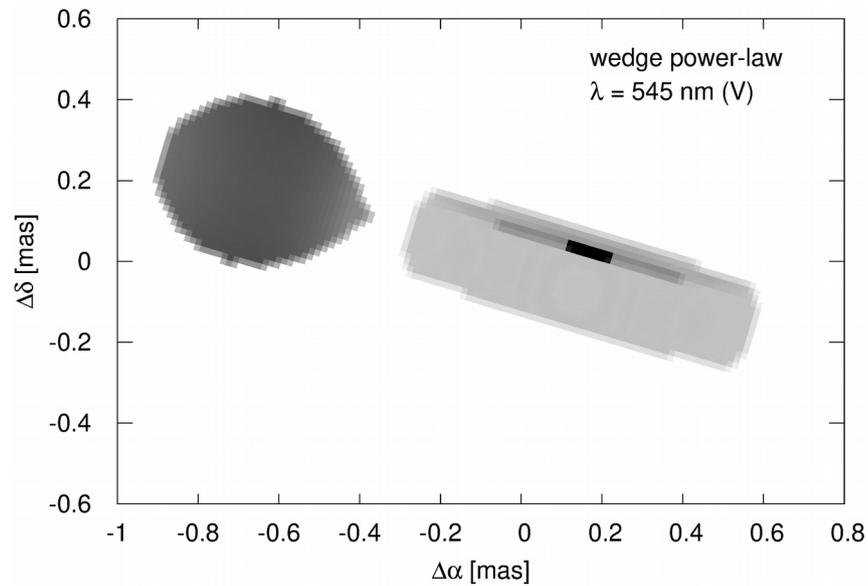
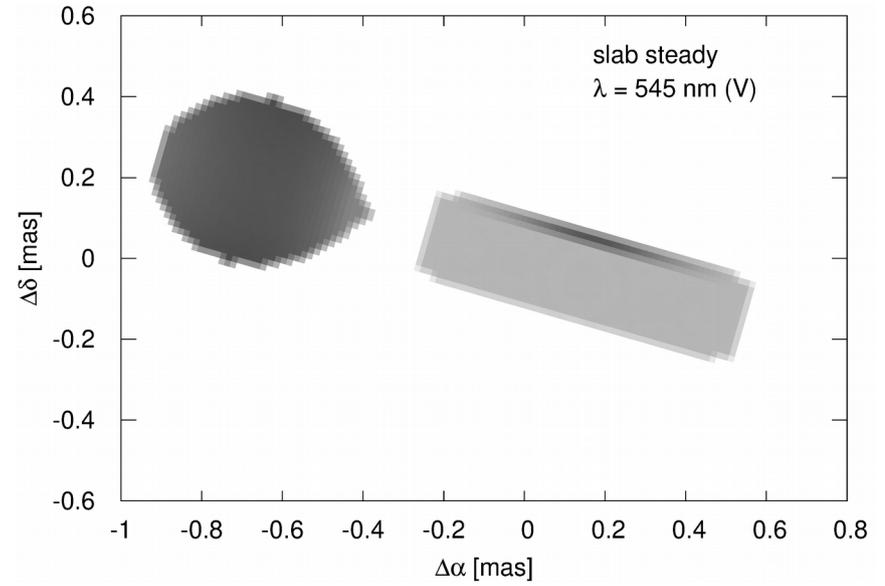
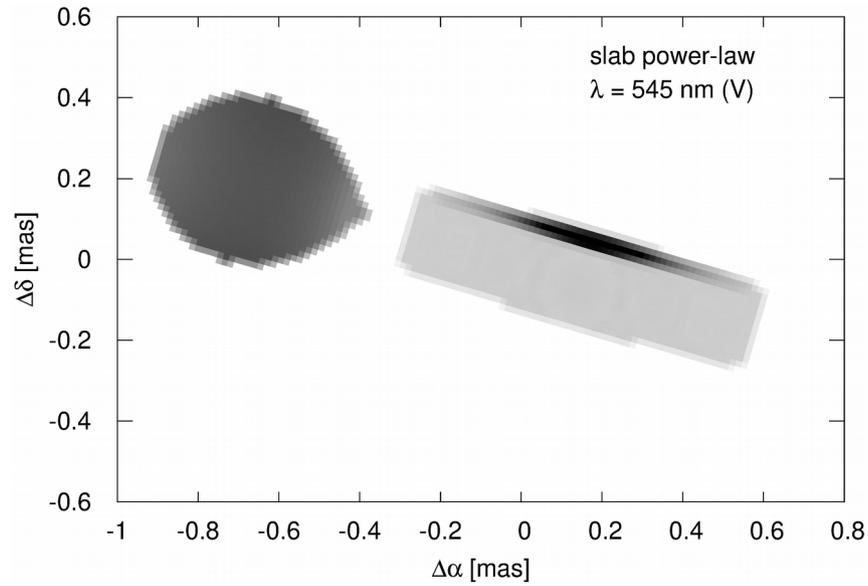
Triple product (T3)



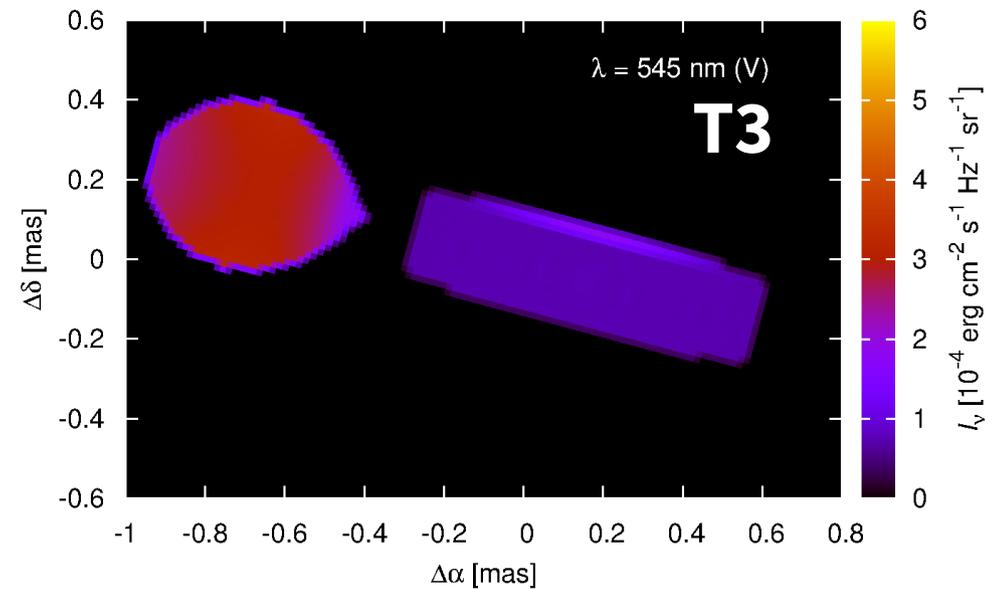
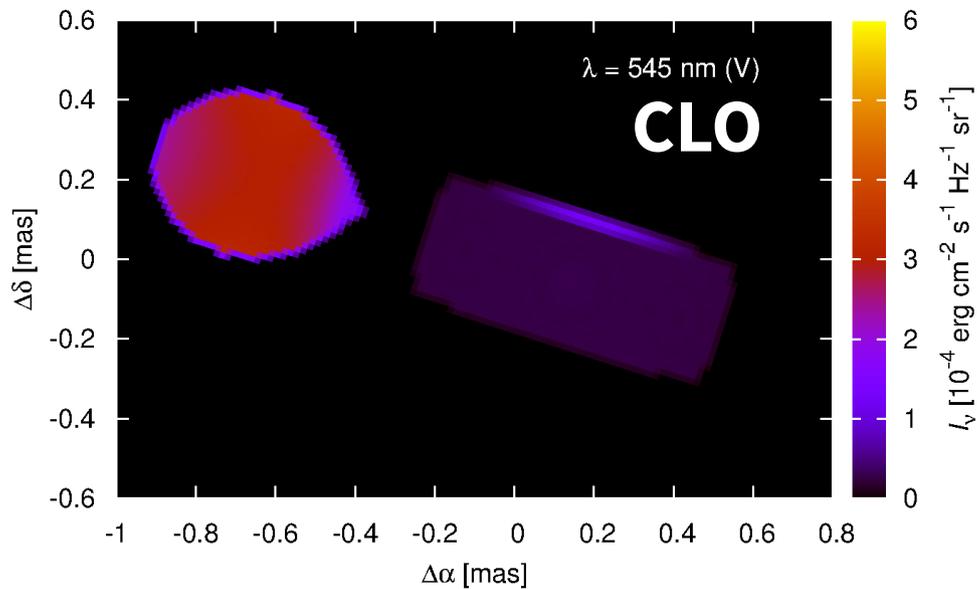
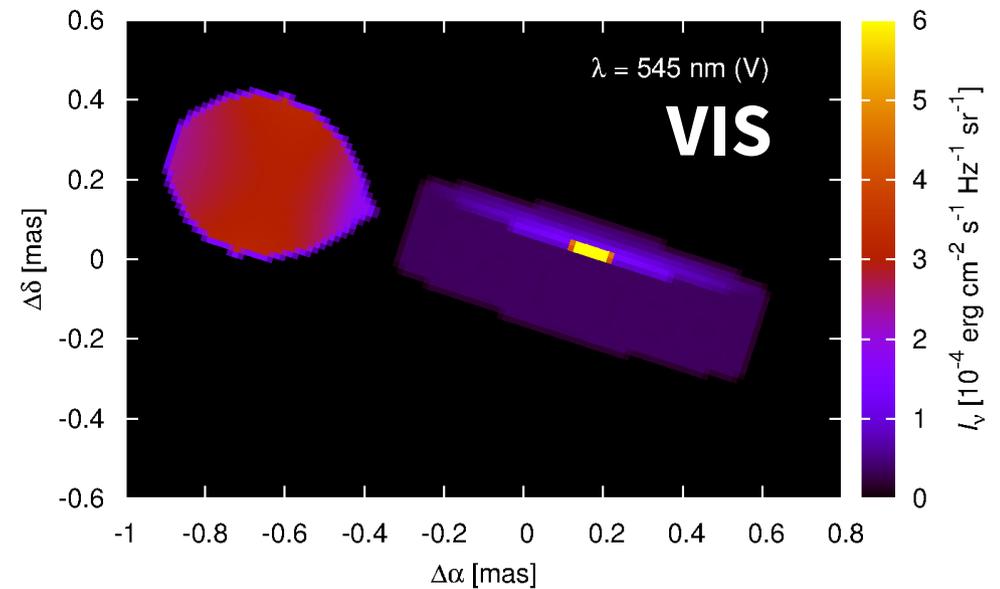
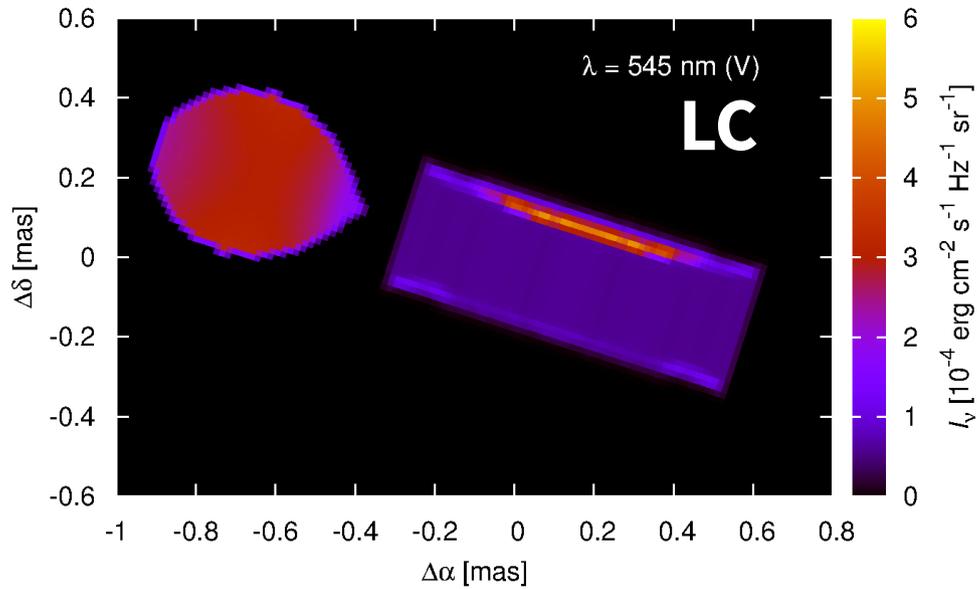
β Lyr A best-fit model



Alternative shapes



Systematic differences

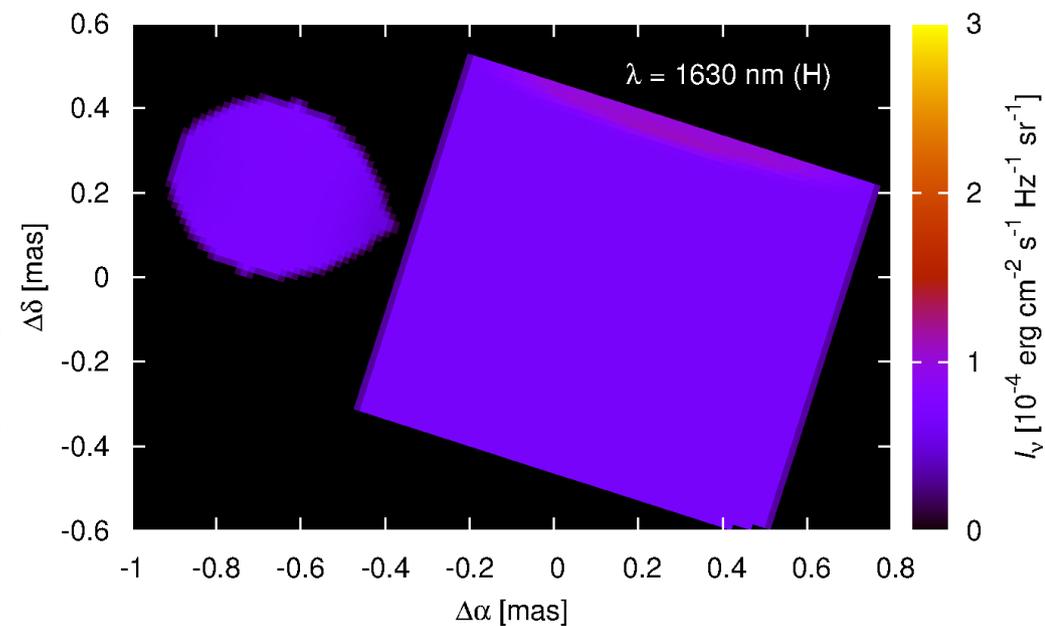
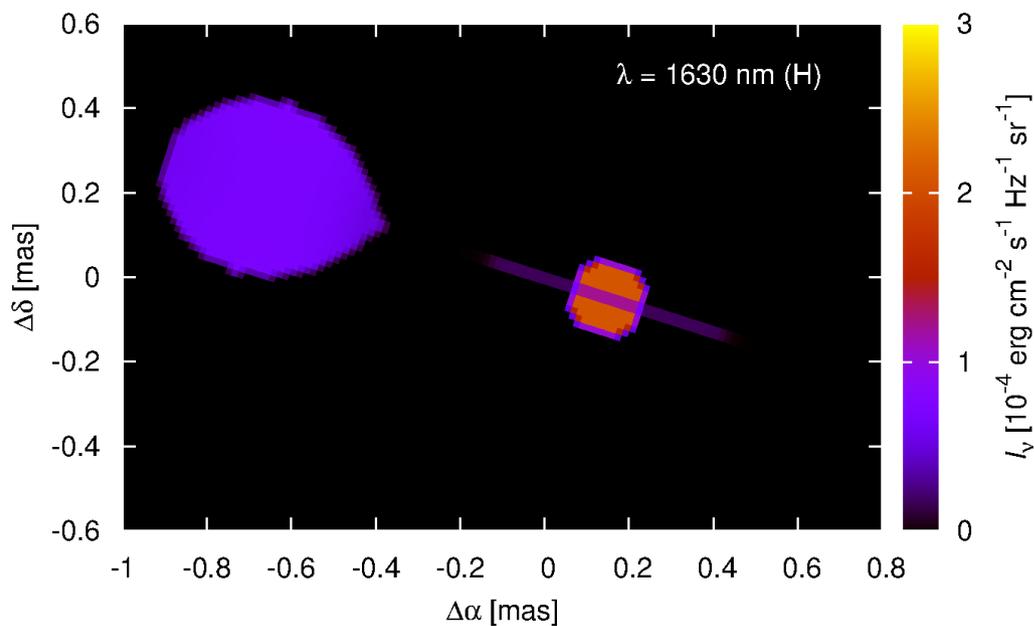


Parameter space

minimum



maximum



Note: A non-negligible part was explored, but some p. were fixed...

Conclusion: One can be never sure the model is sufficient! (trivial)

Hidden problems

- missing scattering (shadowing), better d. atmosphere
- optically thin jets, spot(s), reflection + irradiation
- limited resolution ($\sim 1 R_{\odot}$), discretisation errors, RTE artifacts
- non-LTE?
- systematics between LC & VIS, CLO, T3
- optically thick vs *very* o. t. \leftarrow degenerate problem :(

- missing ΔV^2 , Doppler, SED measurements
- kinematics, missing feedback on HD! \rightarrow dynamical model?
- disk stability, outer edge, precession?

References

Brož & Wolf, *Astronomická měření*, Praha: Matfyzpress, 2017.

ten Brummelaar et al. (2005), *Astrophys. J.* **628**, 453

Budaj (2011), *Astron. Astrophys.* **532**, L12

Chrenko et al. (2017), *Astron. Astrophys.* **295**, 93

Masset (2000), *Astron. Astrophys. Suppl.* **141**, 165

Mourard et al. (submitted), *Astron. Astrophys.*

Rein & Spiegel (2015), *Mon. Not. R. Astron. Soc.* **446**, 1424