Guys, here is a first glimpse of what we may expect from a long-term evolution of Hungarias' spin axes. Just to set things explicit, I consider Hungaria itself as far as the orbit is concerned and the estimated size, $\simeq 13$ km ($H \simeq 11.14$ and I assume $p_V \simeq 0.4$). The simulation includes gravitational torque effect due to the Sun (with, obviously, Hungaria's orbit evolving due to planetary perturbations) and the YORP effect. Since I do not know anything about Hungaria's shape, I just picked one possible value of the YORP strength for a body of this size.

Now, YORP torque is not as "deterministic" as the gravitational torque since it depends on a particular shape of the body; this does not mean that knowing accurately the shape, we would not be able to estimate accurately YORP, just it means that given a possible variety of shapes we may have a variety of YORP results. In particular, YORP may asymptotically drive obliquity ϵ either to be perpendicular to the orbital plane ($\epsilon = 0^\circ$ and 180°) or to lie in the orbital plane ($\epsilon = 90^\circ$); well as a matter of fact the obliquity asymptotic value may also be in between these values, but it seems that happens for a minority of objects. At these end-states of the obliquity the rotation rate may be either accelerated or decelerated. We still seek a firm statistical base to say, which of these cases is most likely, especially when the finite surface thermal conductivity is taken into account. Right now we are in a position to say that the asymptotic obliquity values of 0° and 180° are more likely and the rotation rate may about equally accelerate or decelerate. However, things will evolve when we have faster computers available to scan in a more detail the parametric space.

Well, with this warning I show below the four "canonical" cases for Hungaria: (i) say the more likely cases in Figs. 1 and 2 when ϵ asymptotically tilts perpendicular to the orbital plane, and (ii) perhaps less likely cases in Figs. 3 and 4 when ϵ asymptotically tilts toward the orbital plane. I select integrations, where I assumed an initial rotation period of 5 hr and initial spin axis orientation isotropic in space (each figure shows 18 cases with equidistant step in $\cos\epsilon(0)$ measure).

General comments.– What is characteristic to all cases? Principally the large-amplitude and fast (kyrs to Myrs timescale) oscillations of ϵ due to the gravitational torque. Obviously, even in the space-fixed axis model the obliquity would change for a highly-inclined precessing orbit in space (Hungarias' inclinations are typically $\simeq 22^\circ$) – this is a simple geometric effect. But things get worse when you include the solar gravitational torque because the forcing terms due to the precessing orbit have a crowded spectrum near the proper mode and widths of the associated resonances are large because of high inclination. So even in the gravitational only model, Hungaria's spin axis would wander in a large chaotic zone extending from 0° to $\simeq 140^\circ$, say. Only for higher values it would be more regular. This is something people may know for some time.

Now YORP drives the obliquity evolution throughout this chaotic sea in some way. The chaotic zone may temporarily halt the "net YORP evolution", but eventually it wins and we obtain evolution toward the limiting state. Perhaps least clear is the situation on Fig. 3, but in other cases we recognise the YORP end-states. Interestingly, in the first two cases, Figs. 1 and 2, the evolution toward the $0°$ and $180°$ is not symmetric (in spite of the isotropic initial distribution of obliquity values), and we get more objects asymptotically near the $180°$ value. In the latter two cases, Figs. 3 and 4, we see evolution toward the in-plane direction of the axis. As expected, rotation periods go either small or large at a long term.

A more detailed look at selected runs.– Here I focus in a little more detail on some of the above described cases. See e.g. Fig. 5 where I replot one of the solutions from Fig. 1. In this case the initial rotation period was 5 hr and the initial obliquity was $\simeq 66°$. You see that after the initial wandering of the axis in a large range of obliquity values the evolution then goes along a line typical for the YORP effect (as if it were the single effect in the game). The initial phase is due to capture in librations zone of several overlapping resonances whose proper values of frequency are depicted in the middle plot as straight lines (they range between $\simeq (17-23)$ "/yr). Since the obliquity wanders up and down so wildly, the rotation period in that time is merely halted. But once the orbit escapes from resonance zone, the evolution is more regular. In particular because the rotation period decelerates so much, the precession period goes to zero and the spin axis basically "frozes" with the orbital plane (i.e. in a fixed way follows its evolution in space and aligns with its normal).

The evolution in Fig. 6, that corresponds to one solution in Fig. 2 (initial period 5 hr and obliquity $\simeq 25°$), looks similar, but there is one fundamental difference. After leaving the resonance zone the YORP drives now the rotation period to decrease. In the limit (not seen in this integration since it would take another few Gyrs), the precession period formally diverges and the axis would remain fixed in space (roughly perpendicular to the ecliptic). As such, the final obliquity shows the characteristic variations due to the simple geometric effect described above.

Note that in both examples the initial obliquity was smaller than $90°$, so that if only YORP would be affecting the evolution the asymptotic obliquity would be $0°$. Here both cases finally asymptotically flipped to $180°$, which is due to the resonance effects at the beginning.

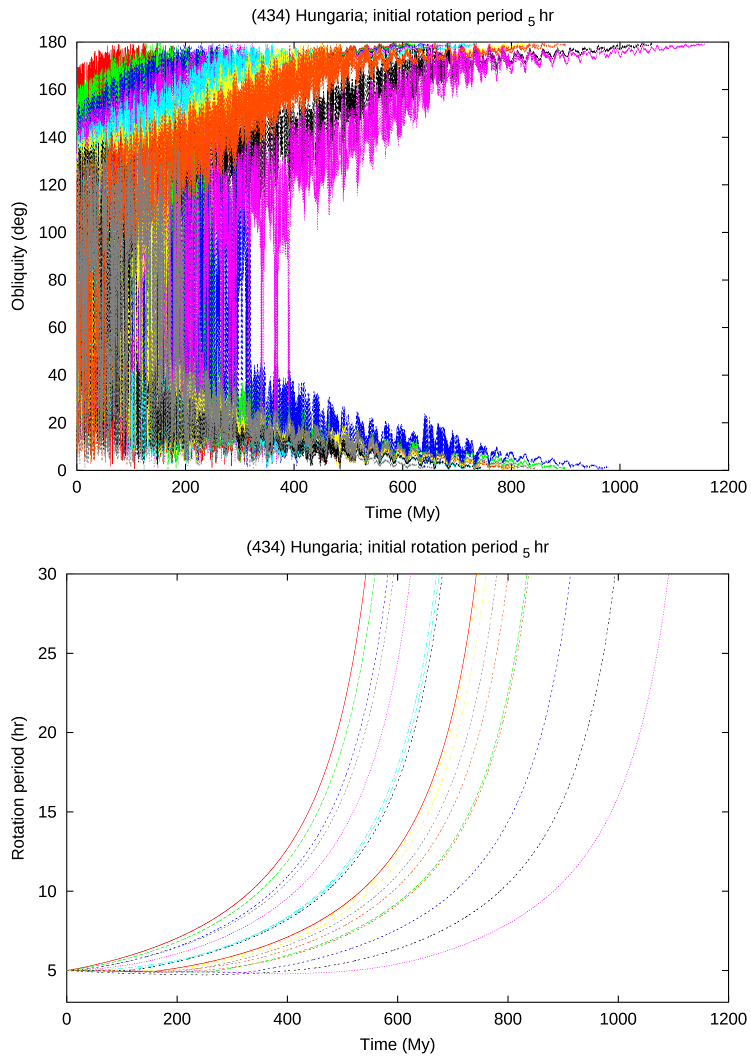

Figure 1: Numerical simulation of the spin state evolution of (434) Hungaria for 18 different values of the the initial obliquity (taken isotropic in space, i.e. equal steps in $\cos\epsilon(0)$). I assume 13 km size and initial rotation period of 5 hr. YORP is tuned "arbitrarily", so that I just took one of many possibilities in strength; asymptotically, this case corresponds to flipping the axis perpendicular to the orbital plane (hence end-state obliquities of 0° and 180°) with deceleration of the rotation rate.

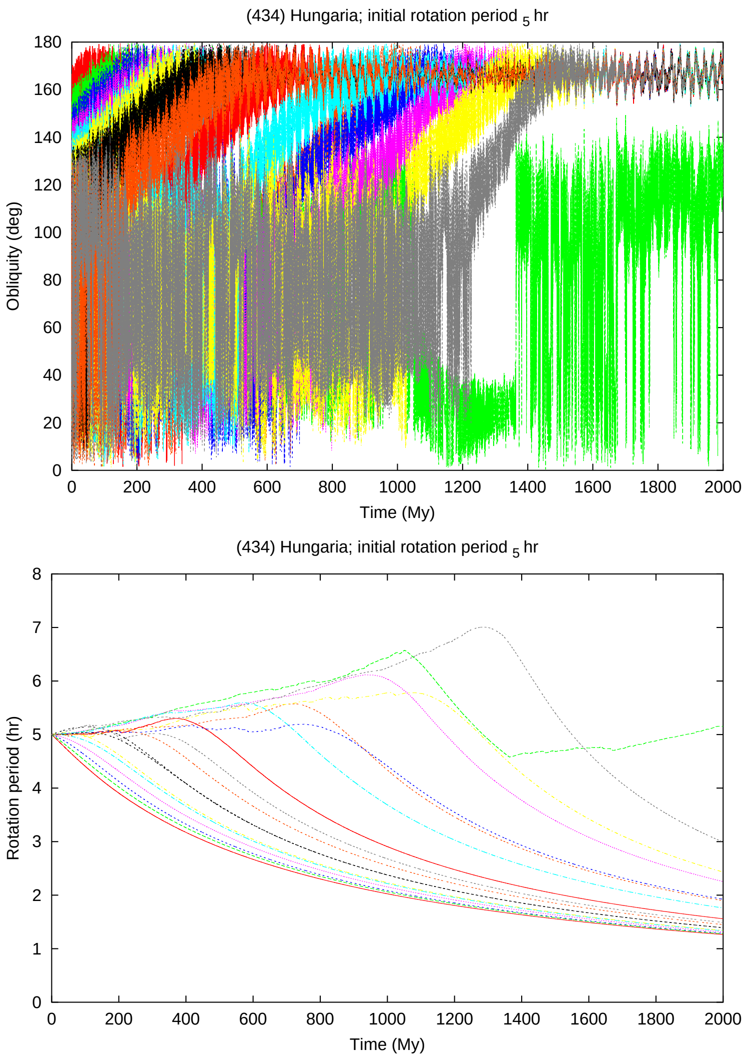

Figure 2: Numerical simulation of the spin state evolution of (434) Hungaria for 18 different values of the the initial obliquity (taken isotropic in space, i.e. equal steps in $\cos\epsilon$). I assume 13 km size and initial rotation period of 5 hr. YORP is tuned "arbitrarily", so that I just took one of many possibilities in strength; asymptotically, this case corresponds to flipping the axis perpendicular to the orbital plane (hence end-state obliquities of 0° and 180°) with acceleration of the rotation rate.

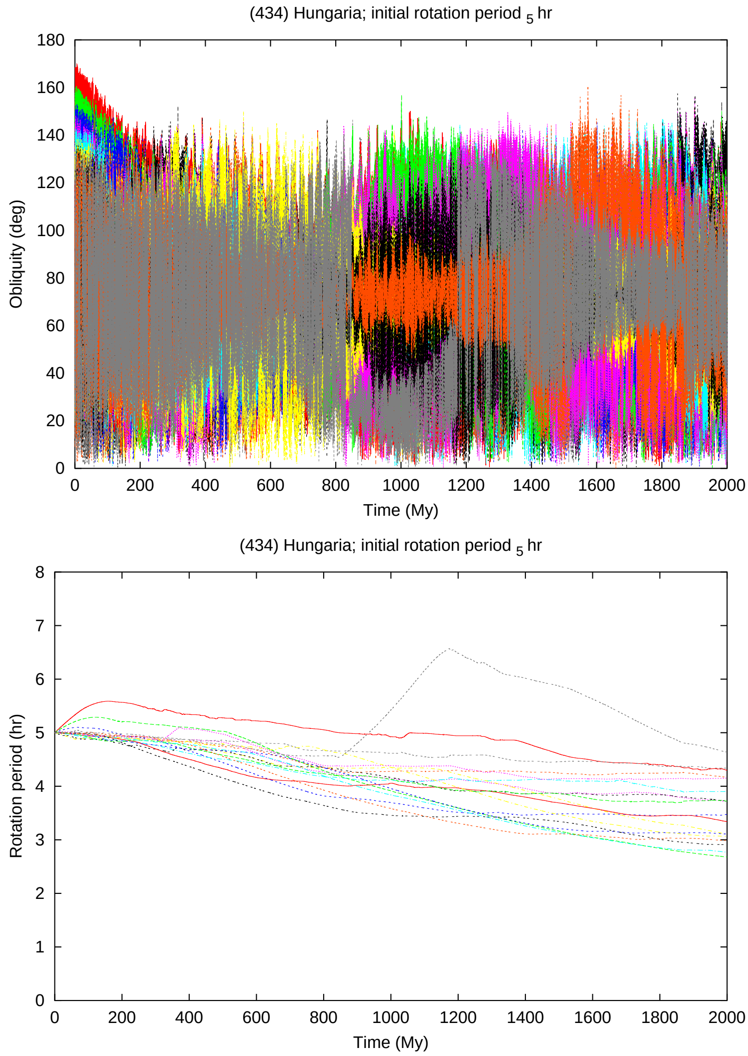

Figure 3: Numerical simulation of the spin state evolution of (434) Hungaria for 18 different values of the the initial obliquity (taken isotropic in space, i.e. equal steps in $\cos\epsilon$). I assume 13 km size and initial rotation period of 5 hr. YORP is tuned "arbitrarily", so that I just took one of many possibilities in strength; asymptotically, this case corresponds to flipping the axis to the orbital plane (hence end-state obliquities of 90°) with acceleration of the rotation rate.

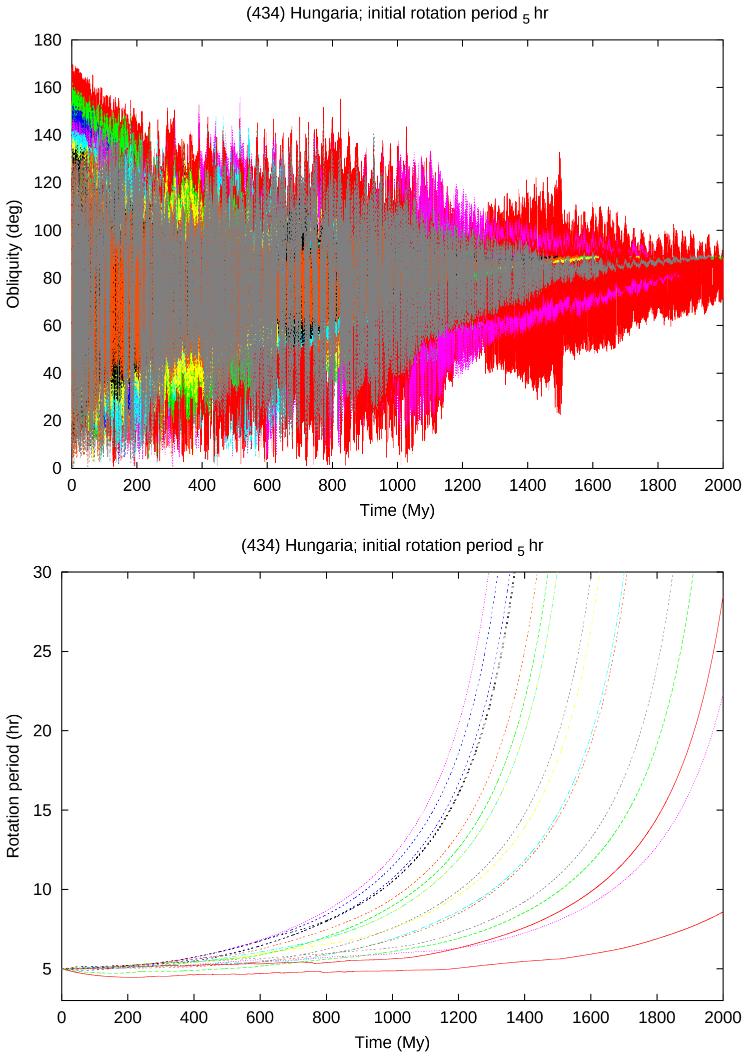

Figure 4: Numerical simulation of the spin state evolution of (434) Hungaria for 18 different values of the the initial obliquity (taken isotropic in space, i.e. equal steps in $\cos\epsilon$). I assume 13 km size and initial rotation period of 5 hr. YORP is tuned "arbitrarily", so that I just took one of many possibilities in strength; asymptotically, this case corresponds to flipping the axis to the orbital plane (hence end-state obliquities of 90°) with deceleration of the rotation rate.

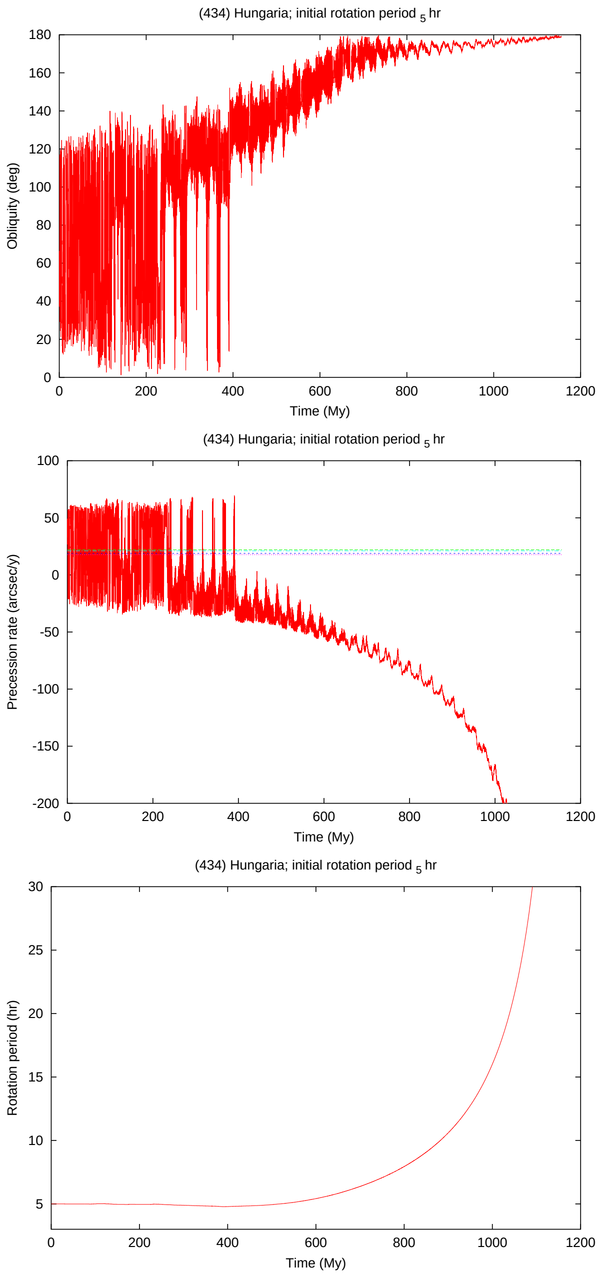

Figure 5: An example of spin axis evolution from Fig. 1; here the initial period is still $P(0) = 5$ hr and the initial obliquity is $\epsilon(0) \simeq 66°$. The middle panel gives now the value of precession rate of the spin axis.

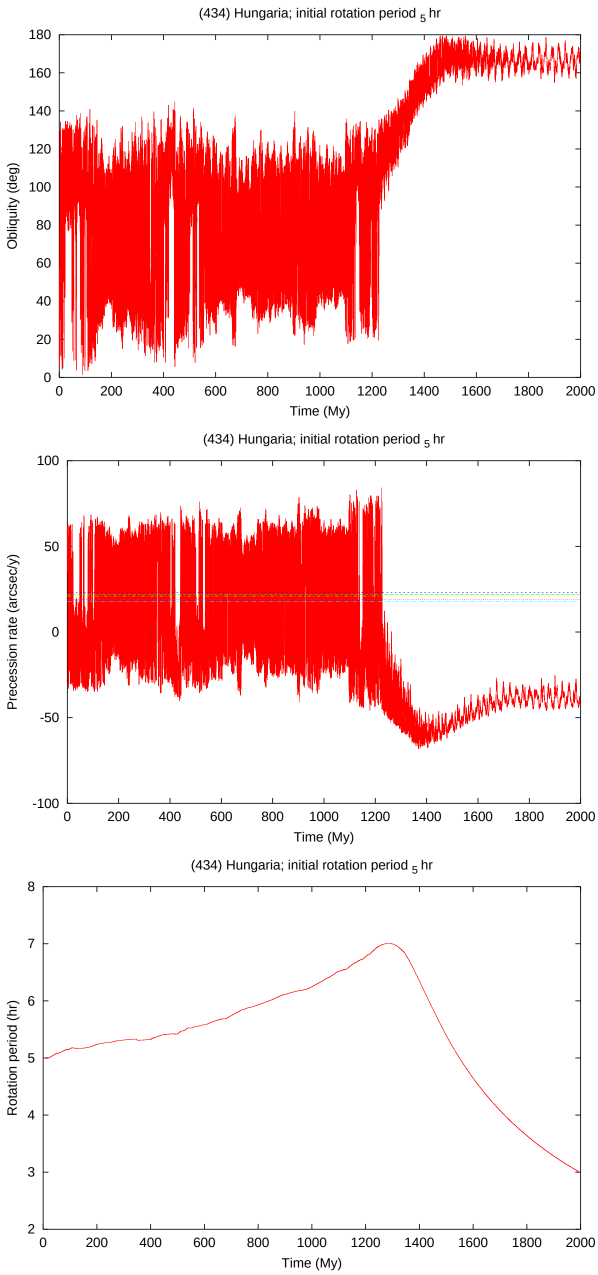

Figure 6: An example of spin axis evolution from Fig. 2; here the initial period is still $P(0) = 5$ hr and the initial obliquity is $\epsilon(0) \simeq 25°$.The middle panel gives now the value of precession rate of the spin axis.