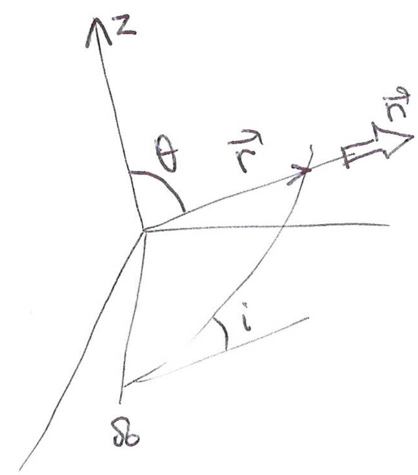
V kapitole o poruchovém poli jsme zjistili, že dlouhoperiodické, či sekulární, změny oskulačních elementů v prvním řádu malého parametru ε lze získat záměnou plné formy poruchového potenciálu R jeho střední hodnotou $\bar{R} = \frac{1}{2\pi}\int_{-\pi}^{\pi} dl\, R(l)$. V této části se věnujeme „technickému výpočtu" $\bar{R}$ pro první dva zonální členy geopotenciálu $n = 2$ a 3:

$$R_{zon} = \frac{\mu}{r}\sum_{n \geq 2}\left(\frac{R}{r}\right)^n J_n P_n(\cos\theta) \quad ; \quad \mu = GM$$

$$\boxed{\begin{aligned} R_2 &= \frac{\mu}{2r}\left(\frac{R}{r}\right)^2 J_2 \left(3\cos^2\theta - 1\right) \\ R_3 &= \frac{\mu}{2r}\left(\frac{R}{r}\right)^3 J_3 \cos\theta \left(5\cos^2\theta - 1\right) \end{aligned}}$$



$$\cos\theta = n_z = \cos f\, e_{Pz} + \sin f\, e_{Qz}$$

Při výpočtu $\bar{R}$ přejdeme od integrační proměnné $l \to f$: $\quad r^2 df = a^2 \eta\, dl \;\to\; dl = df\, \frac{1}{\eta}\left(\frac{r}{a}\right)^2$

▲ kvadrupolní část R_2:

$$\bar{R}_2 = \frac{1}{2\pi}\int_{-\pi}^{\pi} dl\, R_2 = \frac{\mu}{2a}\left(\frac{R}{a}\right)^2 J_2 \frac{1}{\eta}\,\frac{1}{2\pi}\int_{-\pi}^{\pi} df \cancel{\left(\frac{r}{a}\right)^2}\cdot\left(\frac{a}{r}\right)\cancel{\left(\frac{a}{r}\right)^2}\left(3\cos^2\theta - 1\right) =$$

$$= \frac{\mu}{2a}\left(\frac{R}{a}\right)^2 \frac{J_2}{\eta^3}\,\frac{1}{2\pi}\int_{-\pi}^{\pi} df\,(1 + e\cos f)\left[3\left(\cos^2 f\, e_{Pz}^2 + \sin^2 f\, e_{Qz}^2 + 2\sin f\cos f\, e_{Pz}e_{Qz}\right) - 1\right]$$

$\longrightarrow$ dá vždy po středování 0

$$= \frac{\mu}{2a}\left(\frac{R}{a}\right)^2 \frac{J_2}{\eta^3}\left[\frac{3}{2}\left(e_{Pz}^2 + e_{Qz}^2\right) - 1\right]$$

ozn. $\vec{c}$ .. normálový vektor k dráze

pak $\underline{e_{Pz}^2 + e_{Qz}^2 + c_z^2 = 1}$ neboť jde o jednotkový vektor $\vec{e}_z$ projektovaný na ortonormální bázi $(\vec{e}_P, \vec{e}_Q, \vec{c})$

$$\bar{R}_2 = \frac{\mu}{2a}\left(\frac{R}{a}\right)^2 \frac{J_2}{\eta^3}\left[\frac{3}{2}\left(1-\underbrace{c_z^2}_{\cos^2 i}\right)-1\right] = -\frac{\mu}{4a}\left(\frac{R}{a}\right)^2\frac{J_2}{\eta^3}\left(2-3\sin^2 i\right)$$

$$= \boxed{\frac{\mu}{4a}\left(\frac{R}{a}\right)^2\frac{J_2}{\eta^3}\left(1-3\cos^2 i\right)}$$

▲ oktupólní část R_3:

$$\bar{R}_3 = \frac{1}{2\pi}\int_{-\pi}^{\pi} dl\, R_3 = \frac{\mu}{2a}\left(\frac{R}{a}\right)^3\frac{J_3}{\eta}\,\frac{1}{2\pi}\int_{-\pi}^{\pi} df\, \cancel{\left(\frac{r}{a}\right)^2}\left(\frac{a}{r}\right)\left(\frac{a}{r}\right)^{\cancel{3}}\left(5\cos^3\theta - 3\cos\theta\right)$$

$$= \frac{\mu}{2a}\left(\frac{R}{a}\right)^3\frac{J_3}{\eta^5}\,\frac{1}{2\pi}\int_{-\pi}^{\pi} df\,(1+e\cos f)^2\Big[5\left(\cos f\, e_{Pz}+\sin f\, e_{Qz}\right)^3 - 3\left(\cos f\, e_{Pz}+\sin f\, e_{Qz}\right)\Big]$$

$$= \frac{\mu}{2a}\left(\frac{R}{a}\right)^3\frac{J_3}{\eta^5}\,\frac{1}{2\pi}\int_{-\pi}^{\pi} df\,\overbrace{(1+e\cos f)^2}^{\text{sudá fce } f\to -f}\Big[5\cos^3 f\, e_{Pz}^3 + \cancel{15\cos^2 f\sin f\, e_{Pz}^2 e_{Qz}} + 15\cos f\sin^2 f\, e_{Pz} e_{Qz}^2 + \cancel{5\sin^3 f\, e_{Qz}^3} - 3\cos f\, e_{Pz} - \cancel{3\sin f\, e_{Qz}}\Big]$$

jsou liché $f \to -f$

$$= \frac{\mu}{2a}\left(\frac{R}{a}\right)^3\frac{J_3}{\eta^5}\,\frac{1}{2\pi}\int_{-\pi}^{\pi} df\,\left(\cancel{1}+2e\cos f+\cancel{e^2\cos^2 f}\right)\cdot\left(5\cos^3 f\, e_{Pz}^2 + 15\cos f\, e_{Pz}e_{Qz}^2 - 15\cos^3 f\, e_{Pz}e_{Qz}^2 - 3\cos f\, e_{Pz}\right)$$

nulové dávají zdvojce jen liché mocniny $\cos f$

$$= \frac{\mu}{a}\left(\frac{R}{a}\right)^3\frac{J_3 e}{\eta^5}\,\frac{1}{2\pi}\int_{-\pi}^{\pi} df\,\left(5\cos^4 f\, e_{Pz}^3 + 15\cos^2 f\, e_{Pz}e_{Qz}^2 - 15\cos^4 f\, e_{Pz}e_{Qz}^2 - 3\cos^2 f\, e_{Pz}\right)$$

průměrně $\langle \cos^2 f \rangle = \frac{1}{2}$

$\langle \cos^4 f \rangle = \frac{3}{8}$

$$= \frac{\mu}{a}\left(\frac{R}{a}\right)^3 \frac{J_3 e}{\eta^5}\left[\frac{15}{8} e_{pz}^3 + \frac{15}{2} e_{pz} e_{Qz}^2 - \frac{45}{8} e_{pz} e_{Qz}^2 - \frac{3}{2} e_{pz}\right] =$$

$$= \frac{3}{8}\frac{\mu}{a}\left(\frac{R}{a}\right)^2 \frac{J_3 e}{\eta^5} e_{pz}\left[5 e_{pz}^2 + 20 e_{Qz}^2 - 15 e_{Qz}^2 - 4\right] =$$

$$= \frac{3}{8}\frac{\mu}{a}\left(\frac{R}{a}\right)^3 \frac{J_3 e}{\eta^5} e_{pz}\left[5\left(e_{pz}^2 + e_{Qz}^2\right) - 4\right]$$

$1 - c_z^2 = \sin^2 i$

$e_{pz} = \sin i \sin\omega$

$$= \frac{3}{8}\frac{\mu}{a}\left(\frac{R}{a}\right)^3 J_3 \frac{e \sin\omega}{\eta^5} \sin i \left(5\sin^2 i - 4\right)$$

$$= \frac{3}{8}\frac{\mu}{a}\left(\frac{R}{a}\right)^3 J_3 \frac{e \sin\omega}{\eta^5} \sin i \left(1 - 5\cos^2 i\right)$$

$(= \bar{R}_3)$