Lagrangián tedy nabývá tvaru

$$\boxed{L = \frac{1}{2} m_2' \dot{\vec{R}}\cdot\dot{\vec{R}} + \frac{m_2(m_0+m_1)}{R} + X_0 X_1 (m_0+m_1)\left\{ \frac{1}{2}\dot{\vec{r}}\cdot\dot{\vec{r}} + \frac{G(m_0+m_1)}{r} + \frac{1}{2}\frac{Gm_2}{R}\left(\frac{r}{R}\right)^2(3\cos^2\gamma-1) + \frac{1}{2}\ \ldots \right\}} \quad (*)$$

$\vec{r}(t)$ & $\vec{R}(t)$ kuplovány

druhý člen v (*) bude malou poruchou, když

$$\varepsilon \sim \frac{m_2}{m_0+m_1}\left(\frac{r}{R}\right)^3 \ll 1$$

(pro Slunce-Zemi-Měsíc: $\varepsilon \approx 332000\cdot\left(\frac{1}{390}\right)^3 \approx \frac{1}{200}$)

kdežto tentýž člen představuje poruchu řádu

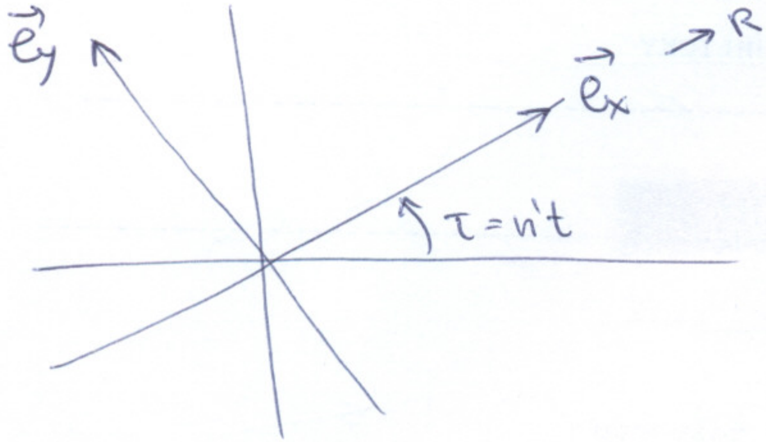
$$\approx X_0 X_1 \left(\frac{r}{R}\right)^2 \approx \frac{1}{82}\left(\frac{1}{390}\right)^2 \approx \frac{1}{10^7}\ !$$

tj. zanedbatelný efekt; lze tedy přijmout aproximaci kdy $\vec{R}(t)$ keplerovské řešení (elipsa či kružnice)

a
$$L' = \frac{1}{2}\dot{\vec{r}}\cdot\dot{\vec{r}} + \frac{G(m_0+m_1)}{r} + \frac{1}{2}\frac{Gm_2}{R}\left(\frac{r}{R}\right)^2(3\cos^2\gamma-1) + \ldots$$

kde $\vec{R}(t)$ (nyní opisuje O)

Zavedeme nyní rotující systém, že $\vec{e}_x$ ve směru $\vec{R}$ (HS



$$\dot{\vec{e}}_x = n'\vec{e}_y$$

$$\dot{\vec{e}}_y = -n'\vec{e}_x$$

$$n'^2 R^3 = G(m_0 + m_1 + m_2)$$

$$\vec{r} \equiv X\vec{e}_x + Y\vec{e}_y \left(+ Z\vec{e}_z\right)$$

$$\dot{\vec{r}} = \dot{X}\vec{e}_x + \dot{Y}\vec{e}_y + \dot{Z}\vec{e}_z + n'X\vec{e}_y - n'Y\vec{e}_x$$

$$= (\dot{X} - n'Y)\vec{e}_x + (\dot{Y} + n'X)\vec{e}_y + \dot{Z}\vec{e}_z$$

$$\dot{\vec{r}}^2 = \dot{X}^2 + \dot{Y}^2 + \dot{Z}^2 + \tfrac{1}{2}n'^2(X^2+Y^2) + 2n'(X\dot{Y} - Y\dot{X})$$

$$\frac{1}{2}\frac{Gm_2}{R^3}\left(3(\vec{r}\cdot\vec{N})^2 - r^2\right) \approx \tfrac{1}{2}n'^2\left[3X^2 - X^2 - Y^2 - Z^2\right]$$

$$\approx \tfrac{1}{2}n'^2\left[2X^2 - Y^2 - Z^2\right]$$

$$L' = \tfrac{1}{2}(\dot{X}^2 + \dot{Y}^2 + \dot{Z}^2) + \frac{G(m_0+m_1)}{r} + 2n'(X\dot{Y} - Y\dot{X})$$

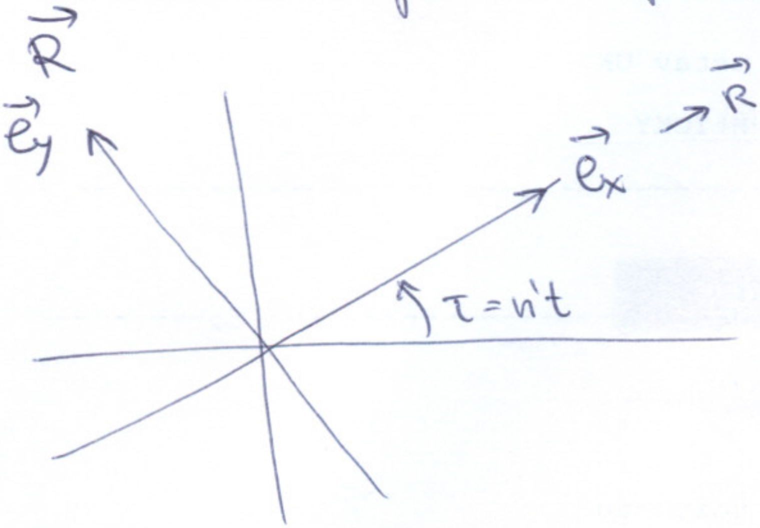
$$+ \tfrac{1}{2}n'^2(X^2 + \cancel{Y^2}) + \tfrac{1}{2}n'^2(2X^2 - \cancel{Y^2} - Z^2)$$

$$\boxed{= \tfrac{1}{2}(\dot{X}^2 + \dot{Y}^2 + \dot{Z}^2) + n'(X\dot{Y} - Y\dot{X}) + \frac{G(m_0+m_1)}{r} + \tfrac{1}{2}n'^2(3X^2 - Z^2)}$$

$$r^2 = X^2 + Y^2 + Z^2$$

nyní zanedbáme pohyb v Z

Zavedeme nyní rotující systém, že $\vec{e}_x$ ve směru (HS



$$\dot{\vec{e}}_x = n'\vec{e}_y$$

$$\dot{\vec{e}}_y = -n'\vec{e}_x$$

$$\underline{n'^2 R^3 = G(m_0+m_1+m_2)}$$

$$\vec{r} \equiv x\vec{e}_x + y\vec{e}_y \left(+ z\vec{e}_z\right)$$

$$\dot{\vec{r}} = \dot{x}\vec{e}_x + \dot{y}\vec{e}_y + \dot{z}\vec{e}_z + n'x\vec{e}_y - n'y\vec{e}_x$$

$$= \underline{(\dot{x}-n'y)\vec{e}_x + (\dot{y}+n'x)\vec{e}_y + \dot{z}\vec{e}_z}$$

$$\dot{\vec{r}}^2 = \dot{x}^2+\dot{y}^2+\dot{z}^2 + \tfrac{1}{2}n'^2(x^2+y^2) + 2n'(x\dot{y}-y\dot{x})$$

$$\frac{1}{2}\frac{Gm_2}{R^3}\left(3(\vec{r}\cdot\vec{N})^2 - r^2\right) \approx \tfrac{1}{2}n'^2\left[3x^2-x^2-y^2-z^2\right]$$

$$\approx \underline{\tfrac{1}{2}n'^2\left[2x^2-y^2-z^2\right]}$$

$$L' = \tfrac{1}{2}(\dot{x}^2+\dot{y}^2+\dot{z}^2) + \frac{G(m_0+m_1)}{r} + 2n'(x\dot{y}-y\dot{x})$$

$$+ \tfrac{1}{2}n'^2(x^2+\cancel{y^2}) + \tfrac{1}{2}n'^2(2x^2-\cancel{y^2}-z^2)$$

$$\boxed{= \tfrac{1}{2}(\dot{x}^2+\dot{y}^2+\dot{z}^2) + n'(x\dot{y}-y\dot{x}) + \frac{G(m_0+m_1)}{r} + \tfrac{1}{2}n'^2(3x^2-z^2)}$$

$r^2 = x^2+y^2+z^2$

nyní zanedbáme pohyb v $\underline{z}$

Tento Lagrangián dává rnice

$$\begin{aligned} \ddot{X} - 2n'\dot{Y} &= \frac{\partial F(X,Y)}{\partial X} \\ \ddot{Y} + 2n'\dot{X} &= \frac{\partial F(X,Y)}{\partial Y} \end{aligned}$$

$$F(X,Y) = \frac{G(m_0+m_1)}{r} + \frac{3}{2}n'^2X^2$$

$$\boxed{\frac{1}{2}V^2 - F(X,Y) = C}$$

Princip ekvivalence a pohyb G (v Hillově aprox.) (H7

$$\overset{\bullet}{i} \longleftrightarrow \overset{\bullet}{j} \qquad G_{ij} \frac{m_i m_j}{\Delta_{ij}}$$

$$G_{ij} = G\,(1 + \delta_i + \delta_j)$$

odpovídá $\boxed{m_i^g = m_i\,(1 + \delta_i)}$

Tj. ze vzh. H3)

$$\frac{G_{02}\, m_0 m_2}{\Delta_{02}} + \frac{G_{12}\, m_1 m_2}{\Delta_{12}} =$$

$$= \underbrace{G\,(1+\delta_2)}_{G_2}\, m_2\, (m_0 + m_1) \left[\frac{X_0 (1+\delta_0)}{\Delta_{02}} + \frac{X_1 (1+\delta_1)}{\Delta_{12}} \right]$$

$$= G_2\, \frac{m_2 (m_0 + m_1)}{R} \left\{ 1 + \cancel{X_0 \delta_0 + X_1 \delta_1} + X_0 X_1 (\delta_1 - \delta_0) \frac{r}{R} \cos\gamma + \dots \right\}$$

$\Rightarrow$ extra potenciál:

$$\Delta V = X_0 X_1 (m_0 + m_1) \cdot \left\{ G m_2 \frac{\delta_1 - \delta_0}{R^2}\, r \cos\gamma \right\}$$

$$\boxed{\delta L' = \frac{G m_2}{R^3} (\delta_1 - \delta_0)(\vec{r} \cdot \vec{R})} =$$

$$\doteq n'^2 (\delta_1 - \delta_0)\, R\, X$$ v souřadnicích synodického systému.

Princip ekvivalence a pohyb G (v Hillově aprox.) (H7

$$i \longleftrightarrow j \qquad G_{ij}\frac{m_i m_j}{\Delta_{ij}}$$

$$G_{ij} = G(1+\delta_i+\delta_j)$$

odpovídá $\boxed{m_i^g = m_i(1+\delta_i)}$

Tj. ze str. H3)

$$\frac{G_{02}m_0m_2}{\Delta_{02}} + \frac{G_{12}m_1m_2}{\Delta_{12}} =$$

$$= \underbrace{G(1+\delta_2)}_{G_2} m_2(m_0+m_1)\left[\frac{X_0(1+\delta_0)}{\Delta_{02}} + \frac{X_1(1+\delta_1)}{\Delta_{12}}\right]$$

$$= G_2\frac{m_2(m_0+m_1)}{R}\left\{1+\cancel{X_0\delta_0+X_1\delta_1} + X_0X_1(\delta_1-\delta_0)\frac{r}{R}\cos\gamma+\dots\right\}$$

$\Rightarrow$ extra potenciál:

$$\Delta V = X_0X_1(m_0+m_1)\cdot\left\{Gm_2\frac{\delta_1-\delta_0}{R^2}r\cos\gamma\right\}$$

$$\boxed{\delta L' = \frac{Gm_2}{R^3}(\delta_1-\delta_0)(\vec{r}\cdot\vec{R})} =$$

$$\doteq n'^2(\delta_1-\delta_0)RX$$ v souřadnicích synodického systému.