

of the existence of the other type only by deduction from the indications of our external senses.

Objection is sometimes raised to the extravagantly important part taken by light-signals and light-propagation in Einstein's discussion of space and time. But Einstein did not invent a space and time depending on light-signals; he pointed out that the space and time already in general use depended on light-signals and equivalent processes, and proceeded to show the consequences of this. Turning from fictitious space and time to the absolute four-dimensional world, we still find the velocity of light playing a very prominent part. It is scarcely necessary to offer any excuse for this. Whether the substratum of phenomena is called *aether* or *world* or *space-time*, one requirement of its structure is that it should propagate light with this velocity.

The resolution of the four-dimensional continuum into a succession of instantaneous spaces is not dictated by anything in the structure of the continuum. Nevertheless, it is convenient, and corresponds approximately to our practical outlook on the world; and it is rarely necessary to go back to the undivided world. We have to go back to the undivided world when a comparison is made between the phenomena experienced by observers with different motions, who make the resolution in different directions. Moreover, a world-wide resolution into a space and time with the familiar properties is possible only when the continuum satisfies certain conditions. Are these conditions rigorously satisfied? They are not; that is Einstein's second great discovery. It is no more possible to divide the universe in this way than to divide the whole sky into squares. We have

tried to make the division, and it has failed; and to cover up the consequences of the failure we have introduced an almost supernatural agency—gravitation. When we cease to strive after this impossibility—a mode of division which there was never any adequate reason for believing to be possible—gravitation as a separate agency becomes unnecessary. Our concern here is with the bearing of this result on time. Time is now not merely relative, but local. The relative time for an observer is a construction extended by astronomers throughout the universe according to mathematical rules; but these rules break down in a region disturbed by the proximity of heavy matter, and cannot be fulfilled accurately. We can preserve our time-partitions only by making up fresh rules as we require them. The local time for a particular observer is always definite, and is the physical representation of the flight of instants of which he is immediately aware; the extended mesh-work of co-ordinates radiating from this is drawn so as to conform roughly to certain rules—so as not to violate too grossly certain requirements which the untutored mind thought necessary at one time. Subject to this, time is merely one of four co-ordinates, and its exact definition is arbitrary.

To sum up, world-wide time is a mathematical system of location of events according to rules which on examination can only be regarded as arbitrary; it has not any structural—and still less any metaphysical—significance. Local time, which for animate beings corresponds to the immediate time-sense, is a type of linear succession of events distinct from a pure spacelike succession; and this distinction is fully recognised in the relativity theory of the world.

### Theory and Experiment in Relativity.<sup>1</sup>

By DR. NORMAN CAMPBELL.

"SPACE" and "time" are the conceptions of theory, not of laws. They are neither necessary nor useful in the statement of the results of any experiment. The experimental concepts with which, like all theoretical ideas, they are connected are such magnitudes as length, area, volume, angle, period (of a system), or time-interval. The numerical laws of experimental geometry involve two or more "spatial" magnitudes and no other magnitudes; for example, the area of a rectangle is proportional to the product of the lengths of its sides. There are no laws relating "temporal" magnitudes only.

Relativity neither adds to nor subtracts from the collection of spatial and temporal laws. The laws which it explains all involve magnitudes that are not spatial or temporal. And this is fortunate. For the subject has been so completely

examined that it is very improbable that any proposed new laws could be true. If relativity predicted anything inconsistent with firmly established experiment, NATURE would not devote a special number to discussing it.

It may be objected that relativity does predict new and strange laws; it predicts that the velocity of light in a region remote from material bodies is always the same; and it predicts unfamiliar experiences of observers travelling at great speeds or in the neighbourhood of concentrated mass. But, it may be replied, the measurement of the velocity of light does not involve only spatial and temporal magnitudes; we do not measure that velocity as we do the velocity of a material body; an element of theory is always involved. Again, we do *not* observe any disturbance of geometrical laws in the neighbourhood of the densest bodies we know. And as for Prof. Eddington's observers in aeroplanes travelling with half the velocity of light, no two human

<sup>1</sup> Since it is impossible to make a short article on a large subject anything but a summary, perhaps I may be permitted to refer any reader who is interested to my "Physics: The Elements" for a fuller discussion of many of the questions raised.

beings have smoked, or, if the doctrines of relativity are true, ever will smoke, cigars—let alone make accurate measurements—in such aeroplanes, and afterwards compared their experiences. If we pretend to talk about experiments, let us be sure that we do talk about experiments, and not about something that cannot possibly happen.

However, it may not be useless to ask what would happen if we did find our spatial laws untrue, in the manner suggested, at speeds that can be realised. I suggest that we should make our laws true once more by changing slightly the meaning of the terms in them. The technical terms of science are labels attached to collections of observations that can be grouped into laws, which those terms are used to describe. If we find that the supposed laws are not true, the terms become meaningless; we might abandon them altogether; but generally we discover that, by a slight regrouping of the facts according to the new laws, we can make once more a collection of the facts to which the old term may be applied appropriately to state the new laws in almost precisely the old form.

Consider, for example, the term "simultaneous." Primarily, two events were judged to be simultaneous by direct perception. Using this test and examining a limited range of experience, we found the law that events that are simultaneous to one observer are simultaneous to another. But later we found that the law was not valid for more extended experience, including the sound and flash from a gun. That discovery made "simultaneous" meaningless, and with it all the temporal magnitudes; there was no longer any way of assigning uniquely a numeral to represent the time-interval between two events. So we changed the meaning of "simultaneous," and introduced a "correction" (very complicated, as sound-rangers know); by this means we made "simultaneous" once more the expression of a law, and reproduced our specifications for measuring time-intervals in exactly the old form, but, of course, with rather different content. If we encountered new difficulties when we extended our observations to events in systems moving with great relative speeds, we could, I think, introduce a new "correction" for speed, and reproduce once more the form of our old laws and our old methods of measurement. At any rate, the resulting change of form need not be so great as to cause any appearance of paradox.

I conclude, therefore, that nothing that the most extravagant imagination has suggested so far could make us diverge appreciably from our present spatial and temporal laws. But it is otherwise with our theories. The experimental physicist has a theory of time and space, although he may not be conscious of it. It is based on Cartesian geometry.<sup>2</sup> It likens "space" to an array of black dots in a cubical lattice, and

<sup>2</sup> It is interesting to notice that, though the theory is sometimes called Euclidean, Euclid had never heard of it. No Greek geometer would have known what you meant if you had told him that space was three-dimensional.

"time" to a series of ticks from a metronome. It connects the position of a body with the individual characteristics of the dots that it "occupies," and the magnitudes length, area, volume with the number of those dots. The time of an event it connects with the individual characteristics of the ticks. The theory explains well some spatial laws, but in some directions it is misleading. Thus it fails to make a distinction between lengths and areas, which (in the last resort) must be measured by the superposition of rigid bodies,<sup>3</sup> and volumes, which cannot be measured by such superposition. It should be noted that the dots and ticks, the "points" and "instants" of mathematically minded philosophers, are purely theoretical ideas. They have no meaning apart from the theory, and, like the position of a hydrogen molecule, cannot be determined by experiment.

Prof. Einstein has altered and expanded this theory. In conjunction with Minkowski, he has altered it by merging the dots and ticks, formerly independent, into a single array of world-points, and by making the arrangement of these points quite different from that of a cubic (or Euclidean) lattice. He has expanded it by introducing the idea of the "natural path" of a body among the points, which enables him to explain the laws of dynamics without the (theoretical) idea of forces. But his propositions still form a theory, and they still contain purely theoretical ideas, which cannot be determined by experiment—the world-point or the infinitesimal "interval," which must be integrated before it can be related to measured magnitudes.

These changes are very disturbing to the experimenter. He wants theories to explain laws. Explanation involves not only the possibility of deducing the laws (for that is easily attained), but also the introduction of satisfactory ideas. In the older types of physical theory this "satisfactoriness" was obtained by means of an analogy between the ideas of the theory and the concepts of some experimental laws. Thus in the older theory of space the points were related in a way analogous to that in which small material bodies can be related. In the new theory this analogy fails. For the mathematician the passage from flat three-dimensional space to curved four-dimensional space is trivial; for the experimenter it is vital, because we do not actually experience any arrangements at all analogous to those of points in such a space. The satisfactoriness of the theory, for those who press it on our attention, is derived, not from material analogy, but from the intrinsic elegance and beauty of the relations involved, the faculty for appreciating which distinguishes the pure mathematician from his fellows.

It is not surprising, therefore, that experimenters have found difficulty in accepting the theory as an ultimate solution of their problem. The old theories explained, because they inter-

<sup>3</sup> "Rigid bodies" is a label attached to a collection of facts grouped in laws, the laws that make possible the measurement of lengths and areas.

preted in terms of familiar ideas; even to the most revolutionary of mankind, familiarity is a source of some satisfaction. The new theory is based on ideas utterly unfamiliar, and it might be urged that anything based on them must be the precise contrary to explanation. But if we ask why we are so ready to accept theories based on material analogy, we shall find our reason in the fact that such theories have actually turned out to possess the amazing property of predicting unsuspected laws. The theory of relativity also possesses that property. Ought we not to extend, so as to include it, our notions of the proper limits of physical theory, and to rid ourselves of the discomfort of unfamiliarity by the simple process of studying its ideas so closely that they become an integral part of our mental equipment?

It may be asked, Do theories, indeed, aim at nothing but satisfactoriness and prediction? Is

not their object rather to discover the true nature of the real world? Such questions must be answered by questions. Do physicists (I say nothing of mathematicians or philosophers) believe that anything is real for any reason except that it is a conception of a true law or of a true theory? Have we any reason to assert that molecules are real except that the molecular theory is true—true in the sense of predicting rightly and interpreting its predictions in terms of acceptable ideas? What reason have we ever had for saying that thunder and lightning really happen at the same time, except that the conception of simultaneity which is such that this statement is true makes it possible to measure time-intervals? When these questions are answered it will be time to discuss whether relativity tells us anything about real time and real space.

### The Relation between Geometry and Einstein's Theory of Gravitation.

By DOROTHY WRINCH and DR. HAROLD JEFFREYS.

THE term "geometry" has been used ever since the time of Euclid to denote two completely distinct subjects; but the formal similarity of their propositions has been so close as to obscure until recently the entire dissimilarity of their status in scientific knowledge. The Greek geometers seem to have been inspired originally by the need for a satisfactory method of surveying; at the same time, their logical turn of mind led them to present their results in the now familiar form of a deductive science. The characteristics of such a science are that a certain number of primitive propositions  $p_1$ , now called postulates, are stated at the beginning, and that from these, by a process of pure logic, further propositions  $q_1$  are one by one developed. But this development is quite a separate process from that of deciding whether the primitive propositions are true or not, and if this is not done it is impossible to assert that the deduced propositions are true.

Different sets of primitive propositions  $p_2, p_3, \dots$  would give different sets of deduced propositions  $q_2, q_3, \dots$  and the complete working out of these is a science in itself; its results are all, therefore, of the form " $p_1$  implies  $q_1$ ," " $p_2$  implies  $q_2$ ," and so on. Euclid actually used in his development several postulates which he never explicitly stated, but which have been made explicit by modern writers; our present object, however, is not to indicate these, but to consider his geometry in the perfectly deductive form it would have had if he had actually stated them. We have noticed that in any other system in which any one of Euclid's postulates is false, many of his deduced propositions are also false. This, however, does not affect his method in the least; all his arguments are independent of the truth of the postulates, and in every case it is possible

to assert—and this is all the modern geometer asserts—that if the postulates are true the propositions are true. A system like Euclid's is, therefore, a part of pure logic; the large division of pure logic that includes it as a very special case is pure geometry. Of the many systems of pure geometry now known, all are on just the same footing, and there is no sense in which any one of them is preferable to any other.

Euclid's contemporaries, however, were not interested merely in his logical method; they wished to identify the furrows in their fields with his lines, and the fields themselves with his surfaces; and to have some justification for this it was necessary to assume that his postulates were true of them. Only one example is needed to show how formidable an assumption this was. In order to prove one of his earliest propositions, Euclid assumes that a triangle can be picked up, transported bodily, and deposited on top of another. Imagine this process carried out when the triangles are fields! The impossibility of carrying it out implies that a most important proposition was not proved for the very case to which they contemplated applying his geometry, and hence that, so far as the knowledge of that day went, there was not the slightest reason for believing that geometry was applicable for its original purpose of earth-measurement. Yet its results, in so far as they were capable of being applied in actual surveying, seem to have been instantly accepted. Why? It may have been due partly to lack of disposition to criticise something that the critics felt they could not have done better themselves, a mental attitude that may perhaps still occasionally exist; but the chief reason was probably that some of the deduced propositions were directly verifiable, such as the proposition that the equality of corresponding