placements, and to investigations of the characteristic behaviour of spectrum lines, as all such data will have a part in solving one of the most absorbing questions in cosmic physics.

Evershed adduces his observations upon the spectrum of Venus as evidence of an "earth-effect" driving the gases from the earth-facing hemisphere of the sun, and he would by this hypothetical action explain the observed displacements of the solar lines, and thus negative the deduction from the Einstein theory. Two series of Venus observations have been made by Dr. S. B. Nicholson and myself. The details will appear in a forthcoming Contribution from the Mount Wilson Observatory. Our observations indicate that the displacements of the Venus lines to the violet relative to skylight are due to non-uniform illumination of the slit when the guiding is done upon the visual image, the effect increasing with the refraction and becoming more evident the smaller the image. The explanation is based upon the observation that spectrograms taken at low altitudes give larger displacements to the violet than those taken on the same night at higher altitudes, and that the displacements correlate with the cotangent of the altitude and the reciprocal of the diameter of the planet at the time of observation.

In respect to the observations at Mount Wilson on the lines of the cyanogen band at $\lambda 3883$, I have as yet found no grounds for considering them seriously in error. The explanation of the results adverse to the theory based upon dissymmetry appears inadequate (*Observatory*, p. 260, July, 1920), and the assumption that the adverse results are due to superposed metallic lines seems to be negatived by the observations of Adams, Grebe, Bachem, and myself that for these lines there is no displacement between the centre and limb of the sun. Metallic lines as a class shift to the red in passing from the centre to the limb. If, then, metallic lines are superposed on these band lines in such a way as to mask the gravitational displacement to the red when observed at the centre of the sun, this should be revealed by a shift to the red at the limb.

The lines of the cyanogen bands are under investigation in the observatory laboratory both as reversed in the furnace and as produced in the arc under varying pressure. The measures show no evidence of a displacement to the red under decreased pressure as indicated by Perot's observations.

The present programme at Mount Wilson aims at an accumulation of varied and extensive data that will furnish a suitable basis from which to approach the general question of the behaviour of Fraunhofer lines relative to terrestrial sources.

Non-Euclidean Geometries.

By PROF. G. B. MATHEWS, F.R.S.

THE ordinary theory of analytical geometry may be extended by analogy as follows: Let $x_1, x_2, \ldots x_n$ be independent variables, each ranging over the complete real (or ordinary complex) continuum. Any particular set $(x_1, x_2, \ldots x_n)$, in that order, is said to be a point, the co-ordinates of which are these x_i; and the aggregate of these points is said to form a point-space of n dimensions (P_n). Taking $r<n$, a set of r equations $\phi_1=0, \phi_2=0, \ldots \phi_r=0$, connecting the co-ordinates, will in general define a space P_{n-r} contained in P_n. Theorems about loci, contact, envelopes, and the principle of duality all hold good for this enlarged domain, and we also have a system of projective geometry analogous to the ordinary one.

Physicists are predominantly interested in metrical geometry. The ordinary metrical formulæ for a P_3 may be extended by analogy to a P_n; there is no logical difficulty, but there is, of course, the psychological fact that our experience (so far) does not enable us to "visualise" a set of rectangular axes for a P_n if $n>3$; not, at least, in any way obviously analogous to the cases $n=2, 3$.

In ordinary geometry, for a P_3 we have the formula

$$ds^2=dx_1^2+dx_2^2+dx_3^2$$

for the linear element called the distance between two points (x), $(x+dx)$. Riemann asked himself the question whether, for every P_n, this was necessarily a typical formula for ds, on the assumption that solid bodies can be moved about in space without distortion of any kind. His result is that we may take as the typical form, referred to orthogonal axes,

$$ds^2=\Sigma dx^2/N^2,$$

where

$$N=1+\tfrac{1}{4}\alpha\Sigma x^2,$$

and α is an arbitrary constant, called the *curvature* of the P_n in question. This curvature is an intrinsic property of the P_n, and should not be considered as a warp or strain of any kind. When $\alpha=0$, we have the Euclidean case. As an illustration of the theory that can be actually realised, take the sphere $x^2+y^2+z^2=r^2$ in the ordinary Euclidean P_3. By putting

$$D=u^2+v^2+4r^2,$$
$$Dx,\ Dy,\ Dz=4r^2u,\ 4r^2v,\ (u^2+v^2-4r^2)r,$$

the equation $x^2+y^2+z^2=r^2$ becomes an identity, and we may regard the surface of the sphere as a P_2 with (u, v) as co-ordinates. The reader will easily verify that

$$ds^2=(du^2+dv^2)\div\{1+\frac{1}{4r^2}\ (u^2+v^2)\}^2;$$

so we have a case of Riemann's formula with $\alpha=r^{-2}$. We *cannot* find a similar formula for the surface of an ellipsoid, because a lamina that "fits" a certain part of the ellipsoid cannot be freely moved about so as to remain in contact with the surface.

To avoid misunderstanding, it should be said that Riemann's expression for ds^2 is not the only one that is taken to be the typical or standard formula. The important thing is that, given any formula for ds^2, in a P_n, we can, by direct calculation, find an expression for the curvature of P_n in the neighbourhood of any assigned point (x). It is only when this curvature is everywhere the same that we have a P_n for which the axiom of free mobility is valid. When the curvature varies from place to place we are not entitled, for instance, to assume that we can carry about an invariable foot-rule for purposes of physical measurement.

In the simpler theory of relativity we have a formula

$$ds^2 = dx^2 + dy^2 + dz^2 - c^2dt^2, \quad . \quad . \quad . \quad (1)$$

where c is a real constant. As it originally presents itself, x, y, z are ordinary rectangular coordinates, t is the time, and c the experimental velocity of light. By a suitable choice of units we can make the value of c any finite constant that we please. Following Minkowski, I shall call (x, y, z, t) a world-point; the aggregate of these points may be provisionally called a space-time world $P(x, y, z, t)$.

When $t = t_0$, a constant, $dt = 0$ and (1) reduces to the ordinary Euclidean formula. We may express this by saying that the sub-world $P(x, y, z, t_0)$ is Euclidean. Actual experiments take time; so we cannot verify this assertion by observation. If, however, two observers, at different places, make measurements which begin and end at the same instants, we may expect their results to be consistent. As Prof. Einstein has pointed out, the question of simultaneity (and, indeed, of time itself, as an *observed* quantity) is a more difficult one than appears at first sight.

The main difficulty about (1), as it seems to me, is that the expression on the right is not a definite form; hence in the neighbourhood of every "real" point (x, y, z, t) there is a real region for which ds^2 is negative. It is possible that the difficulty of interpretation is more apparent than real, as is the case in some well-known examples. For instance, a hyperbola may be analytically defined as an ellipse of semiaxes a, bi, where a, b are real; and, *moreover*, v. Staudt's theory of involution gives an actual geometrical meaning to the algebraic definition.

If, with $i^2 = -1$, we put $ct = i\tau$, the formula (1) becomes

$$ds^2 = dx^2 + dy^2 + dz^2 + d\tau^2, \quad . \quad . \quad . \quad (2)$$

the typical formula for a Euclidean P_4. This makes it very tempting to assume that the successions of phenomena in our world of experience are, so to speak, sections of a *space-world* $P(x, y, z, \tau)$, obtained by giving τ purely imaginary values. This point of view has been taken by Minkowski and others.

The mathematical theories of abstract geometry and kinematics are so complete that physicists have a definite set of hypotheses from which to choose the one most suited to their purpose; and besides this they have to frame axioms and definitions about time, energy, etc., with which the pure mathematician is not concerned.

Whatever may be the ultimate form given to the theory of relativity, the predictive quality of its formulæ gives it a high claim to attention, and it certainly seems probable that, for the sake of what Mach calls economy of thought, we may feel compelled to change our ideas of "actual" space and time.

In an article like this it is impossible to go into detail; the following references may be useful to readers who desire further information:—"The Elements of Non-Euclidean Geometry," by J. L. Coolidge, is rather condensed, but very conscientious and trustworthy; one of the best analytical discussions of the metrical theory is in Bianchi's "Lezioni di Geometria Differenziale," chap. xi.; and Lie's "Theorie der Transformationsgruppen," vol. iii., chaps. xx.–xxiv., contains a most valuable critique of Riemann and Helmholtz. The article "Geometry" in the "Encyclopædia Britannica" (last edition) gives an outline of the theory and numerous references. Finally, there is an elaborate "Bibliography of Non-Euclidean Geometry" by D. M. J. Somerville (see NATURE, May 16, 1912, vol. lxxxix., p. 266).

The General Physical Theory of Relativity.

By J. H. JEANS, Sec. R.S.

THE relativity theory of gravitation, which is at present the centre of so much interest, owes its existence to an earlier physical theory of relativity which had proved to be in accord with all the known phenomena of Nature except gravitation. The gravitational theory is only one branch, although a vigorous and striking branch, of a firmly established parent tree. The present article will deal solely with the main trunk and roots of this tree.

Newton's laws of motion referred explicitly to a state of rest, but also showed that the phenomena to be expected from bodies in a state of rest were precisely identical with those to be expected when the same bodies were moving with constant velocity. Indeed, Newton directed special attention to this implication of his laws of motion in the following words:—

COROLLARY V.: *The motions of bodies included in a given space are the same among themselves, whether that space is at rest, or moves uniformly forwards in a right line without any circular motion.*

"A clear proof of which we have," continues Newton, "from the experiment of a ship, where all motions happen after the same manner whether