Simulation of Magnetic Field Dissipation in Gamma-Ray Bursts

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Abstract

We report on the first steps in 3D simulation of magnetic field dissipation in gamma-ray burst prompt emission (GRB). The model is based on magnetically driven Poynting flux outflow.

Introduction

While the *internal shock* model has had some success in reproducing GRB characteristics, it suffers from an *efficiency problem* (the relatively low efficiency with which the central engine's energy is converted to prompt gamma-rays), and a *field strength problem* (generating the field strength needed to produce efficient synchrotron emission).

In several of the more promising models of the central engine, a rotating relativistic object powers the outflow. These objects naturally have strong magnetic fields, and the transmission of rotational energy to the outflow via the field (Poynting flux) is also the the most effective ways of satisfying the *baryon loading constraint* (the large energy-to-rest-mass ratio needed to explain GRBs).

Such magnetically powered outflows come in two basic varieties: the DC model (axisymmetric, e.g. Blandford and Lytikov 2002), and the AC model (flow generated by a central engine with a nonaxisymmetric magnetic field, e.g. Drenkhahn and Spruit 2003). Their advantage over the internal shock model is that they can produce prompt emission with high (50%) efficiency, naturally provide the strong magnetic field needed for synchrotron emission, and are very effective at accelerating the flow.

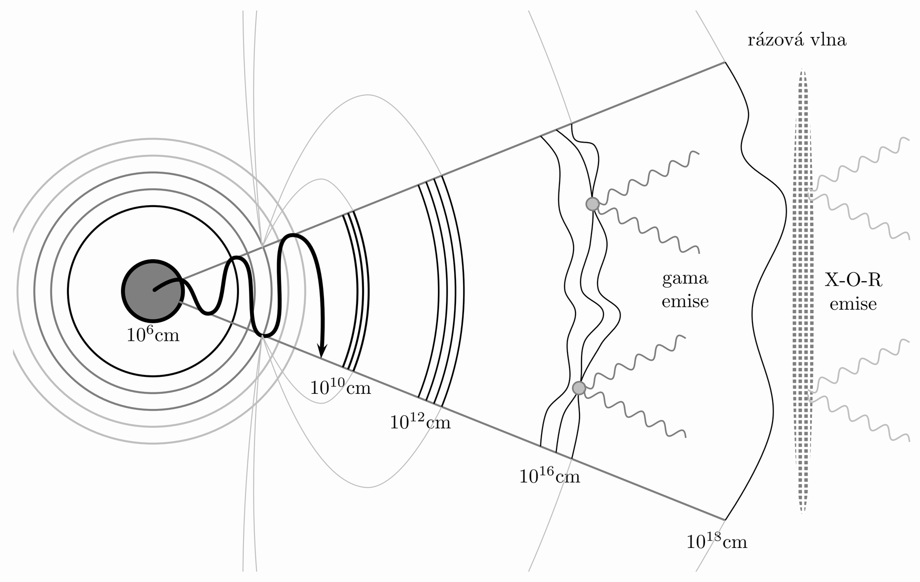

Figure 1: A primitive sketch of the magnetically driven fireball model. Millisecond pulsar in the center of γ-optically thick region, Poynting flux of anti-parallel magnetic field, reconnection of the field in the γ-optically thin region. An afterglow is produced in the framework of relativistic shock wave in the interstellar medium.

Power Density Spectrum

We have done an analysis of 10 long multi-peak GRBs the BATSE triggers 1440, 1676, 2156, 2856, 6472, 7906, 7994, 8001, 8026 and 8036 (BATSE). We subtracted white noise and we show that the slope of Fourier power density spectrum (PDS) of these GRBs is closed to Kolmogorov turbulent spectrum $-5/3$, typical for MHD turbulence and magnetic reconnection. High diversity of peaks in the lightcurve yields that the stochastic process we encounter works near the critical regime.

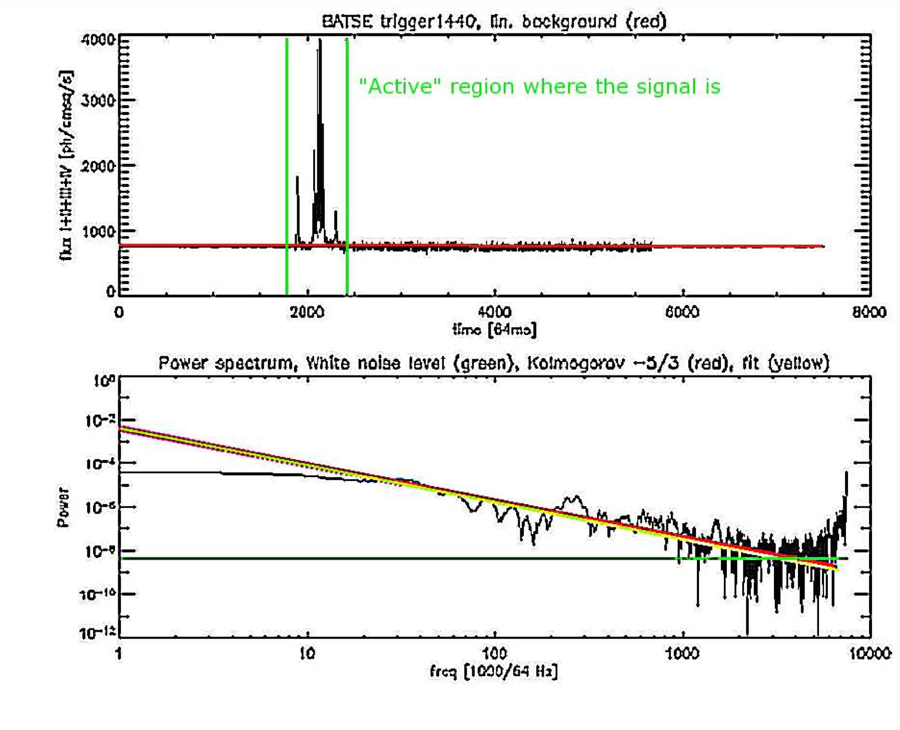

Figure 2: Example of the PDS analysis for BATSE trigger 1676. The fitted slope gives the value of $-5/3$.

Magnetic Reconnection

Magnetic reconnection is a consequence of non-ideal MHD concept where $\mathbf{E} = \mathbf{E}' + \mathbf{v} \times \mathbf{B} = \eta\mathbf{j}$. A basic scheme of magnetic reconnection is drawn in Figure 3. The velocity of the outflow from the X-type null point is controlled by the reconnection rate $\mathcal{M}$ that represents the efficiency as well. In two basic models of 2D reconnection it yields

$$\mathcal{M} \equiv \frac{v}{v_A} \approx \begin{cases} \frac{1}{\mathcal{R}_m^{1/2}} & \text{(Sweet -- Parker)} \\ \frac{\pi}{8\log\mathcal{R}_m} & \text{(Petschek)} \end{cases}$$

where $v_{\rm out} = v_A = B_0/\sqrt{4\pi\rho_0}$ is the Alfven speed and $\mathcal{R}_m \equiv L_0 v_0/\eta$ the magnetic Reynolds number of MHD turbulence.

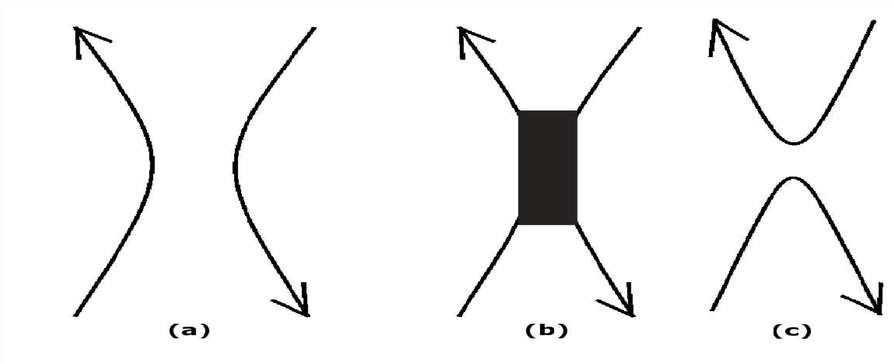

Figure 3: Magnetic reconnection a) anti-parallel magnetic field lines b) entering diffusion region c) change of topology.

Thus the magnetic reconnections change the topology of the field lines. In terms of helicity H it yields

$$\frac{dH}{dt} \equiv \frac{\mathrm{d}}{\mathrm{d}t}\int_V \mathbf{A}\cdot\mathbf{B}dV \neq 0 \qquad \mathbf{B} \equiv \nabla \times \mathbf{A}$$

Naturally question arises, can we achieve a feasible configuration in the case of GRBs? There could be an extreme magnetic field $\sim 10^{14}$ G induced e.g. by the α-Ω dynamo process (Usov 1994). Also the striped magnetic wind could produce anti-parallel magnetic field behind the the light cylinder of a non-axisymmetric pulsar or a magnetar (Coroniti 1990).

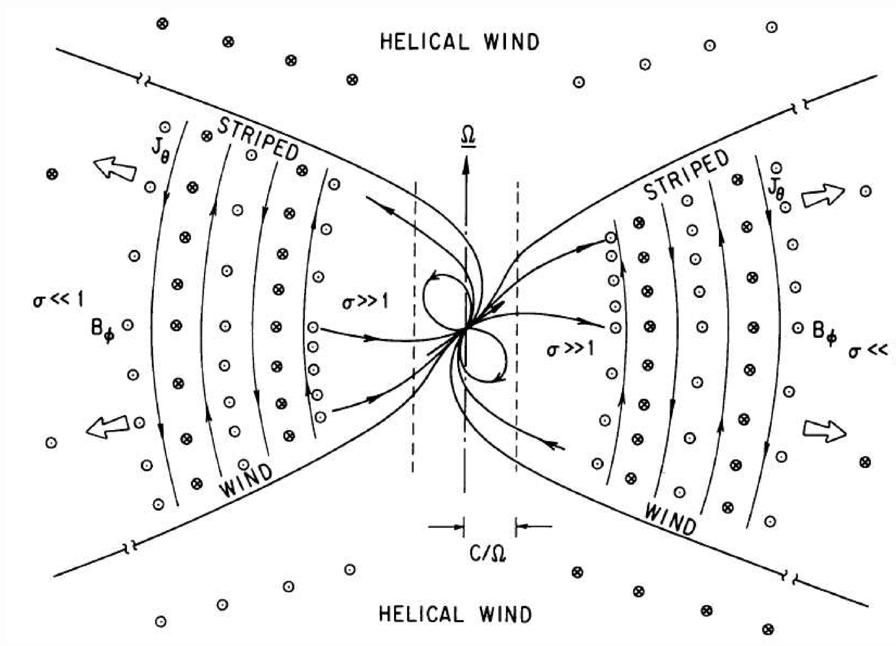

Figure 4: The striped wind configuration of magnetic field around a millisecond pulsar.

Model

We try to run a simulation of the resistive MHD processes in the magnetically driven fireball. We assume a relativistic Poynting flux outflow in the form of a jet where the magnetic reconnections happen.

The interesting part of our model (suggested by Spruit & Drenkhahn 2003) is that magnetic field dissipation can solve both the acceleration of the jet and the radiation mechanism at once. The law of the energy conservation yields

$$\frac{dw}{dt} + \nabla\cdot S = -j\cdot E$$

where $w = \left(E^2/8\pi + B^2/8\pi\right)$ is the electromagnetic energy density and S is the Poynitng flux. It can be seen that the energy trapped in the magnetic field and available to dissipate is not only the magnetic energy density $B^2/8\pi$ but the energy driven by $S = B^2/4\pi$. It is useful to think about it in the term of magnetic enthalpy

$$w_m = u_m + p_m$$

Dissipation of the magnetic energy is converted into the internal energy u_m which can be radiated away through synchrotron radiation mechanism if it happens in the optically thin region above the photosphere, or it heats the plasma and let it expand if it occurs within the photosphere. The dissipation of of magnetic field leads to the pressure losses and gradient of pressure accelerate the flow, its Lorentz factor $\Gamma(t)$.

Simulation

Because the three dimensions allow much more complex topological structures we decided to build up a pure MHD (not PIC) 3D simulator. The general resistive compressible MHD equations

$$\partial_t\rho = -\nabla\cdot(\rho\mathbf{v}) \qquad p\partial_t\mathbf{v} = -\rho\,(\mathbf{v}\cdot\nabla)\,\mathbf{v} - \nabla p + \mathbf{j}\times\mathbf{B}$$

$$\partial_t U = -\nabla\cdot\mathbf{S} \qquad \partial_t\mathbf{B} = \nabla\times(\mathbf{v}\times\mathbf{B}) - \nabla\times(\eta\mathbf{j})$$

$$\mathbf{E} = -\mathbf{v}\times\mathbf{B} + \eta\mathbf{j}$$

where the electric current $\mathbf{j}$, internal energy U and Poynting flux

$$\mathbf{j} = \frac{1}{\mu_0}\nabla\times\mathbf{B} \qquad U = \rho w + \frac{\rho}{2}v^2 + \frac{B^2}{2\mu_0}$$

$$\mathbf{S} = \left(U + p + \frac{B^2}{2\mu_0}\right)\mathbf{v} - (\mathbf{v}\cdot\mathbf{B})\frac{\mathbf{B}}{\mu_0} + \eta\mathbf{j}\times\frac{\mathbf{B}}{\mu_0}$$

with the pressure defined as $p = (\gamma - 1)\,\rho w$ and $\gamma = 5/3$ are written into dimensionless form. We use one-liquid model, but with locally artificially raised anomal resistivity wherever the virtual drift velocity $|v_D \equiv (m_i/e)\mathbf{j}/\rho| \geq v_{\rm cr}$, where $v_{\rm cr}$ is parameter of the model. Then the equations are rewritten into the comoving frame according to $\Delta x \to L\,(t)\,\Delta x$, where the Δx differential size of the length-scale. This resembles the Big Bang expansion. We assume that the reconnection rate (we scan in every step of the simulation) is proportional to the velocity of the bulk motion $\Gamma \sim (1-\alpha)\Delta p_m$ where α represents the fraction of radiated losses in the magnetic field dissipation. We calculate the reconnections in Newtonian framework using Lax-Wendroff 2nd order integration scheme, then at the end recalculating it from the outflowing jet of $\Gamma(t)$ into the observer frame.

We plan to implement in several pseudo-relativistic tricks into the equations to simulate the special relativistic effects (e.g. $\eta \to \eta/\Gamma$, $\rho \to \Gamma\rho$). The sketch of an elementary cell in the simulation is shown in Figure 5.

As initial conditions we have chosen anti-parallel magnetic field multi-layers. The separation between the layers is assumed to be $\lambda = \Gamma 2\pi c/\Omega$, where Ω is the rotational velocity of the magnetic field progenitor, e.g. a magnetar. To simulate the stochastic nature of the magnetic reconnection process we use spatially periodic boundary conditions. Due to the technical limits we are restricted to maximally $512 \times 512 \times 512$ cells in the grid/matrix. Each cell contains ρ, $\mathbf{v}$, $\mathbf{E}$, $\mathbf{B}$, η, u, p the particle density, the velocity, the electric and magnetic field, the resistivity, the internal energy and the pressure located in the cell as it is seen from Figure 5.

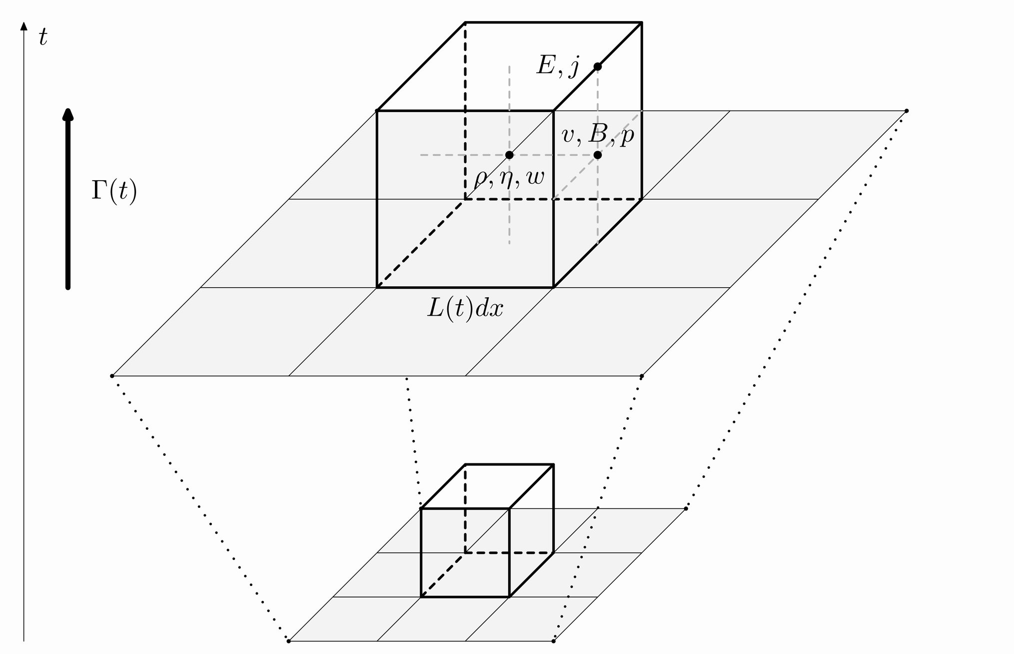

Figure 5: Description of the simulation and an elementary cell in the model.

Outlook to the Future

We will inform you about the progress. Chromo-stereoscopic projection will be written in IDL to visualize the results.

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